Formulating and simulating a hypothesis

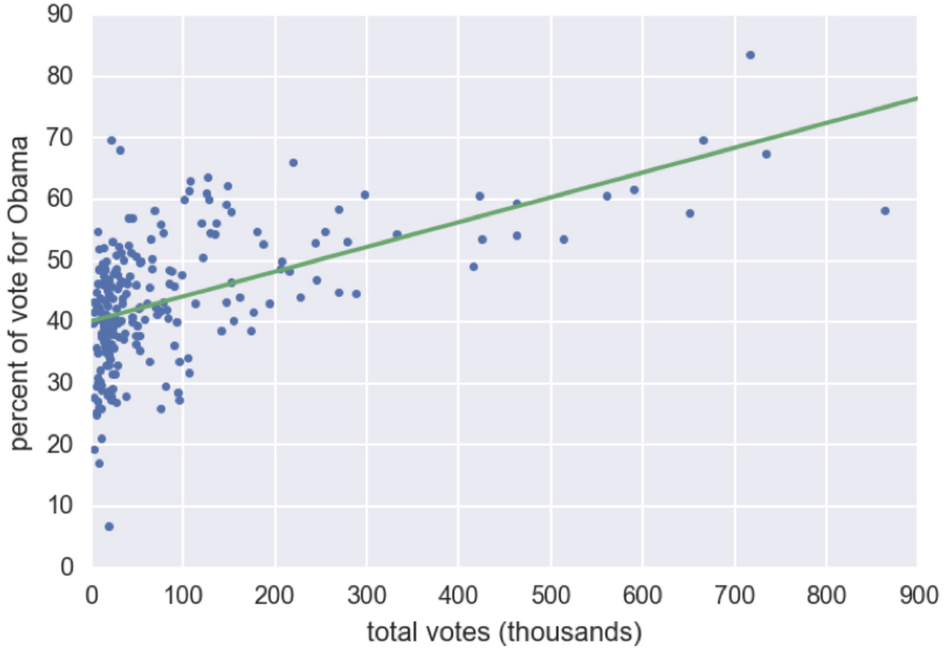
STATISTICAL THINKING IN PYTHON (PART 2)

Justin Bois

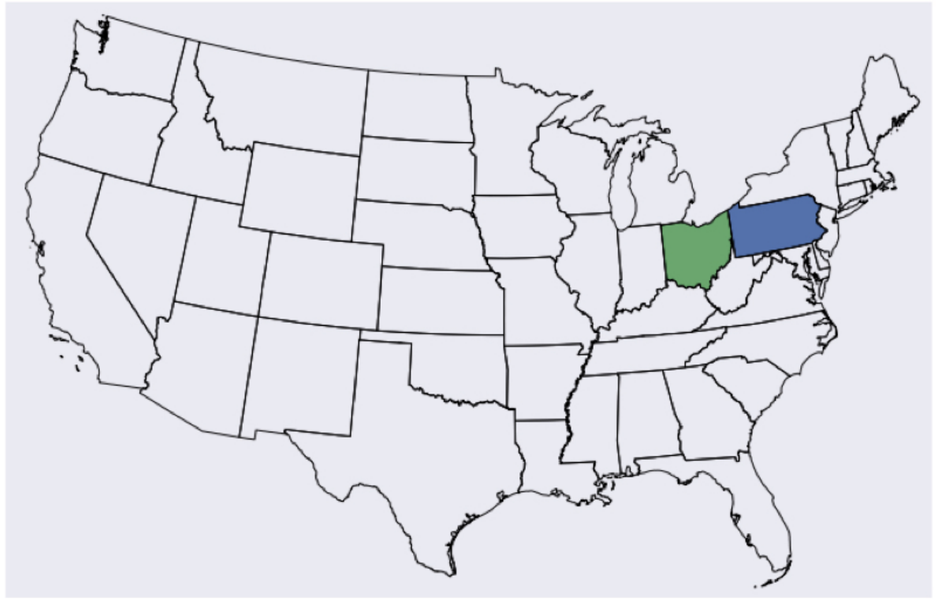
Lecturer at the California Institute of Technology

datacamp

2008 US swing state election results



[1] Data retrieved from Data.gov (https://www.data.gov/)



