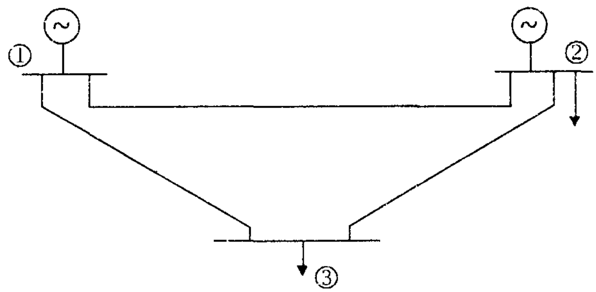
Worked Examples

E 5.1 **A three bus power system is shown in Fig. E5.1. The system parameters are given in Table E5.1 and the load and generation data in Table E5.2. The voltage at bus 2 is maintained at 1.03p.u. The maximum and minimum reactive power limits of the generation at bus 2 are 35 and 0 Mvar respectively. Taking bus 1 as slack bus obtain the load flow solution using Gauss – Seidel iterative method using Y_{Bus}**



E 5.1 A three bus power system

Table E 5.1 Impedance and Line charging Admittances

Bus Code i-k	Impedance (p.u.) Z_{ik}	Line charging Admittance (p.u) y_i
1-2	0.08 + j0 24	0
1-3	0.02 + j0.06	0
2-3	0.06 + j0.018	0

Table E 5.2 Scheduled Generation, Loads and Voltages

Bus No i	Bus voltage V_i	Generation		Load	
		MW	Mvar	MW	Mvar
1	1.05 + j0.0	—	—	0	0
2	1.03 + j0.0	20	—	50	20
3	——	0	0	60	25

Solution :

The line admittance are obtained as

$$y_{12} = 1.25 - j3.75$$

$$y_{23} = 1.667 - j5.00$$

$$y_{13} = 5.00 - j15.00$$

The bus admittance matrix is formed using the procedure indicated in section 2.1 as

$$Y_{Bus} = \begin{bmatrix} 6.25 & -j18.75 & -1.25 & +j3.75 & -5.0 & +j15.0 \\ -1.25 & +j3.73 & 2.9167 & -j8.75 & -j1.6667 & +j5.0 \\ -5.0 & +j15.0 & -1.6667 & +j5.0 & 6.6667 & -j20.0 \end{bmatrix}$$

Gauss – Seidel Iterative Method using Y_{BUS}

The voltage at bus 3 is assumed as 1 + j0. The initial voltages are therefore

$$V_1^{(0)} = 1.05 + j0.0$$

$$V_2^{(0)} = 1.03 + j0.0$$

$$V_3^{(0)} = 1.00 + j0.0$$

Base MVA = 100

Iteration 1 : It is required to calculate the reactive power Q_2 at bus 2, which is a P-V or voltage controlled bus

$$\delta_2^{(0)} = \tan^{-1}\left(\frac{e_2^{a}}{e_2^{'}}\right) = 0\,.$$

$$e'_{2(new)} = |V_2|_{sch} \cos\delta_2 = (1.03)(1.0) = 1.03$$

$$e'_{2(new)} = V_{2\,sch} \sin\delta_2^{(0)} = (1.03)(0.0) = 0.00$$

$$Q_2^{(0)} = \left[\left(e'_{2(new)}\right)^2 B_{22} + \left(e''_{2(new)}\right)^2 B_{22}\right] +$$

$$\sum_{\substack{k=1\\k\neq 2}}^{3} \left[\left(e''_{2(new)} e'_k G_{2k} + e'_k B_{2k}\right) - \left(e'_{2(new)} e''_k G_{2k} - e''_k B_{2k}\right)\right]$$

Substituting the values

$$\begin{aligned} Q_2^{(0)} &= \left[(1.03.)^2 8.75 + (0)^2 8.75\right] + 0(1.05)(-1.25) + 0.(-3.75) \\ &\quad - 1.03[(0)(-1.25) - (1.05)(-3.75)] \\ &\quad + (0)[(1)(-1.6667) + (0)(-5.0)] \\ &\quad - 1.03[(0)(-1.6667) - (1)(-5)] \\ &= 0.07725 \end{aligned}$$

Mvar generated at bus 2

= Mvar injection into bus 2 + load Mvar

= 0.07725 + 0.2 = 0.27725 p.u.

= 27.725 Mvar

This is within the limits specified.

