A Bayesian Approach to Predicting NFL Player Scores in Fanduel Tournaments

STAT 578, Fall 2017, ***Team 5****: Aaron Ray, Kiomars Nassiri, Michael Chan*

December 8, 2017

Purpose

The National Football League, NFL, is a professional American football league consisting of 32 teams. It is the most professional American football league in the world and is one of the most followed sports leagues in North America. Being a football fan, however, does not end with following matches on TV or stadiums or buying your favorite team's jersey. Over the years, football fans have come up with interesting NFL-themed side-activities, the most popular of which is called "Fantasy Football".

Participants in a **Fantasy Football** game act as the managers of a virtual football team and try to maximize their points by picking up the best line-up. Points are given based on actual performance of players in real-world competition. Each team is allowed a pre-determined number of players on its roster, as well as a specified number at each position that can or must be used in each game (the "starters").

For the purpose of this project, we have chosen to work with the data gathered from the **FanDuel** internet company. FanDuel is a web-based fantasy sports game and with 6 million registered users and is the second largest daily fantasy sports company (as measured by entry fees and user base) in the daily fantasy sports industry.

We will leverage a Hierarchical Bayesian approach with the Markov Chain Monte Carlo method to predict the Fantasy points likely to be scored by an NFL quarterback in any given game. The goal is to predict the points scored by each player given certain prior conditions and predictor variables that will assist our model in providing credible posterior prediction intervals. The following are the research questions that will be answered in this project:

- **Interpretation**: What features can help us effectively predict FanDuel points players receive in a future match?
- **Prediction**: How reliable are the predictions for the future performance of players?

Data

Historical data on the performance of the players is extracted from the **RotoGuru** website. Data scraped from RotoGuru includes information about the FanDuel points received by each player, player's position, player's opponent team, Home/Away match indication, etc. for each week.

Data cleaning is performed using R routines. Some data cleaning tasks are needed to calculate Player rank. The code used to get the data from RotoGuru can be found in the Appendices - Data Section.

Response Variables

- `FanDuelPts`: Points position at the end of a single game

Predictor Variables

- `AvgPts5Wks`: The 5 game average points of the player
- `AvgOppPAP7Wks` : The 7 game average Opposing Points Allowed to Position (OppPAP) by the current player's opposing defense. For example, if the Buffalo Bills defense allowed a total of 30 points per game to wide receivers for six games straight, then this number would equal to the average of 30 for any wide receiver facing the Bills defense.
- `Position`: The position the player plays
- `HomeGame`: Whether it is home game.
- `Rank`: The rank of a player based on recent performance

Sample Data

```r
fdp <- read.csv("fdpfinal.csv", sep = ',', header = TRUE)

head(fdp, 2)
```

```
##   Position Year YearWeek Opponent Week PlayerId             Name  Team HomeGame
## 1       QB 2015   201513 Steelers   13     1060 Hasselbeck, Matt Colts        0
## 2       QB 2015   201514  Jaguars   14     1060 Hasselbeck, Matt Colts        0
##   FanDuelPts FanDuelSalary AvgOppPAP7Wks SdOppPAP7Wks OallAvgPAP OallStdevPAP
## 1       6.86          6500         20.95        8.045      17.44        2.909
## 2       9.08          6600         24.16        6.738      17.44        2.909
##   AvgPts5Wks StdevPts5Wks OffRnk5Wks DefRnk7Wks
## 1      14.85        4.824      Rank4      Rank1
## 2      14.82        4.897      Rank4      Rank1
```

Model

At the lowest level, we model the performance (`FanDuelPts`) as normally-distributed around a true value:

$y|\alpha, \beta_{defense}, \beta_{home}, \beta_{away}, \sigma_r^2 \sim N(\alpha + X_{defense} \cdot \beta_{defense} + X_{home} \cdot \beta_{home} + X_{away} \cdot \beta_{away}, \sigma_y^2 I)$

where

$\alpha =$ The average fan duel point of the previous 5 weeks of the player, `AvgPts5Wks`

$\beta_{defense,p} =$ defense coefficient against team t for position p

$\beta_{home,p,r} =$ home coefficient for position p and a rank r player

$\beta_{away,p,r} =$ Away coefficient for position p and a rank r player

$y =$ `FanDuelPts`

$x_p =$ interaction indicator term for opposing team score allowed by position p

$x_{home,p,r} =$ interaction indicator term for rank r, position p, and whether it is home game

At the higher level, we model the defense effect, $\beta_{defense}$, as how well a particular team's defense has performed against the player's position. We pool the effect based on the position of the player. That is, the defense coefficient is normally distributed from the same position specific distribution.

$\beta_{defense,p} \sim N(\delta_p, \sigma_\delta^2)$

where σ_δ is constant $= 1000$

For the home and away game effect, β_{home} and β_{away}, we model the effect for player of the same rank has the same distribution. We model the home and away game effect to be the same for players of the same position.

$\beta_{home,p,r} \sim N(\eta_r, \sigma_\eta^2)$

$\beta_{away,p,r} \sim N(\rho_r, \sigma_\rho^2)$

where σ_η, σ_ρ are constant $= 1000$

We will approximate non informative prior using:

$\sigma_y \sim Inv - gamma(0.0001, 0.0001)$

$\delta \sim N(0, 10000^2)$

$\eta \sim N(0, 10000^2)$

$\rho \sim N(0, 10000^2)$

Note that in this project, we model the performance (`FanDuelPts`) as normally-distributed around a true value. Alternatively, we could have use Poisson distribution instead to avoid predicting the `FanDuelPts` less than zero. However, for the purpose of predicting player's `FanDuelPts`, we mainly only need the relative strength of players and the normal assumption produces good enough model. The portion of predicted `FanDuelPts` should be relatively small.

Here is the JAGS model:

```
#sink("fdp.bug")
#cat("
model {
  for (i in 1:length(y)) {
    y[i] ~ dnorm(alpha[i] + inprod(X.defense[i, ], beta.defense)
                 + inprod(X.home[i, ], beta.home)
                 + inprod(X.away[i, ], beta.away), sigmasqinv)
  }

  # The entry of the beta.defense corresponds to Opponent:Position
  # In our model, we pool the beta.defense based on position.
  # i.e. All defense effects of the same position are drawn from the same distribution
  for (p in 1:Num.Position) {
    for (f in 1:Num.fixed.pred) {
      beta.defense[(f-1) * Num.Position + p] ~ dnorm(delta[p], 1/1000^2)
      delta[(f-1) * Num.Position + p] ~ dnorm(0, 1/100000^2)
    }
  }

  # The entry of the beta.home and beta.away corresponds to Rank:Position
  # In our model, we pool the beta.home/away based on rank
  for (r in 1:Num.Rank) {
    for (t in 1:Num.Position) {
      beta.home[(t-1) * Num.Rank + r] ~ dnorm(eta[r], 1/1000^2)
      beta.away[(t-1) * Num.Rank + r] ~ dnorm(rho[r], 1/1000^2)
    }
    eta[r] ~ dnorm(0, 1/100000^2)
    rho[r] ~ dnorm(0, 1/100000^2)
  }

  sigmasqinv ~ dgamma(0.0001, 0.0001)
  sigmasq <- 1/sigmasqinv
```

```
}
#    ",fill = TRUE)
#sink()
```

Here is the DAG model:

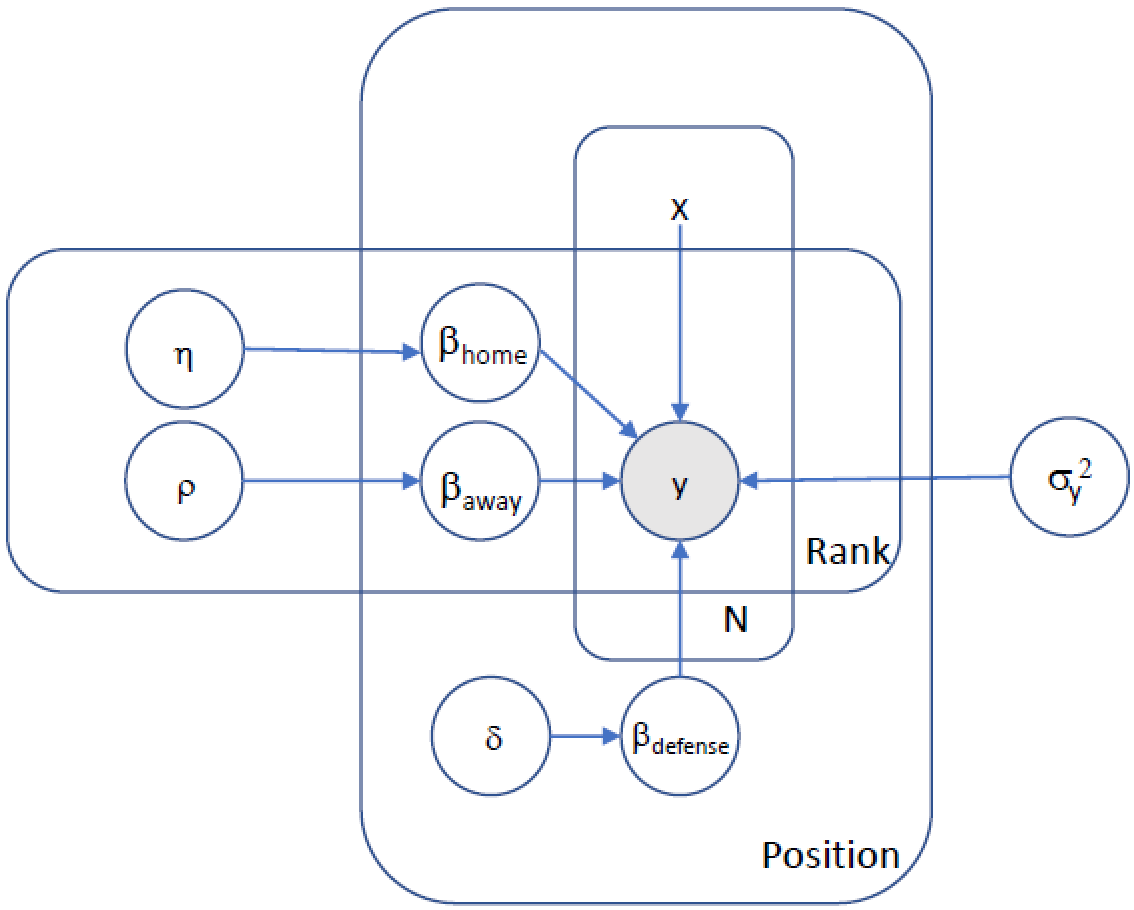


Figure 1: DAG model

Computation

We use overdispersed starting points and 4 chains to initialized. We have seen slow mix in and hence we use thinning of 5 to get a smaller number of data points. Convergence diagnostics are performed graphically as well as using Gelman Statistics. We make sure sample size are > 400. The Monte Carlo error of the β are less than 0.06, but the ones of the hyper parameters are much higher at around 5.

Training Data Setup

Due to significant roster change usually happens in the off-season, we believe it is the best to not use data across seasons. We use 2016 data, `fdp_train`, as it is the most recent year with a full season. We set aside the last week for verifying prediction accuracy of the model, `fdp_test`.

This session shows some sample X used in the computation. `X.defense` captures the interaction terms between the defensive power (higher `AvgOppPAP7Wks` means the player is facing a weaker team, since they allows players score more point on them) and position.

Sample `X.defense`

```
##    AvgOppPAP7Wks:PositionPK AvgOppPAP7Wks:PositionQB
## 28                        0                    19.19
```

`X.home` and `X.away` are the interaction terms between Position and Rank. Together `X.home` and `X.away` will always sum to one. The intercept is implicitly included.

Sample `X.home`

```
##    RankRank1:PositionPK RankRank2:PositionPK RankRank3:PositionPK
## 28                    0                    0                    0
##    RankRank4:PositionPK RankRank1:PositionQB RankRank2:PositionQB
## 28                    0                    0                    0
##    RankRank3:PositionQB RankRank4:PositionQB
## 28                    0                    0
```

The code used to set up the data used in the model computation is listed in *Appendices - Computation* section.