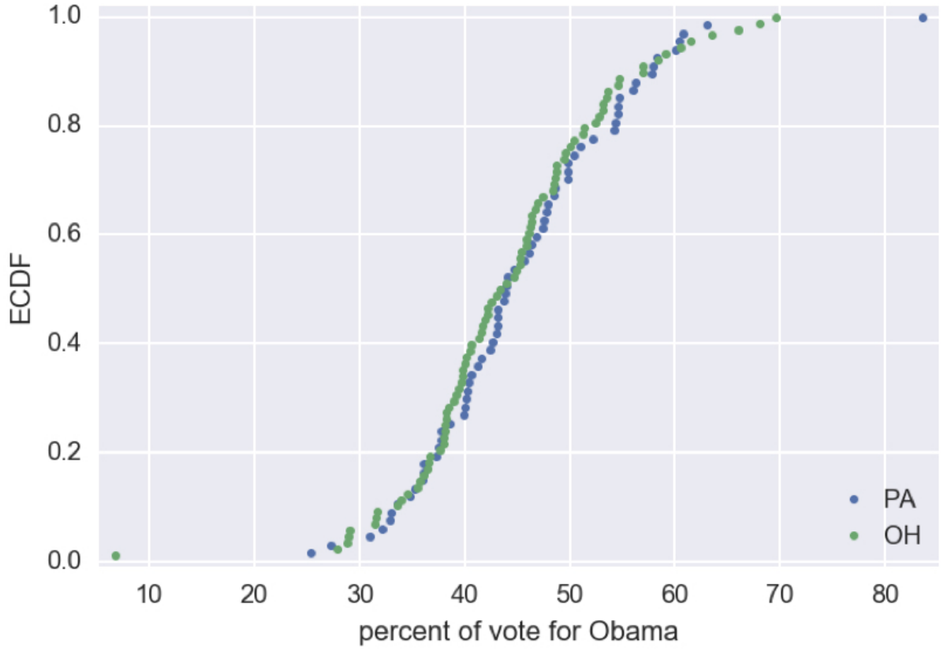
Hypothesis testing

- Assessment of how reasonable the observed data are assuming a hypothesis is true

Null hypothesis

- Another name for the hypothesis you are testing

ECDFs of swing state election results



[1] Data retrieved from Data.gov (https://www.data.gov/)

Percent vote for Obama

	PA	OH	PA — OH difference
mean	45.5%	44.3%	1.2%
median	44.0%	43.7%	0.4%
standard deviation	9.8%	9.9%	−0.1%

[1] Data retrieved from Data.gov (https://www.data.gov/)

Simulating the hypothesis

Pennsylvania

Ohio

[1] Data retrieved from Data.gov (https://www.data.gov/)

