Inferential Statistics Concepts

INTRODUCTION TO LINEAR MODELING IN PYTHON

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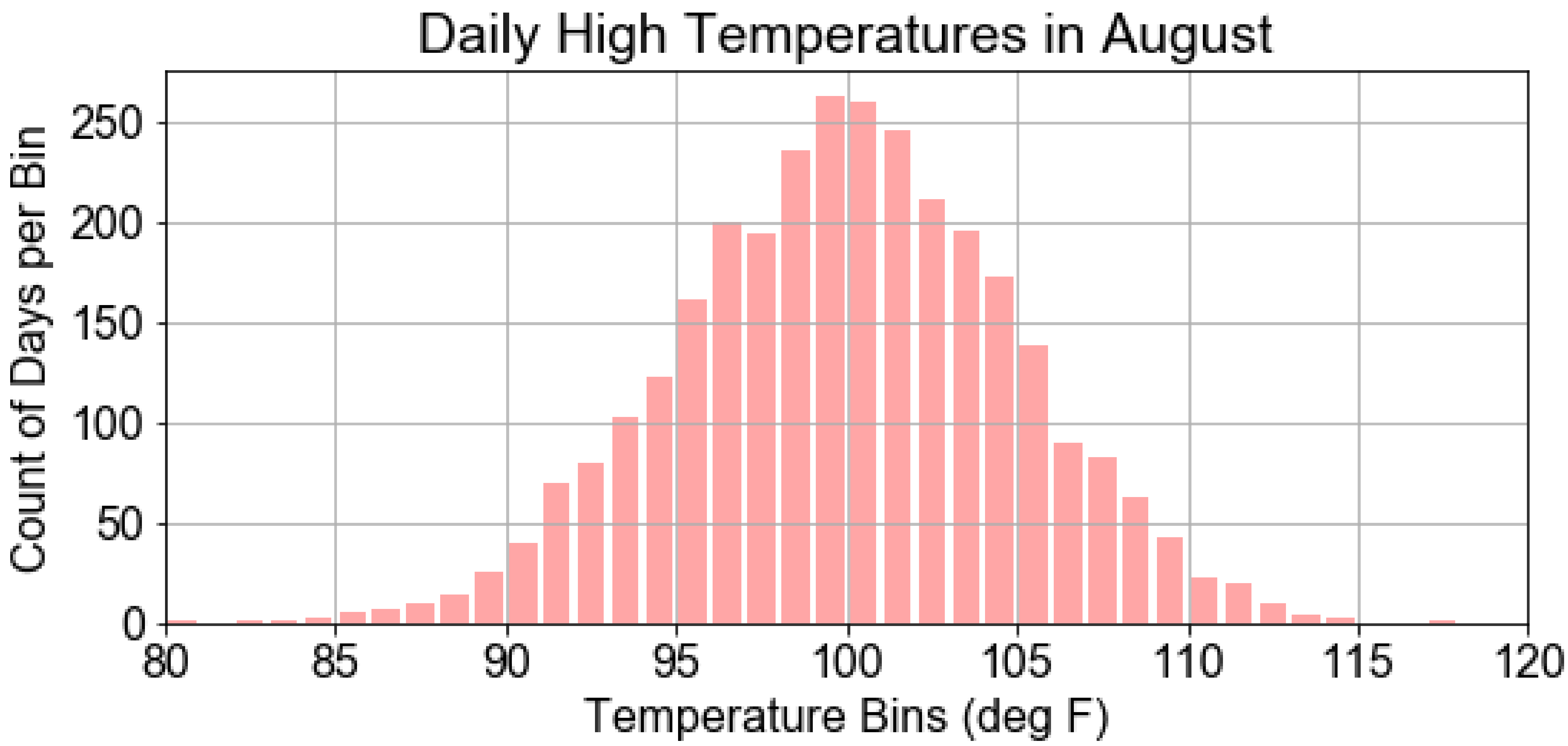
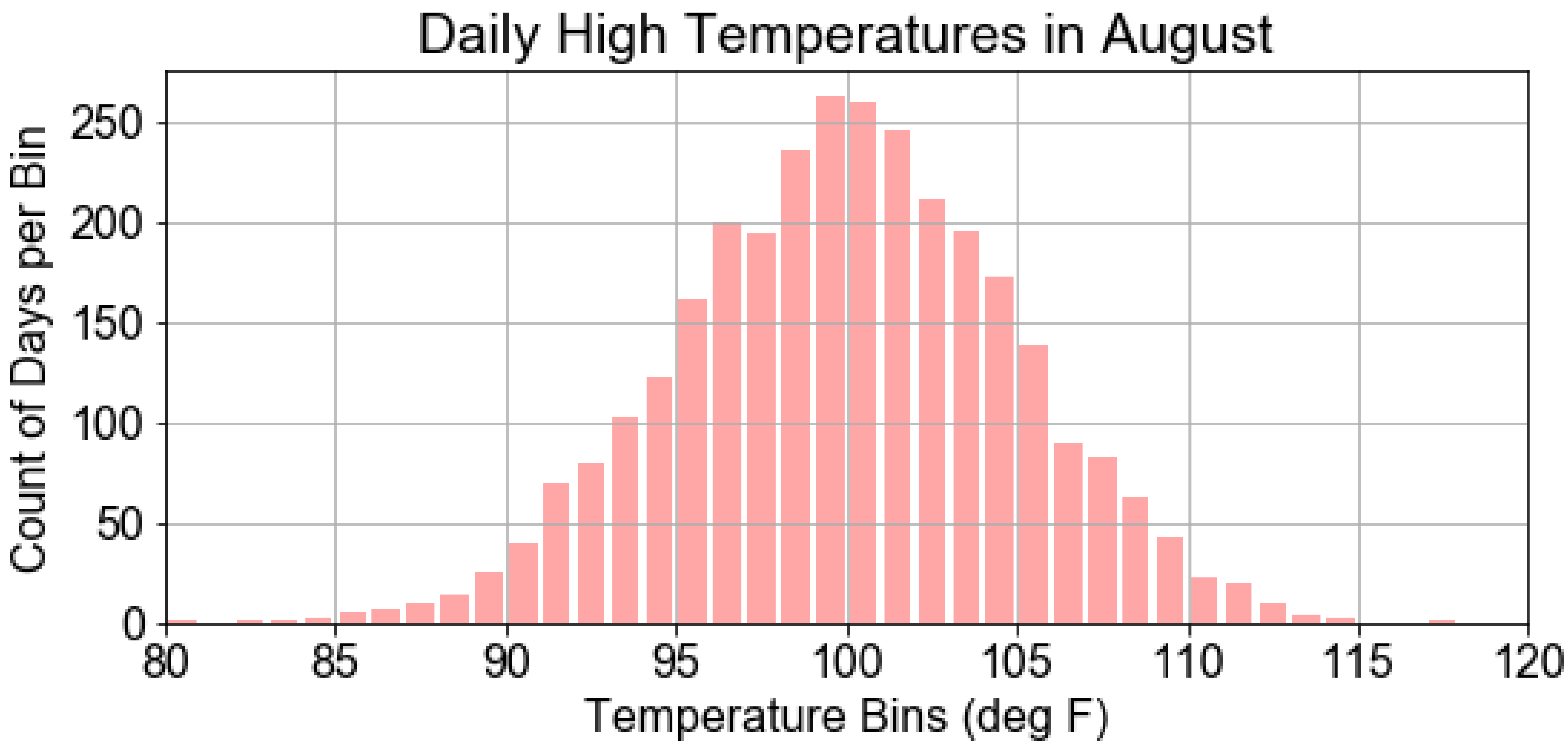
Probability Distribution



