

UDEMY COURSE  
ROBOT OPERATING SYSTEM  
(PART II)  
NAVIGATION AND SLAM

PROF. ANIS KOUBAA

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# Section 1. Frames

<https://www.udemy.com/user/anis-koubaa/>

## LEARNING OUTCOME

- ▶ Recognize the concepts of pose and coordinate frames
- ▶ Represent the position and orientation of the robot in space

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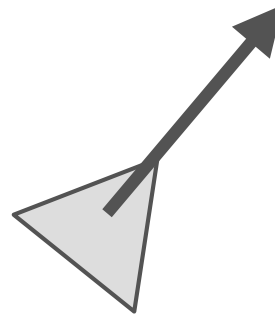
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Pose of a Robot in a Frame

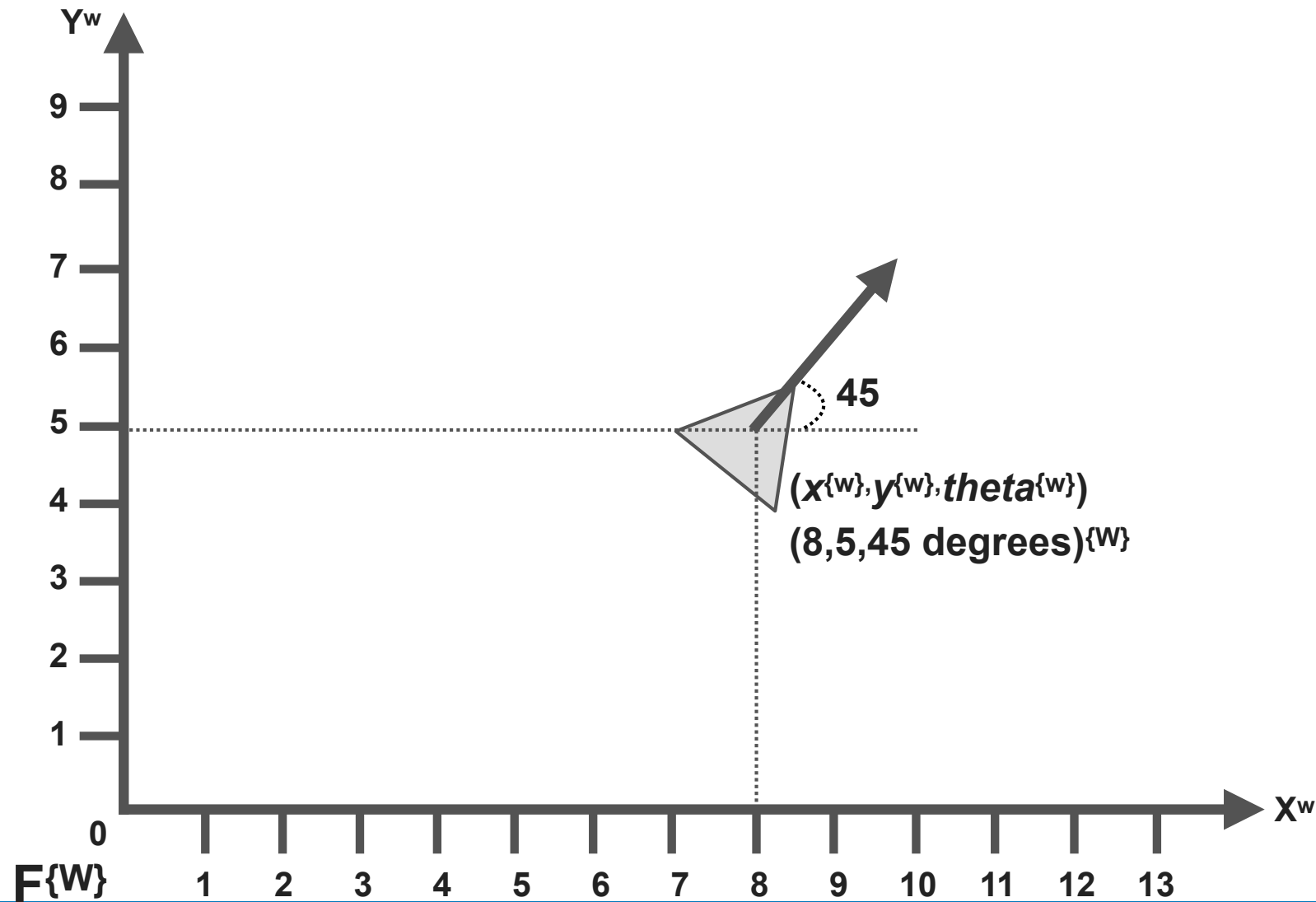
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## COORDINATE FRAME