Initialization

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 746
##    Unobserved stochastic nodes: 29
##    Total graph size: 18345
##
## Initializing model
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 746
##    Unobserved stochastic nodes: 29
##    Total graph size: 18345
##
## Initializing model
```

Convergence diagnostics

Trace Plots

The trace plots of all parameters show good distribution convergence. Please see *Appendices - Converage diagnostics* for all trace plots and the Gelman Statistics summary, and MCMC Summary

Gelman Statistics

Converged as `gelman.R.max` $= 1.002 < 1.1$ and the plot also looks good.

Effective Sample Size

```
(eff.size = effectiveSize(result$coda.sam[, ]))
```

```
##    beta.away[1]    beta.away[2]    beta.away[3]    beta.away[4]    beta.away[5]
##            3711            3557            4146            3985            1718
##    beta.away[6]    beta.away[7]    beta.away[8] beta.defense[1] beta.defense[2]
##            1736            1787            1849            2988            1497
##    beta.home[1]    beta.home[2]    beta.home[3]    beta.home[4]    beta.home[5]
##            3792            4202            3943            3788            1767
##    beta.home[6]    beta.home[7]    beta.home[8]        delta[1]        delta[2]
##            1773            1805            1817           29040           30000
##          eta[1]          eta[2]          eta[3]          eta[4]          rho[1]
##           30088           29956           30305           30853           29666
##          rho[2]          rho[3]          rho[4]         sigmasq
##           29489           30000           30441           29940
```

The effective sample sizes of all parameters are greater than 400. See appendics for full output.

Model Assessment

All code in this sections are listed in appendices

General posterior model assumption check

Probabiliy of players should perform better at home than away

Rank:Position	Prob.home.bt.away
RankRank1:PositionPK	0.8093
RankRank2:PositionPK	0.9147
RankRank3:PositionPK	0.5072
RankRank4:PositionPK	0.7559
RankRank1:PositionQB	0.6288
RankRank2:PositionQB	0.9248
RankRank3:PositionQB	0.9508
RankRank4:PositionQB	0.7928

The above table shows that players perform better at home than away as expected.

Beta defense

If a player is facing a team which gives up more points to players on average, we expect the player will score more points.

beta.defense.position	pct025	pct975	median	mean
AvgOppPAP7Wks:PositionPK	-0.0663	0.5435	0.2350	0.2347
AvgOppPAP7Wks:PositionQB	-0.2931	0.1377	-0.0741	-0.0750

We observe that the median beta.defense for PK is positive as expected. But for QB, it is negative, that implies QB actually scores less against bad defensive team.

DIC

```
## Mean deviance:  4700
## penalty 19
## Penalized deviance: 4719
```

The effective number of parameters("penalty") is 19, and the Plummer's DIC ("Penalized deviance") is 4719. Not that we have 29 parameters in our model, 10 of them were shrunk away.

Posterior Predictive Check

Error correlation Check

A posterior predictive p-value using the following test quantity

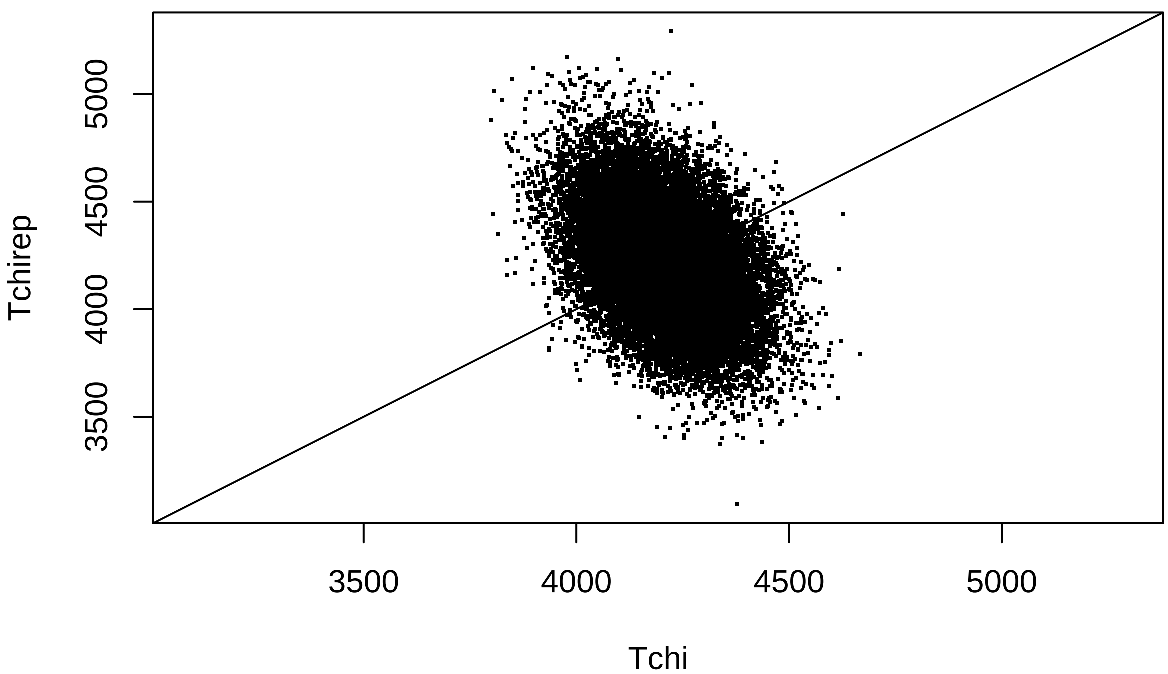
$$T(y, X, \theta) = |\hat{cor}(\epsilon, \text{time})|$$

where $\hat{cor}(\epsilon, \text{time})$ is sample correlation between the error vector ϵ and the year week in the data. The larger this quantity is for the model, the less well it fits the data as that would mean the error is correlated with time.

The p.value, `mean(T.rep.sim >= T.sim)` , is 0.1162, > 0.05, which does not indicate any evidence of problem.

Chi-square Discrepancy Check

Chi-square discrepancy check is used to check for general model issues like mis-specified means, mis-specified variances, and over-concentrated prior.



The posterior predictive p-value using the chi-square discrepancy is `p.value.Tchi`=0.5057. The p-value is $>$ 0.05. Hence, it does not indicate any evidence of problems.

Individual Data Point Discrepancy Check

Using $Pr(y^{rep} \geq y|y)$ as posterior predictive p-value

The posterior predictive p-value using individual data point is `p.value.y.rep.all` = 0.5076, which is $>$ 0.05. This shows no evidence of problem.

Non Negative Check

As discussed in the model section, we use normal distribution, instead of Poisson distribution, to simplify the model. Here we'll check the portion of predicted value < 0.

The percentage of predicted values that are less than zero is `perct.lt.zero` = 0.0463, which is relatively small. This justify the decision to use normal to simplify our model.

Prediction

We use the last week of data to check the prediction effectiveness of the model. This is essentially a cross validation analysis.

Prediction of Individual Player Performance

A measure of the effectiveness of this model is to predict individual player performance. Consider an example data point, `fdp_test[1,]`. The real FanDuelPts is 7.42

The predicted value has the following 95% interval

```
##   2.5%  97.5%
##  4.944 27.266
```

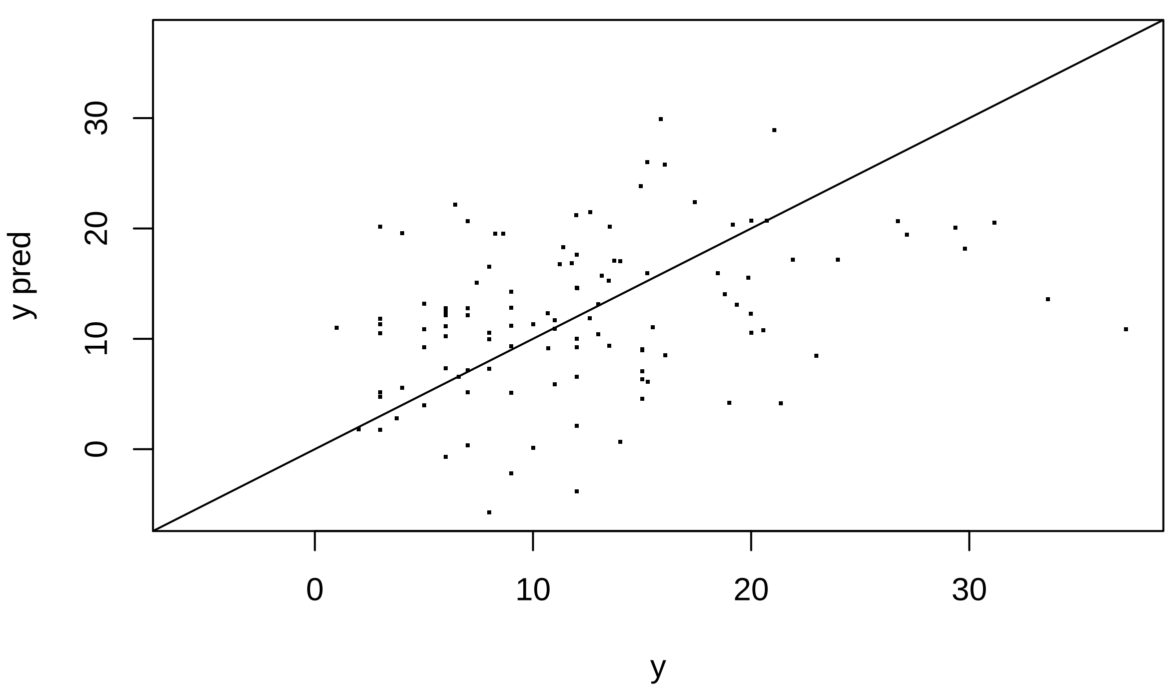
which does contain the actual data value of 7.42

Overall Prediction Effectiveness

To measure the overall prediction effectiveness, we can look at $Pr(y_{pred} >= y)$ as a posterior p-value.

The probability of $y_{pred} >= y$ is 0.4683, close to 0.5, which implies a relatively good predictive value.

A look at a cross section of how one simulation of a prediction of the whole test set



Results

This report tries to answer the following two questions:

- **Interpretation**: What features can help us effectively predict FanDuel points players receive in a future match?
- **Prediction**: How reliable are the predictions for the future performance of players?

Interpretation

In our model section, we defined the mixed hierarchical linear regression model that depends on the defensive effectiveness of the opposing team, ($\beta_{defense}$), home and away effectiveness (β_{home} and β_{away}) and the average offense performance (α), with pooling based on position and/or rank. In the *General posterior model assumption check* section, we have shown that the β_{home} and β_{away} have the expected interpretation in that players enjoy home advantage in general ($\beta_{home} > \beta_{away}$)

Prediction

In the *Prediction* subsection under the *Model Assessment* section, we have shown that the model has reasonable prediction ability. In the *Prediction of Individual Player Performance* section, we have shown that the model predicted the `FanduelPoint` of a player within the 95% posterior credible interval. In the *Overall Prediction Effectiveness* section, we have shown how the model predicted the `FanduelPoint` of all players relatively well.

Analysis Note

It should be noted that we only picked two positions to predict due to computation resource constraint. In the pursue of creating this model, we have tried multiple routes:

1) We have tried to include all positions. However, the model took 5 hours to finish the MCMC simulation. Therefore, we elected to include a smaller subset.
2) We have tried to include `FanDuelSalary` as a predictor. However, that does not improve the model, and added sufficient time to compute - About 30 minutes to compute.

We believe the current model is a comprise that give reasonable balance of interpretability(a simpler model) and predictability (which may require more parameters)

Contributions

All three members contribute roughly the same amount of works. While each member owns certain pieces of the project, all members contribute idea, and review all parts of the project. The following list summarizes the main contribution of individual member:

- **Aaron Ray** (aaronwr2@illinois.edu) - Came up with the project idea, the primary goals, data discovery and attribution.
- **Kiomars Nassiri** (nassiri2@illinois.edu) - Presentation and various documentation.
- **Michael Chan** (mhchan3@illinois.edu) - Finalize data cleansing, drive the design, implementation, and the analysis of the model.

Reference

The analysis is inspired by the study presented in the article, **Bayesian Hierarchical Modeling Applied to Fantasy Football Projections for Increased Insight and Confidence**, by Scott Rome.

Appendices

Data

Data Gathering and Processing Code

```
library(rvest)
```

```
## Loading required package: xml2
```

```
library(rjags)
library(MASS)
library(lattice)
library(magrittr)
library(qdap)
```

```
## Loading required package: qdapDictionaries

## Loading required package: qdapRegex

## Loading required package: qdapTools

## Loading required package: RColorBrewer

##
## Attaching package: 'qdap'

## The following object is masked from 'package:magrittr':
##
##     %>%

## The following object is masked from 'package:rvest':
##
##     %>%

## The following object is masked from 'package:base':
##
##     Filter
```

```
library(TTR)
library(moments)
```

Getting the Data

This section is dedicated to gathering statistics for 2017 FanDuel Football Stats. For the source of this data see http://rotoguru1.com/cgi-bin/fstats.cgi?pos=0&sort=4&game=f&colA=0&daypt=0&xavg=3&inact=0&maxprc=99999&outcsv=0.

Get Raw Data

```
#Initiate primary dataframe with 10 column names
d = data.frame(matrix(ncol=10,nrow=1))
cnames = c("Week","Year","PlayerId","Name","Position","Team","HomeGame","Opponent","FanDuelPts","FanDuel
colnames(d) <- cnames

#Scrape rotoguru1 site for weekly FanDuel stats and bind each week's data to 'd'
for(year in 2014:2017){
  for(week in 1:16){
    page = read_html(
      gsub(" ","",
           paste("http://rotoguru1.com/cgi-bin/fyday.pl?week=",week,"&year=",year ,"&game=fd&scsv=1")
      ))
    dtext = page %>% html_nodes("pre") %>% html_text(trim = TRUE)
    dtable = read.table(text=dtext, sep = ";", header=TRUE, col.names = cnames, quote=NULL)
    d = rbind(d,dtable)
  }

}

write.csv(d, file = "rawfdp.csv",row.names=FALSE, na="")
```

```
d <- read.csv("rawfdp.csv", sep = ',', header = TRUE)
d$Opponent <- lapply(d$Opponent, as.character)
```

Basic clean up