Populations and Statistics



Sampling the Population

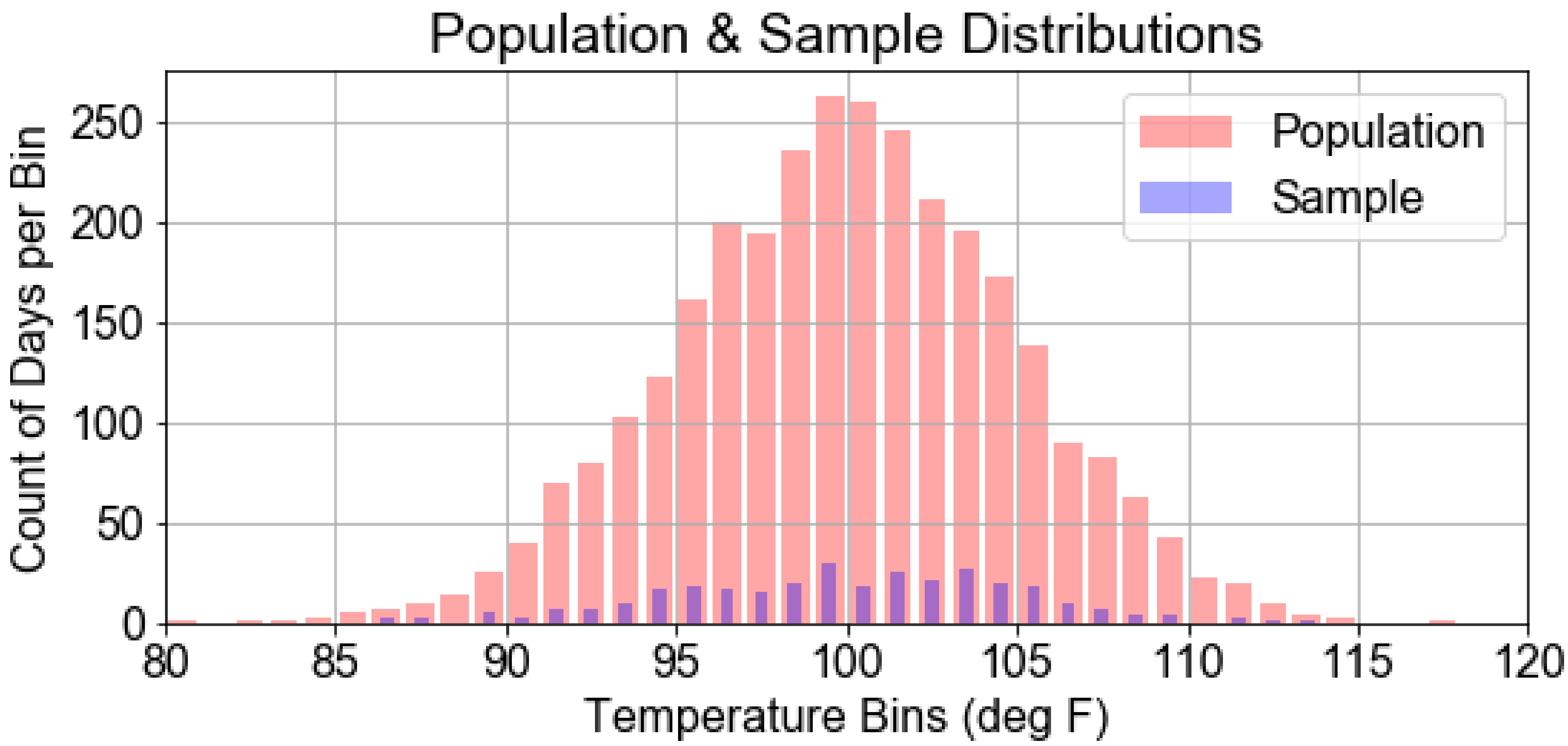
Population statistics vs Sample statistics

```
print( len(month_of_temps), month_of_temps.mean(), month_of_temps.std() )
print( len(decade_of_temps), decade_of_temps.mean(), decade_of_temps.std() )
```

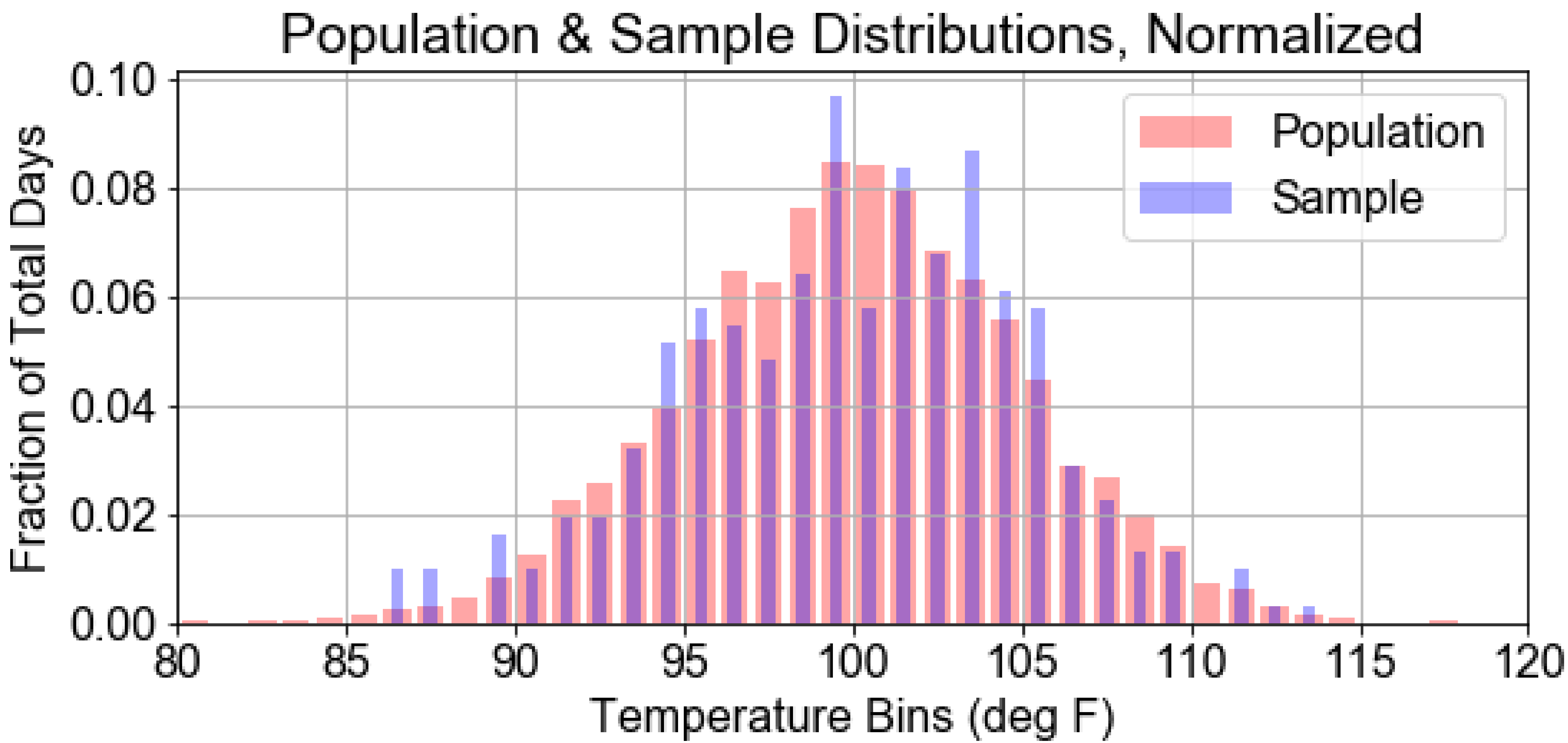
Draw a Random Sample from a Population

```
month_of_temps = np.random.choice(decade_of_temps, size=31)
```

