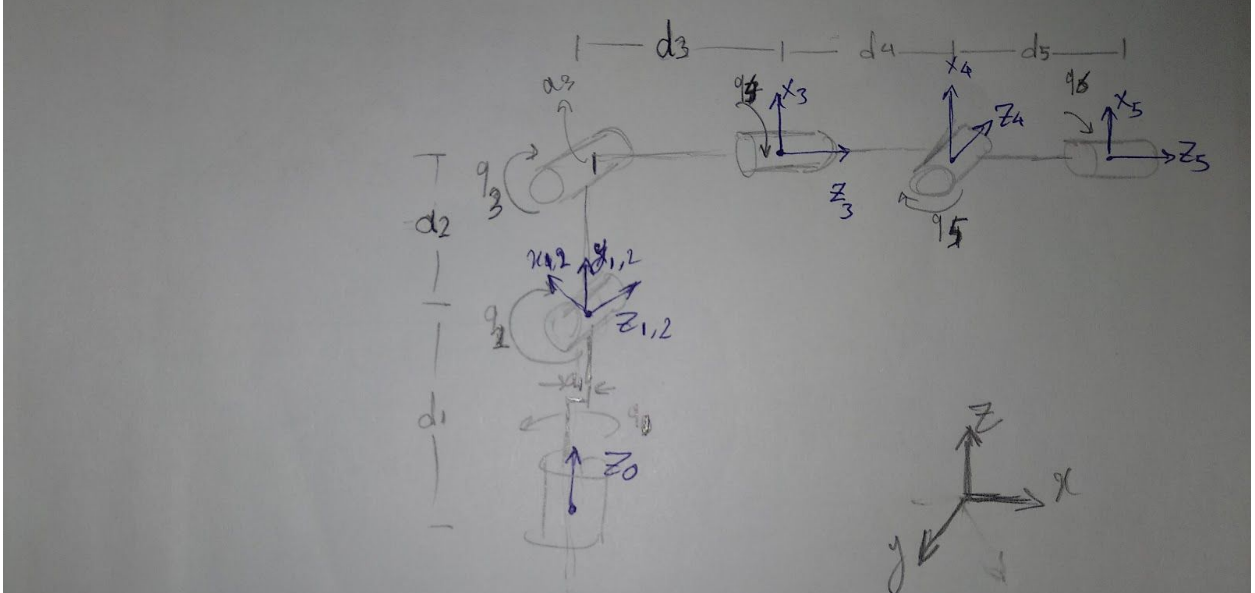
DoNR

DoNR HW02

Forward and Inverse Kinematics of Kuka KR 10 R11000-2

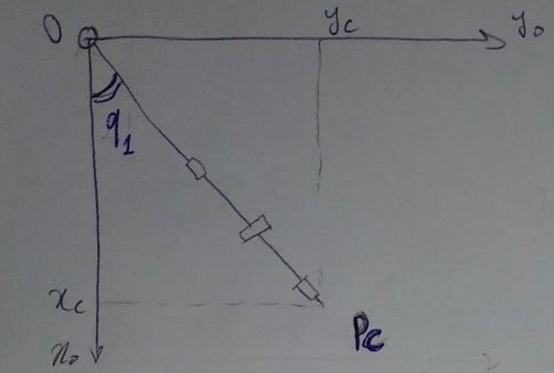
Github link: https://github.com/mhd-medfa/Kuka-KR-10-Forward-and-Inverse-Kinematics

Forward Kinematics:



$$T = R_z(q_1) . T_z(d_1) T_x(a_1) . R_y(q_2) . T_z(a_2) . R_y(q_3) . T_x(a_3) . T_z(d_3) . R_x(q_4) .$$
$$. T_x(d_4) . R_y(q_5) . T_x(d_5) . R_x(q_6)$$

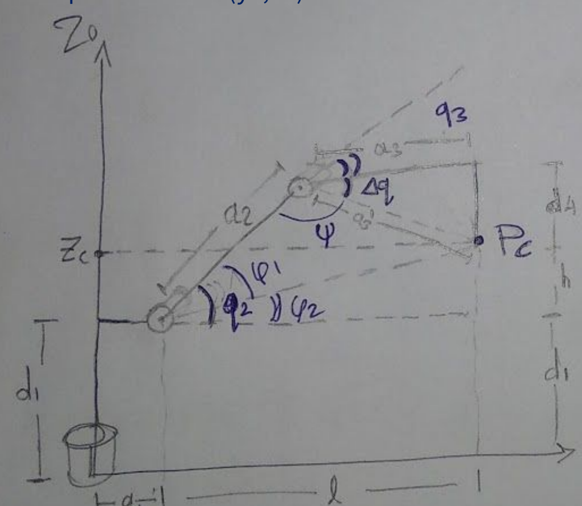
Inverse Kinematics



P_c : center of the end-effector.

