1. Radiative Transfer Fundamentals

- **Specific Intensity I_ν:**
$$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$
- **Radiative Transfer Equation (RTE):**
$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$
where τ_ν is the optical depth and S_ν is the source function.
- **Optical Depth τ_ν:**
$$\tau_\nu = \int \alpha_\nu \, ds$$
where α_ν is the absorption coefficient per unit length.
- **General Solution to RTE:**
$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(t)e^{-(\tau_\nu - t)} \, dt$$
For constant S_ν:
$$I_\nu = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$
- **Source Function S_ν:**
$$S_\nu = \frac{j_\nu}{\alpha_\nu}$$
where j_ν is the emissivity per unit volume, and α_ν is the absorption coefficient per unit length.
- **Limiting Cases:**
  - **Optically Thin ($\tau_\nu \ll 1$):**
$$I_\nu \approx I_\nu(0)(1 - \tau_\nu) + S_\nu \tau_\nu$$
  - **Optically Thick ($\tau_\nu \gg 1$):**
$$I_\nu \approx S_\nu$$

2. Blackbody Radiation

- **Planck Function:**
$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/(kT)} - 1}$$
- **Energy Density of Blackbody Radiation:**
$$u = aT^4$$
where $a = \dfrac{8\pi^5 k^4}{15h^3c^3} \approx 7.5657 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$ is the radiation constant.
- **Radiation Pressure:**
$$P_{\mathrm{rad}} = \frac{1}{3}u = \frac{a}{3}T^4$$
- **Stefan-Boltzmann Law:**
$$F = \sigma T^4$$
where F is the total energy flux, and $\sigma = \dfrac{2\pi^5 k^4}{15h^3c^2} \approx 5.6704 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$ is the Stefan-Boltzmann constant.
- **Average Photon Energy:**
$$\langle h\nu \rangle \approx 2.7\,kT$$
- **Wien's Displacement Law:**
  - **Wavelength Form:**
$$\lambda_{\max} T = 2.8978 \times 10^{-3}\,\mathrm{m\,K}$$
  - **Frequency Form:**
$$\nu_{\max} = \frac{kT}{h} \times 2.8214$$

3. Kirchhoff's Law for Thermal Emission

- **Kirchhoff's Law:**

$$\frac{j_\nu}{\alpha_\nu} = B_\nu(T)$$

implying that "good absorbers are good emitters."

- **Blackbody Radiation Condition:**

$$a_\nu = 1$$

for an ideal blackbody, where a_ν is the absorptivity at frequency ν.

4. Bremsstrahlung (Free-Free Emission)

- **Emissivity per Unit Volume and Frequency j_ν:**

$$j_\nu = \frac{16}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{e^6}{m_e^2 c^3} Z^2 n_e n_i \frac{1}{\sqrt{kT}} e^{-h\nu/(kT)} \overline{g}_{ff}$$

where $\overline{g}_{ff}$ is the velocity-averaged Gaunt factor.

- **Total Emissivity Integrated over Frequency:**

$$j(T) = C_1 Z^2 n_e n_i T^{1/2}$$

where C_1 is a constant that depends on fundamental constants and the Gaunt factor.

- **Free-Free Absorption Coefficient α_ν^{ff}:**

$$\alpha_\nu^{ff} = \frac{4}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{e^6}{m_e^2 c} Z^2 n_e n_i \frac{1}{\nu^3 \sqrt{kT}} \left(1 - e^{-h\nu/(kT)}\right) \overline{g}_{ff}$$

- **Optical Depth for Free-Free Absorption:**

$$\tau_\nu = \alpha_\nu^{ff} L$$

where L is the path length.

5. Larmor's Formula for Instantaneous Power

- **Instantaneous Power of an Accelerating Charge:**

$$P = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3}$$

6. Kinetic Theory and Thermal Velocities

- **Root-Mean-Square Speed of Particles:**

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

- **Kinetic Energy per Particle:**

$$E_k = \frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} kT$$

- **Hydrostatic Equilibrium:**

$$\frac{dP}{dr} = -\rho \frac{GM(r)}{r^2}$$

- **Virial Theorem for Spherical Systems:**

$$kT = \frac{GMm_p}{3R}$$

where m_p is the proton mass.

