A Bayesian Approach to Predicting NFL Quarterback Scores in Fanduel Tournaments

*STAT 578, Fall 2017, **Team 5**: Aaron Ray, Kiomars Nassiri, Michael Chan*

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Project Description

The National Football League (NFL), being one of the major professional sports leagues in North America, has a wide audience. participates in the NFL craze by competing in fantasy football tournaments organized by the daily fantasy site, "FanDuel.com". Participants in a **Fantasy Football** game act as the managers of a virtual footbal team and try to maximize their points by picking up the best line-up. Points are given based on actual performance of players in real-world competition. For the purpose of this project we have chosen to work with the data gathered from the **FanDuel** internet company. We will leverage a Hierarchical Bayesian approach with the Markov Chain Monte Carlo method to predict the fantasy points likely to be scored by an NFL quarterback in any given game. The goal is to predict the points scored by each player given certain prior conditions and predictor variables that will assist our model in providing credible posterior prediction intervals.

The analysis is inspired by the study presented in the article, **Bayesian Hierarchical Modeling Applied to Fantasy Football Projections for Increased Insight and Confidence**, by Scott Rome.

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Dataset Description

Team has set up a process to gather the historical data from the RotoGuru website. The following is the code used to get the data from RotoGuru:

```
# Scrape rotoguru1 site for weekly FanDuel stats and bind each week's data to the
# pre-defined dataframe, 'd'.

for(year in 2014:2017){
  for(week in 1:16){
    page = read_html(
      gsub(" ","",
           paste("http://rotoguru1.com/cgi-bin/fyday.pl?week=",week,"&year=",
                 year ,"&game=fd&scsv=1")
      ))
    dtext = page %>% html_nodes("pre") %>% html_text(trim = TRUE)
    dtable = read.table(text=dtext, sep = ";", header=TRUE, col.names = cnames,
```

```
                              quote=NULL)
    d = rbind(d,dtable)
  }
}
```

Data cleaning is performed using R routines. Some data cleaning tasks are needed to calculate Player rank.

Response Variables

- `FanDuelPts`: Points position at the end of a single game

Predictor Variables

- `AvgPts5Wks`: The 5 game average points of the player
- `AvgOppPAP7Wks` : The 7 game average Opposing Points Allowed to Position (OppPAP) by the current player's opposing defense. For example, if the Buffalo Bills defense allowed a total of 30 points per game to wide receivers for six games straight, then this number would equal to the average of 30 for any wide receiver facing the Bills defense.
- `Position`: The position the player plays
- `HomeGame`: Whether it is home game.
- `Rank`: The rank of a player based on recent performance

Analysis Ideas

Model

At the lowest level, we model the performance (`FanDuelPts`) as normally-distributed around a true value:

$y|\alpha, \beta_{defense}, \beta_{home}, \beta_{away}, \sigma_r^2 \sim N(\alpha + X_{defense}.\beta_{defense} + X_{home}.\beta_{home} + X_{away}.\beta_{away}, \sigma_y^2 I)$

where

$\alpha =$ The average fan duel point of the previous 5 weeks of the player, `AvgPts5Wks`

$\beta_{defense,p} =$ defense coefficient against team t for position p

$\beta_{home,p,r} =$ home coefficient for position p and a rank r player

$\beta_{away,p,r} =$ Away coefficient for position p and a rank r player

$y =$ `FanDuelPts`

$x_p =$ interaction indicator term for opposing team score allowed by position p

$x_{home,p,r} =$ interaction indicator term for rank r, position p, and whether it is home game

At higher level, we model the defense effect, $\beta_{defense}$, as how good(bad) a particular team's defense is against the player's position. We pool the effect based on the position of the player. That is, the defense coefficient is normally distributed from the same position specific distribution.

$\beta_{defense,p} \sim N(\delta_p, \sigma_\delta^2)$

where σ_δ is constant $= 1000$

For the home and away game effect, β_{home} and β_{away}, we model the effect for player of the same rank has the same distribution. We model the home and away game effect to be the same for players of the same position.

$\beta_{home,p,r} \sim N(\eta_r, \sigma_\eta^2)$

$\beta_{away,p,r} \sim N(\rho_r, \sigma_\rho^2)$

where σ_η, σ_ρ are constant $= 1000$

We will approximate non informative prior using:

$\sigma_y \sim Inv - gamma(0.0001, 0.0001)$

$\delta \sim N(0, 10000^2)$

$\eta \sim N(0, 10000^2)$

$\rho \sim N(0, 10000^2)$

Here is the JAGS model:

```
#sink("fdp.bug")
#cat("
model {
  for (i in 1:length(y)) {
    y[i] ~ dnorm(alpha[i] + inprod(X.defense[i, ], beta.defense)
                 + inprod(X.home[i, ], beta.home)
                 + inprod(X.away[i, ], beta.away), sigmasqinv)
  }

  # The entry of the beta.defense corresponds to Opponent:Position
  # In our model, we pool the beta.defense based on position.
  # i.e. All defense effects of the same position are drawn from the same distribution
  for (p in 1:Num.Position) {
    for (f in 1:Num.fixed.pred) {
      beta.defense[(f-1) * Num.Position + p] ~ dnorm(delta[p], 1/1000^2)
      delta[(f-1) * Num.Position + p] ~ dnorm(0, 1/100000^2)
    }
  }

  # The entry of the beta.home and beta.away corresponds to Rank:Position
  # In our model, we pool the beta.home/away based on rank
  for (r in 1:Num.Rank) {
    for (t in 1:Num.Position) {
      beta.home[(t-1) * Num.Rank + r] ~ dnorm(eta[r], 1/1000^2)
      beta.away[(t-1) * Num.Rank + r] ~ dnorm(rho[r], 1/1000^2)
    }
    eta[r] ~ dnorm(0, 1/100000^2)
    rho[r] ~ dnorm(0, 1/100000^2)
  }

  sigmasqinv ~ dgamma(0.0001, 0.0001)
  sigmasq <- 1/sigmasqinv
}
#    ",fill = TRUE)
#sink()
```

```
library(knitr)
```

Sample Data

```r
fdp <- read.csv("fdpfinal.csv", sep = ',', header = TRUE)

head(fdp)
```

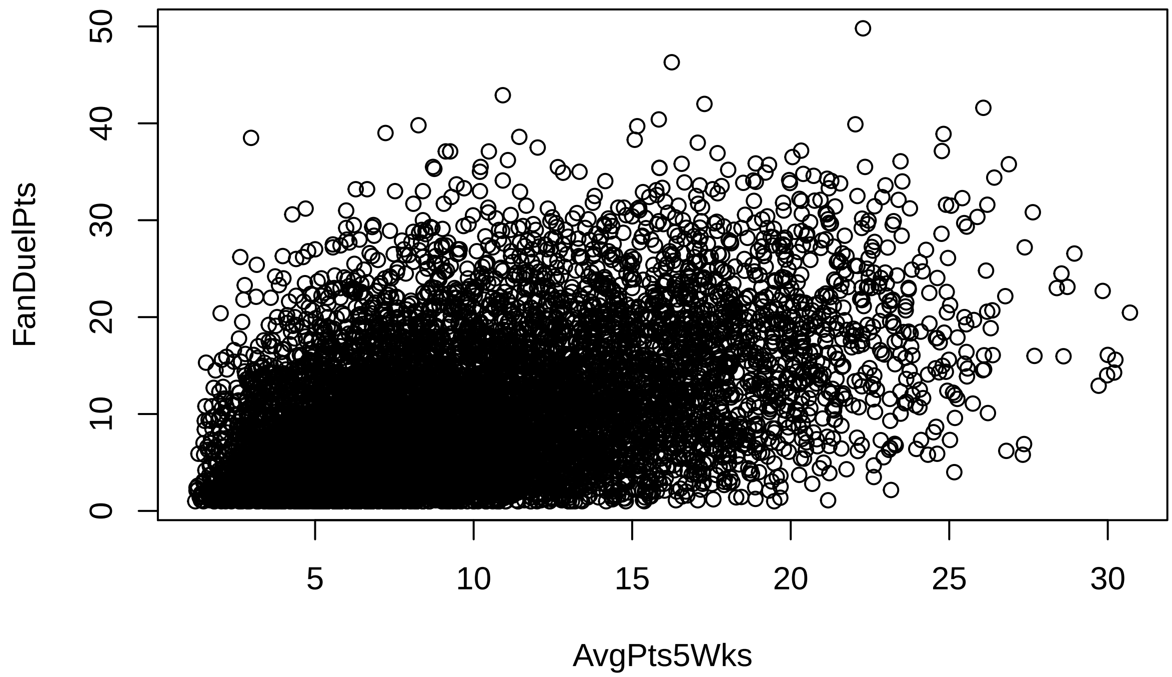
```
##   Position Year YearWeek Opponent Week PlayerId             Name    Team
## 1       QB 2015   201513 Steelers   13     1060 Hasselbeck, Matt   Colts
## 2       QB 2015   201514  Jaguars   14     1060 Hasselbeck, Matt   Colts
## 3       QB 2015   201515   Texans   15     1060 Hasselbeck, Matt   Colts
## 4       QB 2015   201516 Dolphins   16     1060 Hasselbeck, Matt   Colts
## 5       QB 2015   201501   Ravens    1     1081  Manning, Peyton Broncos
## 6       QB 2015   201502   Chiefs    2     1081  Manning, Peyton Broncos
##   HomeGame FanDuelPts FanDuelSalary AvgOppPAP7Wks SdOppPAP7Wks OallAvgPAP
## 1        0       6.86          6500         20.95        8.045      17.44
## 2        0       9.08          6600         24.16        6.738      17.44
## 3        1       8.98          6400         16.36        9.690      17.44
## 4        0       3.96          6000         20.46        6.002      17.44
## 5        1       5.90          9100         18.99       11.697      17.44
## 6        0      21.24          8200         13.85        4.342      17.44
##   OallStdevPAP AvgPts5Wks StdevPts5Wks OffRnk5Wks DefRnk7Wks
## 1        2.909      14.85        4.824      Rank4      Rank1
## 2        2.909      14.82        4.897      Rank4      Rank1
## 3        2.909      13.56        5.490      Rank4      Rank3
## 4        2.909      12.11        5.566      Rank4      Rank2
## 5        2.909      14.96        8.496      Rank3      Rank1
## 6        2.909      10.53        5.011      Rank4      Rank4
```

```r
fdp['Rank'] = fdp$OffRnk5Wks
```

