PEU 218 Assignment 1

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Contents

1 Question 1

1.1 Problem

Given point $P(1, 2, 3)$ and vector $\vec{A} = \hat{\imath} + \hat{\jmath}$, find point Q on the $x-$axis such that $\overrightarrow{PQ}$ & $\vec{A}$ are orthogonal.

1.2 Solution

$$\overrightarrow{Q-P} \cdot \vec{A} = 0$$

$$(q_1 - p_1)a_1 + (q_2 - p_2)a_2 + (q_3 - p_3)a_3 = 0$$

Since Q is on the x-axis, $q_2 = 0$.

$$(q_1 - 1)1 + (0 - 2)1 + \cancelto{0}{(q_3 - 3)0} = 0$$

$$\therefore q_1 = 3$$

$$Q = (3, 0, 0)$$

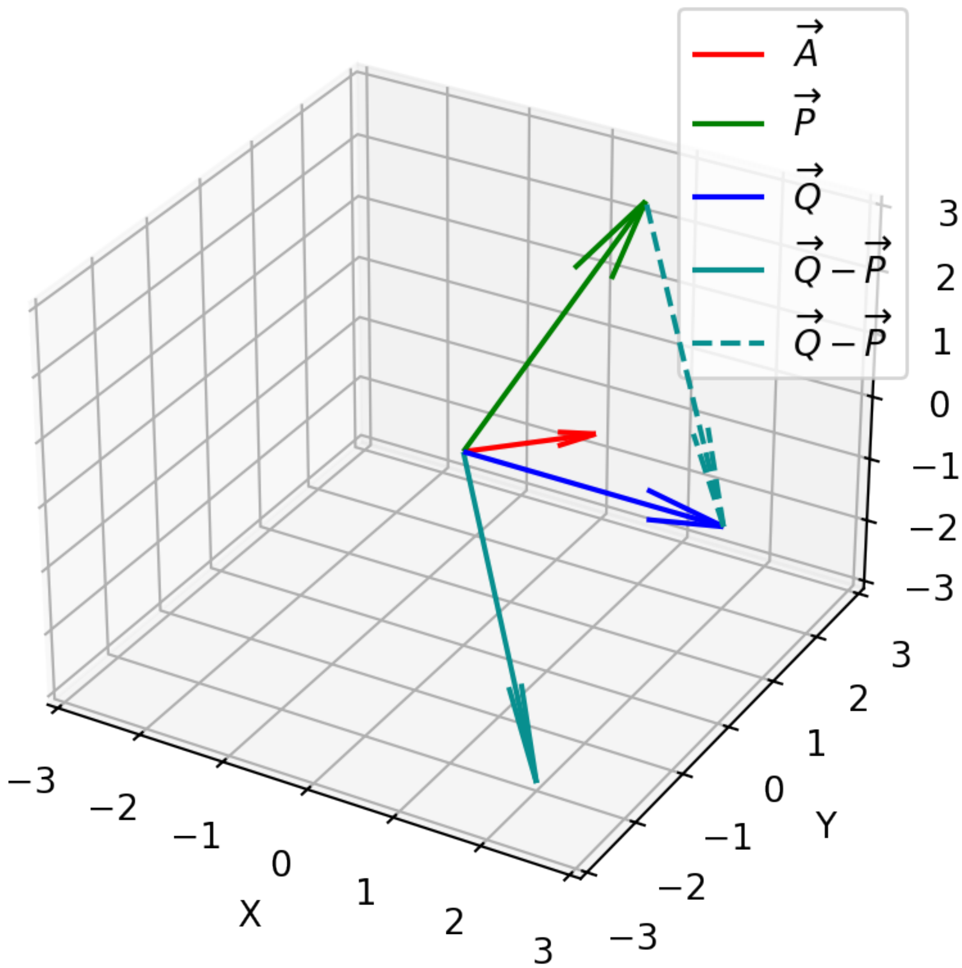

Figure 1: The point Q is on the x-axis.[2]

2 Question 2

2.1 Problem

Find the shortest distance between point $P(3,1,2)$ and the plane given by $x-2y+z=5$.