Simulating the hypothesis

```
60.08,  40.64,  36.07,  41.21,  31.04,  43.78,  44.08,  46.85,
44.71,  46.15,  63.10,  52.20,  43.18,  40.24,  39.92,  47.87,
37.77,  40.11,  49.85,  48.61,  38.62,  54.25,  34.84,  47.75,
43.82,  55.97,  58.23,  42.97,  42.38,  36.11,  37.53,  42.65,
50.96,  47.43,  56.24,  45.60,  46.39,  35.22,  48.56,  32.97,
57.88,  36.05,  37.72,  50.36,  32.12,  41.55,  54.66,  57.81,
54.58,  32.88,  54.37,  40.45,  47.61,  60.49,  43.11,  27.32,
44.03,  33.56,  37.26,  54.64,  43.12,  25.34,  49.79,  83.56,
40.09,  60.81,  49.81,  56.94,  50.46,  65.99,  45.88,  42.23,
45.26,  57.01,  53.61,  59.10,  61.48,  43.43,  44.69,  54.59,
48.36,  45.89,  48.62,  43.92,  38.23,  28.79,  63.57,  38.07,
40.18,  43.05,  41.56,  42.49,  36.06,  52.76,  46.07,  39.43,
39.26,  47.47,  27.92,  38.01,  45.45,  29.07,  28.94,  51.28,
50.10,  39.84,  36.43,  35.71,  31.47,  47.01,  40.10,  48.76,
31.56,  39.86,  45.31,  35.47,  51.38,  46.33,  48.73,  41.77,
41.32,  48.46,  53.14,  34.01,  54.74,  40.67,  38.96,  46.29,
38.25,   6.80,  31.75,  46.33,  44.90,  33.57,  38.10,  39.67,
40.47,  49.44,  37.62,  36.71,  46.73,  42.20,  53.16,  52.40,
58.36,  68.02,  38.53,  34.58,  69.64,  60.50,  53.53,  36.54,
49.58,  41.97,  38.11
```

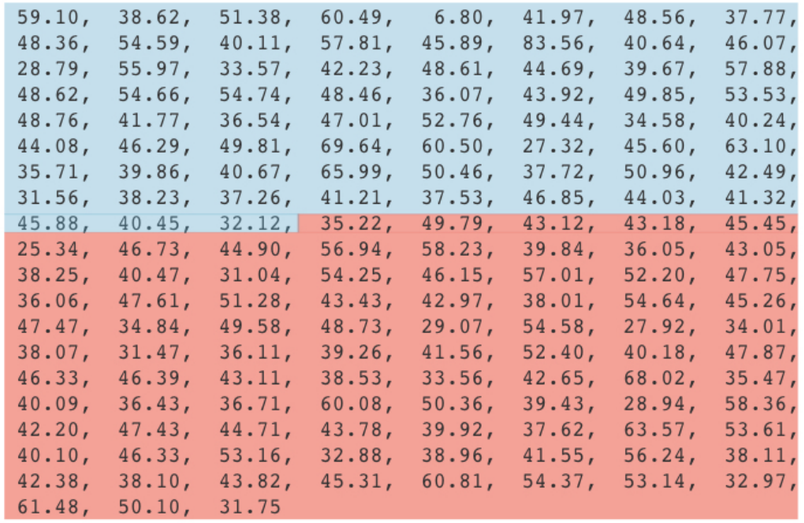
[1] Data retrieved from Data.gov (https://www.data.gov/)

Simulating the hypothesis

```
59.10,  38.62,  51.38,  60.49,   6.80,  41.97,  48.56,  37.77,
48.36,  54.59,  40.11,  57.81,  45.89,  83.56,  40.64,  46.07,
28.79,  55.97,  33.57,  42.23,  48.61,  44.69,  39.67,  57.88,
48.62,  54.66,  54.74,  48.46,  36.07,  43.92,  49.85,  53.53,
48.76,  41.77,  36.54,  47.01,  52.76,  49.44,  34.58,  40.24,
44.08,  46.29,  49.81,  69.64,  60.50,  27.32,  45.60,  63.10,
35.71,  39.86,  40.67,  65.99,  50.46,  37.72,  50.96,  42.49,
31.56,  38.23,  37.26,  41.21,  37.53,  46.85,  44.03,  41.32,
45.88,  40.45,  32.12,  35.22,  49.79,  43.12,  43.18,  45.45,
25.34,  46.73,  44.90,  56.94,  58.23,  39.84,  36.05,  43.05,
38.25,  40.47,  31.04,  54.25,  46.15,  57.01,  52.20,  47.75,
36.06,  47.61,  51.28,  43.43,  42.97,  38.01,  54.64,  45.26,
47.47,  34.84,  49.58,  48.73,  29.07,  54.58,  27.92,  34.01,
38.07,  31.47,  36.11,  39.26,  41.56,  52.40,  40.18,  47.87,
46.33,  46.39,  43.11,  38.53,  33.56,  42.65,  68.02,  35.47,
40.09,  36.43,  36.71,  60.08,  50.36,  39.43,  28.94,  58.36,
42.20,  47.43,  44.71,  43.78,  39.92,  37.62,  63.57,  53.61,
40.10,  46.33,  53.16,  32.88,  38.96,  41.55,  56.24,  38.11,
42.38,  38.10,  43.82,  45.31,  60.81,  54.37,  53.14,  32.97,
61.48,  50.10,  31.75
```