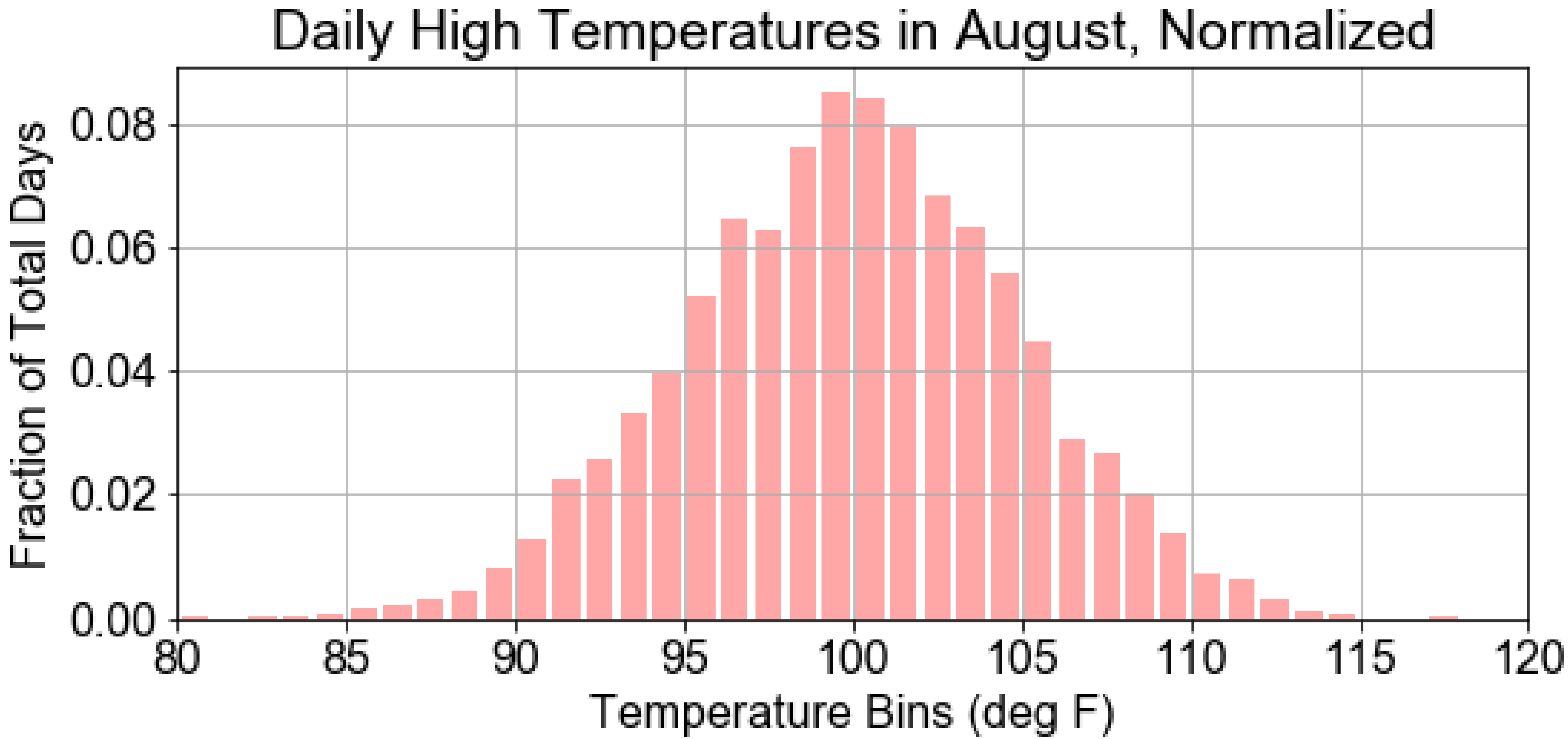
Visualizing Distributions



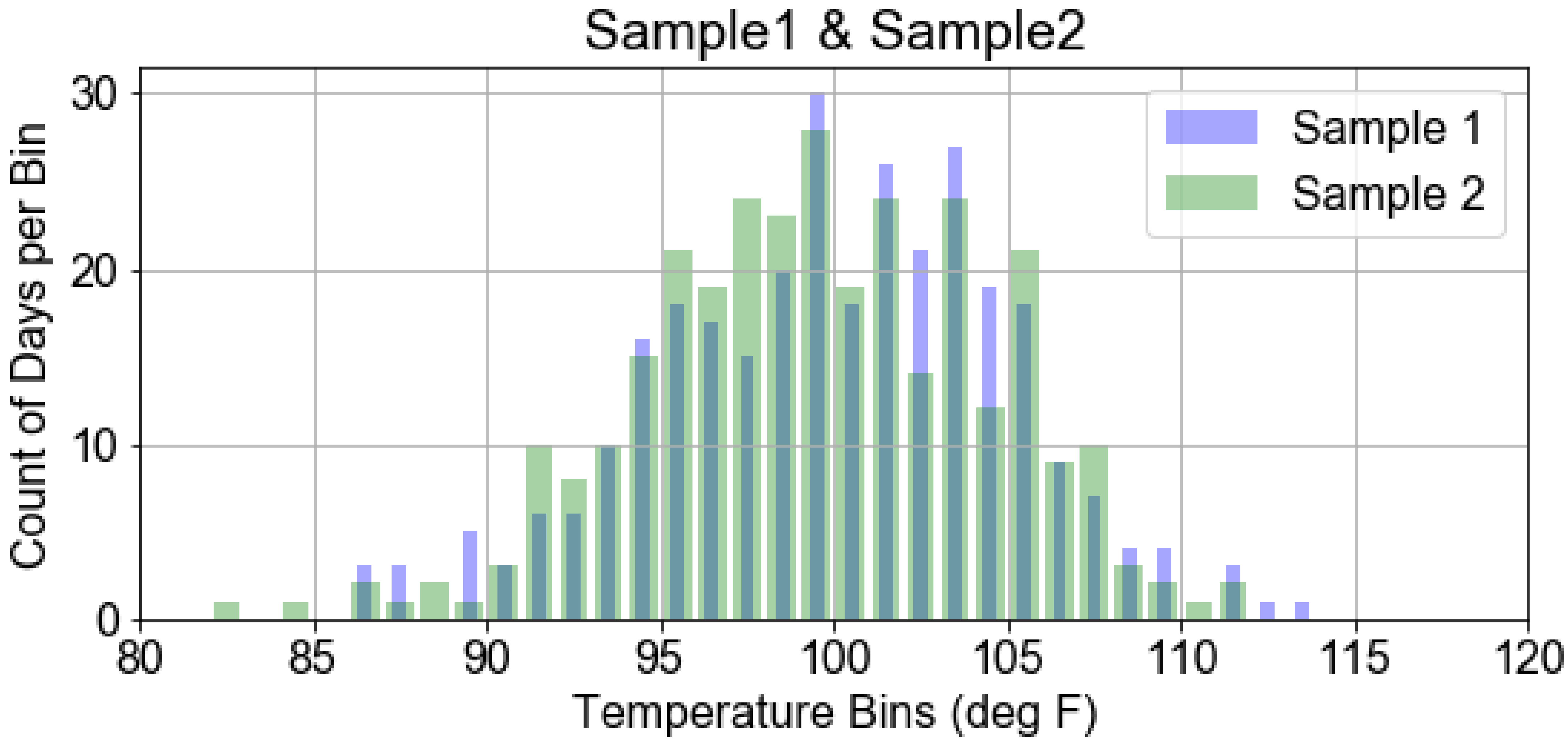
Visualizing Distributions



Probability and Inference



Visualizing Distributions



Resampling

```
# Resampling as Iteration
num_samples = 20
for ns in range(num_samples):
    sample = np.random.choice(population, num_pts)
    distribution_of_means[ns] = sample.mean()
```

```
# Sample Distribution Statistics
mean_of_means = np.mean(distribution_of_means)
stdev_of_means = np.std(distribution_of_means)
```

Let's practice!

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Model Estimation and Likelihood

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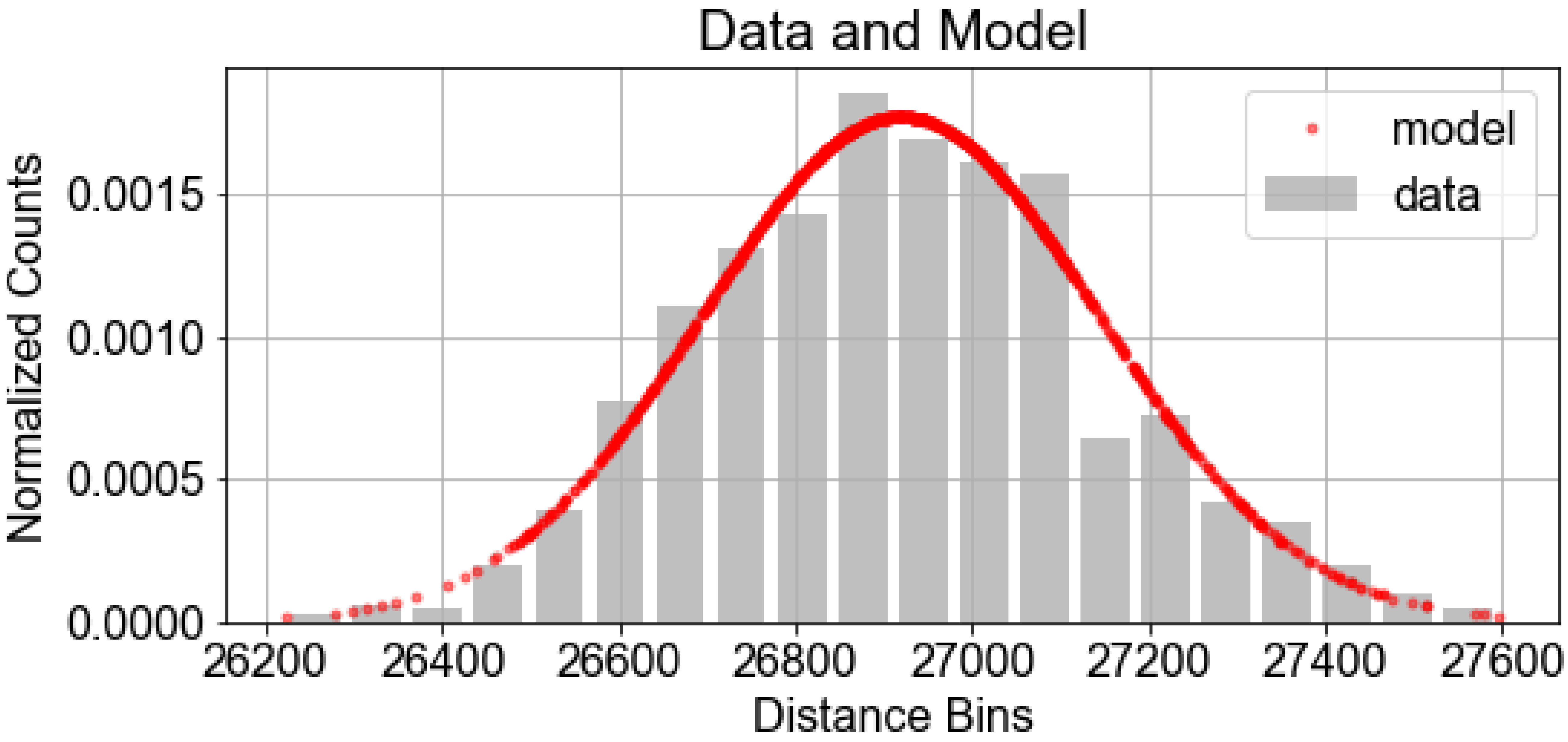
Estimation


Data and Model
model
data
Normalized Counts
0.0015
0.0010
0.0005
0.0000
26200
26400
26600
26800
27000
27200
27400
27600
Distance Bins

Estimation

```python
# Define gaussian model function
def gaussian_model(x, mu, sigma):
    coeff_part = 1/(np.sqrt(2 * np.pi * sigma**2))
    exp_part = np.exp( - (x - mu)**2 / (2 * sigma**2) )
    return coeff_part*exp_part
```