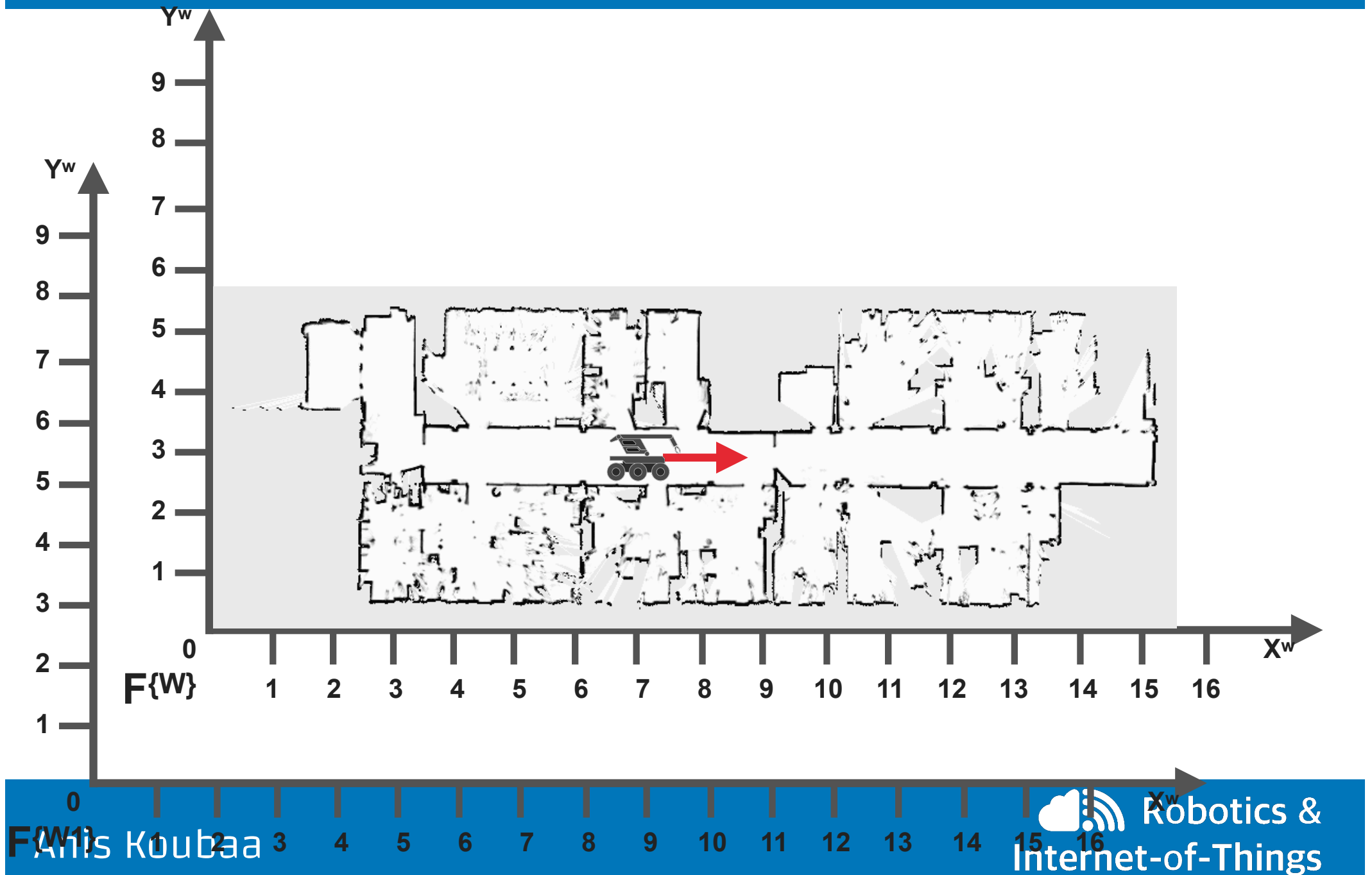


**What is the location of this robot?**

# COORDINATE FRAME AND POSE











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