Abstract (informal)

A lot of the discrete models break the tyre down to nodes elements interconnected with springs and dampers. Then solve the dynamics of the whole system. We propose an equivalent model to the interconnected springs-damper models, where the node element is made infinitely small, and the differential equation is integrated in close form. We show that the model is sufficiently accurate for predicting loads over irregular road profiles. Experimental studies are carried out to show the ability of the model to predict contact patch pressure distribution in driving over a step profile. Due to the closed form and simple nature of model, it lends itself very well to real-time simulation applications such as driver-in-the-loop. The dynamic stability of the model in full vehicle simulations is discussed.

In order to solve the shape of the tyre after contact, we only need the homogenous equation, because we want to know shape of the free pieces fore and aft of the contact. The pressure distribution comes from the full deflection, not just the contact patch.

We write out the equations for deflection of a 2d ring, assuming the ring is large enough that the equations of the differential element are not very different. This can be shown by adding the radial component of the circumferential force (tension in the element).

The steps of the simulation are as follows:

1. Detect penetration: find the road points in contact with the ring. The road is represented as discretized triangles (line segments in 2d). The boundaries of shape of the road at the contact boundary is very important.
2. Solve tyre shape: road shape in the contact patch (especially its boundaries) determines the boundary conditions of the equations.
3. Solve pressure distribution in the contact patch. This depends on stiffness and damping values of the tyre. The inclusion of the damping values will be discussed further.
4. Update tyre position given the forces. Required for vehicle simulation.

## Penetration detection

Use a KD-Tree to find the closest triangles and loop through them to find the ones in contact.

## Solve Tyre Shape

The methodology behind the tyre beam model.

We start with a simple beam on elastic foundation model where the beam has stiffness E and surface moment of inertia I and the foundation has stiffness K. The beam is under normal force q(x) (tangential force to come later) . A differential element of the beam is shown in figure:

A picture containing text, clock

Description automatically generated

For the above differential element:

The above equation implicitly assumes that curvature is roughly equal to second derivative of *w* at low values of curvature, which is why bending moment equation is second order. We can show this is true on a curved beam too.

General solution of homogenous equation:

Where

Which obviously leads to C1 and C2 being 0.