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

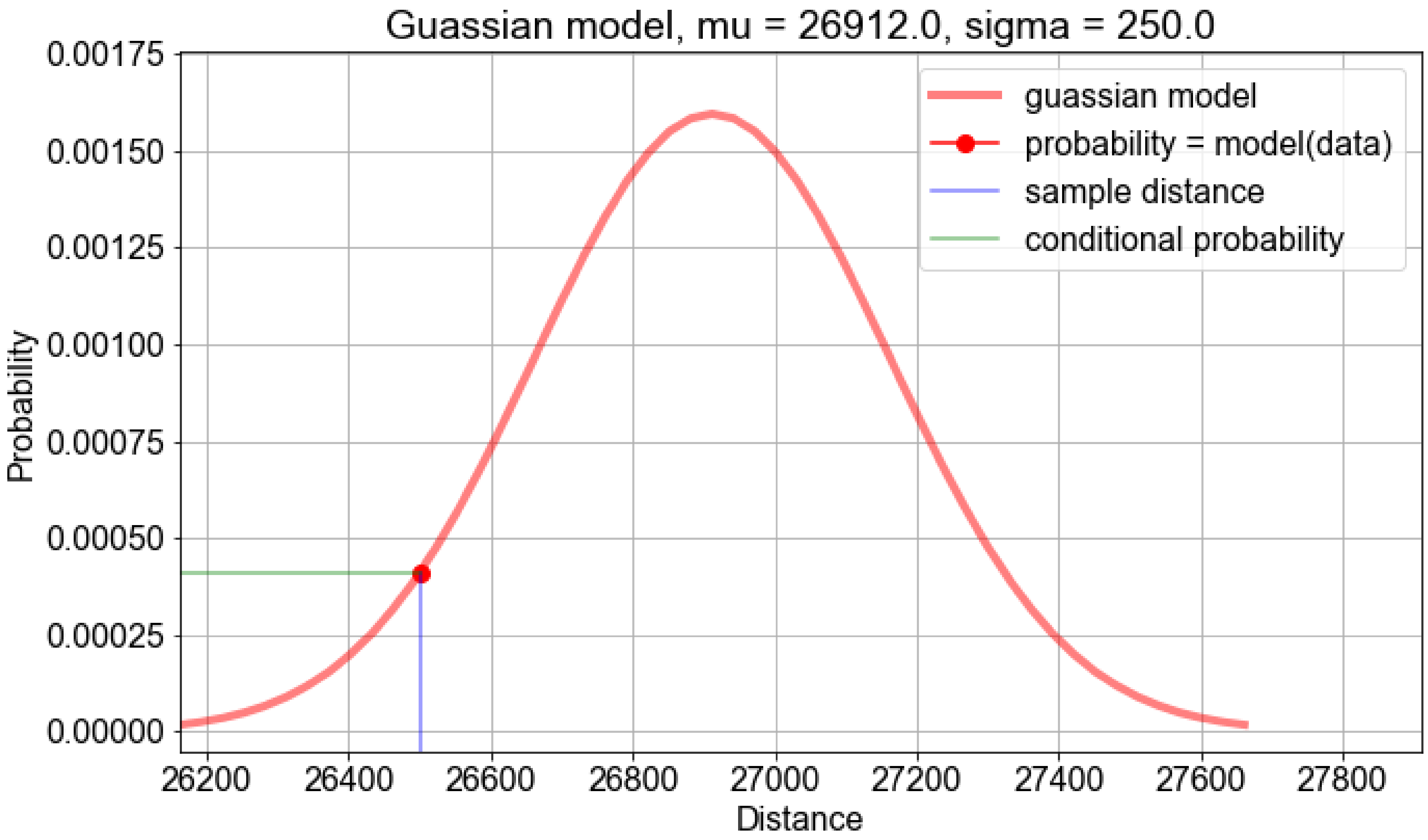
```python
# Compute sample statistics
mean = np.mean(sample)
stdev = np.std(sample)
```

```python
# Model the population using sample statistics
population_model = gaussian(sample, mu=mean, sigma=stdev)
```

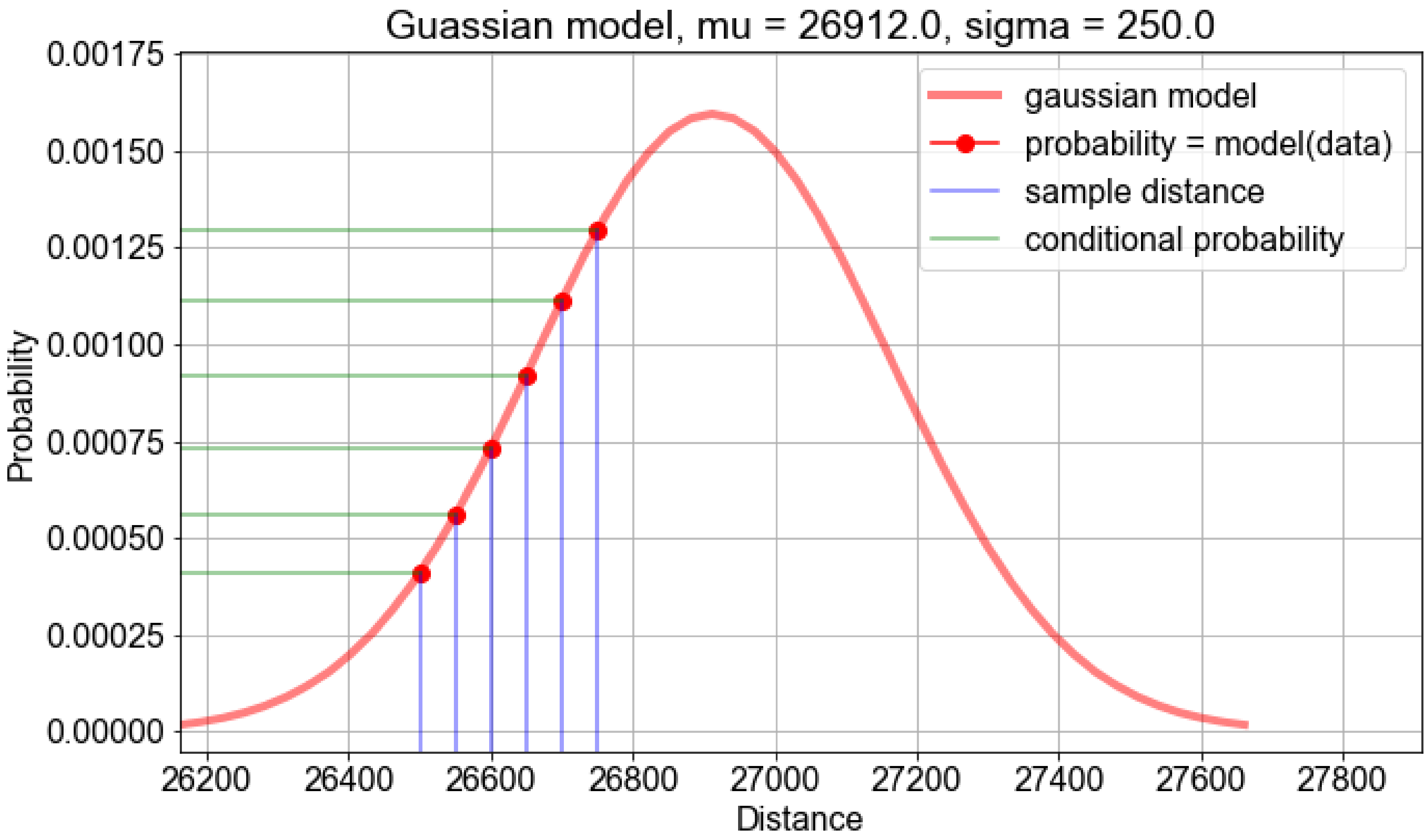
Likelihood vs Probability

- Conditional Probability: $P(\text{outcome A}|\text{given B})$
- Probability: $P(\text{data}|\text{model})$
- Likelihood: $L(\text{model}|\text{data})$

Computing Likelihood



Computing Likelihood



Likelihood from Probabilities

```
# Guess parameters
mu_guess = np.mean(sample_distances)
sigma_guess = np.std(sample_distances)
```

```
# For each sample point, compute a probability
probabilities = np.zeros(len(sample_distances))
for n, distance in enumerate(sample_distances):
    probabilities[n] = gaussian_model(distance, mu=mu_guess, sigma=sigma_guess)
```

```
likelihood = np.product(probs)
loglikelihood = np.sum(np.log(probs))
```