2.2 Solution

$$\boldsymbol{d} = |(\vec{P} - \vec{A}) \cdot \widehat{\boldsymbol{n}}|$$

$$\vec{P} = \langle 3, 1, 2 \rangle$$

We could substitute in the plane equation to get $\vec{A}$.

$$\vec{A} = \langle 0, 0, 5 \rangle$$

$$\widehat{\boldsymbol{n}} = \frac{\vec{n}}{|\vec{n}|} = \frac{\langle 1, -2, 1 \rangle}{\sqrt{6}}$$

$$\boldsymbol{d} = \frac{1}{\sqrt{6}} \left| \sum_i (P_i - A_i) n_i \right| = \frac{|1(3-0) + -2(1-0) + 1(2-5)|}{\sqrt{6}} = \frac{2}{\sqrt{6}}$$

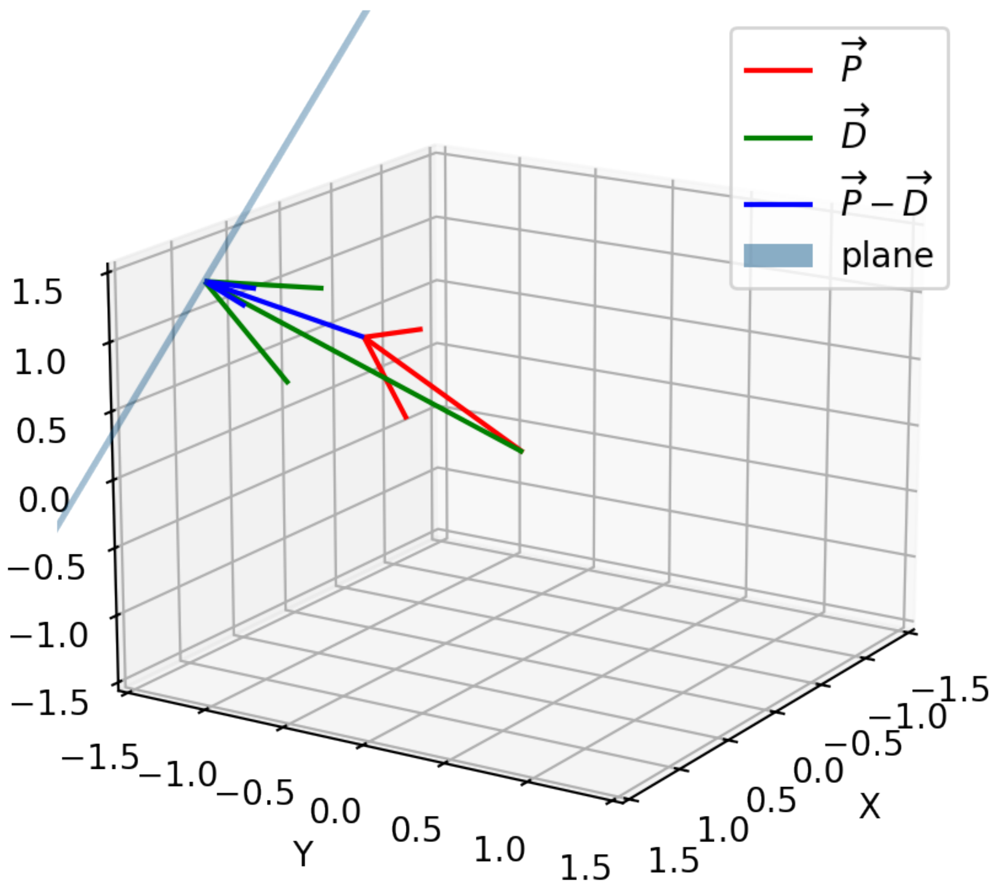

Figure 2: The shortest distance between the point P and the plane.[2]

3 Question 3

3.1 Problem

Find a unit vector that is orthogonal to the line

$$x = 2t - 1, y = -t - 1, z = t + 2$$

and the vector $\hat{\imath} - \hat{\jmath}$.