Simple Ideas

$y|\alpha \sim N(\alpha, \sigma_y^2 I)$

```r
mod.classic = lm(FanDuelPts ~ AvgPts5Wks, data = fdp)

plot(FanDuelPts ~ AvgPts5Wks, data = fdp)
```



All over the place, let's add the team defense

`X.defense` is the indicator matrix

** Set up train data **

```
#fdp_train=fdp[fdp$Year == 2015, ]
fdp_train=fdp[fdp$Year == 2016  & ((fdp$Position == "QB"& fdp$FanDuelSalary > 6500 & !is.na(fdp$FanDuelS
#fdp_train=fdp[fdp$Year == 2015 & (fdp$Position == "QB" | fdp$Position == "RB") & fdp$FanDuelSalary > 65

fdp_test =fdp_train[fdp_train$YearWeek >= 201615, ]
fdp_train=fdp_train[fdp_train$YearWeek < 201615, ]
fdp_train = droplevels(fdp_train)
```

```
Use.Rank = TRUE
Num.Opponent = length(unique(fdp_train[, "Opponent"]))
Num.Position = length(unique(fdp_train[, "Position"]))
#Num.fixed.pred=2 #AvgOppPAP7Wks + FanDuelSalary
Num.fixed.pred=1 #AvgOppPAP7Wks
if (Use.Rank) {
  Num.Rank = length(unique(fdp_train[, "Rank"]))
  Num.HomeAwayInit = Num.Rank
  Model.File.Ext = ""
} else {
  Num.HomeAwayInit = 1
  Model.File.Ext = ".norank"
}
```

```r
if (Num.Position == 1) {
  #X.defense = model.matrix(~ 0 + AvgOppPAP7Wks + FanDuelSalary, data=fdp_train)
  X.defense = model.matrix(~ 0 + AvgOppPAP7Wks, data=fdp_train)
  if (Use.Rank) {
    X.home = model.matrix(~ 0 + Rank , data=fdp_train)
    X.away = model.matrix(~ 0 + Rank , data=fdp_train)
  } else {
    X.home = rep(1, nrow(fdp_train))
    X.away = rep(1, nrow(fdp_train))
  }
} else {
  #X.defense = model.matrix(~ 0 + AvgOppPAP7Wks:Position + FanDuelSalary:Position, data=fdp_train)
  X.defense = model.matrix(~ 0 + AvgOppPAP7Wks:Position, data=fdp_train)
  if (Use.Rank) {
    X.home = model.matrix(~ 0 + Rank:Position , data=fdp_train)
    X.away = model.matrix(~ 0 + Rank:Position , data=fdp_train)
  } else {
    X.home = model.matrix(~ 0 + Position , data=fdp_train)
    X.away = model.matrix(~ 0 + Position , data=fdp_train)
  }
}


X.home = X.home * fdp_train$HomeGame
X.away = X.away * (1- fdp_train$HomeGame)
X = cbind(X.defense, X.home, X.away)
```

```r
library(rjags)
set.seed(20171008)
```

```r
# Initialization List for the 4 chains
jags.inits=list(
  list( sigmasqinv=    0.01,  delta = rep(-100000, Num.Position * Num.fixed.pred),
        eta = c(100000, -100000, 100000, -100000)[1:Num.HomeAwayInit],
        rho = c(-100000, 100000, -100000, 100000)[1:Num.HomeAwayInit],
        .RNG.name = "base::Mersenne-Twister", .RNG.seed = 20171008 ),
  list( sigmasqinv=    0.01,  delta = rep(100000, Num.Position * Num.fixed.pred),
        eta = c(100000, -100000, -100000, 100000)[1:Num.HomeAwayInit],
        rho = c(-100000, 100000, 100000, -100000)[1:Num.HomeAwayInit],
        .RNG.name = "base::Mersenne-Twister", .RNG.seed = 20171008 + 1 ),
  list( sigmasqinv=0.000001,  delta = rep(-100000, Num.Position * Num.fixed.pred),
        eta = c(-100000, 100000, -100000, 100000)[1:Num.HomeAwayInit],
        rho = c(100000, -100000, 100000, -100000)[1:Num.HomeAwayInit],
        .RNG.name = "base::Mersenne-Twister", .RNG.seed = 20171008 + 2 ),
  list( sigmasqinv=0.000001,  delta = rep(100000, Num.Position * Num.fixed.pred),
        eta = c(-100000, 100000, 100000, -100000)[1:Num.HomeAwayInit],
        rho = c(100000, -100000, -100000, 100000)[1:Num.HomeAwayInit],
        .RNG.name = "base::Mersenne-Twister", .RNG.seed = 20171008 + 3 )
)

data.jags <- list(
  y= fdp_train$FanDuelPts,
```

```
  alpha = fdp_train$AvgPts5Wks,
  X.defense = X.defense,
  X.home = X.home,
  X.away = X.away,
  Num.fixed.pred=Num.fixed.pred,
  Num.Position=Num.Position,
  #Num.Opponent=Num.Opponent,
  Num.Rank=Num.Rank
)
```

```
burnAndSample = function(m, N.burnin, N.iter, show.plot, mon.col, n.thin=1) {
  update(m, N.burnin) # burn-in

  x <- coda.samples(m, mon.col, n.iter=N.iter, n.thin)

  if(show.plot) {
    plot(x, smooth=FALSE)
  }

  gelman.R = gelman.diag(x, autoburnin=FALSE, multivariate = FALSE)
  print(gelman.R)

  result <- list(
    coda.sam = x,
    gelman.R.max=max(gelman.R$psrf[, 1])
  )

  return(result)
}
```

```
runModel=TRUE
runSample=TRUE

mon.col <- c("delta", "eta", "rho", "beta.defense", "beta.home", "beta.away", "sigmasq")

NSim = 30000
NChain = 4
NThin = 5
NTotalSim = NSim * NChain / 5
if (runModel) {
  m <- jags.model("fdp.bug", data.jags, inits = jags.inits, n.chains=NChain, n.adapt = 1000)
  save(file=paste("fdp.jags.model.init", Model.File.Ext, ".Rdata", sep=""), list="m")
} else {
  load(paste("fdp.jags.model.init", Model.File.Ext, ".Rdata", sep=""))
  m$recompile()
}
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 746
##    Unobserved stochastic nodes: 29
```

```
##    Total graph size: 18345
##
## Initializing model
```

```r
load.module("dic")
```