It`s Useful to take the log because it`s has numerical statistics Better

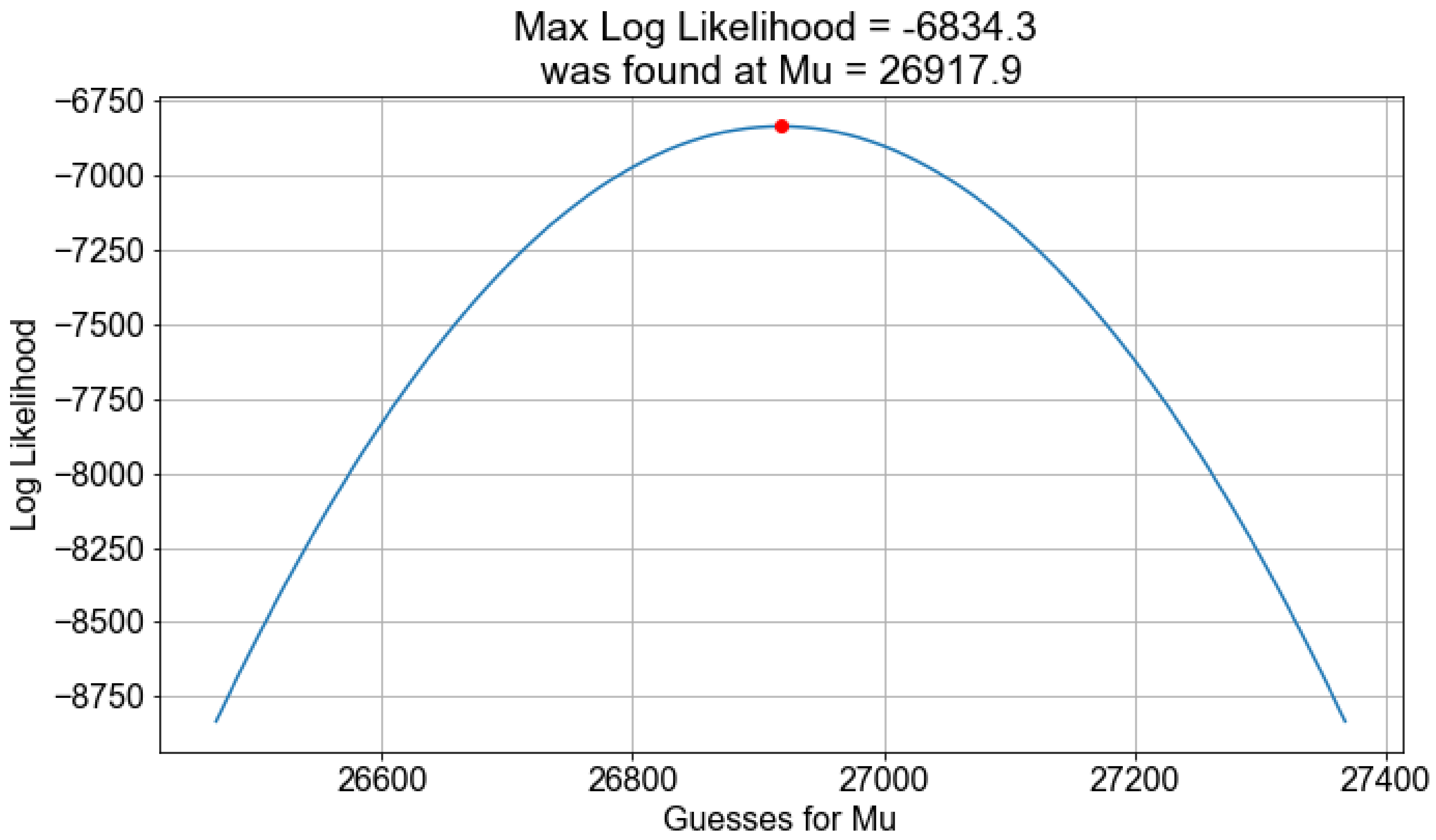
Maximum Likelihood Estimation

```
# Create an array of mu guesses
low_guess = sample_mean - 2*sample_stdev
high_guess = sample_mean + 2*sample_stdev
mu_guesses = np.linspace(low_guess, high_guess, 101)
```

```
# Compute the loglikelihood for each guess
loglikelihoods = np.zeros(len(mu_guesses))
for n, mu_guess in enumerate(mu_guesses):
    loglikelihoods[n] = compute_loglikelihood(sample_distances, mu=mu_guess, sigma=sample_stdev)