```
#Clean up
##Remove null row
d2=d[-1,]
##Remove invalid entries
d2=d2[d2$Opponent>"-",]
##Bring all negative scores up to 0, these scores are typically very close to 0 anyway
d2[d2$FanDuelPts<0,]$FanDuelPts = 0
##Remove all non-defense player scores that are less than 1 since these players most likely didn't play
d2[d2$FanDuelPts < 1 & d2$Position != "def",]$Year = 0
d2 = d2[d2$Year > 0,]
##Convert home and away abbreviations to 1's and 0's
d2$HomeGame = as.integer(mgsub(c("h","a"),c(1,0),d2$HomeGame))


#Replace Team/Opponent abbreviated locations with team names
tmabbs = c("ari", "atl", "bal", "buf", "car", "chi", "cin", "cle", "dal", "den", "det", "gnb", "hou", "
tmnames = c("Cardinals", "Falcons", "Ravens", "Bills", "Panthers", "Bears", "Bengals", "Browns", "Cowbo
d2$Team = mgsub(tmabbs,tmnames,d2$Team)
```

```r
d2$Opponent = mgsub(tmabbs,tmnames,d2$Opponent)

#Create YearWeek for each year/week combo
d2=d2[order(d2$Opponent,d2$Position,d2$Year,d2$Week),]
d2$YearWeek = d2$Year*100+d2$Week
```

Generate moving 7 week team defense statistics

```r
#Calculate the 6 week running mean, median, and std deviation of total points scored against each defen
dPAP = aggregate(FanDuelPts~Year+YearWeek+Position+Opponent, data=d2, FUN=sum)
dPAP = dPAP[order(dPAP$Opponent,dPAP$Position,dPAP$YearWeek),]

# Calculate AvgOppPAP7Wks - The 7 weeks average points that were scored on by players who play a certai
opponent_position_pairs = unique(dPAP[,c('Opponent','Position')])
for (i in 1:nrow(opponent_position_pairs)) {
  dPAP.which = which(dPAP$Opponent == opponent_position_pairs[i, 'Opponent'] & dPAP$Position == opponent
  dPAP[dPAP.which, 'AvgOppPAP7Wks'] = runMean(dPAP[dPAP.which, 'FanDuelPts'],n=7)
  dPAP[dPAP.which, 'SdOppPAP7Wks'] = runSD(dPAP[dPAP.which, 'FanDuelPts'],n=7)

  # shift down by 1 row, as it should be the average score of LAST n weeks(excluding current week)
  dtemp = dPAP[dPAP.which, ]

  dtemp = rbind(NA, dtemp[1:nrow(dtemp)-1, ])
  dPAP[dPAP.which, 'AvgOppPAP7Wks'] = dtemp$AvgOppPAP7Wks
  dPAP[dPAP.which, 'SdOppPAP7Wks'] = dtemp$SdOppPAP7Wks
}

dPAP = dPAP[c("Year","YearWeek","Position","Opponent","AvgOppPAP7Wks", "SdOppPAP7Wks")]
```

```r
#Calculate mean and std deviation of MedOppPAP7Wks for all defenses
dPAP2 = data.frame(aggregate(dPAP$AvgOppPAP7Wks ~ Position, data=dPAP, function(x) c(mean=mean(x), sd=sd
dPAP2 = data.frame(Position = dPAP2$Position, OallAvgPAP = dPAP2$dPAP.AvgOppPAP7Wks[,1], OallStdevPAP =

#Join the running stats to the full dataset
d3 = merge(d2,dPAP,by=c("Year","YearWeek","Position","Opponent"))
d3 = merge(d3,dPAP2,by="Position")
```

Generate moving 5 week player offense statistic

```r
#Calculate 5 week median and std deviations for each player
d4=d3[order(d3$PlayerId,d3$YearWeek),]

# Calculate AvgOppPAP7Wks - The 7 weeks average points that were scored on by players who play a certai
playerIds = unique(d4[,c('PlayerId')])
n = 5
for (i in 1:length(playerIds)) {
  d4.which = which(d4$PlayerId == playerIds[i])
  if (length(d4.which) > n) {
    d4[d4.which, 'AvgPts5Wks'] = runMean(d4[d4.which, 'FanDuelPts'],n=n)
    d4[d4.which, 'StdevPts5Wks'] = runSD(d4[d4.which, 'FanDuelPts'],n=n)
```

```
    # shift down by 1 row, as it should be the average score of LAST n weeks(excluding current week)
    dtemp = d4[d4.which, ]

    dtemp = rbind(NA, dtemp[1:nrow(dtemp)-1, ])
    d4[d4.which, 'AvgPts5Wks'] = dtemp$AvgPts5Wks
    d4[d4.which, 'StdevPts5Wks'] = dtemp$StdevPts5Wks
  }
}
```

```
# remove na
d4 = d4[!is.na(d4$AvgPts5Wks), ]
#Remove 2014 which was only used to calculate the first 6 running stats of 2015
d4 = d4[d4$Year>2014,]
```

Player Offense and defense Rank

```
createRankColumn <- function(data, rank_column) {
  year_week = unique(data[,c('YearWeek')])
  position = unique(data[,c('Position')])
  result_column = 'result_column'
  data[result_column] = NA
  for (i in 1:length(year_week)) {
    for (j in 1:length(position)) {
      #data_year_week.which = which(data$YearWeek == year_week[i] & data$Position == position[j] )
      data_year_week = data[data$YearWeek == year_week[i] & data$Position == position[j], ]
      data_year_week_quantile = quantile(data_year_week[rank_column],
                                         c(0.25, 0.5, 0.75), na.rm = TRUE)

      data[data$YearWeek == year_week[i]& data$Position == position[j]
          & data[rank_column] < data_year_week_quantile[1],
          result_column] = 'Rank4'
      data[data$YearWeek == year_week[i] & data$Position == position[j]
          & data[rank_column] >= data_year_week_quantile[1]
          & data[rank_column] < data_year_week_quantile[2],
          result_column] = 'Rank3'
      data[data$YearWeek == year_week[i] & data$Position == position[j]
          & data[rank_column] >= data_year_week_quantile[2]
          & data[rank_column] < data_year_week_quantile[3],
          result_column] = 'Rank2'
      data[data$YearWeek == year_week[i] & data$Position == position[j]
          & data[rank_column] >= data_year_week_quantile[3],
          result_column] = 'Rank1'
    }
  }
  data[result_column]
}

#Rank player based on current AvgPts5Wks
d4['OffRnk5Wks'] = createRankColumn(d4, 'AvgPts5Wks')
d4['DefRnk7Wks'] = createRankColumn(d4, 'AvgOppPAP7Wks')
```

```r
# Alternative approach
#Rank defense against position based on rounded # of standard deviations from the overall average
#d4$DefRnk7Wks = round(abs((d4$AvgOppPAP7Wks-d4$OallAvgPAP)/d4$OallStdevPAP))+1
#summary(d4$DefRnk7Wks)
```

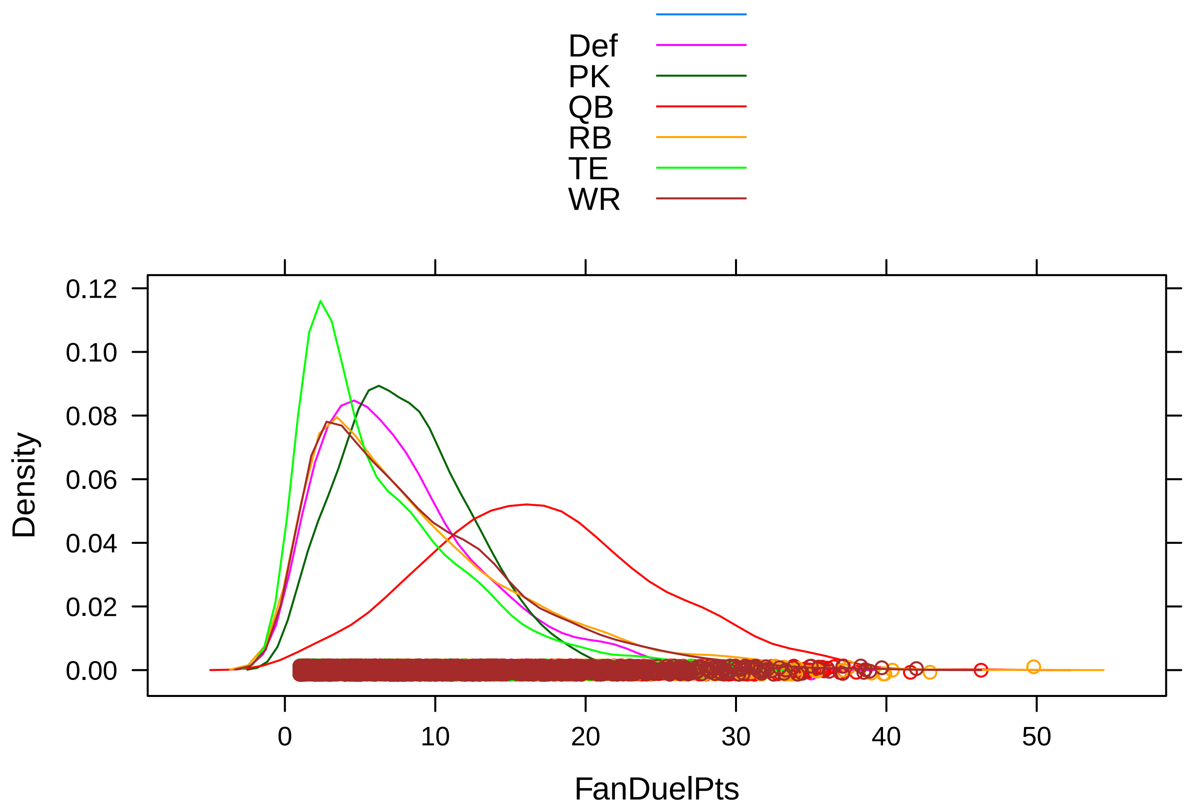
```r
head(d4)
```

```
##      Position Year YearWeek Opponent Week PlayerId             Name    Team
## 3523       QB 2015   201513 Steelers   13     1060 Hasselbeck, Matt   Colts
## 3845       QB 2015   201514  Jaguars   14     1060 Hasselbeck, Matt   Colts
## 3702       QB 2015   201515   Texans   15     1060 Hasselbeck, Matt   Colts
## 4386       QB 2015   201516 Dolphins   16     1060 Hasselbeck, Matt   Colts
## 4301       QB 2015   201501   Ravens    1     1081  Manning, Peyton Broncos
## 3189       QB 2015   201502   Chiefs    2     1081  Manning, Peyton Broncos
##      HomeGame FanDuelPts FanDuelSalary AvgOppPAP7Wks SdOppPAP7Wks OallAvgPAP
## 3523        0       6.86          6500         20.95        8.045      17.44
## 3845        0       9.08          6600         24.16        6.738      17.44
## 3702        1       8.98          6400         16.36        9.690      17.44
## 4386        0       3.96          6000         20.46        6.002      17.44
## 4301        1       5.90          9100         18.99       11.697      17.44
## 3189        0      21.24          8200         13.85        4.342      17.44
##      OallStdevPAP AvgPts5Wks StdevPts5Wks OffRnk5Wks DefRnk7Wks
## 3523        2.909      14.85        4.824      Rank4      Rank1
## 3845        2.909      14.82        4.897      Rank4      Rank1
## 3702        2.909      13.56        5.490      Rank4      Rank3
## 4386        2.909      12.11        5.566      Rank4      Rank2
## 4301        2.909      14.96        8.496      Rank3      Rank1
## 3189        2.909      10.53        5.011      Rank4      Rank4
```

```r
#path = rstudioapi::getSourceEditorContext()$path
#path = gsub(sub("(.*?).*\\/","",path),"fdpfinal.csv",path)
write.csv(d4, file = "fdpfinal.csv",row.names=FALSE, na="")
```

Density Plots

```r
#FanDuelPts distribution by position
densityplot(~FanDuelPts, data=d4,groups=Position,auto.key = TRUE, adjust=1.2)
```



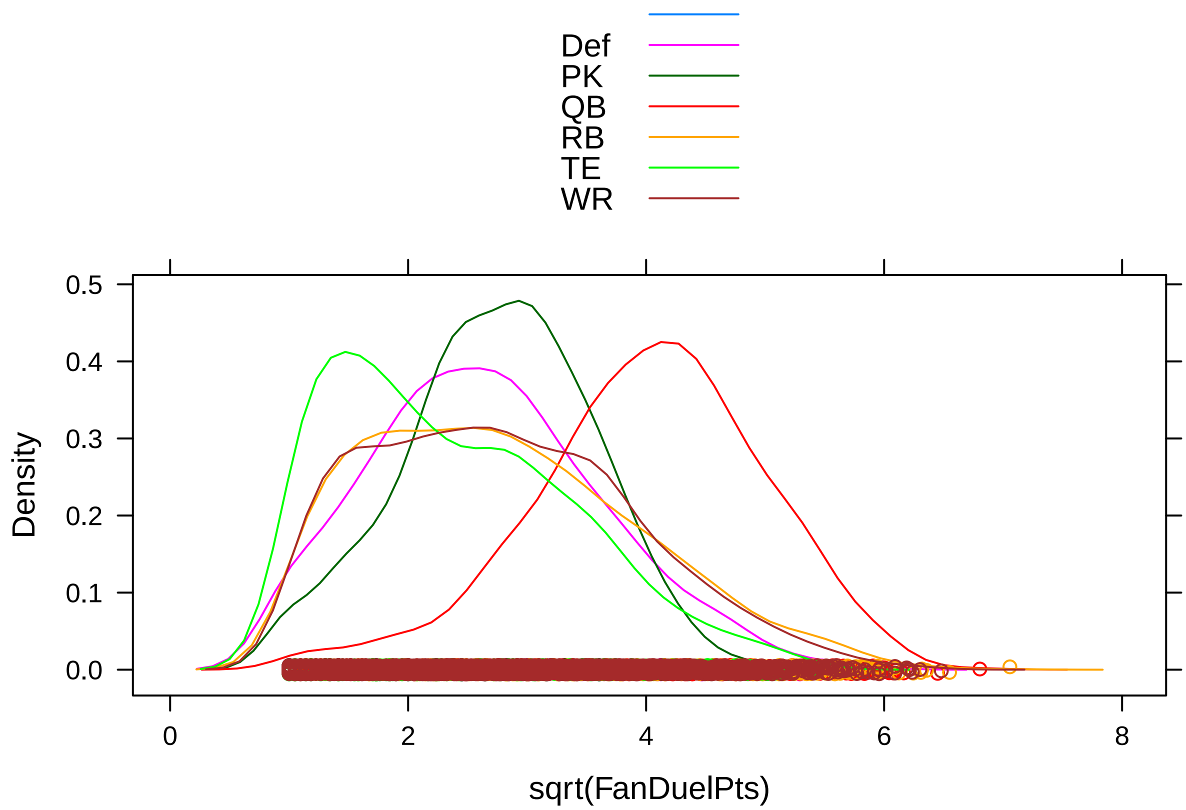