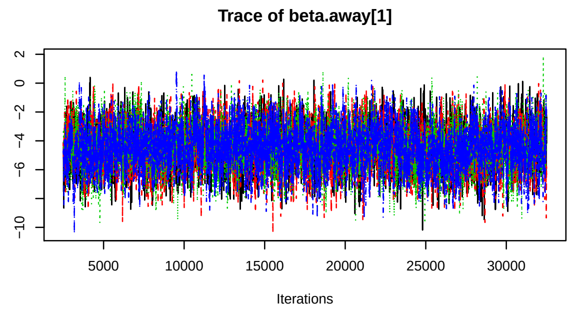
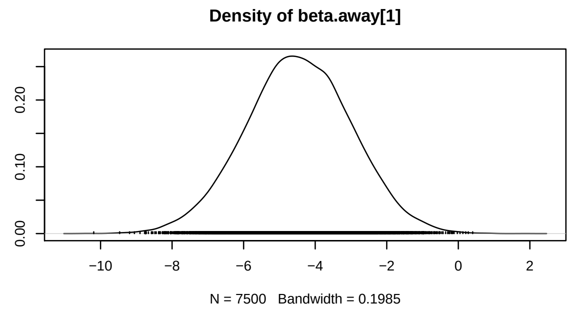
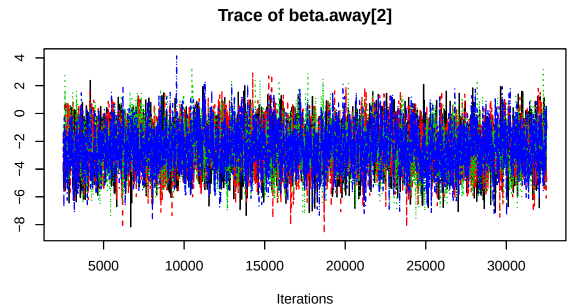
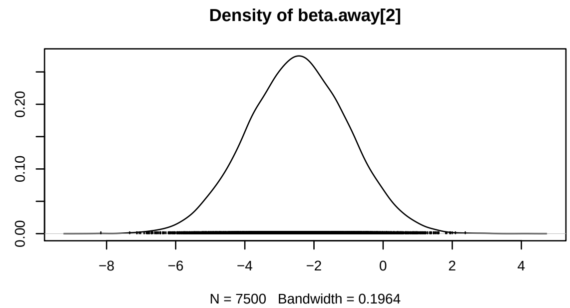
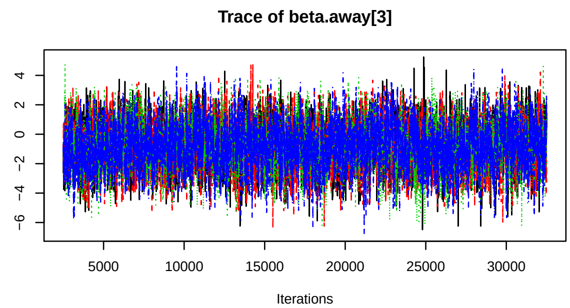
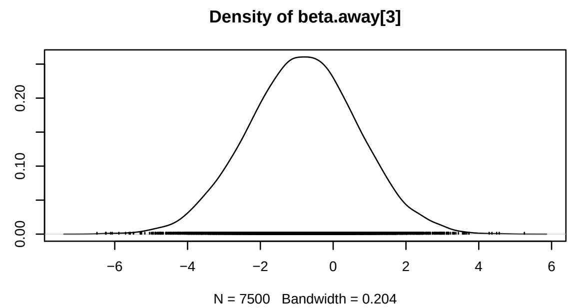
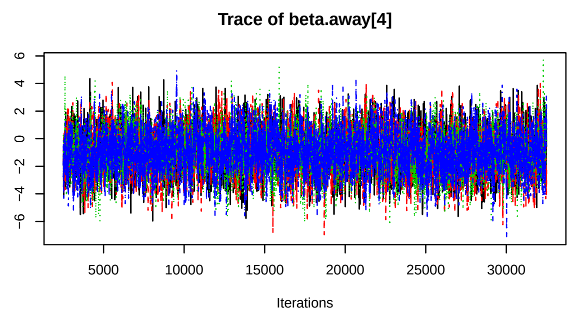
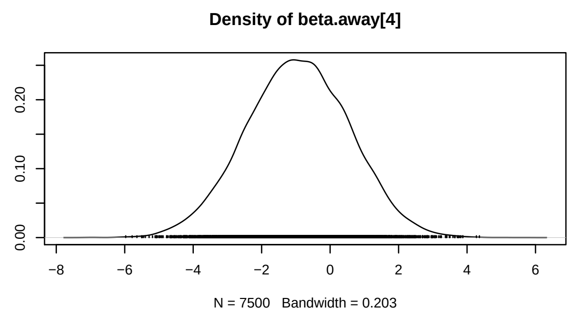
```
## module dic loaded
```

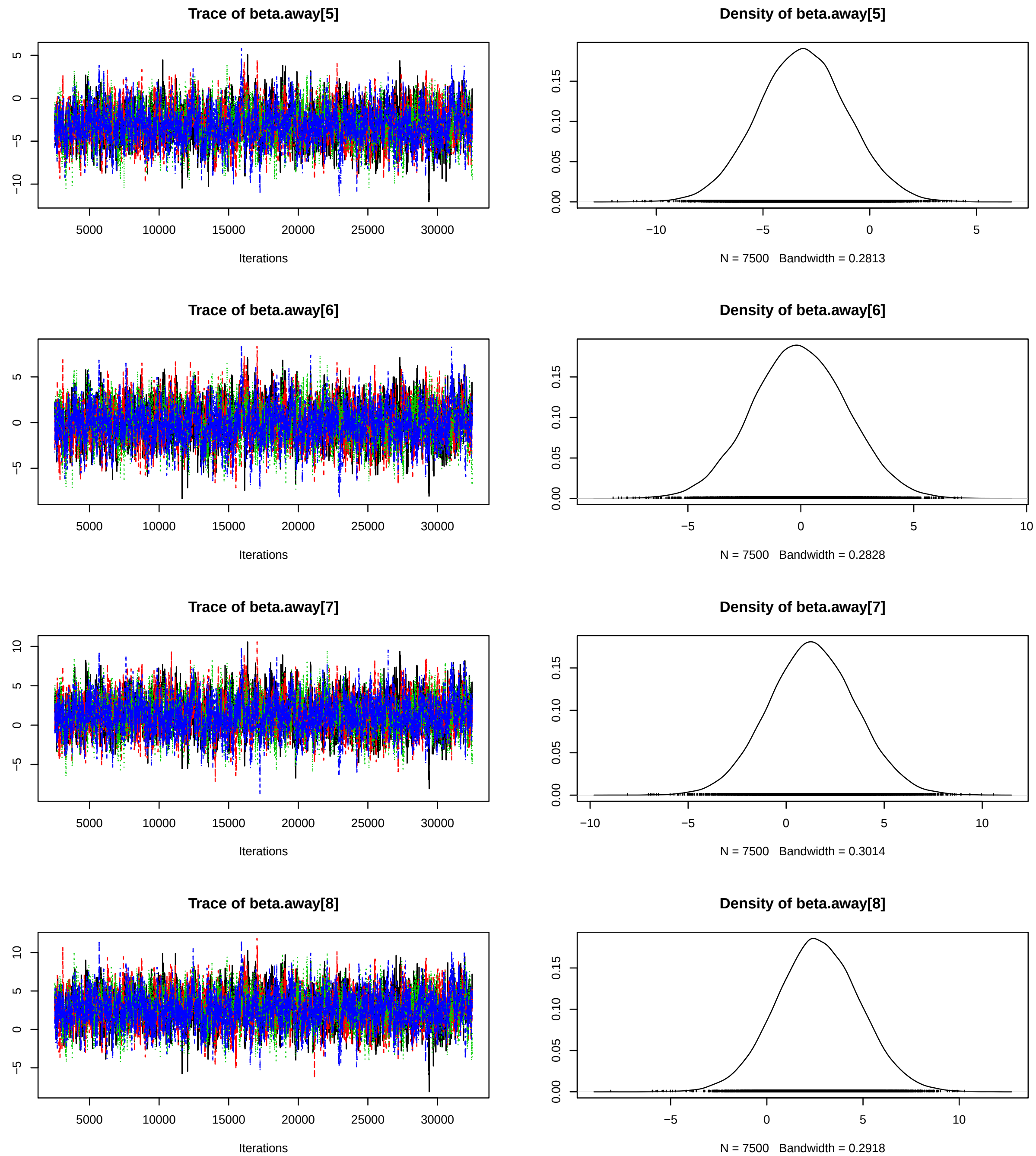
```r
N.Retry.Loop = 1
if (runSample) {
  N.burnin=2500/2
  for (loopIdx in 1:N.Retry.Loop) {
    (start_time <- Sys.time())
    (N.burnin = N.burnin * 2)
    result = burnAndSample(m, N.burnin, NSim, show.plot=FALSE, mon.col = mon.col, n.thin=NChain)
    (end_time <- Sys.time())
    (result$gelman.R.max)
  }
  run.params = paste(".", N.burnin, ".", NChain, ".", NSim, ".", NThin, sep="")
  save(file=paste("fdp.jags.samples", run.params, Model.File.Ext, ".Rdata", sep=""), list="result")
  save(file=paste("fdp.jags.model", run.params, Model.File.Ext, ".Rdata", sep=""), list="m")
} else {
  N.burnin=2500/2 * (2**N.Retry.Loop)
  run.params = paste(".", N.burnin, ".", NChain, ".", NSim, ".", NThin, sep="")
  load(paste("fdp.jags.samples", run.params, Model.File.Ext, ".Rdata", sep=""))
  load(paste("fdp.jags.model", run.params, Model.File.Ext, ".Rdata", sep=""))

  gelman.diag(result$coda.sam, autoburnin=FALSE, multivariate = FALSE)
}
```

