

# Macroeconomic Theory II

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## Assignment 2

Due Friday February 10

Consider the stochastic neoclassical growth model we studied in class. Assume that  $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ ,  $f(k, z) = zk^\alpha$  and  $\log(z') = \rho \log(z) + \epsilon$  where  $\epsilon$  is normally distributed with mean zero and variance  $\sigma_\epsilon^2$ . Set  $\sigma = 2$ ,  $\delta = 0.1$ ,  $\beta = 0.95$ ,  $\rho = 0.95$ ,  $\sigma_\epsilon = 0.01$  and  $\alpha = 0.33$ .

1. Compute the stochastic steady-state values for capital, consumption, output and investment.
2. Linearize the model around the steady-state (if you feel more comfortable you can use the log-linearization method). Show your equations.
3. Solve the policy function for capital and consumption using linearized (or log-linearized) model. The solution of linearized model will give you the rules for consumption and capital as deviations from steady-states. To find the policy functions in levels, convert these rules using the definitions for the deviations from steady-state.
4. Solve the model using value function iteration method.
  - (a) To do this you need to convert the AR(1) process into Markov process. I uploaded a code which does it for you. The method is called tauchen method. It takes the inputs  $N$ , number of grid points for the shock,  $\mu$ , mean value of the shock,  $\rho$ , persistency of the shock,  $\sigma$ , standard deviation of the shock, and  $m$ , the maximum distance to the mean in terms of multiples of the standard deviation. You can set  $N = 7$  and  $m = 3$ . The other inputs are the parameters of the model. The output of the code is  $Z$ , the discretized values for the shock,  $Zprob$ , the transition matrix for the shock, where entry  $(i, j)$  of the matrix is showing the transition probability from state  $z_i$  to state  $z_j$ . Be careful that the levels you obtained using the matlab code,  $Z$ , are the log values of the shock. To covert them into actual levels, take the exponent of  $Z$ .
  - (b) When discretizing the capital, center the grid points around  $k^*$ , steady-state capital level, and set the lower bound of capital to  $\underline{k} = (1 - \kappa)k^*$  and upper bound to  $\bar{k} = (1 + \kappa)k^*$ . You can start by using  $N_k = 200$  grid points for  $k$  and set  $\kappa = 0.6$ .
  - (c) Solve the model using value function iteration method. Compute the associated policy functions and value functions. Plot them against capital for different values of the shock.

- (d) Report the robustness of your results with respect to  $N_k$  and  $\kappa$ . Try two other values, say  $N_k \in 100, 500$  and  $\kappa \in 0.3, 0.8$  and plot the policy function for capital for the mean value of the shock.
  - (e) Compare the policy functions for capital obtained using linearized model and value function iteration method. Discuss the goodness of approximation in the linearized model.
  - (f) Do the same comparison of policy functions for  $\sigma = 5$  and  $\sigma = 10$ . Discuss the sensitivity of approximation with respect to  $\sigma$ .
5. To check the accuracy of your solutions for both methods, I suggest you to use the model when  $\sigma = 1$  and  $\delta = 1$ . In Homework 1, you already obtained the policy functions and value functions for this model analytically. You can use that analytical solution to check the accuracy of your solutions.