```r
##Example tight end kurtosis and skew
kurtosis(d4[d4$Position=="TE",]$FanDuelPts)
```

```
## [1] 4.706
```

```r
skewness(d4[d4$Position=="TE",]$FanDuelPts)
```

```
## [1] 1.365
```

```r
#sqrt(FanDuelPts) distribution by position to improve kurtosis and skew
densityplot(~sqrt(FanDuelPts), data=d4,groups=Position,auto.key = TRUE, adjust=1.2)
```




```
##Example tight end kurtosis and skew
kurtosis(sqrt(d4[d4$Position=="TE",]$FanDuelPts))
```

```
## [1] 2.633
```

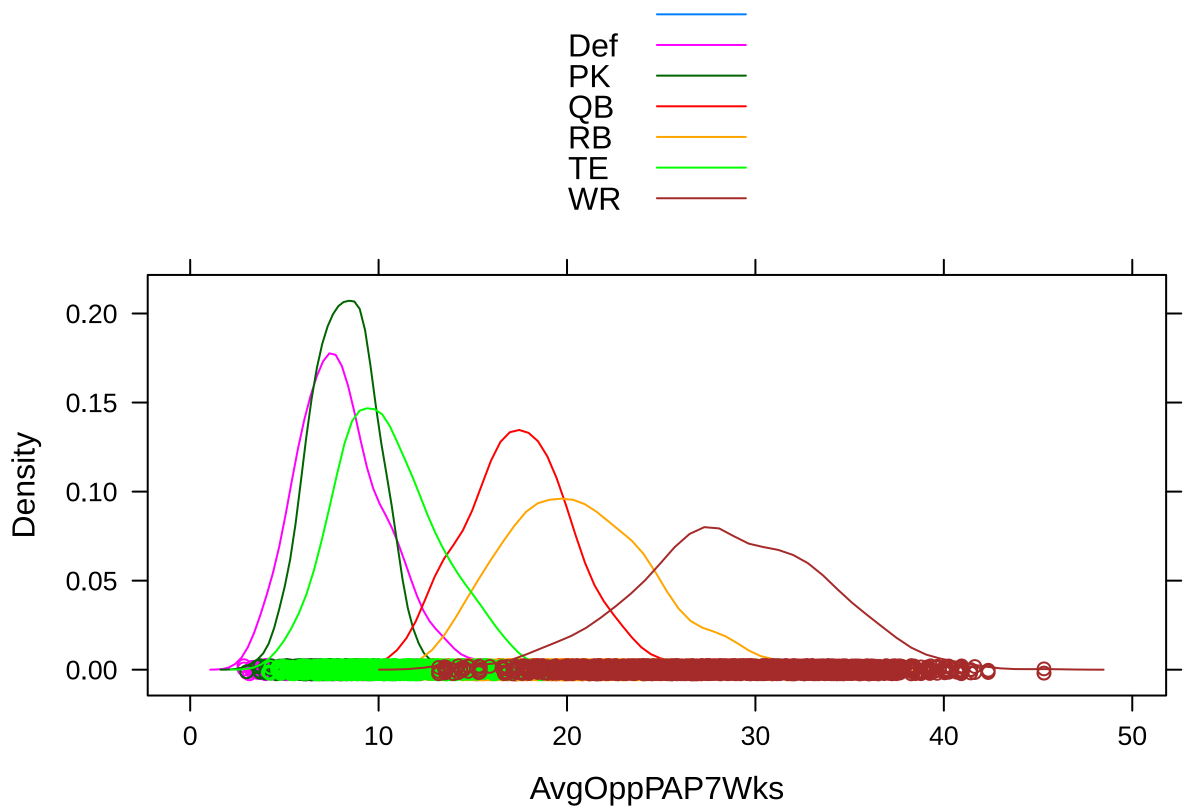
```
skewness(sqrt(d4[d4$Position=="TE",]$FanDuelPts))
```

```
## [1] 0.6202
```

```
#plotting individual players FanDuelPts
densityplot(~FanDuelPts, data=d4[d4$PlayerId>2920 & d4$PlayerId<2930 & d4$Year==2016,],groups=Name,auto
```


```
#plotting individual players AvgPts5Wks
densityplot(~AvgPts5Wks, data=d4[d4$PlayerId>2920 & d4$PlayerId<2930,],groups=Name,auto.key = TRUE, adj
```


```r
#plotting AvgOppPAP7Wks by position
densityplot(~AvgOppPAP7Wks, data=d4,groups=Position,auto.key = TRUE, adjust=1.2)
```



Computation

Training Data Setup

```
fdp_train=fdp[fdp$Year == 2016  & ((fdp$Position == "QB"& fdp$FanDuelSalary > 6500 & !is.na(fdp$FanDuelS
```

```
fdp_test =fdp_train[fdp_train$YearWeek >= 201615, ]
fdp_test = droplevels(fdp_test)
fdp_train=fdp_train[fdp_train$YearWeek < 201615, ]
fdp_train = droplevels(fdp_train)
```

```
Num.Opponent = length(unique(fdp_train[, "Opponent"]))
Num.Position = length(unique(fdp_train[, "Position"]))
#Num.fixed.pred=2 #AvgOppPAP7Wks + FanDuelSalary
Num.fixed.pred=1 #AvgOppPAP7Wks
Num.Rank = length(unique(fdp_train[, "Rank"]))
Num.HomeAwayInit = Num.Rank
Model.File.Ext = ""

X.defense = model.matrix(~ 0 + AvgOppPAP7Wks:Position, data=fdp_train)
X.home = model.matrix(~ 0 + Rank:Position , data=fdp_train)
X.away = model.matrix(~ 0 + Rank:Position , data=fdp_train)
```

```r
X.home = X.home * fdp_train$HomeGame
X.away = X.away * (1- fdp_train$HomeGame)
X = cbind(X.defense, X.home, X.away)
```

```r
library(rjags)
set.seed(20171008)
```

Initialization

```r
# Initialization List for the 4 chains
jags.inits=list(
  list( sigmasqinv=    0.01,  delta = rep(-100000, Num.Position * Num.fixed.pred),
        eta = c(100000, -100000, 100000, -100000)[1:Num.HomeAwayInit],
        rho = c(-100000, 100000, -100000, 100000)[1:Num.HomeAwayInit],
        .RNG.name = "base::Mersenne-Twister", .RNG.seed = 20171008 ),
  list( sigmasqinv=    0.01,  delta = rep(100000, Num.Position * Num.fixed.pred),
        eta = c(100000, -100000, -100000, 100000)[1:Num.HomeAwayInit],
        rho = c(-100000, 100000, 100000, -100000)[1:Num.HomeAwayInit],
        .RNG.name = "base::Mersenne-Twister", .RNG.seed = 20171008 + 1 ),
  list( sigmasqinv=0.000001,  delta = rep(-100000, Num.Position * Num.fixed.pred),
        eta = c(-100000, 100000, -100000, 100000)[1:Num.HomeAwayInit],
        rho = c(100000, -100000, 100000, -100000)[1:Num.HomeAwayInit],
        .RNG.name = "base::Mersenne-Twister", .RNG.seed = 20171008 + 2 ),
  list( sigmasqinv=0.000001,  delta = rep(100000, Num.Position * Num.fixed.pred),
        eta = c(-100000, 100000, 100000, -100000)[1:Num.HomeAwayInit],
        rho = c(100000, -100000, -100000, 100000)[1:Num.HomeAwayInit],
        .RNG.name = "base::Mersenne-Twister", .RNG.seed = 20171008 + 3 )
)

data.jags <- list(
  y= fdp_train$FanDuelPts,
  alpha = fdp_train$AvgPts5Wks,
  X.defense = X.defense,
  X.home = X.home,
  X.away = X.away,
  Num.fixed.pred=Num.fixed.pred,
  Num.Position=Num.Position,
  #Num.Opponent=Num.Opponent,
  Num.Rank=Num.Rank
)
```

```r
burnAndSample = function(m, N.burnin, N.iter, show.plot, mon.col, n.thin=1) {
  update(m, N.burnin) # burn-in

  x <- coda.samples(m, mon.col, n.iter=N.iter, n.thin)

  if(show.plot) {
    plot(x, smooth=FALSE)
  }

  gelman.R = gelman.diag(x, autoburnin=FALSE, multivariate = FALSE)
```

```
  result <- list(
    coda.sam = x,
    gelman.R.max=max(gelman.R$psrf[, 1]),
    gelman.R = gelman.R
  )

  return(result)
}
```

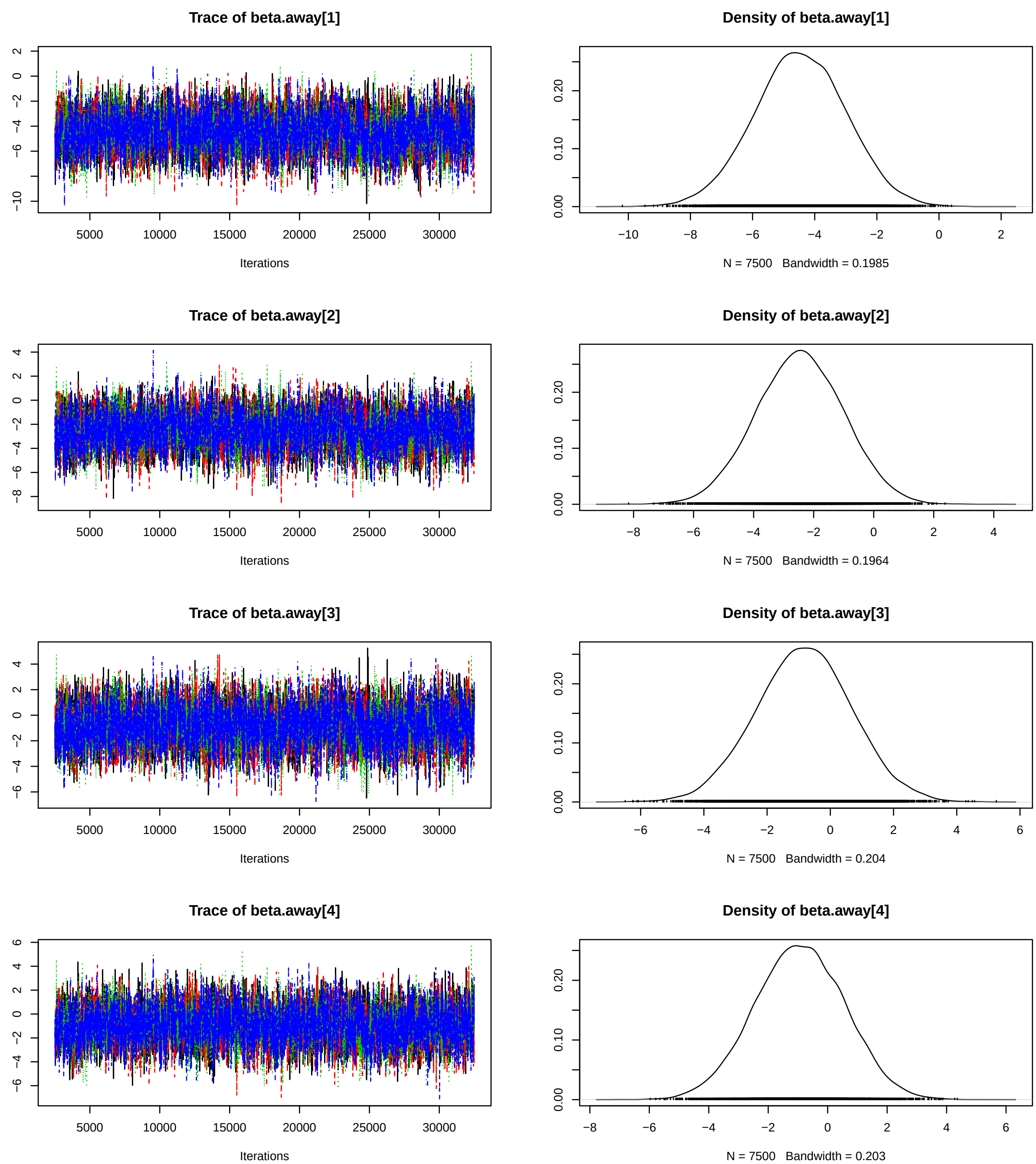
```
runModel=FALSE
runSample=FALSE

mon.col <- c("delta", "eta", "rho", "beta.defense", "beta.home", "beta.away", "sigmasq")