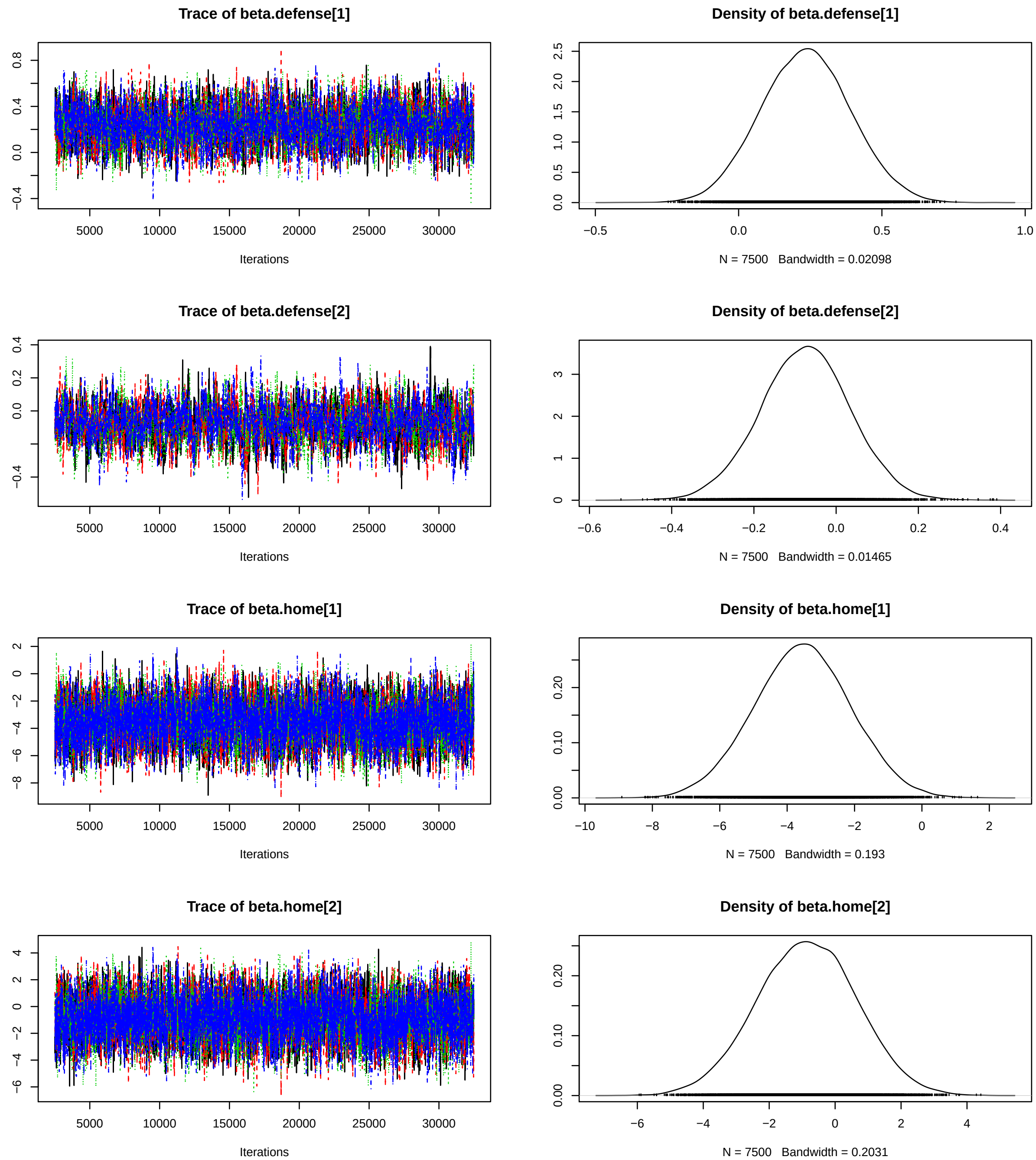
```
## Potential scale reduction factors:
##
##                  Point est. Upper C.I.
## beta.away[1]              1       1.00
## beta.away[2]              1       1.00
## beta.away[3]              1       1.00
## beta.away[4]              1       1.00
## beta.away[5]              1       1.01
## beta.away[6]              1       1.01
## beta.away[7]              1       1.00
## beta.away[8]              1       1.01
## beta.defense[1]           1       1.00
## beta.defense[2]           1       1.01
## beta.home[1]              1       1.00
## beta.home[2]              1       1.00
## beta.home[3]              1       1.00
## beta.home[4]              1       1.00
## beta.home[5]              1       1.00
## beta.home[6]              1       1.01
## beta.home[7]              1       1.01
## beta.home[8]              1       1.01
## delta[1]                  1       1.00
## delta[2]                  1       1.00
```

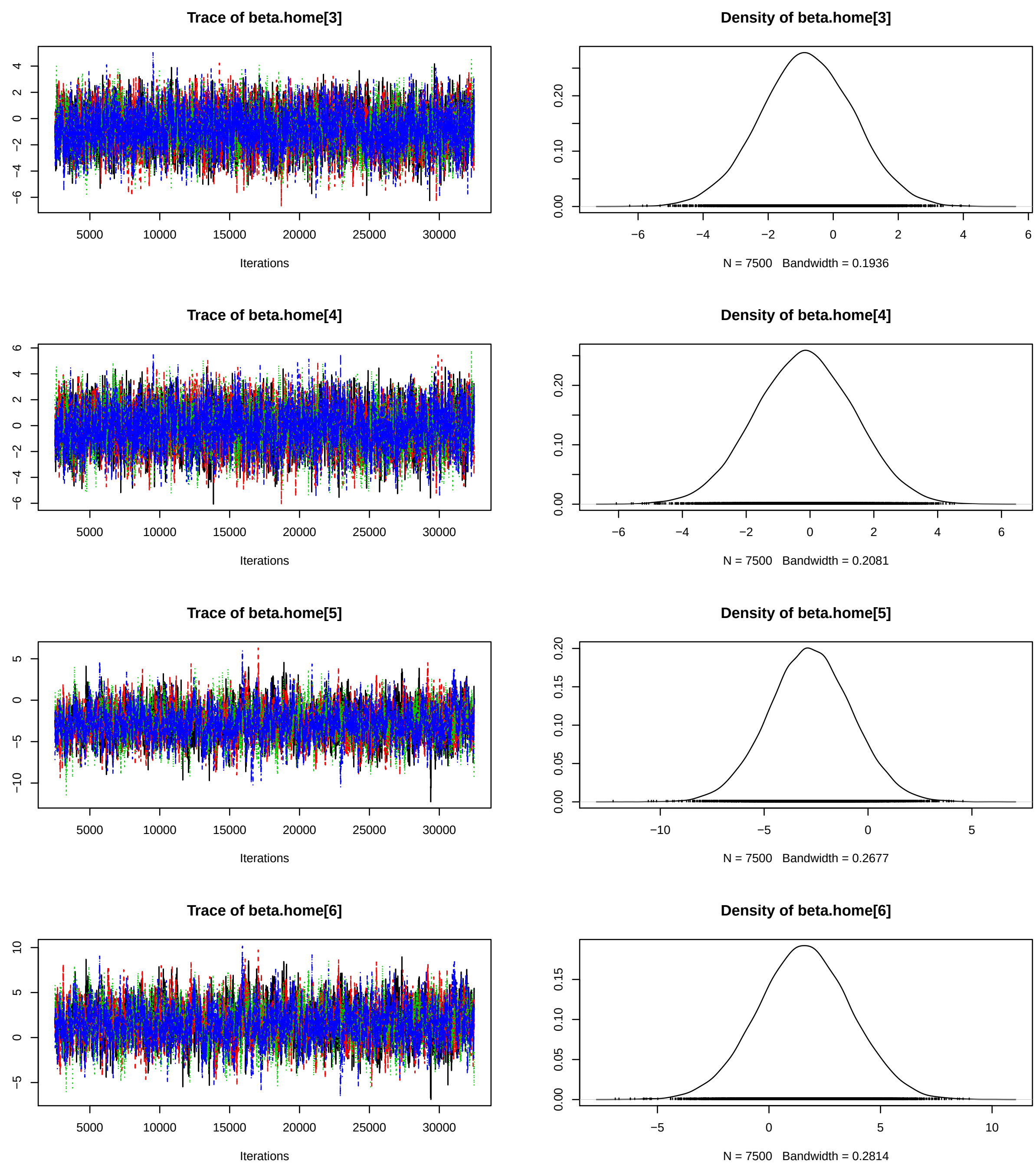
```
## eta[1]                    1       1.00
## eta[2]                    1       1.00
## eta[3]                    1       1.00
## eta[4]                    1       1.00
## rho[1]                    1       1.00
## rho[2]                    1       1.00
## rho[3]                    1       1.00
## rho[4]                    1       1.00
## sigmasq                   1       1.00
```