7. Photon Energy and Frequency Relations

- **Energy per Photon:**

$$E = h\nu$$

- **Wien's Law (High-Frequency Approximation):**

$$I(\nu, T) \approx \frac{2h\nu^3}{c^2} e^{-h\nu/(kT)}$$

- **Rayleigh-Jeans Law (Low-Frequency Approximation):**

$$I(\nu, T) \approx \frac{2\nu^2 kT}{c^2}$$

- **Brightness Temperature T_b:**

$$T_b = \frac{c^2 I_\nu}{2k\nu^2}$$

  Applicable in the Rayleigh-Jeans limit ($h\nu \ll kT$).

8. Emission and Absorption in Plasma

- **Integrated Volume Emissivity:**

$$j(T) = C_1 Z^2 n_e n_i T^{1/2}$$

- **Optical Depth τ_ν:**

$$\tau_\nu = \alpha_\nu L$$

- **Mean Free Path $\lambda_{\mathbf{mfp}}$:**

$$\lambda_{\mathrm{mfp}} = \frac{1}{n\sigma}$$

9. Flux and Luminosity

- **Flux from a Point Source (Inverse Square Law):**

$$F = \frac{L}{4\pi r^2}$$

- **Flux in Terms of Specific Intensity:**

$$F = \int I_\nu \cos\theta \, d\Omega$$

  For an isotropic source:

$$F = \pi I_\nu$$

- **Luminosity of a Spherical Blackbody:**

$$L = 4\pi R^2 \sigma T^4$$

- **Luminosity of Optically Thin Bremsstrahlung Emission:**

$$L = j(T)V = C_1 Z^2 n_e n_i T^{1/2} \times \frac{4}{3}\pi R^3$$

10. Solutions to Radiative Transfer in Specific Cases

- **Optically Thin Medium ($\tau_\nu \ll 1$):**

$$I_\nu \approx I_\nu(0) + j_\nu L$$

  assuming negligible absorption.

- **Optically Thick Medium ($\tau_\nu \gg 1$):**

$$I_\nu \approx S_\nu$$

- **General Solution with No Incident Intensity ($I_\nu(0) = 0$) and Constant S_ν:**

$$I_\nu = S_\nu(1 - e^{-\tau_\nu})$$

11. Thermal Bremsstrahlung Spectrum

- **Power Radiated in a Frequency Interval** $(\nu_1, f\nu_1)$**:**

$$P = \int_{\nu_1}^{f\nu_1} j_\nu(\nu)\, d\nu \propto n_e n_i Z^2 T^{1/2} \left(e^{-h\nu_1/(kT)} - e^{-hf\nu_1/(kT)}\right)$$

- **Frequency of Maximum Emission:**

$$h\nu_{\text{max}} \approx kT$$

12. Units and Constants

- **Constants:**

$h = 6.626 \times 10^{-34}\,\text{J}\,\text{s}$	(Planck's constant)
$c = 3.00 \times 10^{8}\,\text{m}\,\text{s}^{-1}$	(Speed of light)
$k = 1.381 \times 10^{-23}\,\text{J}\,\text{K}^{-1}$	(Boltzmann's constant)
$\varepsilon_0 = 8.854 \times 10^{-12}\,\text{F}\,\text{m}^{-1}$	(Vacuum permittivity)
$e = 1.602 \times 10^{-19}\,\text{C}$	(Elementary charge)
$m_e = 9.109 \times 10^{-31}\,\text{kg}$	(Electron mass)
$m_p = 1.673 \times 10^{-27}\,\text{kg}$	(Proton mass)
$G = 6.674 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$	(Gravitational constant)
$\sigma = 5.6704 \times 10^{-8}\,\text{W}\,\text{m}^{-2}\,\text{K}^{-4}$	(Stefan-Boltzmann constant)

- **Unit Conversions:**

$$1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}$$
$$1\,\text{erg} = 1 \times 10^{-7}\,\text{J}$$
$$1\,\text{ly} = 9.461 \times 10^{15}\,\text{m}$$

13. Additional Relations

- **Density in Terms of Mass and Volume:**

$$\rho = \frac{M}{V}$$

- **Particle Number Density:**

$$n = \frac{\rho}{\mu m_p}$$

where μ is the mean molecular weight.

- **Total Bremsstrahlung Luminosity for Optically Thin Plasma:**

$$L = C_1 Z^2 n_e n_i T^{1/2} V$$

- **Hydrostatic Equilibrium (Isothermal Gas Sphere):**

$$\frac{dP}{dr} = -\rho \frac{GM(r)}{r^2}$$

- **Virial Theorem for Gravitational Systems:**

$$2E_{\text{kin}} + E_{\text{grav}} = 0$$

- **Relation between Temperature and Gravitational Potential:**

$$kT = \frac{GMm_p}{3R}$$

(Derived from the virial theorem for spherical systems.)