PEU 323: Fall 2024
Assignment 5

University of Science and Technology at Zewail City

1. A particle of mass m is subject to an attractive double-delta potential

$$V(x) = -V_0(\delta(x-a) + \delta(x+a)). \tag{1}$$

   Consider only the case of negative energies.

   (a) Find the bound state wave functions.

   Hint: The potential is symmetric. Use this symmetry to find the even/odd eigenstates.

   (b) Show that the eigenvalue equation for even bound states can be expressed as

$$tanh(y) = \frac{\gamma}{y} - 1, \tag{2}$$

   where $y = ka$ and $\gamma = \frac{2maV_0}{\hbar^2}$.

   (c) Show that the eigenvalue equation for odd bound states can be expressed as

$$coth(y) = \frac{\gamma}{y} - 1. \tag{3}$$

   (d) Are there always even bound states? Are there always odd bound states?

   Find an estimate for the energy eigenvalues of both even and odd states. Use this estimation to figure out which one is the ground state.

   A plot of eq.s (2) & (3) would be helpful.

   (e) Estimate the ground state energy for the limit $a \to 0$.

   (f) Estimate the ground state energy for the limit $a \to \infty$.

2. Show that the potential term $V(x)\psi(x)$ in the time independent Schrodinger equation in Configuration space transforms to

$$V(x)\psi(x) \quad \to \quad \int_{-\infty}^{\infty} \tilde{V}(p-p')\, \phi(p')\, dp' \tag{4}$$

in the Schrodinger equation in momentum space, where $\tilde{V}(p)$ and $\phi(p)$ are the Fourier Transforms of the potential and the position-space wave function, respectively.

3. Find the transmission and reflection coefficient for particles incident on a potential barrier defined by

$$V = \begin{cases} V_0 & , \quad 0 \leqslant x \leqslant a \\ 0 & , \quad otherwise, \end{cases} \tag{5}$$

where $V_0 > 0$.

   (a) Analyze both cases $E > V_0$. and $E < V_0$.

   (b) Find the condition for resonant transmission in the case $E > V_0$.

4. Particles with energy E are incident from the left on the potential

$$V = \begin{cases} 0 & , \quad -\infty < x \leqslant -x_0, \\ V_0 & , \quad |x| < x_0, \\ \frac{V_0}{2} & , \quad x > x_0. \end{cases} \tag{6}$$

Find the transmission and reflection coefficients if $\frac{V_0}{2} < E < V_0$, and compare them to those of the potential barrier of problem 3.