PEU 438 Assignment 1

Mohamed Hussien El-Deeb (201900052)

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1 Problem 1

1.1 Blackbody Radiation Energy Inside the Eye

Radius of the Eye (r):

$$r = 1.5\,\text{cm} = 0.015\,\text{m}$$

Volume of the Eye (V_{eye}):

$$V_{\text{eye}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.015\,\text{m})^3 = 1.4137 \times 10^{-5}\,\text{m}^3$$

Energy Density of Blackbody Radiation (u):

$$u = aT^4$$

Where $a = 7.5657 \times 10^{-16}\,\text{J} \cdot \text{m}^{-3} \cdot \text{K}^{-4}$ is the radiation constant, and $T = 37°\text{C} = 310.15\,\text{K}$.

$$T^4 = (310.15\,\text{K})^4 = 9.254 \times 10^9\,\text{K}^4$$

Then,

$$u = (7.5657 \times 10^{-16}\,\text{J} \cdot \text{m}^{-3} \cdot \text{K}^{-4})(9.254 \times 10^9\,\text{K}^4) = 7.0013 \times 10^{-6}\,\text{J/m}^3$$

Total Energy Inside the Eye (E_{eye}):

$$E_{\text{eye}} = u \times V_{\text{eye}} = (7.0013 \times 10^{-6}\,\text{J/m}^3)(1.4137 \times 10^{-5}\,\text{m}^3) = 9.9 \times 10^{-11}\,\text{J}$$

1.2 Energy Entering the Eye from Light Bulb

Intensity at 1 Meter (I):

$$I = \frac{\text{Power}}{4\pi r^2} = \frac{100\,\text{W}}{4\pi(1\,\text{m})^2} = 7.9577\,\text{W/m}^2$$

Area of the Pupil (A_{pupil}):

$$A_{\text{pupil}} = 0.1\,\text{cm}^2 = 1 \times 10^{-5}\,\text{m}^2$$

Power Entering the Eye (P_{eye}):

$$P_{\text{eye}} = I \times A_{\text{pupil}} = (7.9577\,\text{W/m}^2)(1 \times 10^{-5}\,\text{m}^2) = 7.9577 \times 10^{-5}\,\text{W}$$

Energy Entering the Eye in 1 Second (E_{in}):

$$E_{\text{in}} = P_{\text{eye}} \times t = (7.9577 \times 10^{-5}\,\text{W})(1\,\text{s}) = 7.9577 \times 10^{-5}\,\text{J}$$

1.3 Comparison

$$\frac{E_{\text{in}}}{E_{\text{eye}}} = \frac{7.9577 \times 10^{-5}\,\text{J}}{9.9 \times 10^{-11}\,\text{J}} \approx 8 \times 10^5$$

1.4 Answer

It is dark when we close our eyes because the energy of the blackbody photons inside the eye is below the threshold needed to stimulate the photoreceptors.

2 Problem 2

2.1 General Solution

$$I_\nu = I_\nu(\tau_\nu = 0)e^{-\tau_{\lambda,0}} + \int_0^{\tau_{\lambda,0}} S_\nu(\tau_\nu)e^{-(\tau_{\lambda,0}-\tau_\nu)}\, d\tau_\nu$$

$$I_\nu(\tau_\nu = 0) = 0, \quad S_\nu(\tau_\nu) = S_\nu$$

$$I_\nu = S_\nu e^{-\tau_{\lambda,0}} \int_0^{\tau_{\lambda,0}} e^{\tau_\nu}\, d\tau_\nu$$

$$I_\nu = S_\nu(1 - e^{-\tau_{\lambda,0}})$$

2.2 $\tau_{\lambda,0} \ll 1$

$$e^{-\tau_{\lambda,0}} \approx 1 - \tau_{\lambda,0}$$

$$I_\nu = S_\nu \tau_{\lambda,0}$$

$$S_\nu = \frac{j_\nu}{\alpha_\nu}, \quad \tau_{\lambda,0} = \alpha_\nu L$$

$$I_\nu = j_\nu L$$

Where j_ν is the emission function of a specific wavelength.

2.3 $\tau_{\lambda,0} \gg 1$

$$I_\nu = S_\nu$$

$$I_\nu = B_\lambda(T) = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

3 Problem 3

$$I_\nu = I_\nu(\tau_\nu = 0)e^{-\tau_{\lambda,0}} + \int_0^{\tau_{\lambda,0}} S_\nu(\tau_\nu)e^{-(\tau_{\lambda,0}-\tau_\nu)}\,d\tau_\nu$$

$$I_\nu(\tau_\nu = 0) = I_{\lambda,0}, \quad S_\nu(\tau_\nu) = S_\nu$$

$$I_\nu = I_{\lambda,0}e^{-\tau_{\lambda,0}} + S_\nu e^{-\tau_{\lambda,0}} \int_0^{\tau_{\lambda,0}} e^{\tau_\nu}\,d\tau_\nu$$

$$I_\nu = I_{\lambda,0}e^{-\tau_{\lambda,0}} + S_\nu(1 - e^{-\tau_{\lambda,0}})$$

3.1 $\tau_{\lambda,0} \ll 1$

$$e^{-\tau_{\lambda,0}} \approx 1 - \tau_{\lambda,0}$$

$$I_\nu = (1 - \tau_{\lambda,0})I_{\lambda,0} + \tau_{\lambda,0}S_\nu$$

$$I_\nu = I_{\lambda,0} + (S_\nu - I_{\lambda,0})\tau_{\lambda,0}$$

$$S_\nu = \frac{j_\nu}{\alpha_\nu}, \quad \tau_{\lambda,0} = \alpha_\nu L$$

$$I_\nu = I_{\lambda,0}(1 - \alpha_\nu L) + j_\nu L$$

$$I_\nu = I_{\lambda,0} + (j_\nu - \alpha_\nu I_{\lambda,0})L$$

Where j_ν is the emission function of a specific wavelength and a_ν is the absorption coefficient for a specific wavelength.

3.2 $\tau_{\lambda,0} \gg 1$

$$I_\nu = S_\nu$$

$$I_\nu = B_\lambda(T) = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

4 Problem 4

5 Problem 5

References

[1] M.H. El-Deeb. PEU-438 Assignments.