3.2 Solution

From the parametric equations of the line, we can get the direction vector of the line.

$$\mu = \langle 2, -1, 1 \rangle$$

$$\mu \times \langle 1, -1, 0 \rangle = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= \hat{\imath} + \hat{\jmath} + -\hat{k}$$

$$\vec{x} = \pm \langle 1, 1, -1 \rangle$$

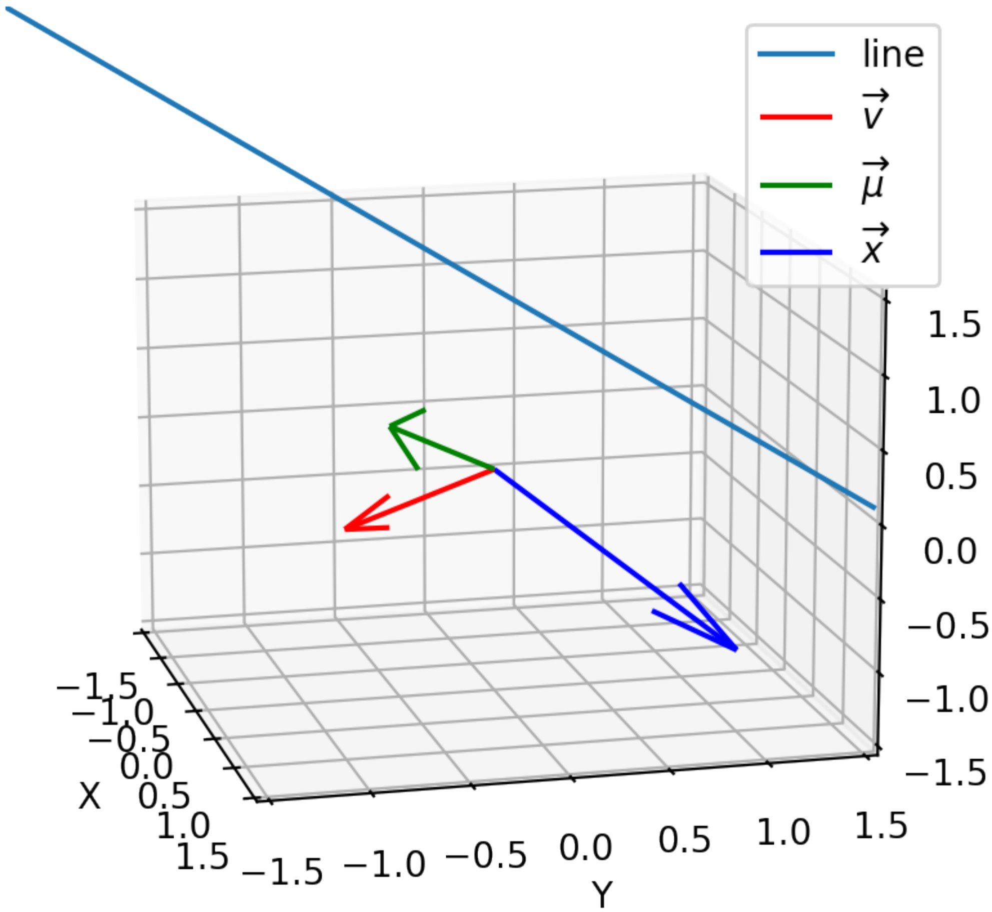

Figure 3: The unit vector that is orthogonal to the line and the vector $\hat{i} - \hat{j}$.[2]

4 Question 4

4.1 Problem

A line is given by $\vec{r} = \vec{a} + \lambda\vec{b}$ where $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 4\hat{i} + 5\hat{j} + 6\hat{k}$. Find the coordinates of the point at which the line intersects the plane $2x + y + 3z = 6$.

4.2 Solution

Plane normal is

$$\vec{n} = \langle 2, 1, 3 \rangle$$

We can use point on the line that satisfies the plane equation and solve for λ.

$$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n} + \lambda \vec{b} \cdot \vec{n}$$

$$= (1 * 2 + 2 * 1 + 3 * 3) + \lambda(4 * 2 + 5 * 1 + 6 * 3) = 13 + 31\lambda = 6$$

$$\lambda = -\frac{7}{31}$$

$$\vec{r} = \vec{a} + \lambda \vec{b} = \langle 1, 2, 3 \rangle - \frac{7}{31} \langle 4, 5, 6 \rangle = \left\langle \frac{3}{31}, \frac{22}{31}, \frac{48}{31} \right\rangle$$

