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Problem 1

$P \propto H^a Q^b w^c \longrightarrow P = K H^a Q^b w^c$

$ML^2T^{-3} = K (L)^a (L^3T^{-1})^b (ML^{-2}T^{-2})^c$

$M: c = 1$

$L: 2 = a + 3b - 2c \rightarrow 4 = a + 3b$

$T: -3 = -b - 2c \rightarrow b = 1$

$\therefore a = 1$

$\therefore P = K H Q w$

Problem 2

$H \rightarrow$ geometric property

$V \rightarrow$ kinematic property

$\rho \rightarrow$ dynamic property

$\pi_1 = H^{a_1} V^{b_1} \rho^{c_1} \Omega$

$\pi_2 = H^{a_2} V^{b_2} \rho^{c_2} \mu$

$\pi_3 = H^{a_3} V^{b_3} \rho^{c_3} g$

For π_1:

$1 = (L)^{a_1} (LT^{-1})^{b_1} (ML^{-3})^{c_1} L$

$L: 0 = a_1 + b_1 - 3c_1 + 1$

$M: 0 = c_1$

$T: 0 = -b_1$

$\therefore a_1 = -1$

$\therefore \pi = \frac{\Omega}{H}$

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Problem 2 cont.

For Π_2:

$1 = (L)^{a_2} (LT^{-1})^{b_2} (ML^{-3})^{c_2} (ML^{-1}T^{-1})$

$L: 0 = a_2 + b_2 - 3c_2 - 1$

$M: 0 = c_2 + 1 \longrightarrow c_2 = -1$

$T: 0 = -b_2 - 1 \rightarrow b_2 = -1$

$\therefore a_2 = -1$

$\therefore \Pi_2 = \dfrac{\mu}{HV\rho}$

For Π_3:

$1 = (L)^{a_3} (LT^{-1})^{b_3} (ML^{-3})^{c_3} (LT^{-2})$

$M: 0 = c_3$

$T: 0 = -b_3 - 2 \rightarrow b_3 = -2$

$L: 0 = a_3 + b_3 - 3c_3 + 1 \rightarrow a_3 = 1$

$\therefore \Pi_3 = \dfrac{gH}{V^2}$

$\therefore 0 = f(\Pi_1, \Pi_2, \Pi_3)$

$\therefore 0 = f\left(\dfrac{D}{H}, \dfrac{\mu}{\rho VH}, \dfrac{gH}{V^2}\right)$

$V = \sqrt{2gH}\ \phi\left[\dfrac{D}{H}, \dfrac{\mu}{\rho VH}\right]$

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Problem 3

$D \rightarrow$ geometric property

$V \rightarrow$ kinematic property

$\rho \rightarrow$ dynamic property

$\pi_1 = D^{a_1} V^{b_1} \rho^{c_1} \Delta P$, $\pi_2 = D^{a_2} V^{b_2} \rho^{c_2} L$

$\pi_3 = D^{a_3} V^{b_3} \rho^{c_3} \mu$, $\pi_4 = D^{a_4} V^{b_4} \rho^{c_4} k$

For π_1:

$1 = (L)^{a_1} (LT^{-1})^{b_1} (ML^{-3})^{c_1} (ML^{-1}T^{-2})$

$M: 0 = c_1 + 1 \rightarrow c_1 = -1$

$T: 0 = -b_1 - 2 \rightarrow b_1 = -2$

$L: 0 = a_1 + b_1 - 3c_1 - 1 \rightarrow a_1 = 0$

$\therefore \pi_1 = \dfrac{\Delta P}{V^2 \rho}$

For π_2:

$1 = (L)^{a_2} (LT^{-1})^{b_2} (ML^{-3})^{c_2} L$

$M: 0 = c_2$

$T: 0 = -b_2 \rightarrow b_2 = 0$

$L: 0 = a_2 + b_2 - 3c_2 + 1 \rightarrow a_2 = -1$

$\therefore \pi_2 = \dfrac{L}{D}$

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Problem 3 cont.