[1] Data retrieved from Data.gov (https://www.data.gov/)

Simulating the hypothesis



59.10, 38.62, 51.38, 60.49, 6.80, 41.97, 48.56, 37.77,
48.36, 54.59, 40.11, 57.81, 45.89, 83.56, 40.64, 46.07,
28.79, 55.97, 33.57, 42.23, 48.61, 44.69, 39.67, 57.88,
48.62, 54.66, 54.74, 48.46, 36.07, 43.92, 49.85, 53.53,
48.76, 41.77, 36.54, 47.01, 52.76, 49.44, 34.58, 40.24,
44.08, 46.29, 49.81, 69.64, 60.50, 27.32, 45.60, 63.10,
35.71, 39.86, 40.67, 65.99, 50.46, 37.72, 50.96, 42.49,
31.56, 38.23, 37.26, 41.21, 37.53, 46.85, 44.03, 41.32,
45.88, 40.45, 32.12, 35.22, 49.79, 43.12, 43.18, 45.45,
25.34, 46.73, 44.90, 56.94, 58.23, 39.84, 36.05, 43.05,
38.25, 40.47, 31.04, 54.25, 46.15, 57.01, 52.20, 47.75,
36.06, 47.61, 51.28, 43.43, 42.97, 38.01, 54.64, 45.26,
47.47, 34.84, 49.58, 48.73, 29.07, 54.58, 27.92, 34.01,
38.07, 31.47, 36.11, 39.26, 41.56, 52.40, 40.18, 47.87,
46.33, 46.39, 43.11, 38.53, 33.56, 42.65, 68.02, 35.47,
40.09, 36.43, 36.71, 60.08, 50.36, 39.43, 28.94, 58.36,
42.20, 47.43, 44.71, 43.78, 39.92, 37.62, 63.57, 53.61,
40.10, 46.33, 53.16, 32.88, 38.96, 41.55, 56.24, 38.11,
42.38, 38.10, 43.82, 45.31, 60.81, 54.37, 53.14, 32.97,
61.48, 50.10, 31.75

"Pennsylvania"

"Ohio"

Permutation

- Random reordering of entries in an array

Generating a permutation sample

```
import numpy as np
dem_share_both = np.concatenate(
                        (dem_share_PA, dem_share_OH))
dem_share_perm = np.random.permutation(dem_share_both)
perm_sample_PA = dem_share_perm[:len(dem_share_PA)]
perm_sample_OH = dem_share_perm[len(dem_share_PA):]
```

Let's practice!

STATISTICAL THINKING IN PYTHON (PART 2)