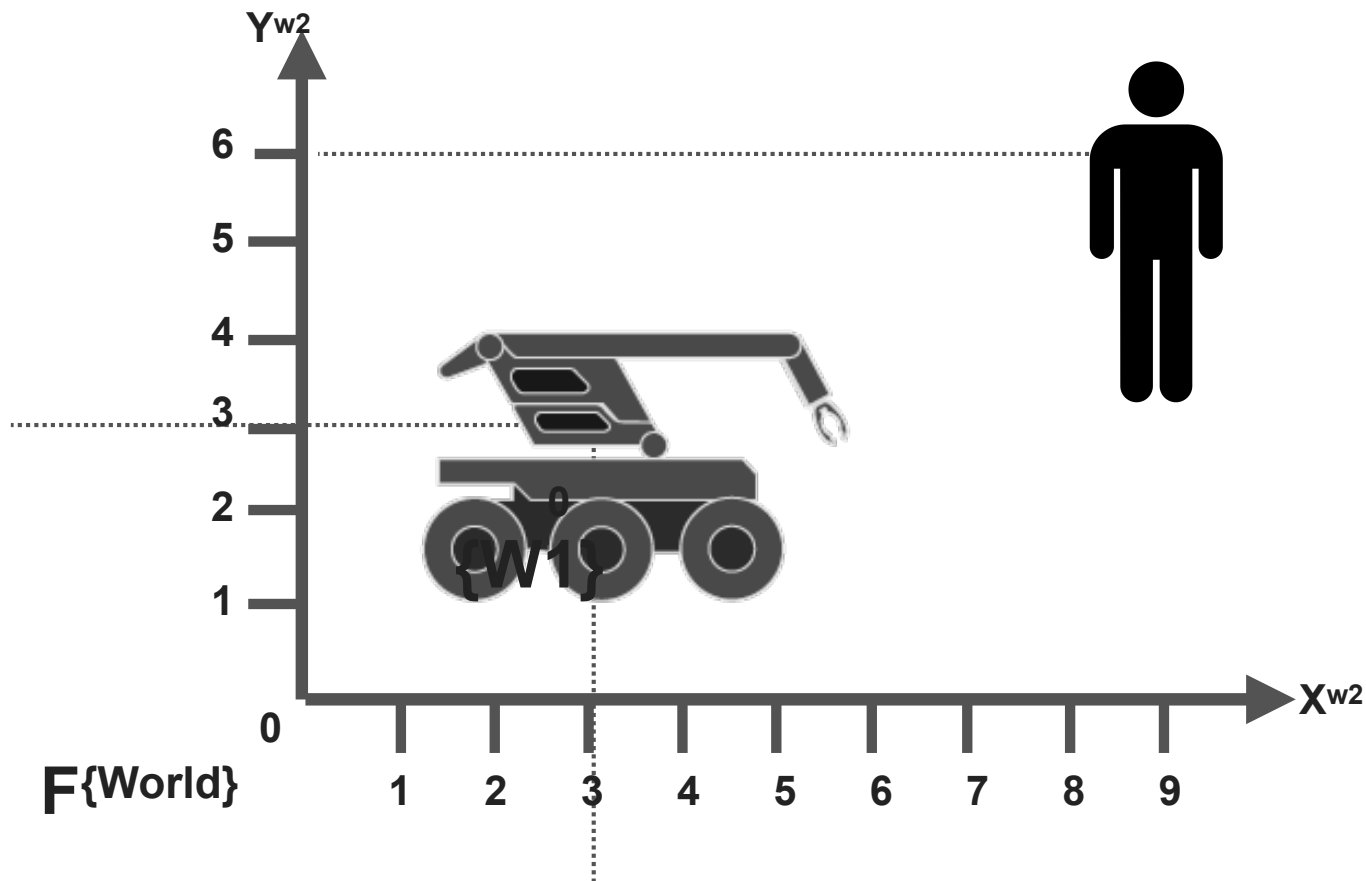
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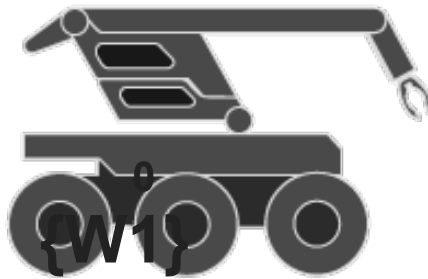
# Dealing with Multiple Frames