For π_3.

$$1 = (L)^{a_3} (LT^{-1})^{b_3} (ML^{-3})^{c_3} (ML^{-1}T^{-1})$$

$M: 0 = c_3 + 1 \longrightarrow c_3 = -1$

$T: 0 = -b_3 - 1 \longrightarrow b_3 = -1$

$L: 0 = a_3 + b_3 - 3c_3 - 1 \longrightarrow a_3 = -1$

$$\therefore \pi_3 = \frac{\mu}{\rho V D}$$

For π_4:

$$1 = (L)^{a_4} (LT^{-1})^{b_4} (ML^{-3})^{c_4} L$$

(as solved in π_2)

$a_4 = -1$, $b_4 = c_4 = 0$

$$\therefore \pi_4 = \frac{k}{D}$$

$$0 = F(\pi_1, \pi_2, \pi_3, \pi_4)$$

$$\therefore 0 = F\left(\frac{\Delta P}{\rho V^2}, \frac{L}{D}, \frac{\mu}{\rho V D}, \frac{k}{D}\right)$$

$$\therefore \frac{\Delta P}{\rho V^2} = \phi\left(\frac{L}{D}, Re, \frac{k}{D}\right) \text{ where } Re = \frac{\mu}{\rho V D}$$

$$\therefore \Delta P = \rho V^2 \phi\left(\frac{L}{D}, Re, \frac{k}{D}\right)$$

Problem 4

Prototype: $D_p = 1.5\,m$

$\mu_p = 3 \times 10^{-2}$ Poise

(specific gravity) $\rho = 0.9$

$Q_p = 3.0\, m^3/s$

Model:

$D_m = 0.15\, m$

$\mu_m = 0.01$ Poise

$(S.g)_m = 1$

$$V_p = \frac{Q_p}{A_p} = \frac{4\, Q_p}{\pi D_p^2} = \frac{4 \times 3}{\pi (1.5)^2} \simeq 1.698\, m/s$$

By equating Reynolds numbers

$Re_m = Re_p$

$$V_m = \frac{\rho_p}{\rho_m}\,\frac{D_p}{D_m}\,\frac{\mu_m}{\mu_p}\,V_p = 0.9 \times 10 \times 0.3 \times 1.698$$

$$\simeq 5.093\, m/s$$

$$Q_m = A_m V_m = \frac{1}{4}\pi D_m^2 V_m \simeq 0.09\, m^3/s$$

$$= 90\ \text{Liters}/s$$

Problem 5

$$\lambda = \frac{L_m}{L_p} = \frac{1}{40}$$

By equating Froude number

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}} \longrightarrow V_p = \sqrt{\frac{L_p}{L_m}}\, V_m$$

$$= \sqrt{40} \times 2\, m/s$$

$$\simeq 12.65\, m/s$$

$$\frac{Q_p}{Q_m} = \frac{A_p V_p}{A_m V_m} = (\lambda)^{-2} (\lambda)^{-\frac{1}{2}} = \lambda^{-\frac{5}{2}}$$

$$\therefore Q_p = \lambda^{-\frac{5}{2}} Q_m = 40^{\frac{5}{2}} \times 2.5\, m^3/s \simeq 25298\, m^3/s$$

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Problem 6

$$\lambda = \frac{L_m}{L_p} = \frac{1}{20}$$

$$\frac{H_p}{H_m} = \frac{1}{\lambda} \rightarrow H_p = \frac{1}{\lambda} H_m = 20 \times 0.2\,m$$
$$= 4\,m$$

since $\frac{Q_p}{Q_m} = \left(\frac{1}{\lambda}\right)^{\frac{5}{2}}$ as caldulated P5

$$\frac{E_p}{E_m} = \frac{\rho g Q_p H_p}{\rho g\, H_m} = \frac{Q_p H_p}{Q_m H_m} = \left(\frac{1}{\lambda}\right)^{\frac{7}{2}}$$

$$\therefore E_p = 20^{7/2} \times \frac{1}{10}\ KW = 3578\ kW$$