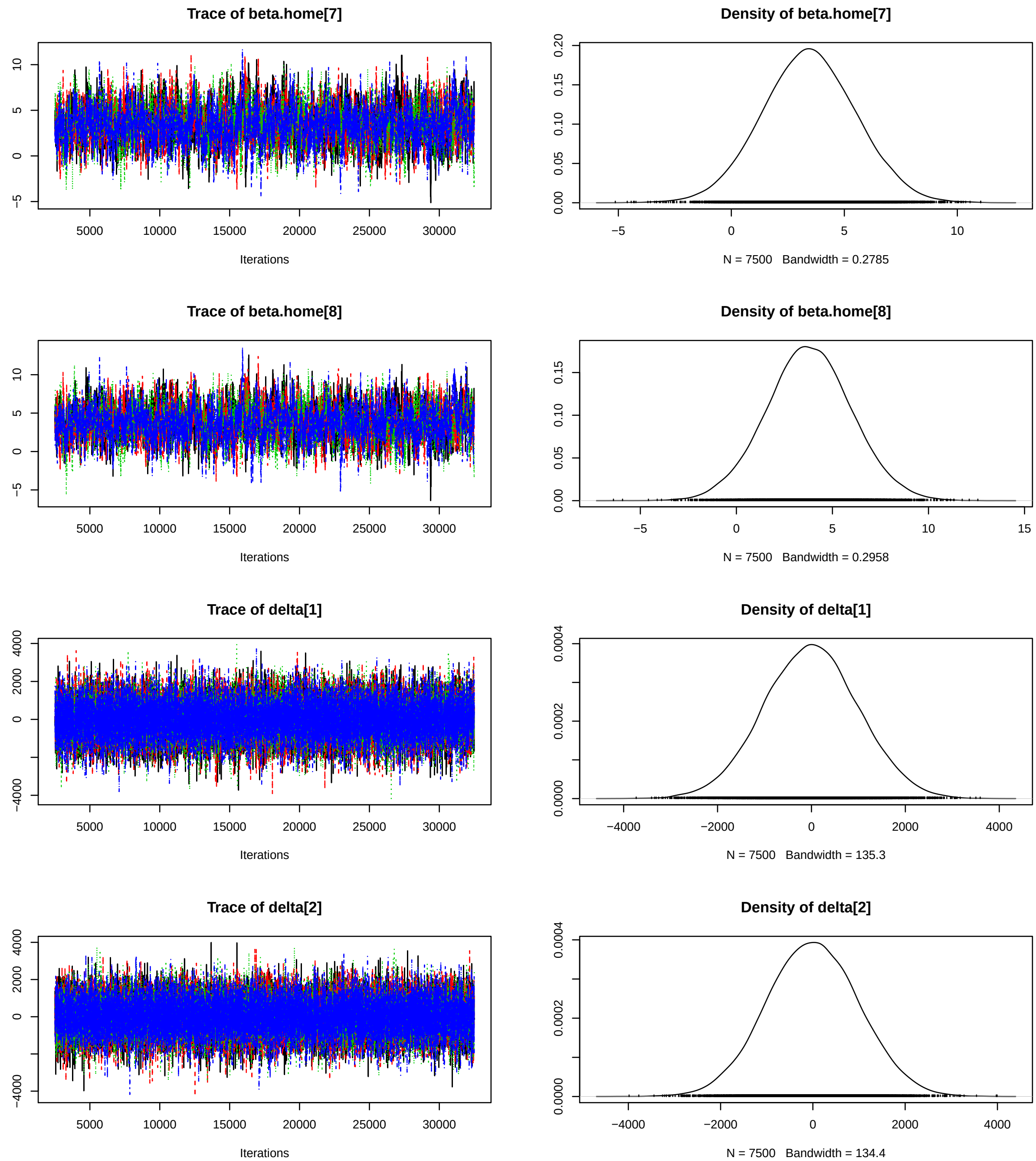
```r
plot(result$coda.sam, smooth=FALSE)
```