```

```
# Find the best guess
max_loglikelihood = np.max(loglikelihoods)
best_mu = mu_guesses[loglikelihoods == max_loglikelihood]
```

Maximum Likelihood Estimation



when model is gaussian the mean matches the answer from least-Square

Let's practice!

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Model Uncertainty and Sample Distributions

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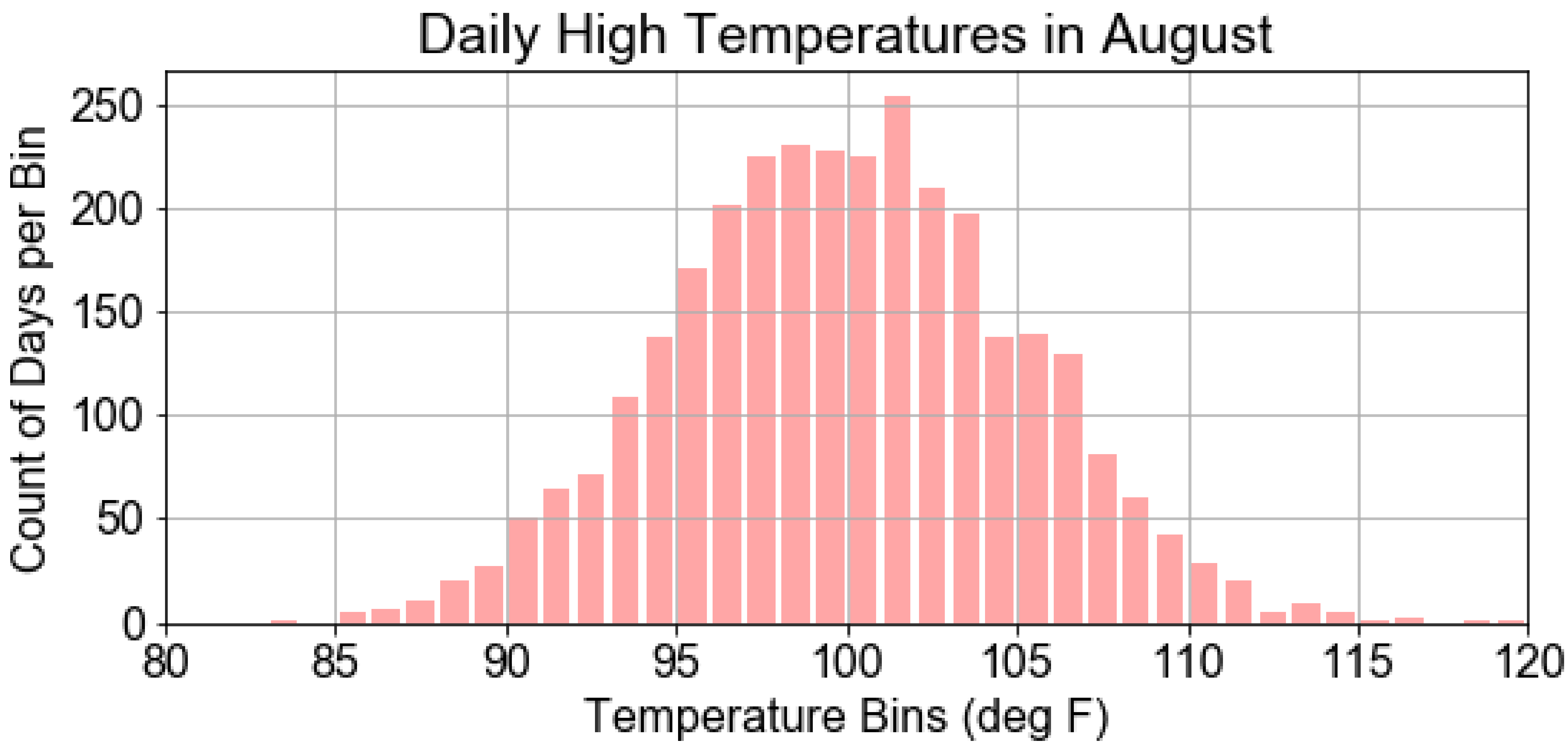
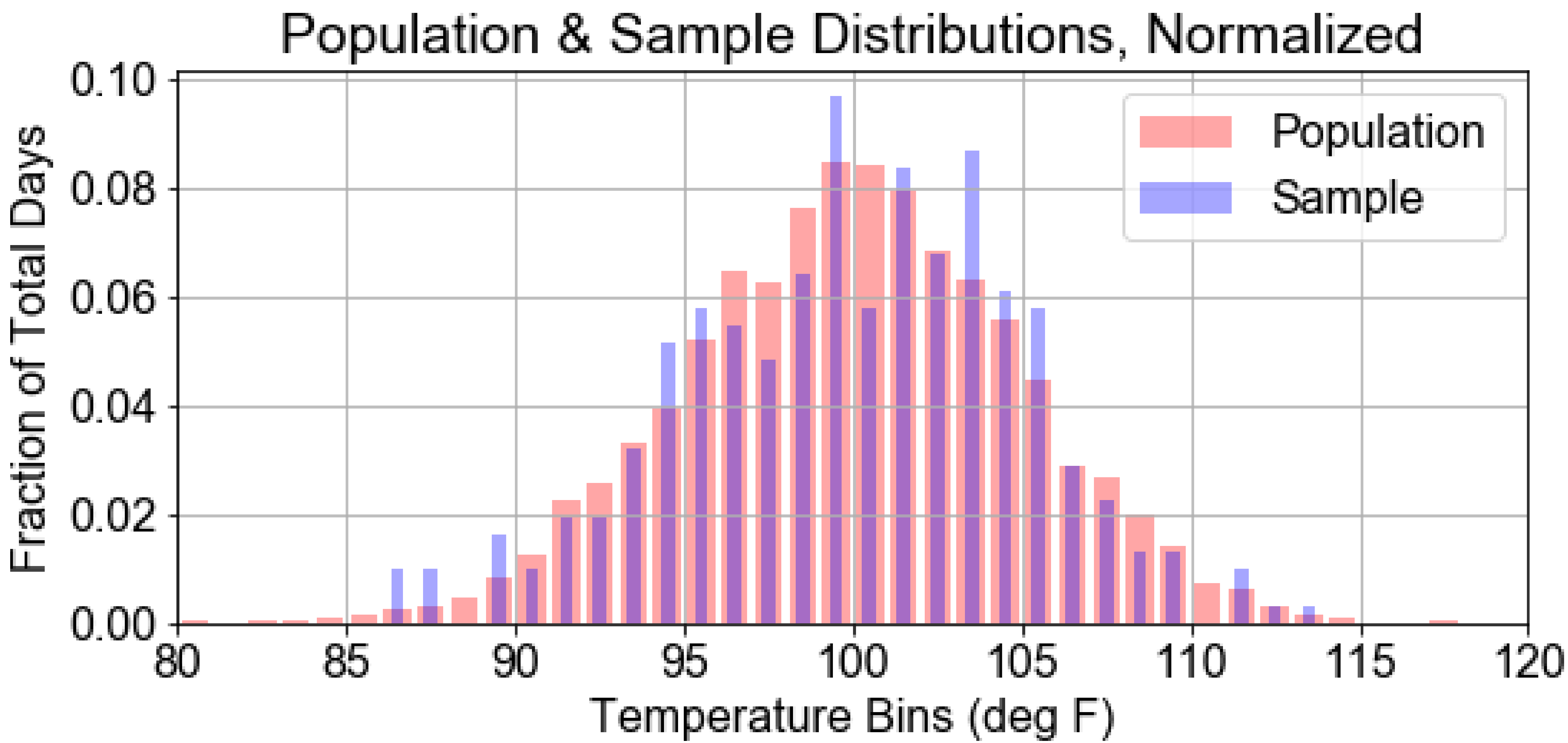
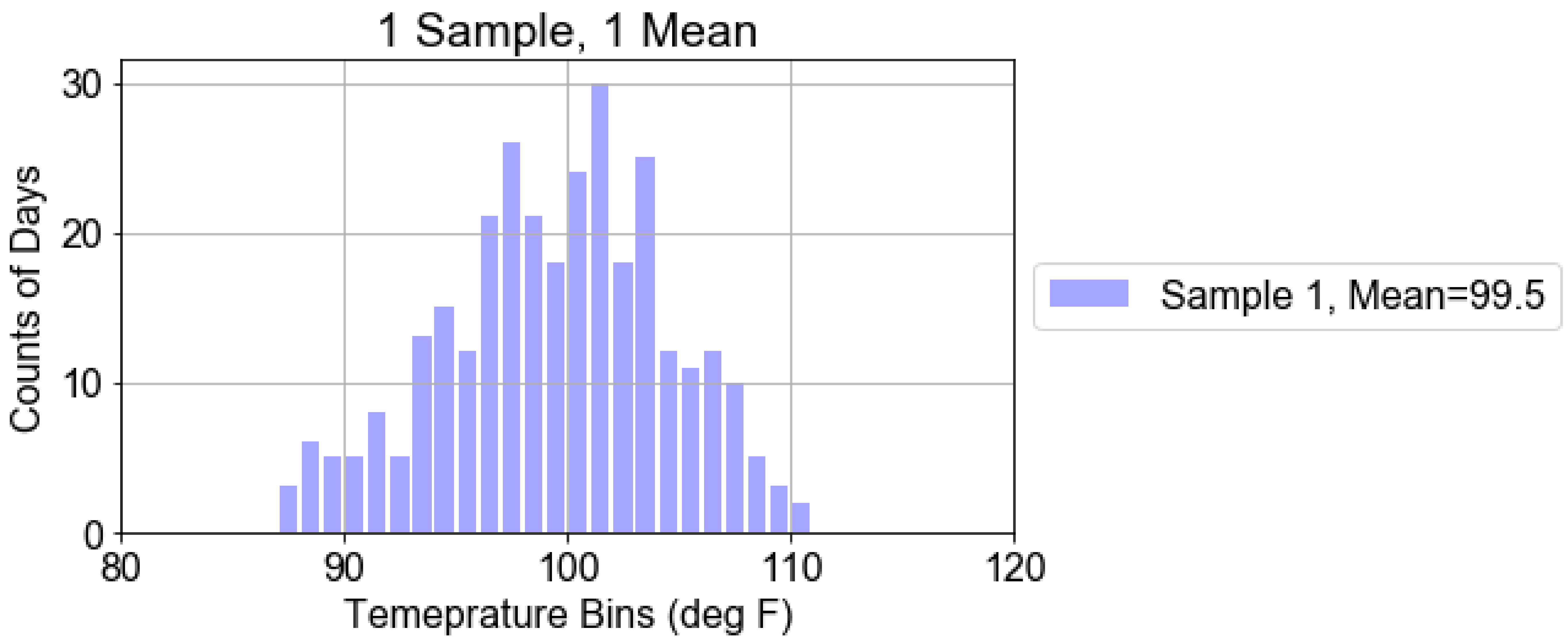
Population Unavailable



