PEU 323 Assignment 2

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1 Problem 1

1.1 Problem

Consider a particle which is free to be anywhere, with equal probability, on a line segment of length L.

(a) Find the normalized probability distribution for such a particle.

(b) Calculate $\langle x \rangle$ and σ_x.

(c) Calculate the probability of finding the particle within σ_x of $\langle x \rangle$.

(d) What are the dimensions of the probability density?

1.2 Solution

2 Problem 2

2.1 Problem

Buffon's Needle: A needle of length l is dropped at random on a sheet of paper with parallel lines a distance l apart. What is the probability that it crosses a line?

2.2 Solution

3 Problem 3

3.1 Problem

Find the probability distribution for the momentum of a harmonic oscillator with angular frequency ω.

3.2 Solution

4 Problem 4

4.1 Problem

Consider the probability density for the location of the electron inside the Hydrogen atom:

$$\rho(r) = Ae^{-2r/a_0}, \tag{1}$$

where a_0 is the Bohr radius.

(a) Find A which normalizes this probability distribution.

(b) *Hint:*

$$\int_0^\infty e^{-x} x^n dx = n!. \tag{2}$$

(c) Calculate the probability for the electron to be found in a sphere, centered about the origin, of radius b_0, with $b_0 << a_0$.

P.S.: You can do this calculation exactly or approximately. The approximate one is much easier.

4.2 Solution

5 Problem 5

5.1 Problem

Consider the map

$$f : \mathbb{C} \to \mathbb{R}_{2\times 2} \tag{3}$$

$$x + iy \mapsto \begin{pmatrix} x & -y \\ y & x \end{pmatrix}$$

from the complex numbers to the set of 2×2 real matrices.

(a) Show that this map is an isomorphism. That is, show that it is invertible and that for $z_1, z_2 \in \mathbb{C}$,

$$f\left(z_1 z_2\right) = f\left(z_1\right) f\left(z_2\right) \tag{4}$$

and hence prove that, in two dimensions, rotations commute.

(b) Prove De Moivre's formula for complex numbers:

$$(r(\cos(\theta) + i\sin(\theta)))^n = r^n(\cos(n\theta) + i\sin(n\theta)) \tag{5}$$

and hence prove that

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}^n = \begin{pmatrix} \cos(n\theta) & -\sin(n\theta) \\ \sin(n\theta) & \cos(n\theta) \end{pmatrix} \tag{6}$$

5.2 Solution

References

[1] M.H. El-Deeb. PEU-323 Assignments.