The voltage at bus i is

$$V_i^{(m+1)} = \frac{+1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^{(m)*}} - \sum_{k=1}^{i-1} y_{ik} V_k^{(m+1)} - \sum_{k=i+1}^{n} y_{ik} V_k^{(m)}\right]$$

$$V_2^{(1)} = \frac{1}{Y_{22}}\left[\frac{P_1 - jQ_2}{V_i^{(0)*}} - Y_{21}V_1 - Y_{23}V_3^{(0)}\right]$$

$$= \frac{1}{(2.9167 - j8.75)}$$

$$\left[\frac{-0.3 - 0.07725}{1.03 - j0.0} - (-1.25 + j3.75)(1.05 + j0.0) + (-1.6667 + j5.0)(1 + j0.0)\right]$$

$$V_2^{(1)} = 1.01915 - j0.032491$$

$$= 1.0196673\angle -1.826^0$$

An acceleration factor of 1.4 is used for both real and imaginary parts.

The accelerated voltages is obtained using

$$v_2' = 1.03 + 1.4(1.01915 - 1.03) = 1.01481$$
$$v_2'' = 0.0 + 1.4(-0.032491 - 0.0) = -0.0454874$$

$$V_2^{(1)}(\text{accelerated}) = 1.01481 - j0.0454874$$

$$= 1.01583\angle -2.56648^0$$

The voltage at bus 3 is given by

$$V_2^{(1)} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{(0)*}} - Y_{31}V_1 - Y_{32}V_2^{(1)}\right]$$

$$= \frac{1}{6.6667 - j20}$$

$$\left[\left(\frac{-0.6 + j0.25}{1 - j0}\right) - (-5 + j15)(1.05 + j0) - (-1.6667 + j5)(1.01481 - j0.0454874)\right]$$

$$= 1.02093 - j0.0351381$$

The accelerated value of $V_3^{(1)}$ obtained using

$$v_3' = 1.0 + 1.4(1.02093 - 1.0) = 1.029302$$

$$v_3'' = 0 + 1.4(-0.0351384 - 0) = -0.0491933$$

$$V_3^{(1)} = 1.029302 - j0.049933$$

$$= 1.03048\angle - 2.73624^0$$

The voltages at the end of the first iteration are

$$V_1 = 1.05 + j0.0$$

$$V_2^{(1)} = 1.01481 - j0.0454874$$

$$V_3^{(1)} = 1.029302 - j0.0491933$$

Check for convergence : An accuracy of 0.001 is taken for convergence

$$\left[\Delta v_2'\right]^{(0)} = \left[v_2'\right]^{(1)} - \left[v_2'\right]^{(0)} = 1.01481 - 1.03 = -0.0152$$

$$\left[\Delta v_2''\right]^{(0)} = \left[v_2''\right]^{(1)} - \left[v_2''\right]^{(0)} = -0.0454874 - 0.0 = -0.0454874$$

$$\left[\Delta v_3''\right]^{(0)} = \left[v_3'\right]^{(1)} - \left[v_3'\right]^{(0)} = 1.029302 - 1.0 = 0.029302$$

$$\left[\Delta v_2''\right]^{(0)} = \left[\Delta v_2''\right]^{(1)} - \left[\Delta v_2''\right]^{(0)} = -0.0491933 - 0.0 = -0.0491933$$

The magnitudes of all the voltage changes are greater than 0.001.

Iteration 2 : The reactive power Q_2 at bus 2 is calculated as before to give

$$\delta_2^{(1)} = \tan^{-1}\frac{\left[v_2''\right]^{(1)}}{\left[v_2'\right]^{(1)}} = \tan^{-1}\left[\frac{-0.0454874}{1.01481}\right] = -2.56648^0$$

$$\left[v_2'\right]^{(1)} = \left|V_{2sch}\right| \cdot \cos\delta_2^{(1)} = 1.03\cos(-2.56648^0) = 1.02837$$

$$\left[v_2''\right]^{(1)} = \left|V_{2sch}\right| \cdot \sin\delta_2^{(1)} = 1.03\sin(-2.56648^0) = -0.046122$$