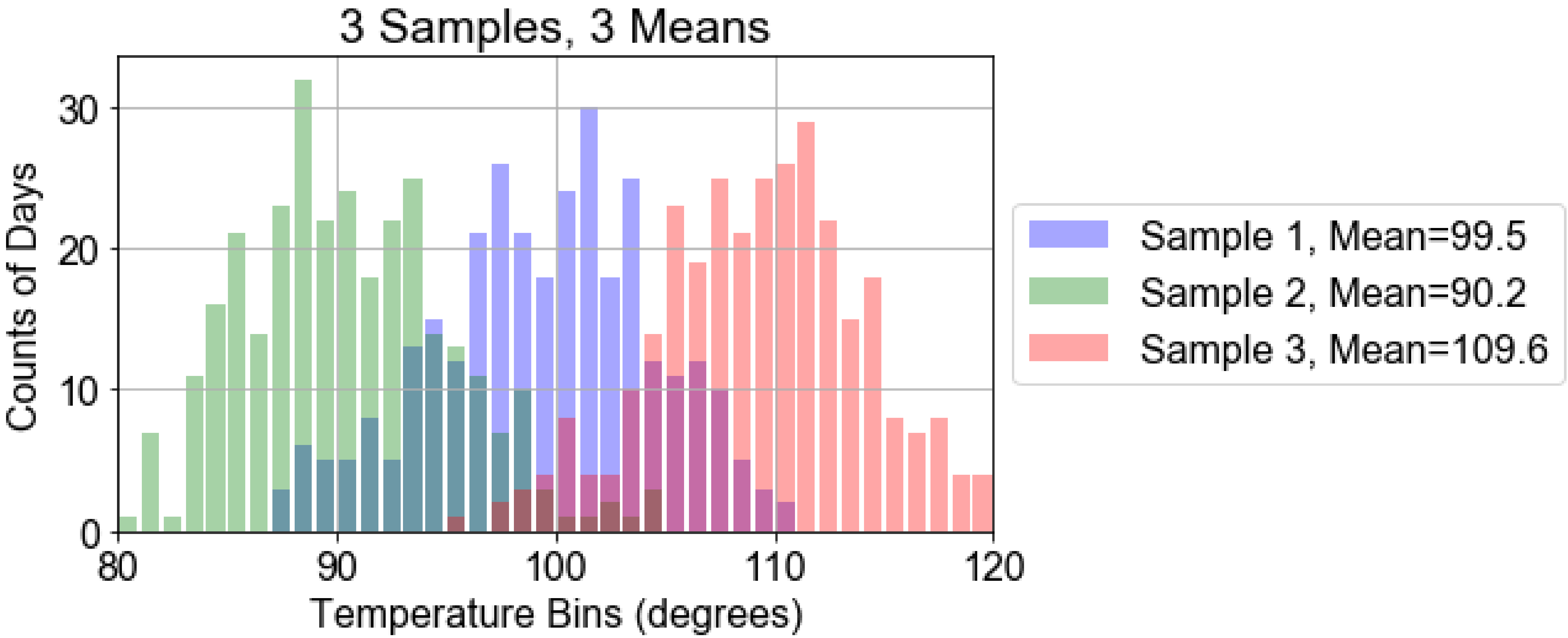
Sample as Population Model



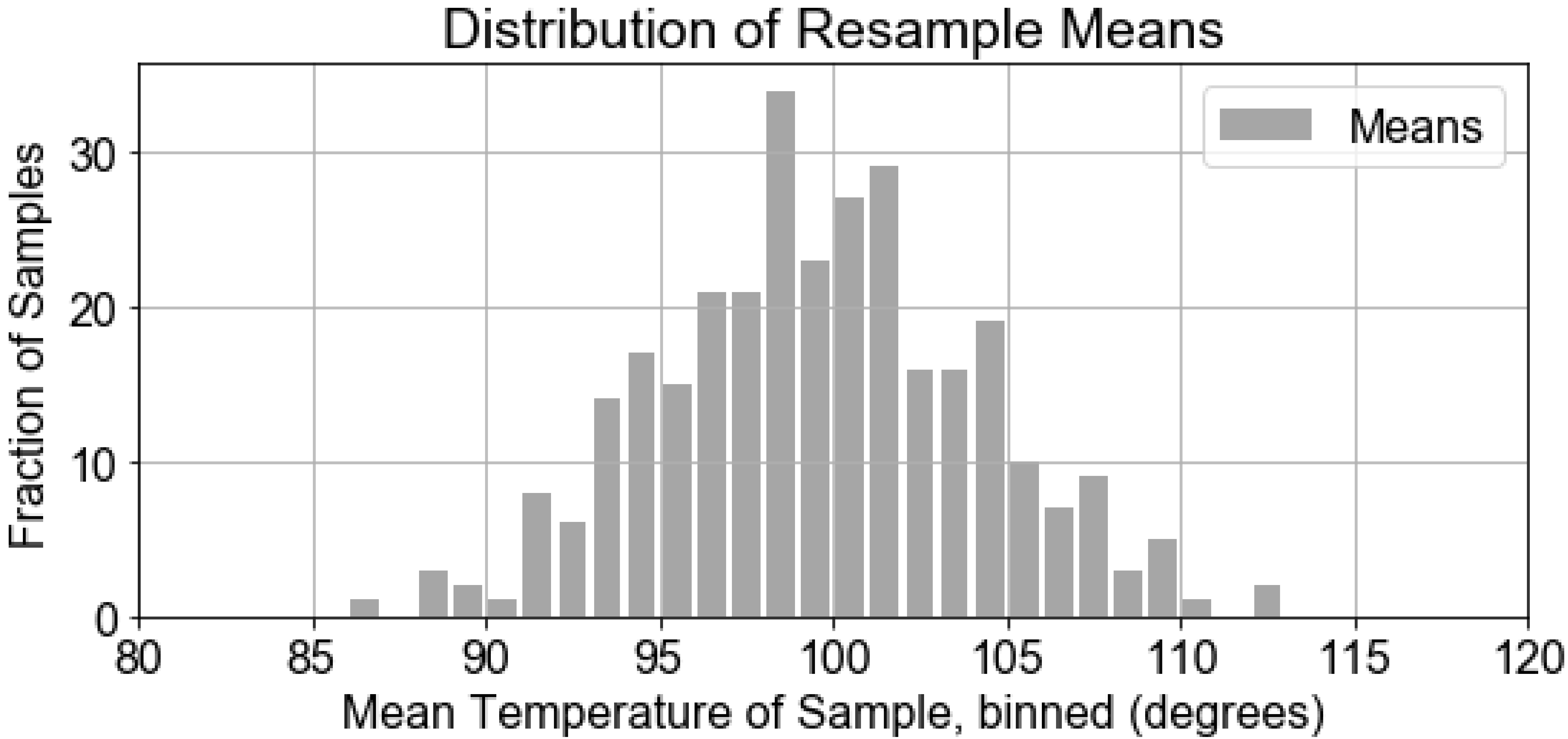
Sample Statistic



Bootstrap Resampling



Resample Distribution



Bootstrap in Code

```
# Use sample as model for population
population_model = august_daily_highs_for_2017
```

```
# Simulate repeated data acquisitions by resampling the "model"
for nr in range(num_resamples):
    bootstrap_sample = np.random.choice(population_model, size=resample_size, replace=True)
    bootstrap_means[nr] = np.mean(bootstrap_sample)
```

Sampling With Replacement

```
# Compute the mean of the bootstrap resample distribution
estimate_temperature = np.mean(bootstrap_means)
```

```
# Compute standard deviation of the bootstrap resample distribution
estimate_uncertainty = np.std(bootstrap_means)
```

Replacement

```python
# Define the sample of notes
sample = ['A', 'B', 'C', 'D', 'E', 'F', 'G']
```

```python
# Replace = True, repeats are allowed
bootstrap_sample = np.random.choice(sample, size=4, replace=True)
print(bootstrap_sample)
```

```
C C F G
```

Replacement

```
# Replace = False
bootstrap_sample = np.random.choice(sample, size=4, replace=False)
print(bootstrap_sample)
```

Sampling Without Replacement Problems :
1- never get repeated
2- sampling changes the model after every draw

```
C G A F
```

```
# Replace = True, more lengths are allowed
bootstrap_sample = np.random.choice(sample, size=16, replace=True)
print(bootstrap_sample)
```

```
C C F G C G A E F D G B B A E C
```

Let's practice!

Model Errors and Randomness

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Types of Errors

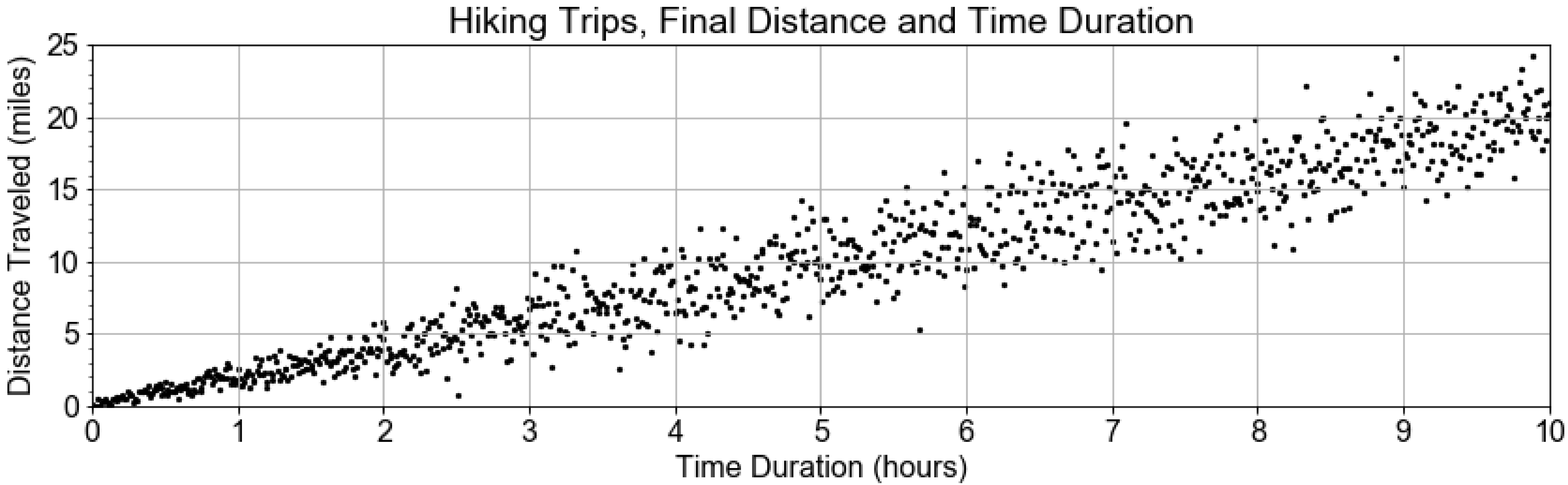
1. Measurement error Mistakes made when collection/recording the data ex: broken sensor
   - e.g.: broken sensor, wrongly recorded measurements
2. Sampling bias Taking draws from one small portion of the population not representing of the rest ex : drawing temperatures only from august when days hottest
   - e.g: temperatures only from August, when days are hottest
3. Random chance variation due to random chance ex : how do we know that the mean slope from model fit is not just due to noise ?

Null Hypothesis

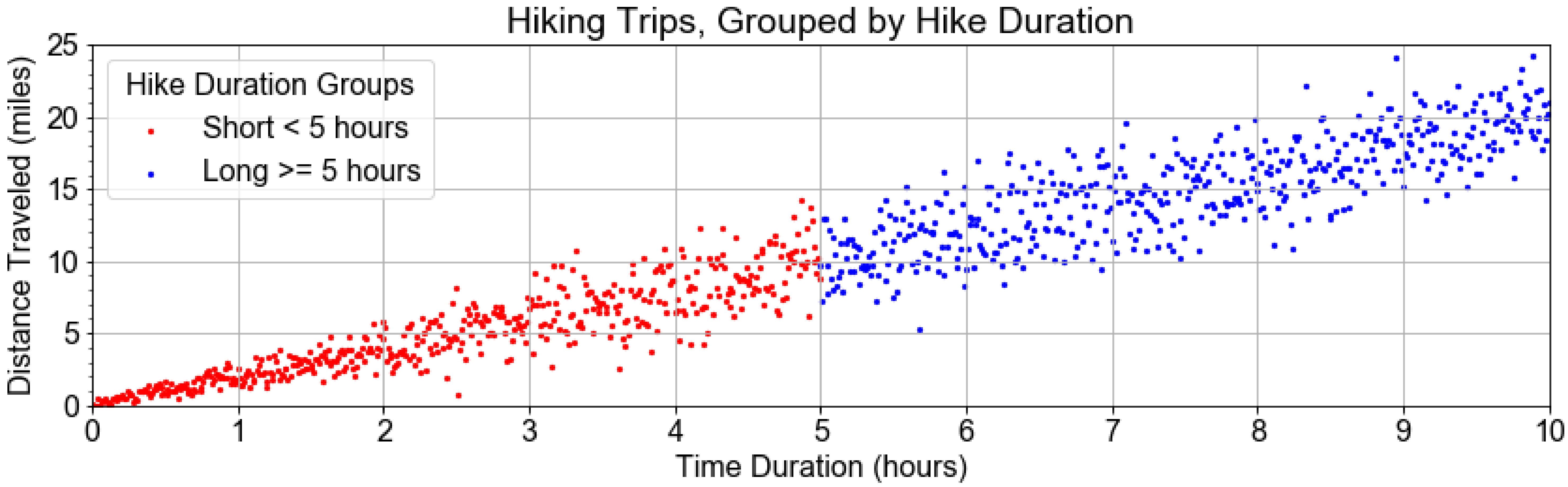
Question: Is our effect due a relationship or due to random chance?

Answer: check the Null Hypothesis.