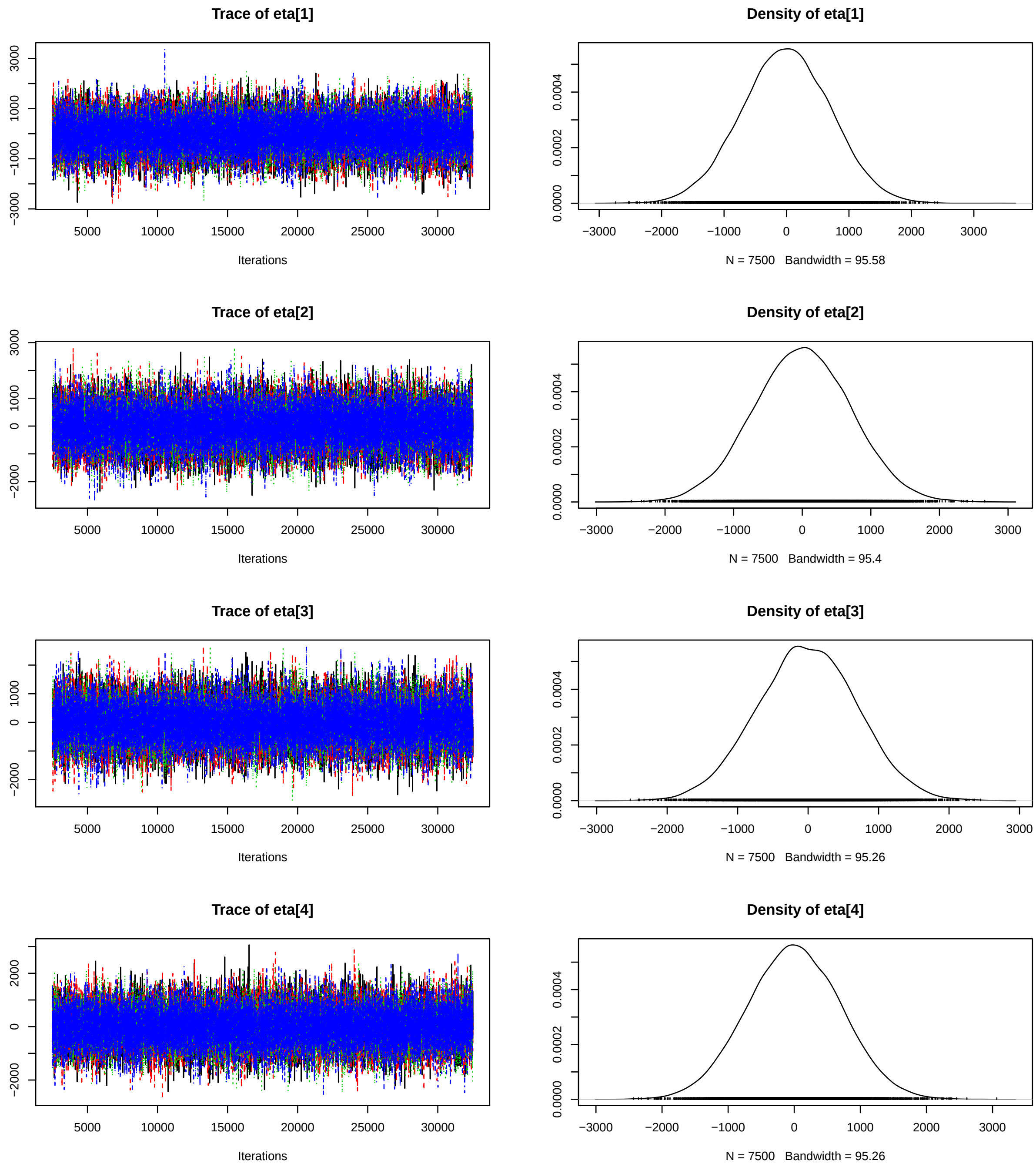

Trace of beta.away[1]
Iterations

Density of beta.away[1]
N = 7500 Bandwidth = 0.1985

Trace of beta.away[2]
Iterations

Density of beta.away[2]
N = 7500 Bandwidth = 0.1964

Trace of beta.away[3]
Iterations

Density of beta.away[3]
N = 7500 Bandwidth = 0.204

Trace of beta.away[4]
Iterations

Density of beta.away[4]
N = 7500 Bandwidth = 0.203

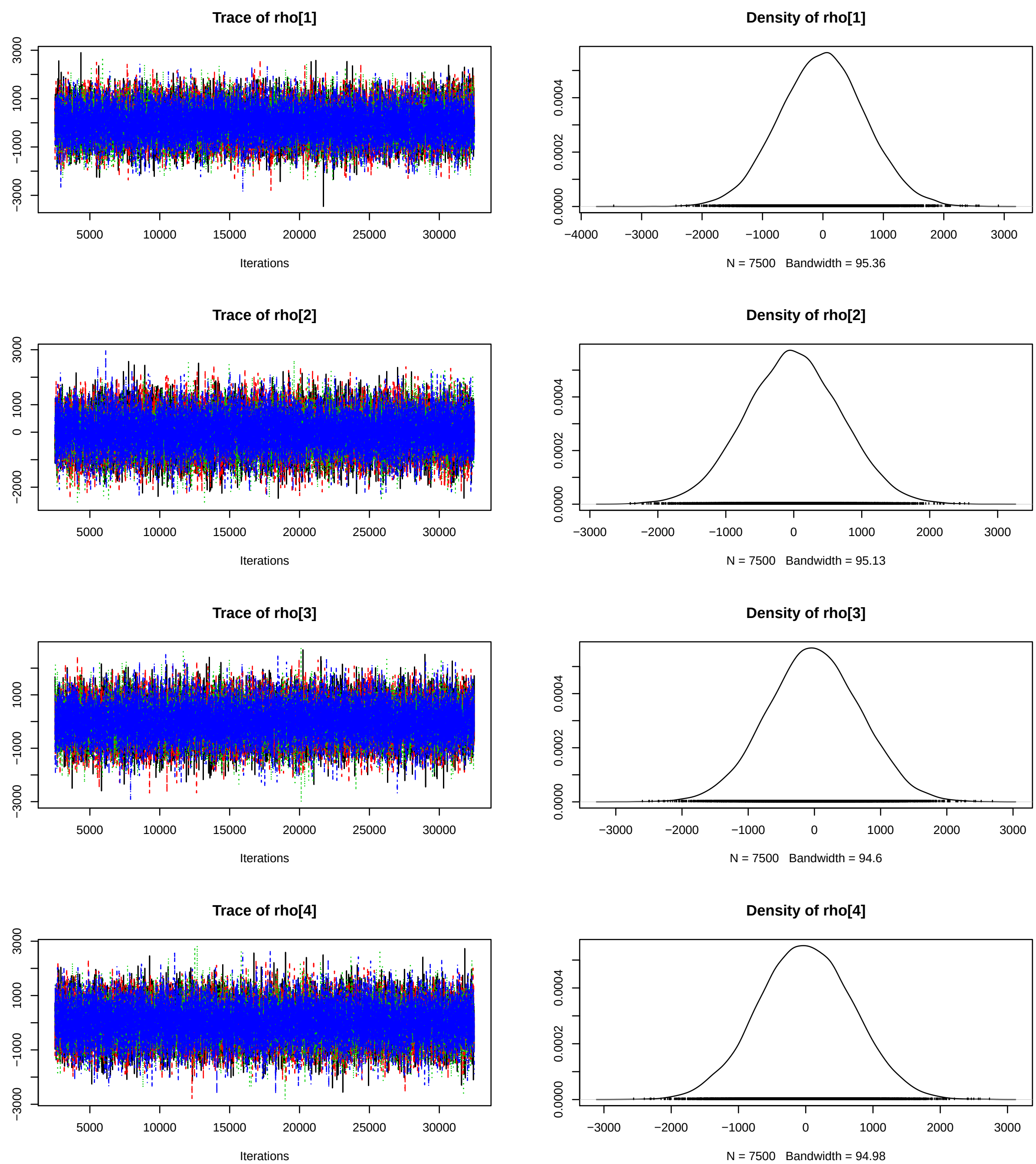

Trace of beta.away[5]
Iterations
Density of beta.away[5]
N = 7500 Bandwidth = 0.2813
Trace of beta.away[6]
Iterations
Density of beta.away[6]
N = 7500 Bandwidth = 0.2828
Trace of beta.away[7]
Iterations
Density of beta.away[7]
N = 7500 Bandwidth = 0.3014
Trace of beta.away[8]
Iterations
Density of beta.away[8]
N = 7500 Bandwidth = 0.2918

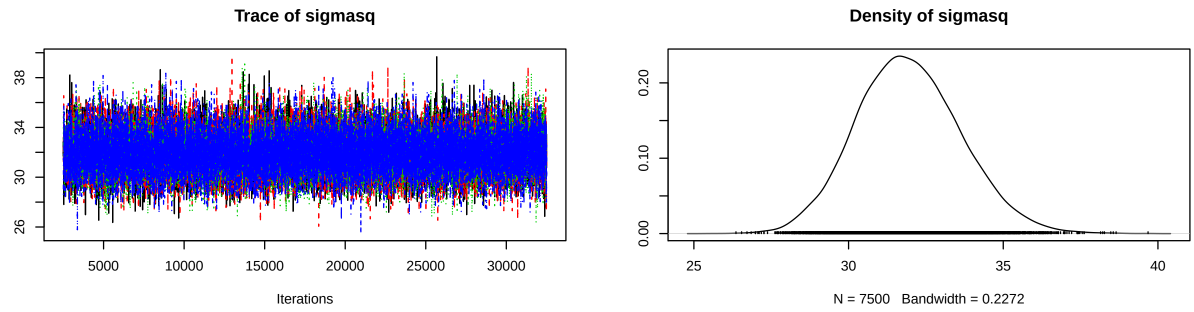

Trace of beta.defense[1]
Iterations
Density of beta.defense[1]
N = 7500 Bandwidth = 0.02098
Trace of beta.defense[2]
Iterations
Density of beta.defense[2]
N = 7500 Bandwidth = 0.01465
Trace of beta.home[1]
Iterations
Density of beta.home[1]
N = 7500 Bandwidth = 0.193
Trace of beta.home[2]
Iterations
Density of beta.home[2]
N = 7500 Bandwidth = 0.2031


Trace of beta.home[3]
Iterations
Density of beta.home[3]
N = 7500 Bandwidth = 0.1936
Trace of beta.home[4]
Iterations
Density of beta.home[4]
N = 7500 Bandwidth = 0.2081
Trace of beta.home[5]
Iterations
Density of beta.home[5]
N = 7500 Bandwidth = 0.2677
Trace of beta.home[6]
Iterations
Density of beta.home[6]
N = 7500 Bandwidth = 0.2814


Trace of beta.home[7]
Iterations
Density of beta.home[7]
N = 7500 Bandwidth = 0.2785
Trace of beta.home[8]
Iterations
Density of beta.home[8]
N = 7500 Bandwidth = 0.2958
Trace of delta[1]
Iterations
Density of delta[1]
N = 7500 Bandwidth = 135.3
Trace of delta[2]
Iterations
Density of delta[2]
N = 7500 Bandwidth = 134.4


Trace of eta[1]
Iterations
Density of eta[1]
N = 7500 Bandwidth = 95.58
Trace of eta[2]
Iterations
Density of eta[2]
N = 7500 Bandwidth = 95.4
Trace of eta[3]
Iterations
Density of eta[3]
N = 7500 Bandwidth = 95.26
Trace of eta[4]
Iterations
Density of eta[4]
N = 7500 Bandwidth = 95.26


Trace of rho[1]
Iterations
Density of rho[1]
N = 7500 Bandwidth = 95.36
Trace of rho[2]
Iterations
Density of rho[2]
N = 7500 Bandwidth = 95.13
Trace of rho[3]
Iterations
Density of rho[3]
N = 7500 Bandwidth = 94.6
Trace of rho[4]
Iterations
Density of rho[4]
N = 7500 Bandwidth = 94.98