$$\left[V_{2new}\right]^{(1)} = 1.02897 - j0.046122$$

$$Q_2^{(1)} = (1.02897)^2(8.75) + (-0.046122)^2(8.75)$$

$$+(-0.046122)[1.05(-1.25) + (0)(-3.75)]$$

$$-(1.02897)[(0)(-1.25) - (1.05)(-3.75)] +$$

$$-(1.02897)[(-0.0491933)(-1.6667) - (1.029302)(-5)]$$

$$= -0.0202933$$

Mvar to be generated at bus 2

= Net Mvar injection into bus 2 + load Mvar

= – 0.0202933 + 0.2 = 0.1797067 p.u. = 17.97067Mvar

This is within the specified limits. The voltages are, therefore, the same as before

$$V_1 = 1.05 + j0.0$$

$$V_2^{(1)} = 1.02897 - j0.0.46122$$

$$V_3^{(1)} = 1.029302 - j0.0491933$$

The New voltage at bus 2 is obtained as

$$V_2^{(2)} = \frac{1}{2.9167 - j8.75}\left[\frac{-0.3 + j0.0202933}{1.02827 + j0.046122}\right.$$

$$-(-1.25 + j3.75)(1 - 05 + j0)$$

$$\left.-(-1.6667 + j5)\cdot(1.029302 - j0.0491933)\right]$$

$$= 1.02486 - j0.0568268$$

The accelerated value of $V_2^{(2)}$ is obtained from

$$v_2' = 1.02897 + 1.4(1.02486 - 1.02897) = 1.023216$$

$$v_2'' = -0.046122 + 1.4(-0.0568268) - (-0.046122 = -0.0611087)$$

$$v_2^{(2)'} = 1.023216 - j0.0611087$$

The new voltage at bus 3 is calculated as

$$V_3^{(2)} = \frac{1}{6.6667 - j20}\left[\frac{-6.6 + j0.25}{1.029302 + j0.0491933}\right.$$

$$-(-5 + j15)(1.05 + j0.0)$$

$$\left.-(-1.6667 + j5.0)\cdot(1.023216 - j0.0611)\right]$$

$$= 1.0226 - j0.0368715$$

The accelerated value of $V_2^{(2)}$ obtained from

$$v_3' = 1.029302 + 1.4(1.0226 - 1.029302) = 1.02$$

$$v_3'' = (-0.0491933) + 1.4(-0.0368715) + (0.0491933) = -0.03194278$$

$$V_3^{(2)} = 1.02 - j0.03194278$$

The voltages at the end of the second iteration are

$$V_1 = 1.05 + j0.0$$

$$V_2^{(2)} = 1.023216 - j0.0611087$$

$$V_3^{(2)} = 1.02 - j0.03194278$$

The procedure is repeated till convergence is obtained at the end of the sixth iteration. The results are tabulated in Table E5.1(a)

Table E5.1 (a) Bus Voltage

Iteration	Bus 1	Bus 2	Bus 3
0	1.05 + j0	1.03 + j0	1.0 + j0
1	1.05 + j0	1.01481 – j0.04548	1.029302 – j0.049193
2	1.05 + j0	1.023216 – j0.0611087	1.02 – j0.0319428
3	1.05 + j0	1.033476 – j0.0481383	1.027448 – j0.03508
4	1.05 + j0	1.0227564 – j0.051329	1.0124428 – j0.0341309
5	1.05 + j0	1.027726 – j0.0539141	1.0281748 – j0.0363943
6	1.05 + j0	1.029892 – j0.05062	1.020301 – j0.0338074
7	1.05 + j0	1.028478 – j0.0510117	1.02412 – j0.034802

Line flow from bus 1 to bus 2

$$S_{12} = V_1 \left(V_1^* - V_2^*\right) Y_{12}^* = 0.228975 + j0.017396$$

Line flow from bus 2 to bus 1

$$S_{21} = V_2 \left(V_2^* - V_1^*\right) Y_{21}^* = -0.22518 - j0.0059178$$

Similarly, the other line flows can be computed and are tabulated in Table E5.1(b). the slack bus power obtained by adding the flows in the lines terminating at the slack bus, is

$$P_1 + jQ_1 = 0.228975 + j0.017396 + 0.684006 + j0.225$$

$$= (0.912981 + j0.242396)$$

Table E5.1(b) Line Flows

Line	P	Power Flow	Q
1-2	+ 0.228975		0.017396
2-1	– 0.225183		0.0059178
1-3	0.68396		0.224
3-1	– 0.674565		–0.195845
2-3	– 0.074129		0.0554
3-2	0.07461		–0.054