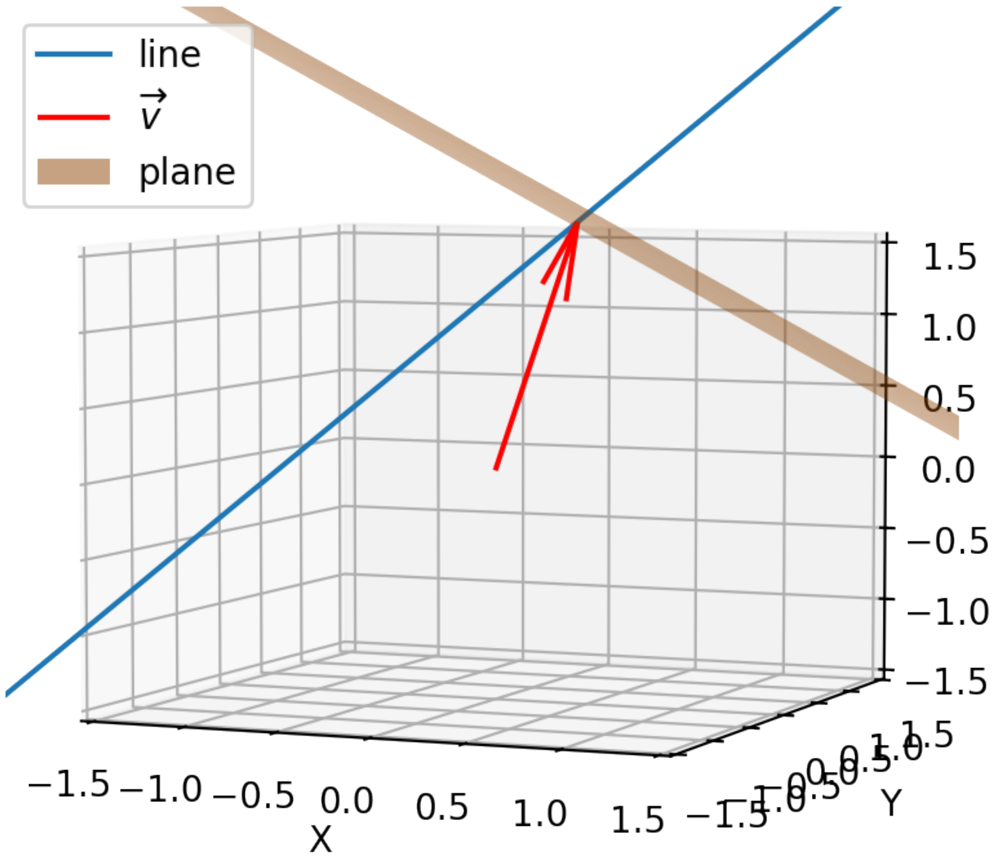

Figure 4: The point at which the line intersects the plane.[2]

5 Question 5

5.1 Problem

Starting with the vector quadruple product

$$(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D})$$

Show that $\vec{D}$ can be expressed as a linear combination of $\vec{A}, \vec{B}, \vec{C}$.

5.2 Solution

let $\vec{V} = (\vec{A} \times \vec{B})$

$$\therefore \vec{X} = \vec{V} \times (\vec{C} \times \vec{D})$$

Using $[BAC - CAB]$ identity

$$\vec{V} \times (\vec{C} \times \vec{D}) = \vec{C}(\vec{V} \cdot \vec{D}) - \vec{D}(\vec{V} \cdot \vec{C})$$

$$\vec{X} = \vec{C}(\vec{A} \times \vec{B} \cdot \vec{D}) - \vec{D}(\vec{A} \times \vec{B} \cdot \vec{C})$$

Similarly we could express $\vec{X}$ as a linear combination of $\vec{A}$ and $\vec{B}$ as

$$\vec{X} = \vec{B}(\vec{C} \times \vec{D} \cdot \vec{A}) - \vec{A}(\vec{C} \times \vec{D} \cdot \vec{B})$$

Equating the two expressions for $\vec{X}$ we get

$$\vec{D} = \frac{\vec{C}(\vec{A} \times \vec{B} \cdot \vec{D}) - \vec{B}(\vec{C} \times \vec{D} \cdot \vec{A}) + \vec{A}(\vec{C} \times \vec{D} \cdot \vec{B})}{\vec{A} \times \vec{B} \cdot \vec{C}}$$

References

[1] M.H. El-Deeb. PEU-218 Assignments.

[2] M.H. El-Deeb. PEU-218 Assignments (Python).