Assignment 3

1- Prove that $\vec{b} \times (\vec{\nabla} \times \vec{b}) = \vec{\nabla}\left(\frac{1}{2}b^2\right) - (\vec{b} \cdot \vec{\nabla})\vec{b}$

2- Let $\vec{F}(x, y, z) = \langle x^2, x^2 y, z + zx \rangle$

a) Verify that $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = 0$

b) Can there exist a function f such that $\vec{F} = \nabla f$? Explain.

3- Suppose that a particle is ejected from the surface $x^2 + y^2 - z^2 = -1$ at the point $(1,1,\sqrt{3})$ along the normal to the surface directed toward the xy plane at $t = 0$ with speed of 10 units per second. When and where does it cross the xy plane?

4- Find the two points on the hyperboloid $x^2 + 4y^2 - z^2 = 4$, where the tangent plane is parallel to the plane $2x + 2y + z = 5$.