$\boxed{q_1 = \text{atan2}(y_c, x_c)}$ ... ① ; $x_c = y_c = 0$ is a singularity point

Or q1 = Pi + atan2(yc,xc)



$$l = \sqrt{x_c^2 + y_c^2} - a_1$$

$$h = z_c - d_1$$

$$q_3' = q_3 + \Delta q$$

$$\psi = 180^\circ - q_3'$$

$$\boxed{\Delta q = \text{atan2}(d_4, a_3)} \; \cdots ②$$

$$a_3' = \sqrt{a_3^2 + d_4^2}$$

$$\cos\psi = \frac{a_2 + a_3'^2 - h^2 - l^2}{2.h.l} = A \; \cdots (*)$$

$$\psi = \text{atan2}(\mp\sqrt{1 - A^2}, A)$$

$$\boxed{q_3 = 180^\circ - \psi - \Delta q} \; \cdots ③$$

Since: $q_2 = \varphi_1 + \varphi_2$

let's find φ_1:

$$\frac{\sin\varphi_1}{d_3'} = \frac{\sin\psi}{\sqrt{l^2+h^2}}$$

$$\Rightarrow \sin\varphi_1 = \frac{d_3' \cdot \sin\psi}{\sqrt{l^2+h^2}} = B \;;\quad \sin\psi = \sin(\pi - q_3') = \sin(q_3')$$

$$= \frac{\tan(q_3')\cdot\cos(q_3')\cdot d_3'}{\sqrt{l^2+h^2}}$$

$$\cancel{= \frac{\sqrt{1-A^2}}{A}\cdot A \cdot \frac{d_3'}{\sqrt{l^2+h^2}}}$$

$$\cancel{\boxed{\sin\varphi_1 = \sqrt{1-A^2}\cdot\frac{d_3'}{\sqrt{l^2+h^2}} = B}}$$

$$\varphi_1 = \operatorname{atan2}(B, \mp\sqrt{1-B^2})$$

$$\varphi_2 = \operatorname{atan2}(h, l)$$

$$\boxed{q_2 = \varphi_1 + \varphi_2 = \operatorname{atan2}(B, \mp\sqrt{1-B^2}) + \operatorname{atan2}(h, l)} \quad (4)$$

```
if abs(T345_bis[2,2]) != 1:
    q3 = arctan2(np.float(T345_bis[0,2]), -np.float(T345_bis[1,2]))
    q5 = arctan2(np.float(T345_bis[2,0]), np.float(T345_bis[2,0]))
    q4 = arctan2(np.sqrt(np.float(T345_bis[0,2])**2 + np.float(T345_bis[1,2])**2), np.float(T345_bis[2,2]))

else:
    print("Singularity case")
```

```
# Solution 2      You, a minute ago • Uncommitted changes
if abs(T345_bis[2,2]) != 1:
    q3 = arctan2(-np.float(T345_bis[0,2]), np.float(T345_bis[1,2]))
    q5 = arctan2(-np.float(T345_bis[2,0]), -np.float(T345_bis[2,0]))
    q4 = arctan2(-np.sqrt(np.float(T345_bis[0,2])**2 + np.float(T345_bis[1,2])**2), np.float(T345_bis[2,2]))

else:
    print("Singularity case")
```

Let's discuss how many solutions we have from IK:

First we notice that we have two solutions for q1:

q1 = atan2(yc,xc)

Or

q1 = π + atan2(yc, xc)

For each case of q1 we have two solutions for q2 and q3 (Elbow up & Elbow down) that we studied in the case of the planar robot.

So far we have 2x2 solutions.

Now we have the wrist which gave us two solutions as well:

When q4 = atan2(y,x)

⇒ q5 = θ

Or

q4 = π + atan2(y, x)

⇒ q5 = - θ