
PEU438-Fall2024
Compact Objects and High Energy Astrophysics

Lecturer: Karim Hammam **Due Date:** October 26, 2024, 11:30 PM
TA: Abdelrahman Attia **Assignment:** Number 2

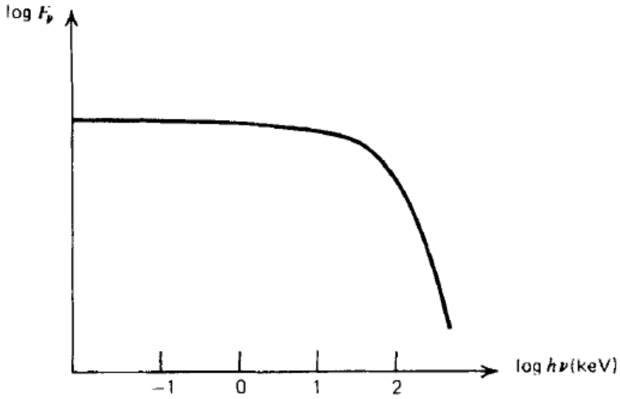
Problem 1 Consider a sphere of ionized hydrogen plasma that is undergoing spherical gravitational collapse. The sphere is held at constant isothermal temperature T_0, uniform density and constant mass M_0 during the collapse, and has decreasing radius $R(t)$. The sphere cools by emission of bremsstrahlung radiation in its interior. At $t = t_0$ the sphere is optically thin.

- a. What is the total luminosity of the sphere as a function of $M_0, R(t)$ and T_0 while the sphere is optically thin?
- b. What is the luminosity of the sphere as a function of time after it becomes optically thick?
- c. Give an implicit relation, in terms of $R(t)$, for the time t_1 when the sphere becomes optically thick.

Problem 2 Suppose X-rays are received from a source of known distance L with a flux $F\,(\mathrm{erg cm^{-2}\ s^{-1}})$. The X-ray spectrum has the form of the Figure. It is conjectured that these X-rays are due to bremsstrahlung from an optically thin, hot, plasma cloud, which is in hydrostatic equilibrium around a central mass M. Assume that the cloud thickness ΔR is roughly its radius $R, \Delta R \sim R$.

- Find R and the density of the cloud, ρ, in terms of the known observations and conjectured mass M.

Hint: Hydrostatic equilibrium gives constraint on ρ and R. From the virial theorem we know that $2\times$ (kinetic energy/particle) = - (gravitational energy/particle)



Problem 3

- (a) Determine the root-mean-square speed, v_{rms}, and corresponding kinetic energy, E_k, of an electron in an HII region of electron temperature, $\mathrm{T_e} = 10^4$ K.
- (b) If the electron lost all of its kinetic energy (came to a stop) during an encounter with a nucleus without recombining, what would be the frequency of the resulting emission and in what part of the spectrum does this frequency occur?
- (c) Compare the frequency of part (b) with a typical thermal Bremsstrahlung photon at a radio frequency of $\nu = 5 \times 10^9$ Hz from an HII region. Based on this comparison, what fraction of an electron's kinetic energy has actually been lost during an encounter with a nucleus?

Problem 4

- (a) Consider the volume emissivity of an optically thin plasma emitting thermal bremsstrahlung radiation at temperature T. Integrate the expression for $j_\nu(\nu)$ (37) from frequency ν_1 to $f\nu_1$ to obtain the power radiated in a fixed logarithmic frequency interval; if $f = 10$, your result would give the power in one decade as a function of frequency. Demonstrate that the power drops rapidly at both $h\nu \ll kT$ and $h\nu \gg kT$. Let $g = Z = 1$.
- (b) Find the frequency at which the power in a fixed logarithmic interval is at a maximum as a function of ν. To obtain a final solution, let the interval factor f become infinitely close to unity, $f = 1 + \varepsilon$ for $\varepsilon \ll 1$. How does $h\nu$ compare with kT at the maximum as $\varepsilon \to 0$?

Problem 5

- (a) Verify the result of the integration of $j_\nu(\nu, T)$ to obtain $j(T)$ for $g = 1$; see (39).
- (b) The Orion nebula, an H II region, is radiating by thermal bremsstrahlung. Consider it to be spherical (radius $R = 8\mathrm{LY}$), optically thin, and at temperature $T = 8000$ K throughout. Let $Z = 1, g = 1$, and $n_{\mathrm{e}} = n_{\mathrm{i}} = 6 \times 10^8\ \mathrm{m}^{-3}$. Find the luminosity (W) of the entire nebula in terms of solar luminosities.
- (c) In what wavelength bands will the power from the Orion nebula be radiated?