Converged as `gelman.R.max` $= 1.002 < 1.1$ and the plot also looks good.

```
(m.summary = summary(result$coda.sam))
```

```
##
## Iterations = 2504:32500
## Thinning interval = 4
## Number of chains = 4
## Sample size per chain = 7500
##
```

```
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
##                    Mean       SD Naive SE Time-series SE
## beta.away[1]    -4.467    1.472 0.008499        0.02417
## beta.away[2]    -2.461    1.456 0.008408        0.02444
## beta.away[3]    -0.832    1.513 0.008733        0.02360
## beta.away[4]    -0.942    1.505 0.008691        0.02398
## beta.away[5]    -3.132    2.096 0.012100        0.05062
## beta.away[6]    -0.051    2.097 0.012108        0.05033
## beta.away[7]     1.343    2.235 0.012903        0.05291
## beta.away[8]     2.670    2.175 0.012558        0.05066
## beta.defense[1]  0.235    0.156 0.000898        0.00285
## beta.defense[2] -0.075    0.109 0.000632        0.00283
## beta.home[1]    -3.547    1.431 0.008264        0.02330
## beta.home[2]    -0.859    1.506 0.008694        0.02329
## beta.home[3]    -0.805    1.435 0.008287        0.02291
## beta.home[4]    -0.162    1.543 0.008909        0.02516
## beta.home[5]    -2.767    2.006 0.011579        0.04770
## beta.home[6]     1.605    2.086 0.012045        0.04957
## beta.home[7]     3.449    2.074 0.011976        0.04888
## beta.home[8]     3.784    2.221 0.012826        0.05233
## delta[1]        -6.780 1003.390 5.793077        5.89868
## delta[2]         1.764  996.816 5.755122        5.75533
## eta[1]          -7.966  708.771 4.092093        4.08607
## eta[2]           2.218  707.396 4.084150        4.08699
## eta[3]          -9.512  706.400 4.078403        4.05872
## eta[4]          -1.529  706.367 4.078209        4.02323
## rho[1]          -6.188  707.088 4.082376        4.10605
## rho[2]          -3.301  705.424 4.072770        4.11118
## rho[3]          -1.886  701.548 4.050388        4.05048
## rho[4]           1.520  704.309 4.066333        4.04433
## sigmasq         31.947    1.685 0.009727        0.00974
##
## 2. Quantiles for each variable:
##
##                     2.5%      25%      50%       75%    97.5%
## beta.away[1]     -7.3717   -5.453  -4.4688  -3.46981   -1.611
## beta.away[2]     -5.3105   -3.446  -2.4545  -1.46614    0.366
## beta.away[3]     -3.8050   -1.845  -0.8234   0.18547    2.160
## beta.away[4]     -3.8988   -1.965  -0.9387   0.09424    1.969
## beta.away[5]     -7.2309   -4.542  -3.1370  -1.74709    1.026
## beta.away[6]     -4.1378   -1.461  -0.0690   1.36573    4.063
## beta.away[7]     -3.0093   -0.157   1.3282   2.84138    5.776
## beta.away[8]     -1.5793    1.223   2.6498   4.12313    6.979
## beta.defense[1]  -0.0663    0.129   0.2350   0.33995    0.543
## beta.defense[2]  -0.2931   -0.148  -0.0741  -0.00196    0.138
## beta.home[1]     -6.3655   -4.514  -3.5414  -2.58559   -0.756
## beta.home[2]     -3.8301   -1.887  -0.8491   0.16783    2.062
## beta.home[3]     -3.6558   -1.769  -0.8066   0.17242    2.007
## beta.home[4]     -3.1788   -1.211  -0.1571   0.88799    2.859
## beta.home[5]     -6.6877   -4.098  -2.7803  -1.43849    1.176
## beta.home[6]     -2.4894    0.204   1.5977   3.00227    5.706
## beta.home[7]     -0.5994    2.064   3.4377   4.83181    7.517
```

```
## beta.home[8]        -0.6143    2.315   3.7747   5.25484    8.181
## delta[1]         -1978.4088 -688.037  -4.1092 662.68424 1965.061
## delta[2]         -1940.6629 -675.923  -2.0463 671.92029 1962.165
## eta[1]           -1413.8970 -482.575  -5.4483 474.92512 1370.112
## eta[2]           -1394.8328 -476.146   1.7122 481.47216 1388.076
## eta[3]           -1396.0830 -483.186  -9.9967 470.21005 1376.652
## eta[4]           -1382.7089 -479.828  -1.6793 479.87624 1378.586
## rho[1]           -1387.9001 -483.225  -2.6988 464.93505 1388.009
## rho[2]           -1382.8577 -480.359  -6.9693 470.32306 1373.230
## rho[3]           -1376.5466 -471.454  -6.5531 468.53942 1366.468
## rho[4]           -1383.8694 -474.965  -1.9381 476.58238 1379.023
## sigmasq             28.7819   30.779  31.8832  33.05067   35.406
```

Effective Sample Size

```r
(eff.size = effectiveSize(result$coda.sam[, ]))
```

```
##    beta.away[1]    beta.away[2]    beta.away[3]    beta.away[4]    beta.away[5]
##            3711            3557            4146            3985            1718
##    beta.away[6]    beta.away[7]    beta.away[8] beta.defense[1] beta.defense[2]
##            1736            1787            1849            2988            1497
##    beta.home[1]    beta.home[2]    beta.home[3]    beta.home[4]    beta.home[5]
##            3792            4202            3943            3788            1767
##    beta.home[6]    beta.home[7]    beta.home[8]        delta[1]        delta[2]
##            1773            1805            1817           29040           30000
##          eta[1]          eta[2]          eta[3]          eta[4]          rho[1]
##           30088           29956           30305           30853           29666
##          rho[2]          rho[3]          rho[4]         sigmasq
##           29489           30000           30441           29940
```

The effective sample sizes of all parameters are greater than 400.

Probabiliy of players perform better at home than away

```r
post.samp = as.matrix(result$coda.sam)

beta.home = post.samp[, paste("beta.home[",1:(Num.Rank*Num.Position),"]", sep="")]
beta.away = post.samp[, paste("beta.away[",1:(Num.Rank*Num.Position),"]", sep="")]

prob.home.away = rep(0, Num.Rank * Num.Position)
for (r in 1:Num.Rank) {
  for (p in 1:Num.Position) {
    idx = (p-1) * Num.Rank + r
    prob.home.away[idx] = mean(beta.home[, idx] > beta.away[, idx])
  }
}
prob.home.away
```

```
## [1] 0.8093 0.9147 0.5072 0.7559 0.6288 0.9248 0.9508 0.7928
```

```r
prob.home.away.df = data.frame(colnames(X.home))
prob.home.away.df$prob.home.bt.away = prob.home.away
```

```
colnames(prob.home.away.df) = c("Rank:Position", "Prob.home.bt.away")

kable(prob.home.away.df)
```

Rank:Position	Prob.home.bt.away
RankRank1:PositionPK	0.8093
RankRank2:PositionPK	0.9147
RankRank3:PositionPK	0.5072
RankRank4:PositionPK	0.7559
RankRank1:PositionQB	0.6288
RankRank2:PositionQB	0.9248
RankRank3:PositionQB	0.9508
RankRank4:PositionQB	0.7928

Beta defense

If a player is facing a team which gives up more points to players on average, we expect the player will score more points.

```
Num.fixed.size=Num.Position*Num.fixed.pred
beta.defense = post.samp[, paste("beta.defense[",1:Num.fixed.size,"]", sep="")]

beta.defense.int.df = data.frame(colnames(X.defense))
beta.defense.int = matrix(rep(0, Num.fixed.size * 4), nrow=Num.fixed.size, ncol = 4)

int.alpha=0.05
for (p in 1:Num.Position) {
  for (f in 1:Num.fixed.pred) {
    idx = (f-1) * Num.Position + p
      beta.defense.int[idx, 1:3] =  quantile(beta.defense[, idx], c(int.alpha/2, 0.5, 1-int.alpha/2))
      beta.defense.int[idx, 4] = mean(beta.defense[, idx])
  }
}

beta.defense.int.df$pct025 = beta.defense.int[, 1]
beta.defense.int.df$pct975 = beta.defense.int[, 3]
beta.defense.int.df$median = beta.defense.int[, 2]
beta.defense.int.df$mean = beta.defense.int[, 4]
colnames(beta.defense.int.df) = c("beta.defense.position", "pct025", "pct975", "median", "mean")
kable(beta.defense.int.df)
```

beta.defense.position	pct025	pct975	median	mean
AvgOppPAP7Wks:PositionPK	-0.0663	0.5435	0.2350	0.2347
AvgOppPAP7Wks:PositionQB	-0.2931	0.1377	-0.0741	-0.0750

We observe that the median beta.defense for PK is positive as expected. But for QB, it is negative, that implies QB actually scores less against bad defensive team.

DIC

```
(dic.samp = dic.samples(m, NTotalSim))
```

```
## Mean deviance:  4700
## penalty 19
## Penalized deviance: 4719
```

The effective number of parameters("penalty") is 19, and the Plummer's DIC ("Penalized deviance") is 4719. Not that we have 29 parameters in our model, 10 of them were shrunk away.

Model Checking

A posterior predictive p-value based on the no-intercept model. Consider test quantity

$$T(y, X, \theta) = |\hat{cor}(\epsilon, \text{time})|$$

where $\hat{cor}(\epsilon, \text{time})$ is sample correlation between the error vector ϵ and the year week in the data. The larger this quantity is for the model, the less well it fits the data (since, if a regression model actually fits, the errors should ideally be uncorrelated with the predictor).

The simulated error vectors ϵ (as rows of a matrix):

```
error.sim <- matrix(NA, NTotalSim, nrow(fdp_train))
y_hat.sim <- matrix(NA, NTotalSim, nrow(fdp_train))
for(s in 1:NTotalSim) {
  y_hat.sim[s, ] =  fdp_train$AvgPts5Wks + (X.defense %*% beta.defense[s, ]) + (X.home %*% beta.home[s,
  error.sim[s, ] <- (fdp_train$FanDuelPts - y_hat.sim[s, ])
}
```

The simulated replicate error vectors ϵ^{rep} (as rows of a matrix), which are the error vectors computed using replicate response vectors y^{rep}:

```
post.sigma.2.sim <- post.samp[,"sigmasq"]
post.sigma.sim <- sqrt(post.sigma.2.sim)

yreps <- matrix(NA, NTotalSim, nrow(fdp_train))
for(s in 1:NTotalSim) {
  yreps[s, ] <- rnorm(nrow(fdp_train), y_hat.sim[s, ], post.sigma.sim[s])
}

error.rep <- matrix(NA, NTotalSim, nrow(fdp_train))

for(s in 1:NTotalSim) {
  error.rep[s, ] <- (yreps[s, ] - y_hat.sim[s, ])
}
```

The simulated values of $T(y, X, \theta)$

```
T.sim = abs(cor(t(error.sim), fdp_train$FanDuelPts))
head(T.sim)
```

```
##        [,1]
## [1,] 0.7236
## [2,] 0.6846
## [3,] 0.7125
## [4,] 0.6739
## [5,] 0.7069
## [6,] 0.7022
```

