Lab 15: Likelihood Ratio Tests

Let's continue the likelihood ratio test example from the handout on April 20. As a reminder, here is our set up:

$X_1, \ldots, X_5 \sim \text{Normal}(\theta, 5^2)$

We want to test the hypotheses

$H_0 : \theta = 25$ $H_A : \theta = 10$

Suppose we observe a sample mean of $\bar{x} = 20$ based on a sample of $n = 5$ observations.

(a) Suppose the null hypothesis of the test is true. What is the distribution of the sample mean $\bar{X}$?

$\bar{X} \sim \text{Normal}(25, 5)$

The mean is from the null hypothesis specification. Note that the variance was divided by 5.

(b) Find the p-value for the likelihood ratio test.

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pnorm(20, mean = 25, sd = sqrt(5))
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## [1] 0.01267366
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(c) Suppose that the alternative hypothesis is $H_A : \theta = 0$ instead of $H_A : \theta = 10$. Would the p-value for the test change? You don't need to actually calculate a p-value. Justify your answer in a few words.

The p-value would be the same because the form of the p-value calculation for this test is determined only by whether the value of θ in the alternative hypothesis is less than the value from the null hypothesis or greater than it.

(d) How about if the alternative hypothesis was $H_A : \theta = 20$. Would the p-value for the test change? You don't need to actually calculate a p-value. Justify your answer in a few words.

The p-value would be the same because the form of the p-value calculation for this test is determined only by whether the value of θ in the alternative hypothesis is less than the value from the null hypothesis or greater than it.

(e) How about if the alternative hypothesis was $H_A : \theta = 25.1$. Would the p-value for the test change? You don't need to actually calculate a p-value. Justify your answer in a few words.

The p-value would be different because in this case the value of θ specified in the alternative hypothesis is greater than the value of θ specified in the null hypothesis.