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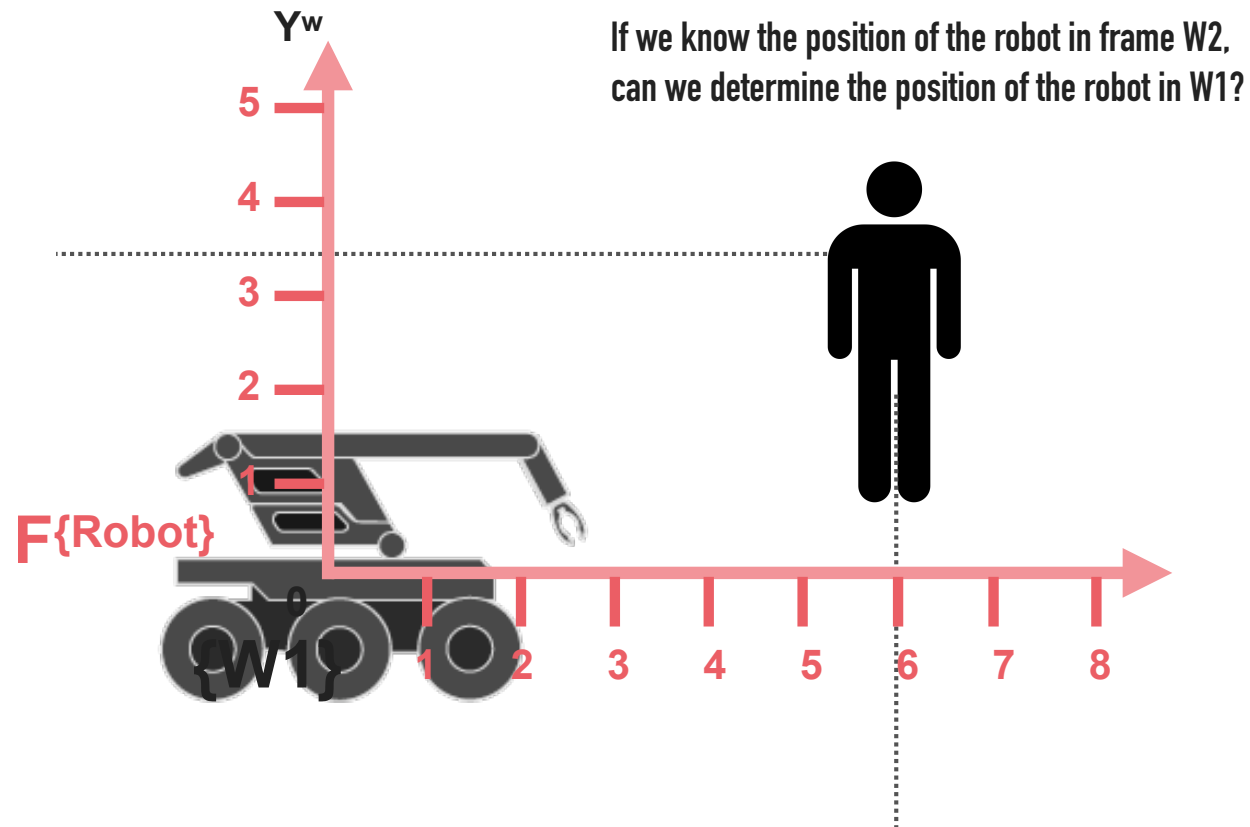
## LOCATION IN WORLD COORDINATE FRAME



