Assignment 4

1- Paraboloidal coordinates u, v, φ are defined in terms of Cartesian coordinates

$$x = \alpha\beta\cos\varphi\,, \qquad y = \alpha\beta\sin\varphi\,, \qquad z = \frac{1}{2}(\alpha^2 - \beta^2)$$

$$where \quad 0 \le \alpha \le \infty, \qquad 0 \le \beta \le \infty, \qquad 0 \le \varphi \le 2\pi$$

Prove that the α-component of $\vec{\nabla} \times \vec{A}$ is

$$\frac{1}{(\alpha^2 + \beta^2)^{1/2}}\left(\frac{A_\varphi}{\beta} + \frac{\partial\varphi}{\partial\beta}\right) - \frac{1}{\alpha\beta}\frac{\partial A_\beta}{\partial\varphi}$$

2- Using spherical coordinates, evaluate

$$\nabla^2\left(\vec{\nabla}\cdot\frac{\vec{r}}{r^2}\right)$$

3- We introduce the so-called spheroidal coordinates (η, θ, φ) by the following equations expressed in rectangular coordinates

$$x = a\sinh\eta\sin\theta\cos\varphi,$$

$$y = a\sinh\eta\sin\theta\sin\varphi,$$

$$z = a\cosh\eta\cos\theta$$

$$where \quad 0 \le \eta \le \infty, \qquad 0 \le \theta \le \pi, \qquad 0 \le \varphi \le 2\pi$$

a) Show that this coordinate system is orthogonal.

b) Find the scale factors.

c) Show that the function $f(\eta, \theta, \varphi) = \ln\tanh\left(\frac{\eta}{2}\right)$ is a solution of Laplace's equation.