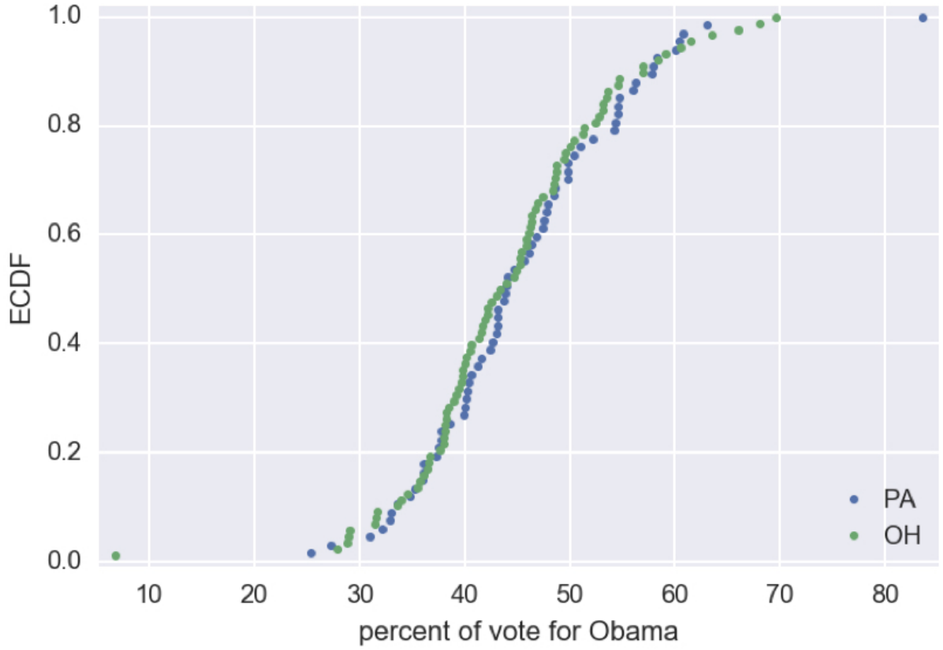
Test statistics and p-values

STATISTICAL THINKING IN PYTHON (PART 2)

Justin Bois

Lecturer at the California Institute of Technology

Are OH and PA different?



[1] Data retrieved from Data.gov (https://www.data.gov/)

Hypothesis testing

- Assessment of how reasonable the observed data are assuming a hypothesis is true

Test statistic

- A single number that can be computed from observed data and from data you simulate under the null hypothesis
- It serves as a basis of comparison between the two

Permutation replicate

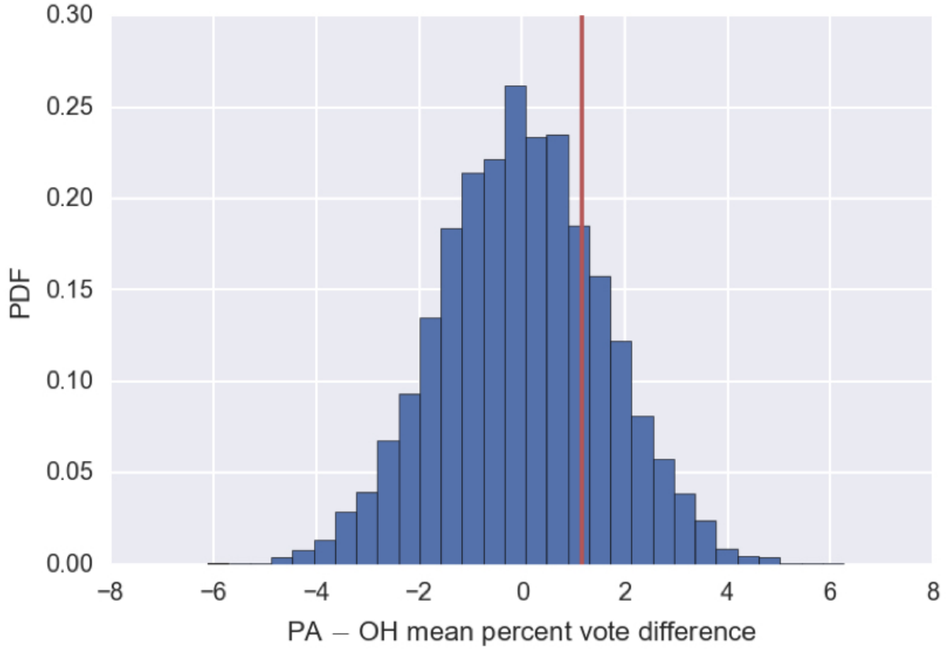
```
np.mean(perm_sample_PA) - np.mean(perm_sample_OH)
```

```
1.122220149253728
```

```
np.mean(dem_share_PA) - np.mean(dem_share_OH) # orig. data
```

