

Exponential function

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Abstract

In this article a power series representation of the exponential function is investigated.

1 Introduction

The exponential function normally denoted as:

$$f(x) = e^x \quad (1)$$

The exponential function can be represented using a power series ¹:

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots \quad (2)$$

This representation only uses which only uses multiplications and divisions.

2 Implementation

The implementation is made in c#. The implementation of the power function representation are done in the following way:

```
static double ex(double x){
    if (x<0)
        return 1/ex(-x);
    if (x>1.0/8)
        return Pow(ex(x/2),2);
    return 1+x*(1+x/2*(1+x/3*(1+x/4*(1+x/5*(1+x/6*
    (1+x/7*(1+x/8*(1+x/9*(1+x/10)))))))));
}
```

Starting from the top, if the $x < 0$ the the function call itself but now rewritten using a positive value of x. if $x > 1/8$ the function the function calls itself but with a smaller argument, which is $x/2$, which gives an better accuracy. To account for halving the argument, the the result from the exponential equation is squared. At last if $0 \leq x \leq 1/8$ then a rewritten version of the power function representation of the exponential function is called.

¹https://en.wikipedia.org/wiki/Exponential_function

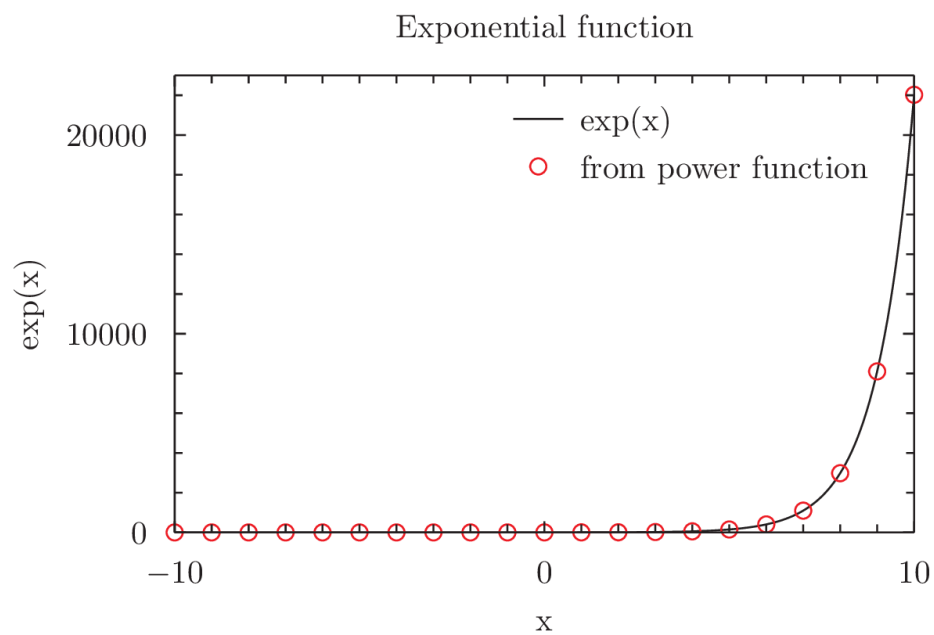


Figure 1: Exponential function plotted together with the power function representation.

3 Results

In figure (1) are plotted together with results from the power function representation equation (??).