NSim = 30000
NChain = 4
NThin = 5
NTotalSim = NSim * NChain / 5
if (runModel) {
  m <- jags.model("fdp.bug", data.jags, inits = jags.inits, n.chains=NChain, n.adapt = 1000)
  save(file=paste("fdp.jags.model.init", Model.File.Ext, ".Rdata", sep=""), list="m")
} else {
  load(paste("fdp.jags.model.init", Model.File.Ext, ".Rdata", sep=""))
  m$recompile()
}

load.module("dic")

N.Retry.Loop = 1
if (runSample) {
  N.burnin=2500/2
  for (loopIdx in 1:N.Retry.Loop) {
    (start_time <- Sys.time())
    (N.burnin = N.burnin * 2)
    result = burnAndSample(m, N.burnin, NSim, show.plot=FALSE, mon.col = mon.col, n.thin=NChain)
    (end_time <- Sys.time())
    (result$gelman.R.max)
  }
  run.params = paste(".", N.burnin, ".", NChain, ".", NSim, ".", NThin, sep="")
  save(file=paste("fdp.jags.samples", run.params, Model.File.Ext, ".Rdata", sep=""), list="result")
  save(file=paste("fdp.jags.model", run.params, Model.File.Ext, ".Rdata", sep=""), list="m")
} else {
  N.burnin=2500/2 * (2**N.Retry.Loop)
  run.params = paste(".", N.burnin, ".", NChain, ".", NSim, ".", NThin, sep="")
  load(paste("fdp.jags.samples", run.params, Model.File.Ext, ".Rdata", sep=""))
  load(paste("fdp.jags.model", run.params, Model.File.Ext, ".Rdata", sep=""))
  m$recompile()
}
```

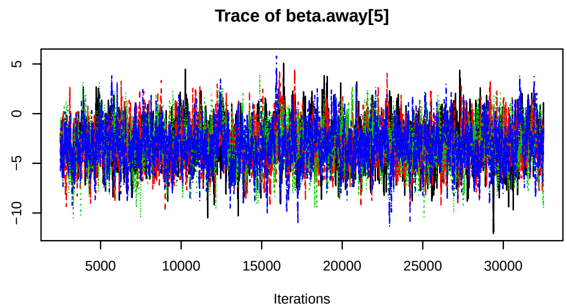
Convergence diagnostics

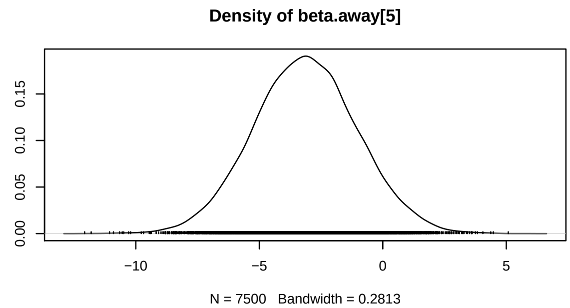
Trace Plots

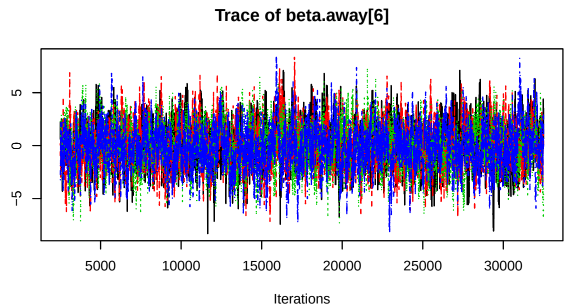
The trace plots of all parameters show good distribution convergence. Please see *Appendices - Converage diagnostics* for all trace plots and the Gelman Statistics summary, and MCMC Summary

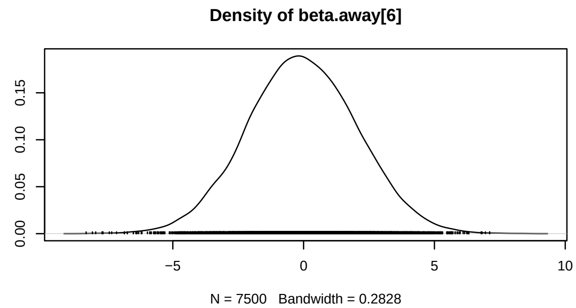
```r
plot(result$coda.sam, smooth=FALSE)
```

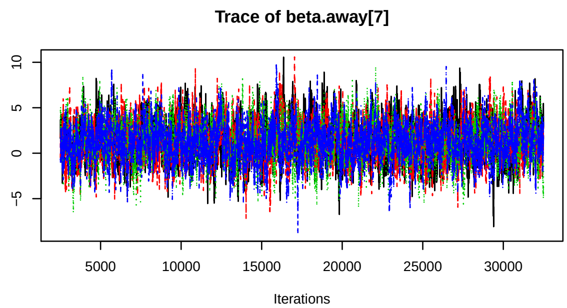


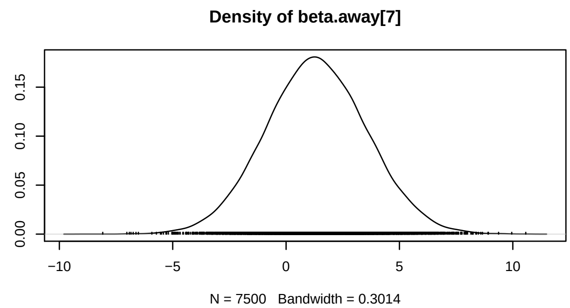

Trace of beta.away[5]
Iterations

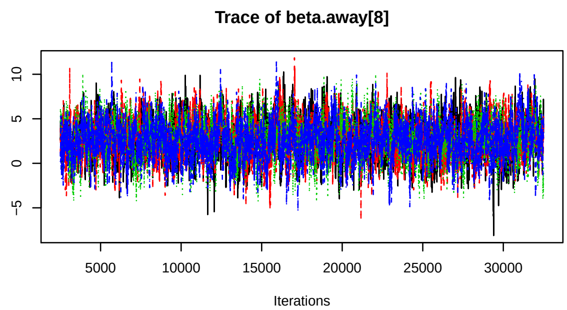

Density of beta.away[5]
N = 7500 Bandwidth = 0.2813

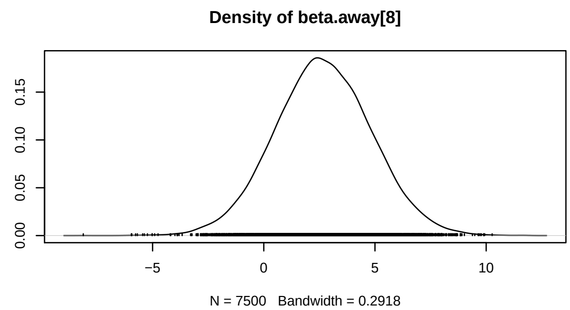

Trace of beta.away[6]
Iterations

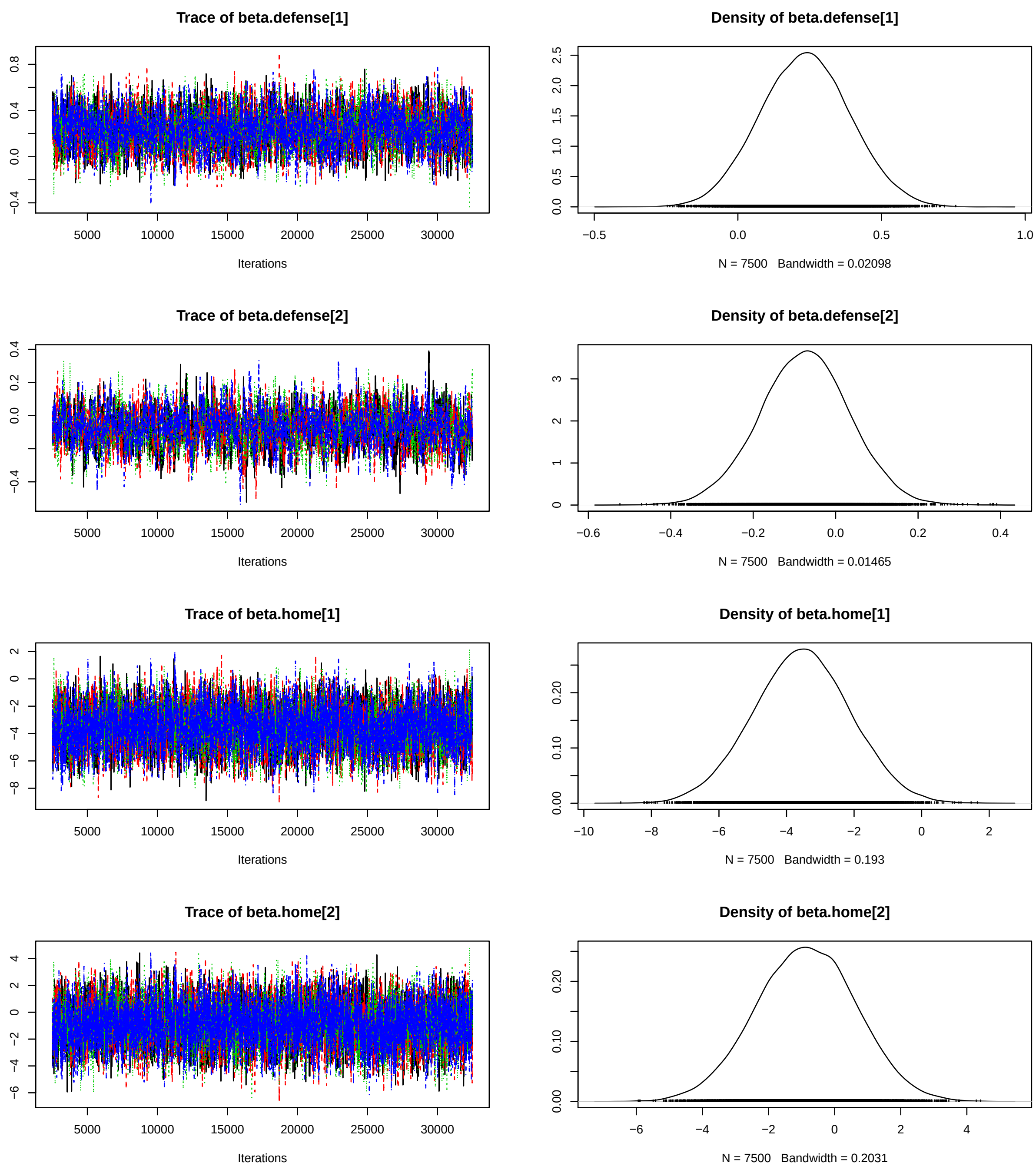

Density of beta.away[6]
N = 7500 Bandwidth = 0.2828

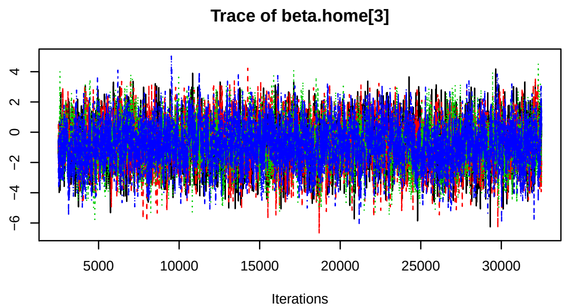

Trace of beta.away[7]
Iterations

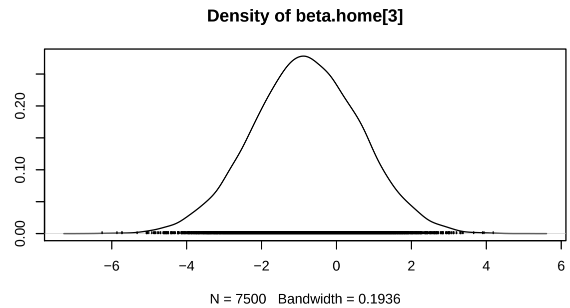

Density of beta.away[7]