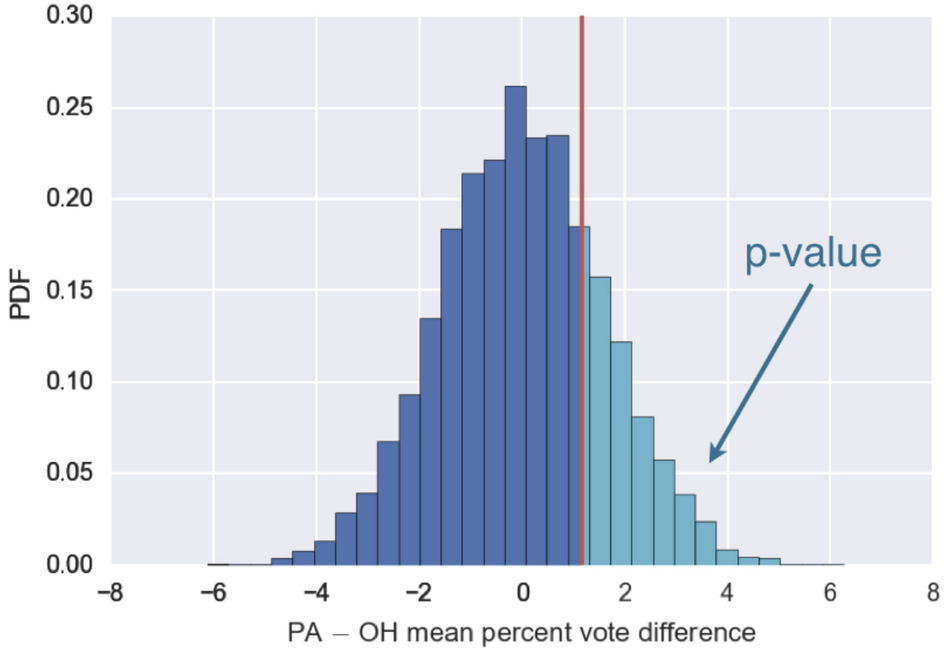
```
1.1582360922659518
```

Mean vote difference under null hypothesis



[1] Data retrieved from Data.gov (https://www.data.gov/)

Mean vote difference under null hypothesis



[1] Data retrieved from Data.gov (https://www.data.gov/)

p-value

- The probability of obtaining a value of your test statistic that is at least as extreme as what was observed, under the assumption the null hypothesis is true
- **NOT** the probability that the null hypothesis is true

The p-value tells you that there is about a 0.6% chance that you would get the difference of means observed in the experiment if frogs were exactly the same. A p-value below 0.01 is typically said to be "statistically significant," but: warning! warning! warning! You have computed a p-value; it is a number. I encourage you not to distill it to a yes-or-no phrase. p = 0.006 and p = 0.000000006 are both said to be "statistically significant," but they are definitely not the same!

Statistical significance

- Determined by the smallness of a p-value

Null hypothesis significance testing (NHST)

- Another name for what we are doing in this chapter

statistical significance ? practical significance

Let's practice!

STATISTICAL THINKING IN PYTHON (PART 2)

Bootstrap hypothesis tests

STATISTICAL THINKING IN PYTHON (PART 2)

Justin Bois

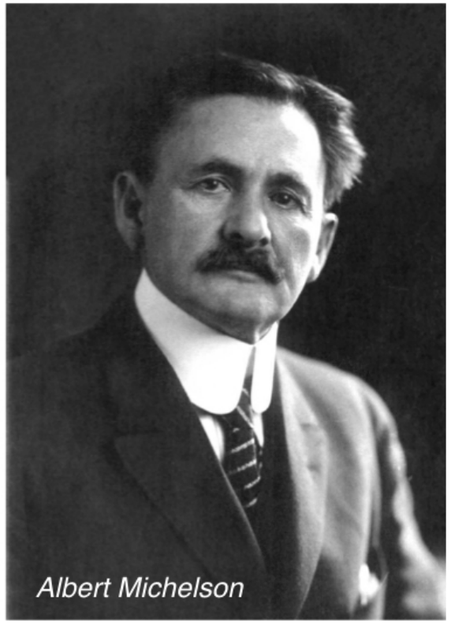
Lecturer at the California Institute of Technology