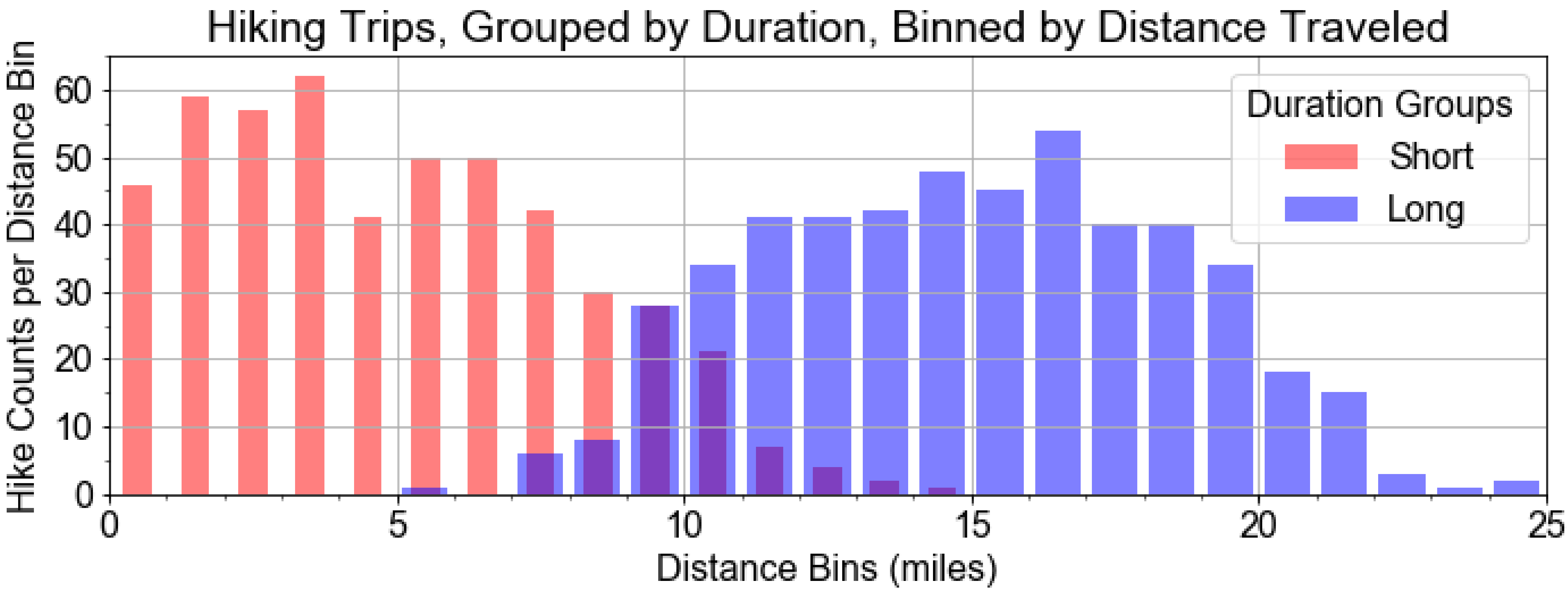
Ordered Data



Grouping Data



Grouping Data



- Short Duration Group, mean = 5

Test Statistic

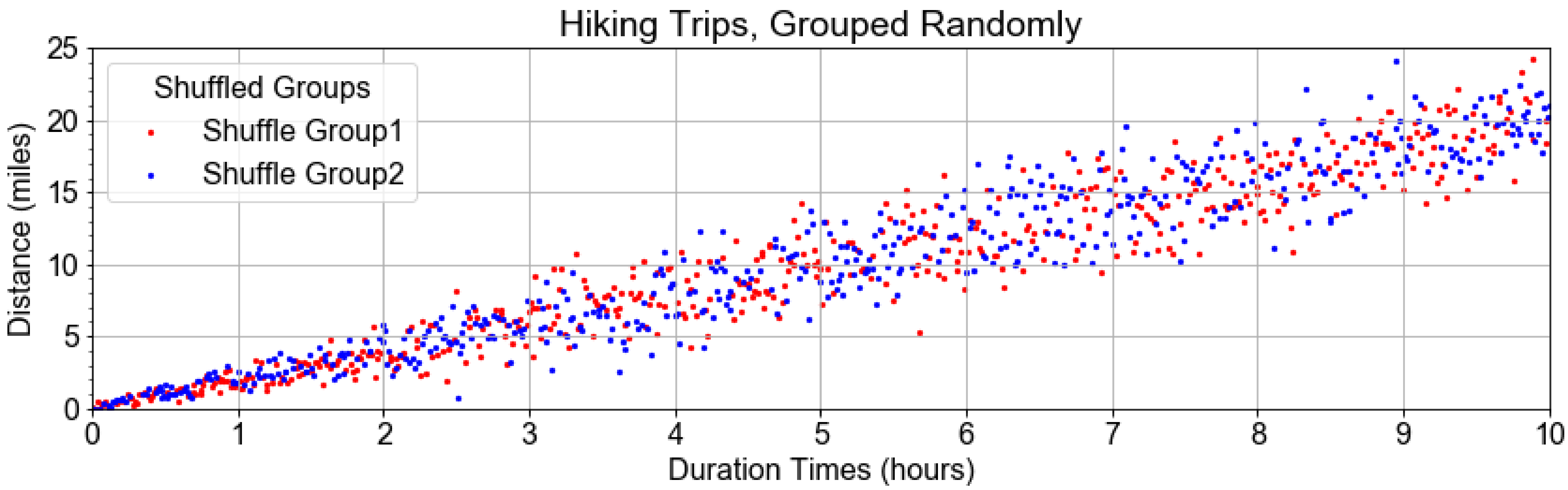
```
# Group into early and late times
group_short = sample_distances[times < 5]
group_long = sample_distances[times > 5]
```

```
# Resample distributions
resample_short = np.random.choice(group_short, size=500, replace=True)
resample_long = np.random.choice(group_long, size=500, replace=True)
```

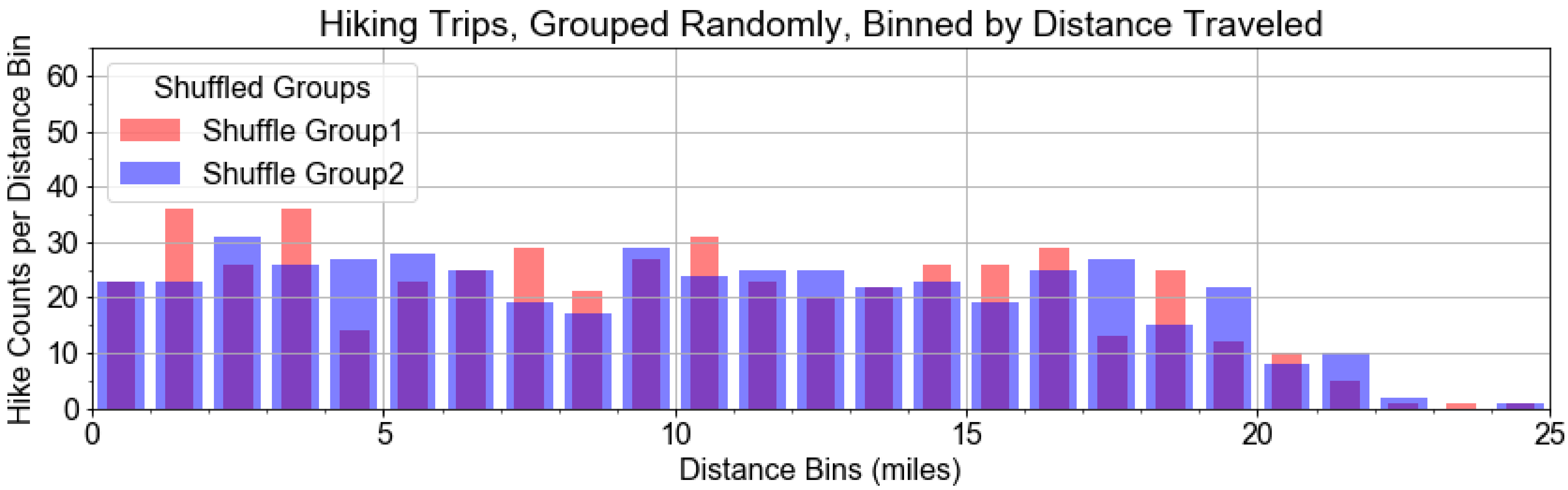
```
# Test Statistic
test_statistic = resample_long - resample_short
```

```
# Effect size as mean of test statistic distribution
effect_size = np.mean(test_statistic)
```

Shuffle and Regrouping



Shuffling and Regrouping



Shuffle and Split

```
# Concatenate and Shuffle
shuffle_bucket = np.concatenate((group_short, group_long))
np.random.shuffle(shuffle_bucket)
```

```
# Split in the middle
slice_index = len(shuffle_bucket)//2
shuffled_half1 = shuffle_bucket[0:slice_index]
shuffled_half2 = shuffle_bucket[slice_index+1:]
```

Resample and Test Again

```
# Resample shuffled populations
shuffled_sample1 = np.random.choice(shuffled_half1, size=500, replace=True)
shuffled_sample2 = np.random.choice(shuffled_half2, size=500, replace=True)
```

```
# Recompute effect size
shuffled_test_statistic = shuffled_sample2 - shuffled_sample1
effect_size = np.mean(shuffled_test_statistic)
```

p-Value

Test Statistic Distibution,
Effect Size = 10.15, p-value = 0.12
Effect Size
Unshuffled
Shuffled
p-value
Bin Counts
Test Statistic Values
50
40
30
20
10
0
−20
−10
0
10
20

Let's practice!

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Looking Back, Looking Forward

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Exploring Linear Relationships

- Motivation by Example Predictions
- Visualizing Linear Relationships
- Quantifying Linear Relationships

Building Linear Models

- Model Parameters
- Slope and Intercept
- Taylor Series
- Model Optimization
- Least-Squares

Model Predictions

- Modeling Real Data
- Limitations and Pitfalls of Predictions
- Goodness-of-Fit

Model Parameter Distributions

- modeling parameters as probability distributions
- samples, populations, and sampling
- maximizing likelihood for parametric shapes
- bootstrap resampling for arbitrary shapes
- test statistics and p-values

Goodbye and Good Luck!

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