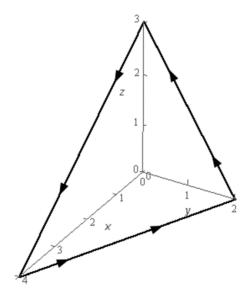
Zewail City of Science and Technology

Physics of Earth and Universe Program



Assignment 5

- 1- Use the Divergence theorem to evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = \sin(\pi x)\,\hat{\imath} + zy^3\hat{\jmath} + (z^2 + 4x)\hat{k}$ and S is the surface of the box with $-1 \le x \le 2, 0 \le y \le 1$ and $1 \le z \le 4$. Note that all six sides of the box are included in S.
- 2- Use Stokes' theorem to evaluate $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ where $\vec{F} = (3yx^2 + z^3)\hat{\imath} + y^2\hat{\jmath} + 4yx^2\hat{k}$ and \mathcal{C} is the triangle with vertices (0,0,3), (0,2,0) and (4,0,0). \mathcal{C} has a counter clockwise rotation if you are above the triangle and looking towards the xy-plane. See the figure below for a sketch of the curve



3- Evaluate the line integral

$$I = \oint_C \left[y(4x^2 + y^2)dx + x(2x^2 + 3y^2)dy \right]$$

around the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.