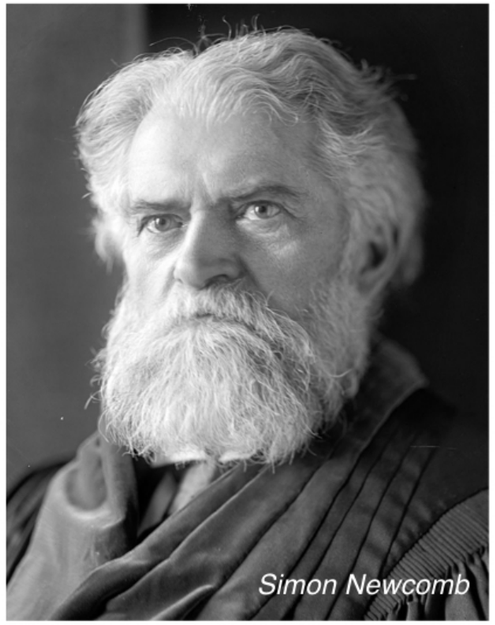
datacamp

Pipeline for hypothesis testing

- Clearly state the null hypothesis
- Define your test statistic
- Generate many sets of simulated data assuming the null hypothesis is true
- Compute the test statistic for each simulated data set
- The p-value is the fraction of your simulated data sets for which the test statistic is at least as extreme as for the real data

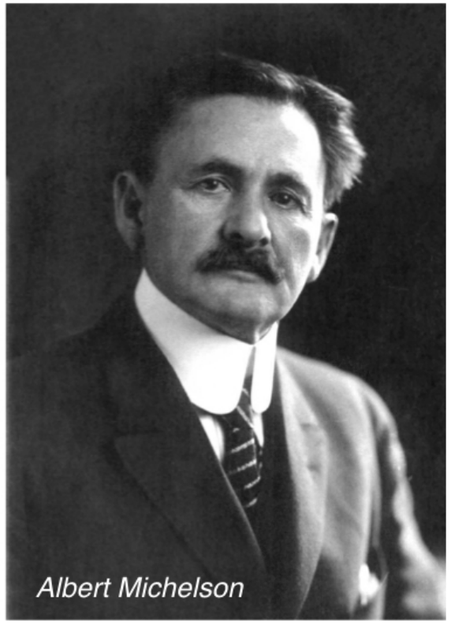
Michelson and Newcomb: speed of light pioneers



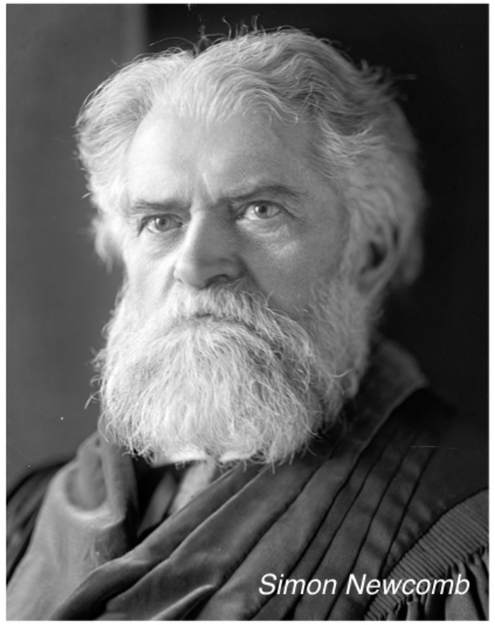


[1] Michelson image: public domain, Smithsonian [2] Newcomb image: US Library of Congress