N = 7500 Bandwidth = 0.3014

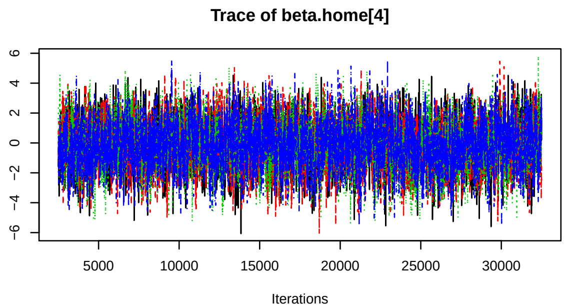

Trace of beta.away[8]
Iterations

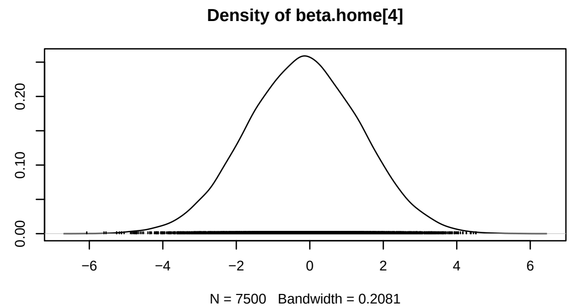

Density of beta.away[8]
N = 7500 Bandwidth = 0.2918

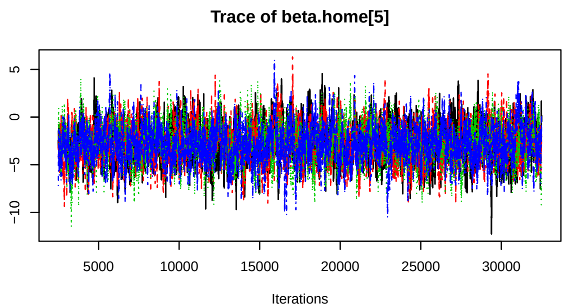

Trace of beta.defense[1]
Iterations
Density of beta.defense[1]
N = 7500 Bandwidth = 0.02098
Trace of beta.defense[2]
Iterations
Density of beta.defense[2]
N = 7500 Bandwidth = 0.01465
Trace of beta.home[1]
Iterations
Density of beta.home[1]
N = 7500 Bandwidth = 0.193
Trace of beta.home[2]
Iterations
Density of beta.home[2]
N = 7500 Bandwidth = 0.2031

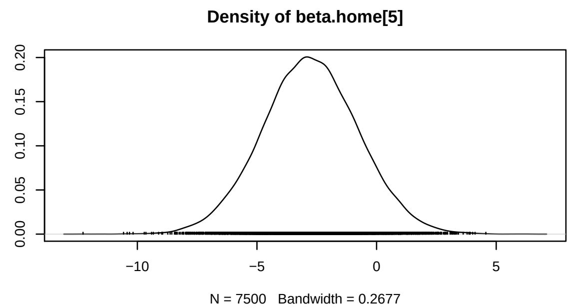

Trace of beta.home[3]
Iterations

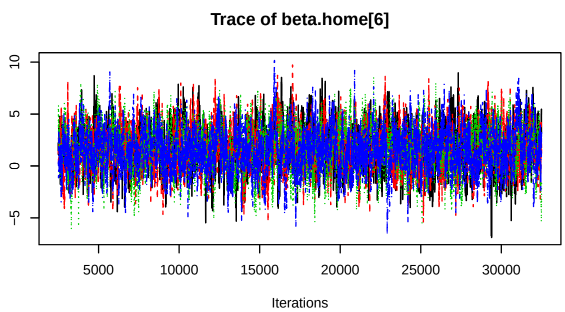

Density of beta.home[3]
N = 7500 Bandwidth = 0.1936

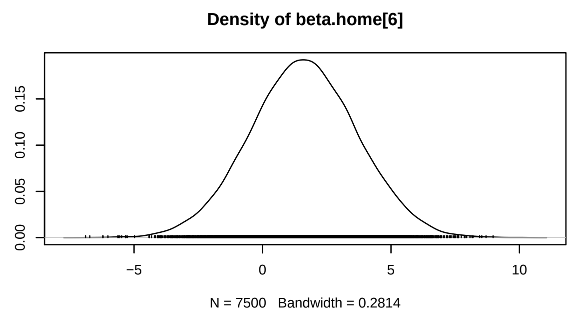

Trace of beta.home[4]
Iterations

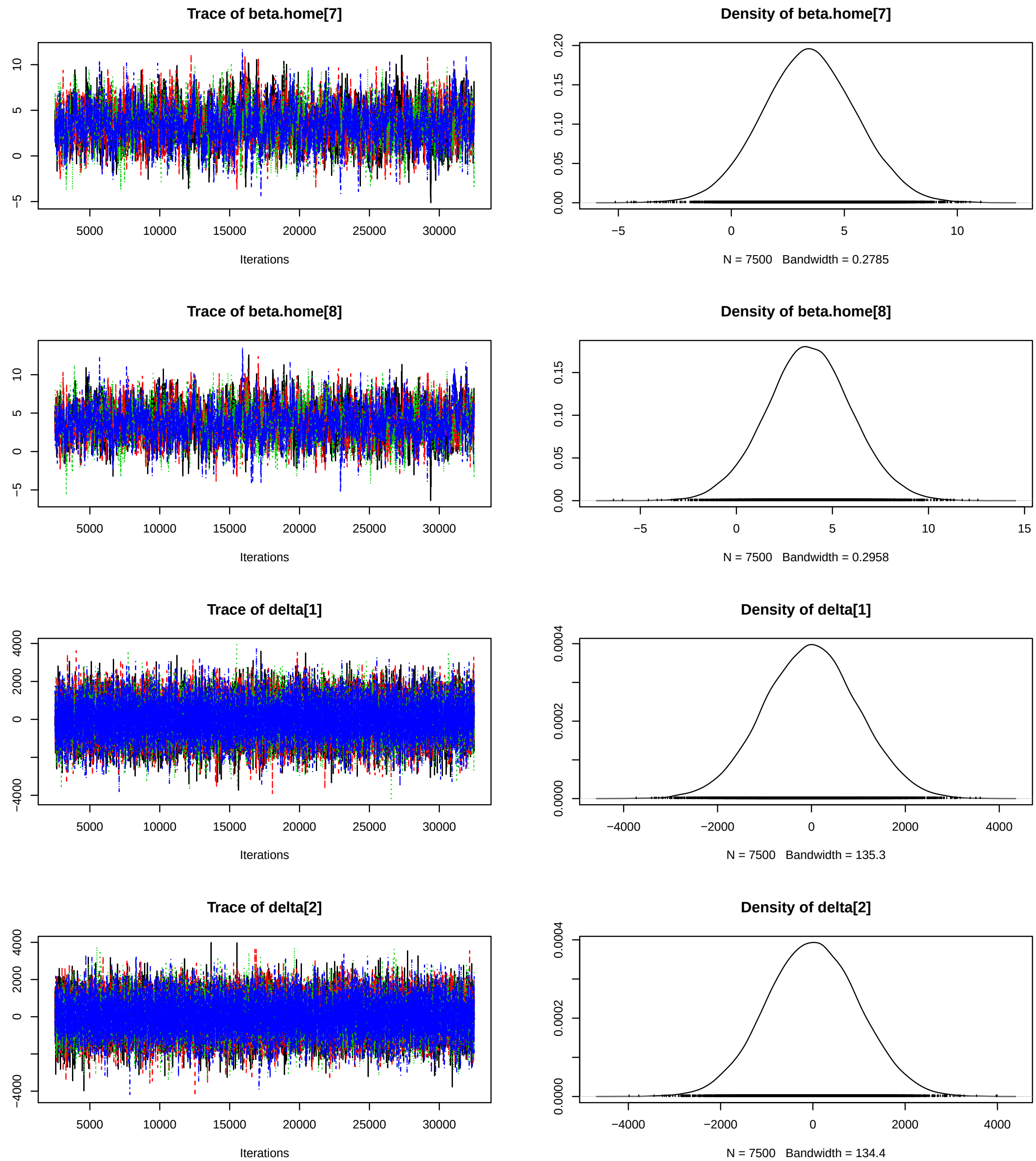

Density of beta.home[4]
N = 7500 Bandwidth = 0.2081

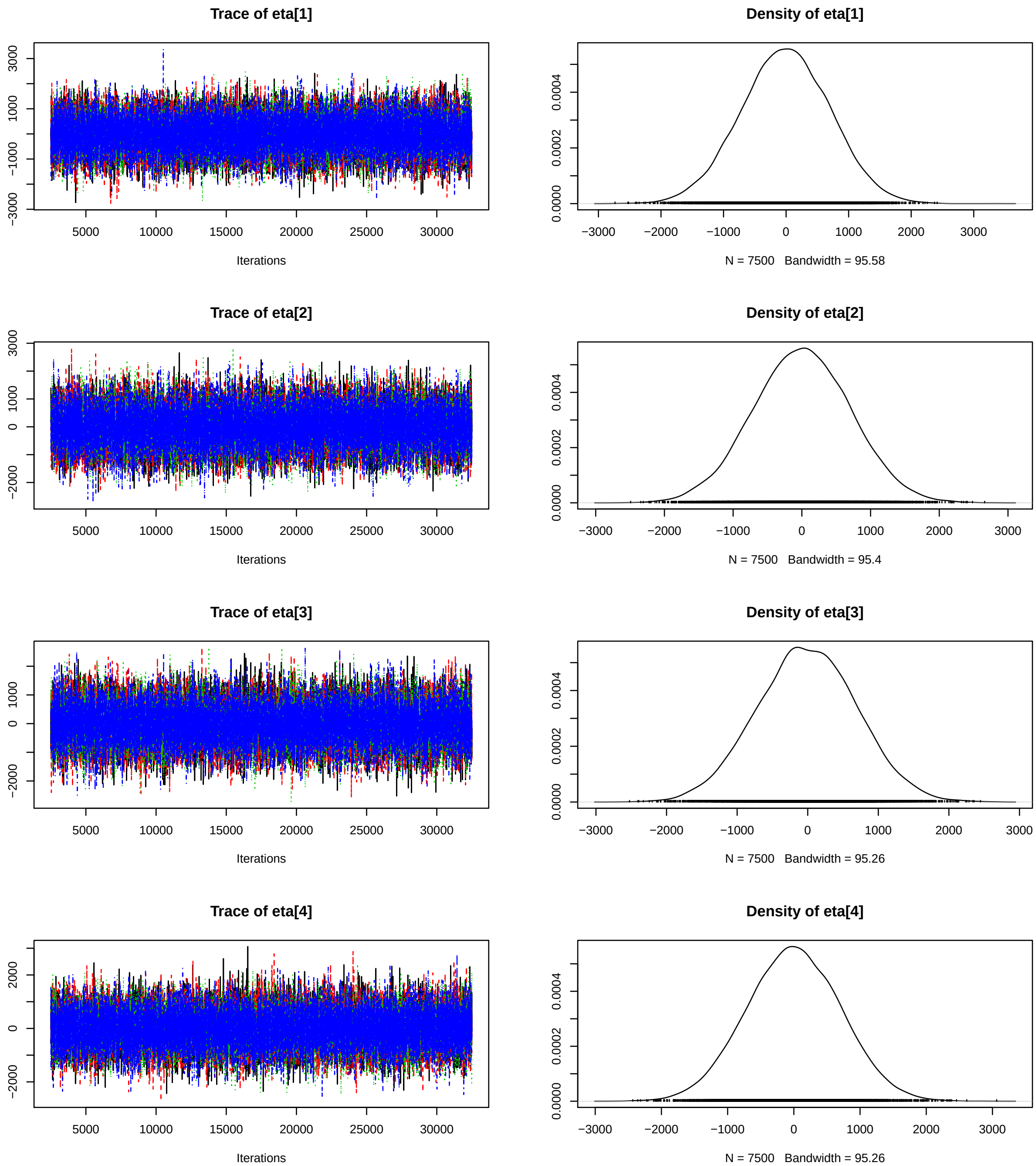

Trace of beta.home[5]
Iterations


Density of beta.home[5]
N = 7500 Bandwidth = 0.2677

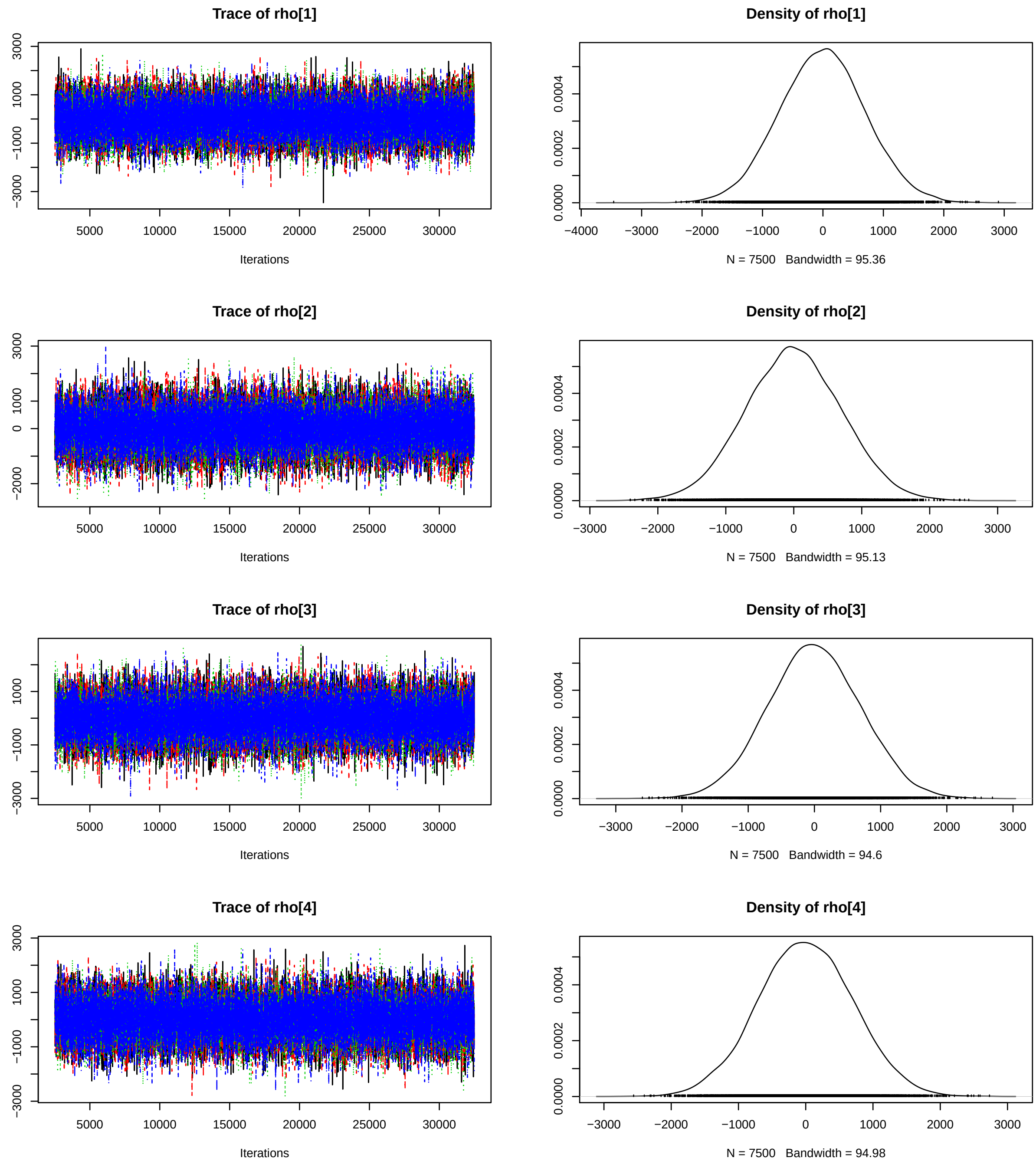

Trace of beta.home[6]
Iterations

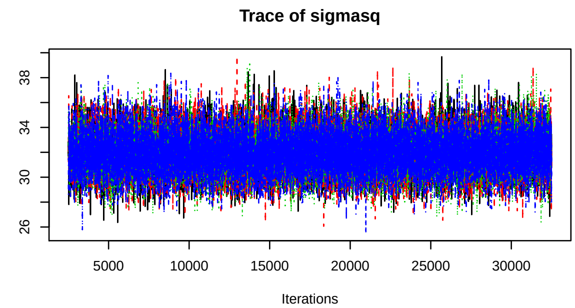

Density of beta.home[6]
N = 7500 Bandwidth = 0.2814

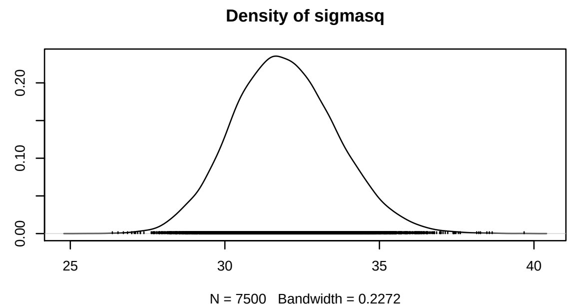

Trace of beta.home[7]
Iterations
Density of beta.home[7]
N = 7500 Bandwidth = 0.2785
Trace of beta.home[8]
Iterations
Density of beta.home[8]
N = 7500 Bandwidth = 0.2958
Trace of delta[1]
Iterations
Density of delta[1]
N = 7500 Bandwidth = 135.3
Trace of delta[2]
Iterations
Density of delta[2]
N = 7500 Bandwidth = 134.4



Trace of eta[1]
Iterations
Density of eta[1]
N = 7500   Bandwidth = 95.58
Trace of eta[2]
Iterations
Density of eta[2]
N = 7500   Bandwidth = 95.4
Trace of eta[3]
Iterations
Density of eta[3]
N = 7500   Bandwidth = 95.26
Trace of eta[4]
Iterations
Density of eta[4]
N = 7500   Bandwidth = 95.26


Trace of rho[1]
Iterations
Density of rho[1]
N = 7500 Bandwidth = 95.36
Trace of rho[2]
Iterations
Density of rho[2]
N = 7500 Bandwidth = 95.13
Trace of rho[3]
Iterations
Density of rho[3]
N = 7500 Bandwidth = 94.6
Trace of rho[4]
Iterations
Density of rho[4]
N = 7500 Bandwidth = 94.98





Gelman Statistics

