Assignment 2: Math. Phys. 3 (PEU 455)

Due October 21st

- **Arfken** (7th edition)
- Exercises: 8.4.1.
- If the photon had a non-zero mass ($m_\gamma \neq 0$), the Coulomb potential would be replaced by the **Yukawa potential**

$$V(\mathbf{r}) = \frac{-e^2}{4\pi\epsilon_0}\frac{e^{-\mu r}}{r},$$

where $\mu = m_\gamma c/\hbar$. With a trial wavefunction of your own devising, estimate the ground state energy of the Hydrogen atom with this potential. Assume $\mu a \ll 1$, and give your answer correct to order $(\mu a)^2$, where a is the Bohr radius.

- Consider a function $f(x,t)$ of the two variable x, t. Now define the new coordinates[1]

$$u = x + t, \qquad v = x - t.$$

(i) Describe the lines of constant u and the lines of constant v

(ii) Show that the lines of constant u are perpendicular to the lines of constant v

(iii) Write df in terms of $\frac{\partial}{\partial u}, \frac{\partial}{\partial v}$

(iv) Now replace t by iy and write df in terms of $\frac{\partial}{\partial z}, \frac{\partial}{\partial \bar{z}}$.

- Write the two-dimensional Laplace equation $\nabla^2\phi(x,y) = 0$ in terms of $\frac{\partial}{\partial z}, \frac{\partial}{\partial z^*}$ and then write it's general solution.

[1] These coordinates are called light-cone coordinates in the context of Minkowski geometry.