Michelson and Newcomb: speed of light pioneers



299,852 km/s

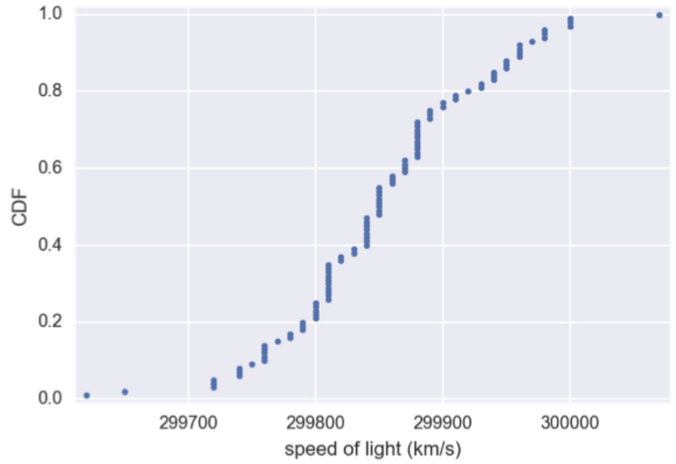


299,860 km/s

[1] Michelson image: public domain, Smithsonian [2] Newcomb image: US Library of Congress

The data we have

Michelson:



Newcomb:

mean = 299,860 km/s

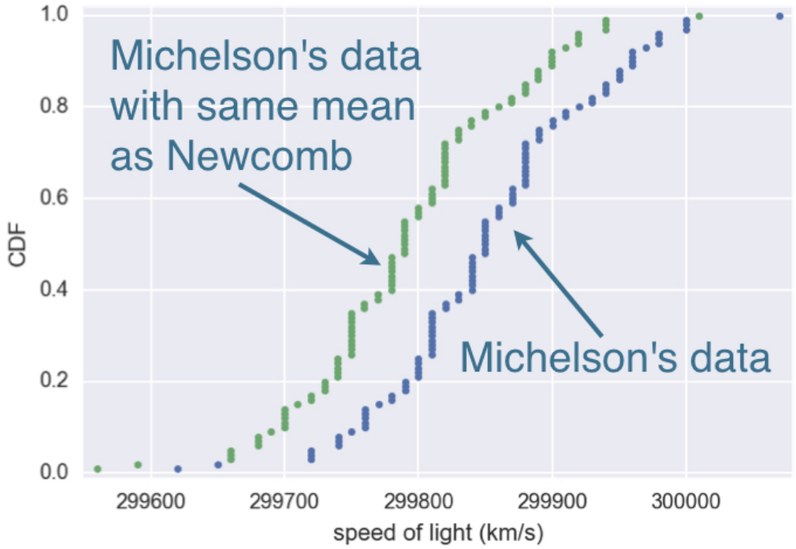
[1] Data: Michelson, 1880

Null hypothesis

- The true mean speed of light in Michelson’s experiments was actually Newcomb's reported value

Shifting the Michelson data

```
newcomb_value = 299860  # km/s
michelson_shifted = michelson_speed_of_light \\
           - np.mean(michelson_speed_of_light) + newcomb_value
```



Calculating the test statistic

```
def diff_from_newcomb(data, newcomb_value=299860):
    return np.mean(data) - newcomb_value
```

```
diff_obs = diff_from_newcomb(michelson_speed_of_light)
diff_obs
```

```
-7.5999999999767169
```

Computing the p-value

```
bs_replicates = draw_bs_reps(michelson_shifted,
                             diff_from_newcomb, 10000)
p_value = np.sum(bs_replicates <= diff_observed) / 10000
p_value
```

```
0.16039999999999999
```

One sample test

- Compare one set of data to a single number

Two sample test

- Compare two sets of data

Let's practice!

STATISTICAL THINKING IN PYTHON (PART 2)