```
result$gelman.R
```

```
## Potential scale reduction factors:
##
##                  Point est. Upper C.I.
## beta.away[1]              1       1.00
## beta.away[2]              1       1.00
```

```
## beta.away[3]             1       1.00
## beta.away[4]             1       1.00
## beta.away[5]             1       1.01
## beta.away[6]             1       1.01
## beta.away[7]             1       1.00
## beta.away[8]             1       1.01
## beta.defense[1]          1       1.00
## beta.defense[2]          1       1.01
## beta.home[1]             1       1.00
## beta.home[2]             1       1.00
## beta.home[3]             1       1.00
## beta.home[4]             1       1.00
## beta.home[5]             1       1.00
## beta.home[6]             1       1.01
## beta.home[7]             1       1.01
## beta.home[8]             1       1.01
## delta[1]                 1       1.00
## delta[2]                 1       1.00
## eta[1]                   1       1.00
## eta[2]                   1       1.00
## eta[3]                   1       1.00
## eta[4]                   1       1.00
## rho[1]                   1       1.00
## rho[2]                   1       1.00
## rho[3]                   1       1.00
## rho[4]                   1       1.00
## sigmasq                  1       1.00
```

Converged as `gelman.R.max` $= 1.002 < 1.1$ and the plot also looks good.

MCMC Summary

```
(m.summary = summary(result$coda.sam))
```

```
##
## Iterations = 2504:32500
## Thinning interval = 4
## Number of chains = 4
## Sample size per chain = 7500
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
##                   Mean       SD Naive SE Time-series SE
## beta.away[1]    -4.467    1.472 0.008499        0.02417
## beta.away[2]    -2.461    1.456 0.008408        0.02444
## beta.away[3]    -0.832    1.513 0.008733        0.02360
## beta.away[4]    -0.942    1.505 0.008691        0.02398
## beta.away[5]    -3.132    2.096 0.012100        0.05062
## beta.away[6]    -0.051    2.097 0.012108        0.05033
## beta.away[7]     1.343    2.235 0.012903        0.05291
## beta.away[8]     2.670    2.175 0.012558        0.05066
```

```
## beta.defense[1]  0.235    0.156 0.000898        0.00285
## beta.defense[2] -0.075    0.109 0.000632        0.00283
## beta.home[1]    -3.547    1.431 0.008264        0.02330
## beta.home[2]    -0.859    1.506 0.008694        0.02329
## beta.home[3]    -0.805    1.435 0.008287        0.02291
## beta.home[4]    -0.162    1.543 0.008909        0.02516
## beta.home[5]    -2.767    2.006 0.011579        0.04770
## beta.home[6]     1.605    2.086 0.012045        0.04957
## beta.home[7]     3.449    2.074 0.011976        0.04888
## beta.home[8]     3.784    2.221 0.012826        0.05233
## delta[1]        -6.780 1003.390 5.793077        5.89868
## delta[2]         1.764  996.816 5.755122        5.75533
## eta[1]          -7.966  708.771 4.092093        4.08607
## eta[2]           2.218  707.396 4.084150        4.08699
## eta[3]          -9.512  706.400 4.078403        4.05872
## eta[4]          -1.529  706.367 4.078209        4.02323
## rho[1]          -6.188  707.088 4.082376        4.10605
## rho[2]          -3.301  705.424 4.072770        4.11118
## rho[3]          -1.886  701.548 4.050388        4.05048
## rho[4]           1.520  704.309 4.066333        4.04433
## sigmasq         31.947    1.685 0.009727        0.00974
##
## 2. Quantiles for each variable:
##
##                      2.5%      25%      50%       75%    97.5%
## beta.away[1]       -7.3717   -5.453  -4.4688  -3.46981   -1.611
## beta.away[2]       -5.3105   -3.446  -2.4545  -1.46614    0.366
## beta.away[3]       -3.8050   -1.845  -0.8234   0.18547    2.160
## beta.away[4]       -3.8988   -1.965  -0.9387   0.09424    1.969
## beta.away[5]       -7.2309   -4.542  -3.1370  -1.74709    1.026
## beta.away[6]       -4.1378   -1.461  -0.0690   1.36573    4.063
## beta.away[7]       -3.0093   -0.157   1.3282   2.84138    5.776
## beta.away[8]       -1.5793    1.223   2.6498   4.12313    6.979
## beta.defense[1]    -0.0663    0.129   0.2350   0.33995    0.543
## beta.defense[2]    -0.2931   -0.148  -0.0741  -0.00196    0.138
## beta.home[1]       -6.3655   -4.514  -3.5414  -2.58559   -0.756
## beta.home[2]       -3.8301   -1.887  -0.8491   0.16783    2.062
## beta.home[3]       -3.6558   -1.769  -0.8066   0.17242    2.007
## beta.home[4]       -3.1788   -1.211  -0.1571   0.88799    2.859
## beta.home[5]       -6.6877   -4.098  -2.7803  -1.43849    1.176
## beta.home[6]       -2.4894    0.204   1.5977   3.00227    5.706
## beta.home[7]       -0.5994    2.064   3.4377   4.83181    7.517
## beta.home[8]       -0.6143    2.315   3.7747   5.25484    8.181
## delta[1]        -1978.4088 -688.037  -4.1092 662.68424 1965.061
## delta[2]        -1940.6629 -675.923  -2.0463 671.92029 1962.165
## eta[1]          -1413.8970 -482.575  -5.4483 474.92512 1370.112
## eta[2]          -1394.8328 -476.146   1.7122 481.47216 1388.076
## eta[3]          -1396.0830 -483.186  -9.9967 470.21005 1376.652
## eta[4]          -1382.7089 -479.828  -1.6793 479.87624 1378.586
## rho[1]          -1387.9001 -483.225  -2.6988 464.93505 1388.009
## rho[2]          -1382.8577 -480.359  -6.9693 470.32306 1373.230
## rho[3]          -1376.5466 -471.454  -6.5531 468.53942 1366.468
## rho[4]          -1383.8694 -474.965  -1.9381 476.58238 1379.023
## sigmasq            28.7819   30.779  31.8832  33.05067   35.406
```

Model Assessment

General posterior model assumption check

Probabiliy of players should perform better at home than away

```
post.samp = as.matrix(result$coda.sam)

beta.home = post.samp[, paste("beta.home[",1:(Num.Rank*Num.Position),"]", sep="")]
beta.away = post.samp[, paste("beta.away[",1:(Num.Rank*Num.Position),"]", sep="")]

prob.home.away = rep(0, Num.Rank * Num.Position)
for (r in 1:Num.Rank) {
  for (p in 1:Num.Position) {
    idx = (p-1) * Num.Rank + r
    prob.home.away[idx] = mean(beta.home[, idx] > beta.away[, idx])
  }
}
prob.home.away
prob.home.away.df = data.frame(colnames(X.home))
prob.home.away.df$prob.home.bt.away = prob.home.away

colnames(prob.home.away.df) = c("Rank:Position", "Prob.home.bt.away")

kable(prob.home.away.df)
```

The above table shows that players perform better at home than away as expected.

Beta defense

If a player is facing a team which gives up more points to players on average, we expect the player will score more points.

```
Num.fixed.size=Num.Position*Num.fixed.pred
beta.defense = post.samp[, paste("beta.defense[",1:Num.fixed.size,"]", sep="")]

beta.defense.int.df = data.frame(colnames(X.defense))
beta.defense.int = matrix(rep(0, Num.fixed.size * 4), nrow=Num.fixed.size, ncol = 4)

int.alpha=0.05
for (p in 1:Num.Position) {
  for (f in 1:Num.fixed.pred) {
    idx = (f-1) * Num.Position + p
      beta.defense.int[idx, 1:3] =  quantile(beta.defense[, idx], c(int.alpha/2, 0.5, 1-int.alpha/2))
      beta.defense.int[idx, 4] = mean(beta.defense[, idx])
  }
}

beta.defense.int.df$pct025 = beta.defense.int[, 1]
beta.defense.int.df$pct975 = beta.defense.int[, 3]
beta.defense.int.df$median = beta.defense.int[, 2]
beta.defense.int.df$mean = beta.defense.int[, 4]
colnames(beta.defense.int.df) = c("beta.defense.position", "pct025", "pct975", "median", "mean")
kable(beta.defense.int.df)
```

We observe that the median beta.defense for PK is positive as expected. But for QB, it is negative, that implies QB actually scores less against bad defensive team.

Posterior Predictive Check

Error correlation Check

A posterior predictive p-value using the following test quantity

$$T(y, X, \theta) = |\hat{cor}(\epsilon, \text{time})|$$

where $\hat{cor}(\epsilon, \text{time})$ is sample correlation between the error vector ϵ and the year week in the data. The larger this quantity is for the model, the less well it fits the data as that would mean the error is correlated with time.

The simulated error vectors ϵ (as rows of a matrix):

```
error.sim <- matrix(NA, NTotalSim, nrow(fdp_train))
y_hat.sim <- matrix(NA, NTotalSim, nrow(fdp_train))
for(s in 1:NTotalSim) {
  y_hat.sim[s, ] =  fdp_train$AvgPts5Wks + (X.defense %*% beta.defense[s, ]) + (X.home %*% beta.home[s,
  error.sim[s, ] <- (fdp_train$FanDuelPts - y_hat.sim[s, ])
}
```

The simulated replicate error vectors ϵ^{rep} (as rows of a matrix), which are the error vectors computed using replicate response vectors y^{rep}:

```
post.sigma.2.sim <- post.samp[,"sigmasq"]
post.sigma.sim <- sqrt(post.sigma.2.sim)

yreps <- matrix(NA, NTotalSim, nrow(fdp_train))
for(s in 1:NTotalSim) {
  yreps[s, ] <- rnorm(nrow(fdp_train), y_hat.sim[s, ], post.sigma.sim[s])
}

error.rep <- matrix(NA, NTotalSim, nrow(fdp_train))

for(s in 1:NTotalSim) {
  error.rep[s, ] <- (yreps[s, ] - y_hat.sim[s, ])
}
```

The simulated values of $T(y, X, \theta)$

```
T.sim = abs(cor(t(error.sim), fdp_train$YearWeek))
head(T.sim)
```

The simulated values of $T(y^{rep}, X, \theta)$

```
T.rep.sim = abs(cor(t(error.rep), fdp_train$YearWeek))
head(T.rep.sim)
```

The simulated values of $T(y^{rep}, X, \theta)$ versus those of $T(y, X, \theta)$, with a reference line indicating where the two values would be equal.

