PEU 323 Assignment 5

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1 Problem 1

(a)

$$V(x) = \begin{cases} -\infty & \text{if } x = -a \\ -\infty & \text{if } x = a \\ 0 & otherwise \end{cases}$$

$$\psi(x) = \begin{cases} \psi(x)_1 & \text{if } 0 < x < a \\ \psi(x)_2 & \text{if } a < x < \infty \\ \pm\psi(-x) & \text{if } -\infty < x < 0 \end{cases}$$

For bound states,

$$-V_0 < E < 0$$

Schrodinger equation:

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = -(E - V_0)\psi$$

For $0 < x < a$:

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi_1}{\partial x^2} = -E\psi_1$$

$$\frac{\partial^2\psi_1}{\partial x^2} = k^2\psi_1$$

$$\text{where } k = \frac{\sqrt{2mE}}{\hbar}$$

$$\psi_1 = Ae^{kx} + Be^{-kx}$$

For $a < x < \infty$:

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi_2}{\partial x^2} = -E\psi_2$$

$$\frac{\partial^2\psi_2}{\partial x^2} = k^2\psi_2$$

$$\psi_2 = Ce^{-kx} + De^{kx}$$

Applying boundary condition at $x = \infty$

$$\psi_2 = Ce^{-kx}$$

Our ψ now is:

$$\psi(x) = \begin{cases} Ae^{kx} + Be^{-kx} & \text{if } 0 < x < a \\ Ce^{-kx} & \text{if } a < x < \infty \\ \pm\psi(-x) & \text{if } -\infty < x < 0 \end{cases}$$

$$\therefore \psi'(x) = \begin{cases} k(Ae^{kx} - Be^{-kx}) & \text{if } 0 < x < a \\ -kCe^{-kx} & \text{if } a < x < \infty \\ \pm\psi'(-x) & \text{if } -\infty < x < 0 \end{cases}$$

Applying boundary condition at $x = a$ for ψ

$$Ae^{2ka} = C - B \tag{1}$$

Applying boundary condition at $x = a$ for ψ'

$$\lim_{\epsilon \to 0} \left[\frac{\hbar^2}{2m} \int_{a-\epsilon}^{a+\epsilon} \frac{\partial^2 \psi}{\partial x^2} \, dx = \int_{a-\epsilon}^{a+\epsilon} V\psi \, dx - E \int_{a-\epsilon}^{a+\epsilon} \psi \, dx \right]$$

$$\frac{\partial \psi}{\partial x}|_{a+} - \frac{\partial \psi}{\partial x}|_{a-} = -\frac{2mV_0}{\hbar^2}\psi(a)$$

$$kCe^{-ka} + k(Ae^{ka} - Be^{-ka}) = \frac{2mV_0}{\hbar^2}Ce^{-ka}$$

$$Ce^{-ka} + Ae^{ka} - Be^{-ka} = \frac{2mV_0}{k\hbar^2}Ce^{-ka}$$

$$(1 - \frac{2mV_0}{k\hbar^2})C - B + Ae^{2ka} = 0$$

$$Ae^{2ka} = (\frac{2mV_0}{k\hbar^2} - 1)C + B \tag{2}$$

$$(\frac{2mV_0}{k\hbar^2} - 1)C + B = C - B$$

$$(1 - \frac{mV_0}{k\hbar^2})C = B$$

$$\psi(x) = \begin{cases} Ae^{kx} + Be^{-kx} & \text{if}\, 0 < x < a \\ Ce^{-kx} & \text{if}\, a < x < \infty \\ \pm\psi(-x) & \text{if}\, -\infty < x < 0 \end{cases}$$

(b)

References

[1] M.H. El-Deeb. PEU-323 Assignments.