PEU 323: Fall 2024
Assignment 4

University of Science and Technology at Zewail City

1. A particle moving in one dimension is in a stationary state whose wave function

$$\psi(x) = A(1 + cos(\frac{\pi}{a}x)) \ , \ -a \leqslant x \leqslant a, \tag{1}$$

and zero otherwise.

   (a) Is this wave function physically acceptable? Explain.

   (b) What is the classically allowed region of this particle?

2. Consider a one-dimensional particle which is confined within the region $0 \leqslant x \leqslant a$ a and whose wave function is

$$\psi(x,t) = sin\left(\frac{n\pi}{a}x\right)e^{-i\omega t} \tag{2}$$

   (a) Find the potential $V(x)$.

   (b) Find the average position of the particle.

3. In problem 2 of Assignment 3, you showed (hopefully) that if a wave function is normalized, then its Fourier Transform is also "normalized". This hinted at some interpretation of the Fourier Transform. In this problem, we push this idea further. We want to show that

$$\langle p \rangle = \int_{-\infty}^{\infty} \hbar k |\phi(k)|^2 \, dk, \tag{3}$$

where $\phi(k)$ is the Fourier Transform of the wave function $\psi(x)$.
Can you use the facts from this problem and the problem of the previous assignment to interpret the F.T. of the wave function?

4. Consider a system that occupies an energy eigenstate of energy E, having a real wave function both at time $t = 0$ and at a later time $t = t_1$.

   (a) Show that there exists a time T for which

$$\psi(x, t + T) = \psi(x, t). \tag{4}$$

(b) Show that for such a system the energy eigenvalues are integer multiples of $\frac{2\pi\hbar}{T}$.

(c) Find this time for a particle in the infinite well extending from $x = 0$ to $x = L$.

5. A particle is in the ground state of a box of length L. Suddenly the box expands (symmetrically) to twice its size, leaving the wave function undisturbed. Find the probability of finding the particle in the ground state of the new box.

6. To get a taste of what you will be doing in your future career, consider an electron in a $V_0 = 20eV$ high square well of width $a = 2a_0$, where $a_0 = 0.529 \times 10^{-10}m$ is the Bohr radius. The energy of the electron$E < V_0$.

(a) Use any numerical technique of your choice to calculate the allowed energy(ies) of the bound electron. Plot (not sketch) the corresponding wave function(s).

(b) If one were to increase the well width a to $a = 6a_0$ and the height of the well to $V_0 = 40eV$, calculate the allowed energy(ies) in this case. Plot all the corresponding wave function(s).