```r
plot(T.sim, T.rep.sim, pch=".", cex=2,
  xlim=c(min(T.sim, T.rep.sim), max(T.sim, T.rep.sim)),
  ylim=c(min(T.sim, T.rep.sim), max(T.sim, T.rep.sim)),
  xlab="T(y,x,theta)", ylab="T(y.rep,x,theta)")
abline(a=0,b=1)
```

The posterior predictive p-value:

```r
(p.value = mean(T.rep.sim >= T.sim))
```

The p.value is 0.1162, $>$ 0.05, which does not indicate any evidence of problem.

Chi-square Discrepancy Check

Chi-square discrepancy check is used to check for general model issues like mis-specified means, mis-specified variances, and over-concentrated prior.

```r
Tchi <- numeric(NTotalSim)
Tchirep <- numeric(NTotalSim)
for(s in 1:NTotalSim){
  Tchi[s] <- sum((fdp_train$FanDuelPts - y_hat.sim[s,])^2 / post.sigma.sim[s])
  Tchirep[s] <- sum((yreps[s,] - y_hat.sim[s,])^2 / post.sigma.sim[s])
}
(p.value.Tchi = mean(Tchirep >= Tchi))
```

```r
plot(Tchi, Tchirep, pch=".", cex=2,
  xlim=c(min(Tchi, Tchirep), max(Tchi, Tchirep)),
  ylim=c(min(Tchi, Tchirep), max(Tchi, Tchirep)),
  xlab="Tchi", ylab="Tchirep")
abline(a=0,b=1)
```

The posterior predictive p-value using the chi-square discrepancy is `p.value.Tchi`=0.5057. The p-value is $>$ 0.05. Hence, it does not indicate any evidence of problems.

Individual Data Point Discrepancy Check

Using $Pr(y^{rep} \geq y|y)$ as posterior predictive p-value

```r
yreps.minus.y <- matrix(NA, NTotalSim, nrow(fdp_train))
for(s in 1:NTotalSim) {
  yreps.minus.y[s, ] <- yreps[s, ] - fdp_train$FanDuelPts
}

(p.value.y.rep.all = mean(yreps.minus.y > 0))
```

The posterior predictive p-value using individual data point is `p.value.y.rep.all` = 0.5076, which is $>$ 0.05. This shows no evidence of problem.

Non Negative Check

As discussed in the model section, we use normal distribution, instead of Poisson distribution, to simplify the model. Here we'll check the portion of predicted value < 0.

```
perct.lt.zero = mean(yreps < 0)
```

The percentage of predicted values that are less than zero is `perct.lt.zero` $= 0.0463$, which is relatively small. This justify the decision to use normal to simplify our model.

Prediction

We use the last week of data to check the prediction effectiveness of the model. This is essentially a cross validation analysis.

```
X.defense.test = model.matrix(~ 0 + AvgOppPAP7Wks:Position, data=fdp_test)
X.home.test = model.matrix(~ 0 + Rank:Position , data=fdp_test)
X.away.test = model.matrix(~ 0 + Rank:Position , data=fdp_test)

X.home.test = X.home.test * fdp_test$HomeGame
X.away.test = X.away.test * (1- fdp_test$HomeGame)
X.test = cbind(X.defense.test, X.home.test, X.away.test)
```

```
y_hat.test <- matrix(NA, NTotalSim, nrow(fdp_test))
for(s in 1:NTotalSim) {
  y_hat.test[s, ] =  fdp_test$AvgPts5Wks + (X.defense.test %*% beta.defense[s, ]) + (X.home.test %*% be
}

y.pred <- matrix(NA, NTotalSim, nrow(fdp_test))
for(s in 1:NTotalSim) {
  y.pred[s, ] <- rnorm(nrow(fdp_test), y_hat.test[s, ], post.sigma.sim[s])
}
```

Prediction of Individual Player Performance

A measure of the effectiveness of this model is to predict individual player performance. Consider an example data point, `fdp_test[1,]`

```
(fdp_test[1,])
```

The real FanDuelPts is 7.42

The predicted value has the following 95% interval

```
quantile(y.pred[, 1], c(0.025, 0.975))
```

which does contain the actual data value of 7.42

Here is the density plot of the posterior density

```
library(lattice)
densityplot(y.pred[, 1])
```

Overall Prediction Effectiveness

To measure the overall prediction effectiveness, we can look at $Pr(y_{pred} >= y)$ as a posterior p-value.

```
y.pred.minus.y <- matrix(NA, NTotalSim, nrow(fdp_test))
for(s in 1:NTotalSim) {
  y.pred.minus.y[s, ] <- y.pred[s, ] - fdp_test$FanDuelPts
}

(p.value.y.pred.all = mean(y.pred.minus.y > 0))
```

The probability of $y_{pred} >= y$ is 0.4683, close to 0.5, which implies a relatively good predictive value.

A look at a cross section of how one simulation of a prediction of the whole test set

```
for (s in 1:1) {
  plot(fdp_test$FanDuelPts, y.pred[s, ], pch=".", cex=2,
  xlim=c(min(y.pred[s, ], fdp_test$FanDuelPts), max(y.pred[s, ], fdp_test$FanDuelPts)),
  ylim=c(min(y.pred[s, ], fdp_test$FanDuelPts), max(y.pred[s, ], fdp_test$FanDuelPts)),
  xlab="y", ylab="y pred")
  abline(a=0,b=1)
}
```

Alternative Model - no rank

```
#sink("fdp.norank.bug")
#cat("
model {
  for (i in 1:length(y)) {
    y[i] ~ dnorm(alpha[i] + inprod(X.defense[i, ], beta.defense)
                 + inprod(X.home[i, ], beta.home)
                 + inprod(X.away[i, ], beta.away), sigmasqinv)
  }

  # The entry of the beta.defense corresponds to Opponent:Position
  # In our model, we pool the beta.defense based on position.
  # i.e. All defense effects of the same position are drawn from the same distribution
  for (p in 1:Num.Position) {
    beta.defense[p] ~ dnorm(delta[p], 1/1000^2)
    delta[p] ~ dnorm(0, 1/100000^2)
  }

  # The entry of the beta.home and beta.away corresponds to Position
  # In our model, we pool the beta.home/away based on Position
  # NO RANK
  for (t in 1:Num.Position) {
    beta.home[t] ~ dnorm(eta, 1/1000^2)
    beta.away[t] ~ dnorm(rho, 1/1000^2)
  }
  eta ~ dnorm(0, 1/100000^2)
  rho ~ dnorm(0, 1/100000^2)

  sigmasqinv ~ dgamma(0.0001, 0.0001)
  sigmasq <- 1/sigmasqinv
}
#    ",fill = TRUE)
#sink()
```

```
Num.Opponent = length(unique(fdp_train[, "Opponent"]))
Num.Position = length(unique(fdp_train[, "Position"]))
#Num.fixed.pred=2 #AvgOppPAP7Wks + FanDuelSalary
Num.fixed.pred=1 #AvgOppPAP7Wks
Num.HomeAwayInit = 1
Model.File.Ext = ".norank"

X.defense = model.matrix(~ 0 + AvgOppPAP7Wks:Position, data=fdp_train)
X.home = model.matrix(~ 0 + Position , data=fdp_train)
X.away = model.matrix(~ 0 + Position , data=fdp_train)


X.home = X.home * fdp_train$HomeGame
X.away = X.away * (1- fdp_train$HomeGame)
X = cbind(X.defense, X.home, X.away)
```

```r
# Initialization List for the 4 chains
jags.inits=list(
  list( sigmasqinv=    0.01,  delta = rep(-100000, Num.Position * Num.fixed.pred),
        eta = c(100000, -100000, 100000, -100000)[1:Num.HomeAwayInit],
        rho = c(-100000, 100000, -100000, 100000)[1:Num.HomeAwayInit],
        .RNG.name = "base::Mersenne-Twister", .RNG.seed = 20171008 ),
  list( sigmasqinv=    0.01,  delta = rep(100000, Num.Position * Num.fixed.pred),
        eta = c(100000, -100000, -100000, 100000)[1:Num.HomeAwayInit],
        rho = c(-100000, 100000, 100000, -100000)[1:Num.HomeAwayInit],
        .RNG.name = "base::Mersenne-Twister", .RNG.seed = 20171008 + 1 ),
  list( sigmasqinv=0.000001,  delta = rep(-100000, Num.Position * Num.fixed.pred),
        eta = c(-100000, 100000, -100000, 100000)[1:Num.HomeAwayInit],
        rho = c(100000, -100000, 100000, -100000)[1:Num.HomeAwayInit],
        .RNG.name = "base::Mersenne-Twister", .RNG.seed = 20171008 + 2 ),
  list( sigmasqinv=0.000001,  delta = rep(100000, Num.Position * Num.fixed.pred),
        eta = c(-100000, 100000, 100000, -100000)[1:Num.HomeAwayInit],
        rho = c(100000, -100000, -100000, 100000)[1:Num.HomeAwayInit],
        .RNG.name = "base::Mersenne-Twister", .RNG.seed = 20171008 + 3 )
)

data.jags <- list(
  y= fdp_train$FanDuelPts,
  alpha = fdp_train$AvgPts5Wks,
  X.defense = X.defense,
  X.home = X.home,
  X.away = X.away,
  Num.fixed.pred=Num.fixed.pred,
  Num.Position=Num.Position
  #Num.Opponent=Num.Opponent,
  #Num.Rank=Num.Rank
)
```

```r
runModel=TRUE
runSample=TRUE

mon.col <- c("delta", "eta", "rho", "beta.defense", "beta.home", "beta.away", "sigmasq")

NSim = 30000
NChain = 4
NThin = 5
NTotalSim = NSim * NChain / 5
if (runModel) {
  bug.file = "fdp.norank.bug"
  m <- jags.model(bug.file, data.jags, inits = jags.inits, n.chains=NChain, n.adapt = 1000)
  save(file=paste("fdp.jags.model.init", Model.File.Ext, ".Rdata", sep=""), list="m")
} else {
  load(paste("fdp.jags.model.init", Model.File.Ext, ".Rdata", sep=""))
  m$recompile()
}

load.module("dic")

N.Retry.Loop = 1
```

```r
if (runSample) {
  N.burnin=2500/2
  for (loopIdx in 1:N.Retry.Loop) {
    (start_time <- Sys.time())
    (N.burnin = N.burnin * 2)
    result = burnAndSample(m, N.burnin, NSim, show.plot=FALSE, mon.col = mon.col, n.thin=NChain)
    (end_time <- Sys.time())
    (result$gelman.R.max)
  }
  run.params = paste(".", N.burnin, ".", NChain, ".", NSim, ".", NThin, sep="")
  save(file=paste("fdp.jags.samples", run.params, Model.File.Ext, ".Rdata", sep=""), list="result")
  save(file=paste("fdp.jags.model", run.params, Model.File.Ext, ".Rdata", sep=""), list="m")
} else {
  N.burnin=2500/2 * (2**N.Retry.Loop)
  run.params = paste(".", N.burnin, ".", NChain, ".", NSim, ".", NThin, sep="")
  load(paste("fdp.jags.samples", run.params, Model.File.Ext, ".Rdata", sep=""))
  load(paste("fdp.jags.model", run.params, Model.File.Ext, ".Rdata", sep=""))
  m$recompile()
  gelman.diag(result$coda.sam, autoburnin=FALSE, multivariate = FALSE)
}
```

Converged as `gelman.R.max` $= 1.002 < 1.1$ and the plot also looks good.

```r
(m.summary = summary(result$coda.sam))
```

Effective Sample Size

```r
(eff.size = effectiveSize(result$coda.sam[, ]))
```

The effective sample sizes of all parameters are greater than 400.

DIC

```r
(dic.samp = dic.samples(m, NTotalSim))
```

The effective number of parameters("penalty") is 7.02, and the Plummer's DIC ("Penalized deviance") is 4781. This model has a higher DIC compared with the original one with rank(4719). Hence, we conclude that the original model (with rank) is better for prediction.