Macroeconomic Theory II

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Assignment 3

Due Friday March 10

You should provide a printed version of your answers to Yongquan on the due date. Do not return the print-out of the code together with your answers. Instead, email the codes for your solutions to Yongquan.

Consider a standard RBC model, where the economy is populated by infinitely many all-identical households and firms. Households live forever, and maximize the expected discounted utility from consumption c and leisure l:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t).$$

Firms produce output using the technology specified by

$$y_t = z_t F(K_t, N_t),$$

where z is a technology shock, K is capital demanded by firms and N is their labor demand. Capital depreciates at the rate δ. Suppose that z follows an AR(1) process:

$$log z_{t+1} = log(z_t) + \epsilon_t,$$

where $0 < \rho < 1$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$

1. Solve for non-stochastic steady state values of Y, C, K, N, I, r, w as a function of parameter values, where Y is output, I is investment, r is net interest rate, and w is the wage rate.

2. Assume that utility takes the following functional form:

$$u(c, l) = \frac{c^{1-\sigma} - 1}{1 - \sigma} + \theta \frac{l^{1-\psi} - 1}{1 - \psi}$$

   Suppose that in the data quarterly interest rate is 1%, the share of capital income in aggregate income is 1/3, aggregate investment is 20% of the aggregate output, and individual labor is 1/3 of the total time available. Given these numbers and assuming $\psi = 1$, calibrate the parameters of the model $\alpha, \beta, \delta, \theta$ such that the non-stochastic steady state of the model implies the same statistics as in the data.

3. Use the above calibration and set $\sigma = 1$, $\rho = 0.95$ and $\sigma_\epsilon = 0.01$. Solve the RBC model either using linearization or non-linear methods to obtain policy functions for consumption, capital and labor.

   (a) Simulate the economy for 1000 periods starting from the non-stochastic steady-state and report the following statistics for output, consumption, investment, labor, real interest rate and wage:

      i. Volatilities (standard deviations)

      ii. Relative volatilities with respect to output

      iii. Contemporaneous correlation with output

      iv. Correlation with one year lag and lead output (Remember your model is assumed to be quarterly. This means you need to check the correlations of the variables with Y_{t-4} and Y_{t+4})

   (b) Plot the impulse responses for output, consumption, investment, real interest rate and wage to a one standard deviation productivity shock. (Plotting them for 10 years (40 quarters) should be enough.)

4. Report the differences in the volatilities and impulse responses of the macroeconomic variables when you change ψ and σ. Specifically, redo part (3) for $\sigma \in \{3, 5\}$ and $\psi \in \{0.5, 2\}$. Here, you have to re-calibrate θ when you change ψ to match the same steady-state value of labor. Comment on the differences with respect to the benchmark case in part (3). Plot the impulse responses together with the one in part (3) to make the comparison easier.

5. Now, you will solve the model of Rogerson (1988). Specifically, assume that utility takes the form $u(c, 1-n) = log(c) - Bn$. Redo part (2) given this specification of utility. Compare your results to part (3) by comparing the change in volatilities and plotting the impulse responses in the same graph for this question and the one in part (3). You again need to re-calibrate B to match the steady-state value of $1/3$ for labor. You should be able to do it by solving the non-stochastic steady-state equations.

6. Now, assume that utility takes the following form: $u(c, 1-n) = log(c - \frac{\theta}{1+\psi} n^{1+\psi})$. Set $\psi = 1$, re-calibrate θ to match steady-state value of labor, and redo part (3). Compare your results to the results you obtained from part (2).