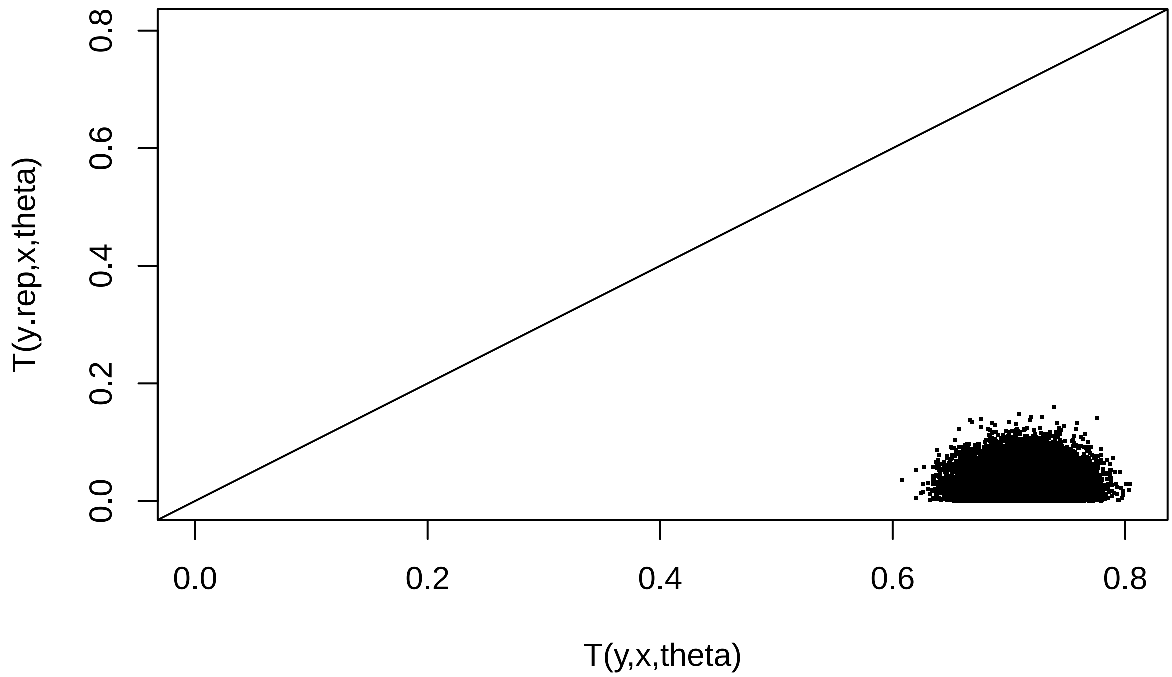
The simulated values of $T(y^{rep}, X, \theta)$

```
T.rep.sim = abs(cor(t(error.rep), fdp_train$FanDuelPts))
head(T.rep.sim)
```

```
##         [,1]
## [1,] 0.06680
## [2,] 0.09912
## [3,] 0.01110
## [4,] 0.01664
## [5,] 0.01113
## [6,] 0.03517
```

The simulated values of $T(y^{rep}, X, \theta)$ *versus those of* $T(y, X, \theta)$*, with a reference line indicating where the two values would be equal.*

```
plot(T.sim, T.rep.sim, pch=".", cex=2,
  xlim=c(min(T.sim, T.rep.sim), max(T.sim, T.rep.sim)),
  ylim=c(min(T.sim, T.rep.sim), max(T.sim, T.rep.sim)),
  xlab="T(y,x,theta)", ylab="T(y.rep,x,theta)")
abline(a=0,b=1)
```



The posterior predictive p-value:

```
(p.value = mean(T.rep.sim >= T.sim))
```

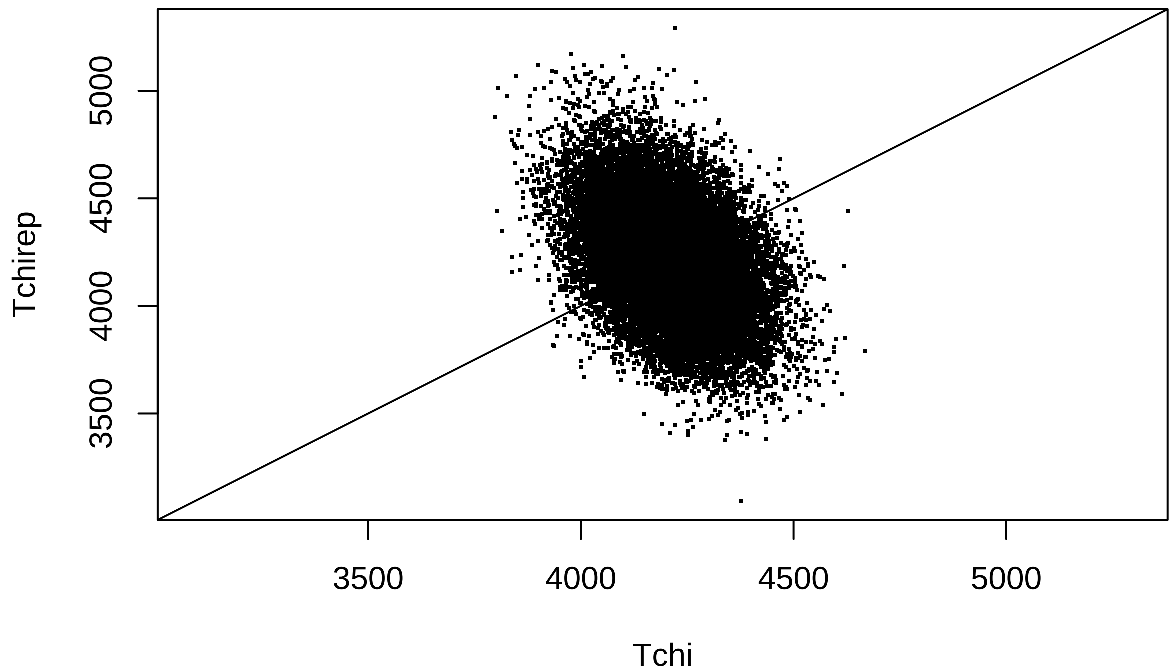
```
## [1] 0
```

The p.value is 0. This indicates the error actually has correlation with the response.

```
Tchi <- numeric(NTotalSim)
Tchirep <- numeric(NTotalSim)
for(s in 1:NTotalSim){
  Tchi[s] <- sum((fdp_train$FanDuelPts - y_hat.sim[s,])^2 / post.sigma.sim[s])
  Tchirep[s] <- sum((yreps[s,] - y_hat.sim[s,])^2 / post.sigma.sim[s])
}
(p.value.Tchi = mean(Tchirep >= Tchi))
```

```
## [1] 0.5057
```

```
plot(Tchi, Tchirep, pch=".", cex=2,
  xlim=c(min(Tchi, Tchirep), max(Tchi, Tchirep)),
  ylim=c(min(Tchi, Tchirep), max(Tchi, Tchirep)),
  xlab="Tchi", ylab="Tchirep")
abline(a=0,b=1)
```



The posterior predictive p-value using the chi-square discrepancy is `p.value.Tchi`=0.5057. The p-value is > 0.05. Hence, it does not indicate any evidence of problems.

Using $Pr(y^{rep} \geq y|y)$ as posterior predictive p-value

```
yreps.minus.y <- matrix(NA, NTotalSim, nrow(fdp_train))
for(s in 1:NTotalSim) {
  yreps.minus.y[s, ] <- yreps[s, ] - fdp_train$FanDuelPts
}

(p.value.y.rep.all = mean(yreps.minus.y > 0))
```

```
## [1] 0.5076
```

The posterior predictive p-value using individual data point is `p.value.y.rep.all` = 0.5076, which is > 0.05. This shows no evidence of problem.

Alternative Model - no rank

```
#sink("fdp.norank.bug")
#cat("
model {
  for (i in 1:length(y)) {
    y[i] ~ dnorm(alpha[i] + inprod(X.defense[i, ], beta.defense)
```

```
                 + inprod(X.home[i, ], beta.home)
                 + inprod(X.away[i, ], beta.away), sigmasqinv)
  }

  # The entry of the beta.defense corresponds to Opponent:Position
  # In our model, we pool the beta.defense based on position.
  # i.e. All defense effects of the same position are drawn from the same distribution
  for (p in 1:Num.Position) {
    beta.defense[p] ~ dnorm(delta[p], 1/1000^2)
    delta[p] ~ dnorm(0, 1/100000^2)
  }

  # The entry of the beta.home and beta.away corresponds to Position
  # In our model, we pool the beta.home/away based on Position
  for (t in 1:Num.Position) {
    beta.home[t] ~ dnorm(eta, 1/1000^2)
    beta.away[t] ~ dnorm(rho, 1/1000^2)
  }
  eta ~ dnorm(0, 1/100000^2)
  rho ~ dnorm(0, 1/100000^2)

  sigmasqinv ~ dgamma(0.0001, 0.0001)
  sigmasq <- 1/sigmasqinv
}
#    ",fill = TRUE)
#sink()
```