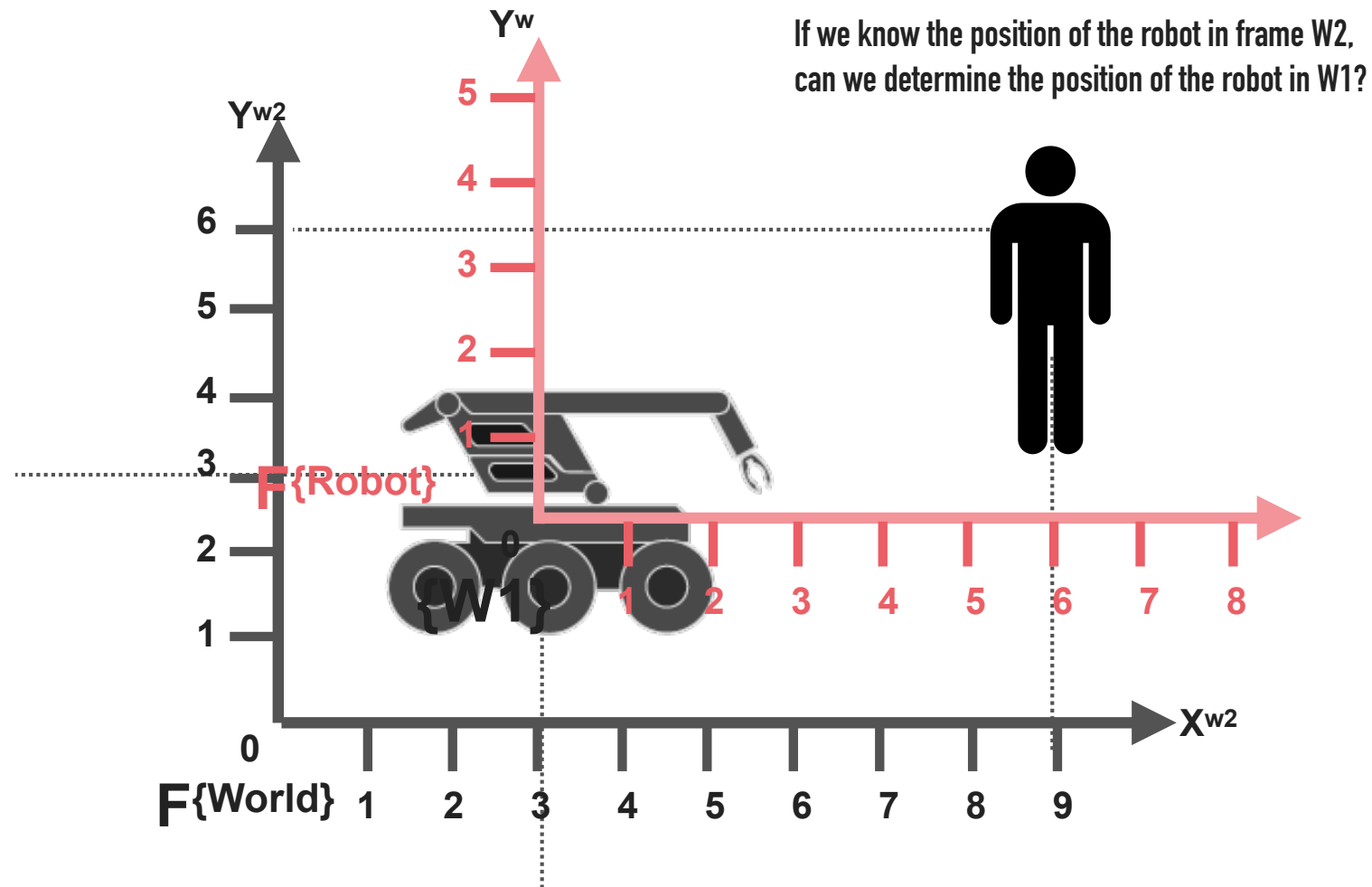
## LOCATION IN THE ROBOT COORDINATE FRAME



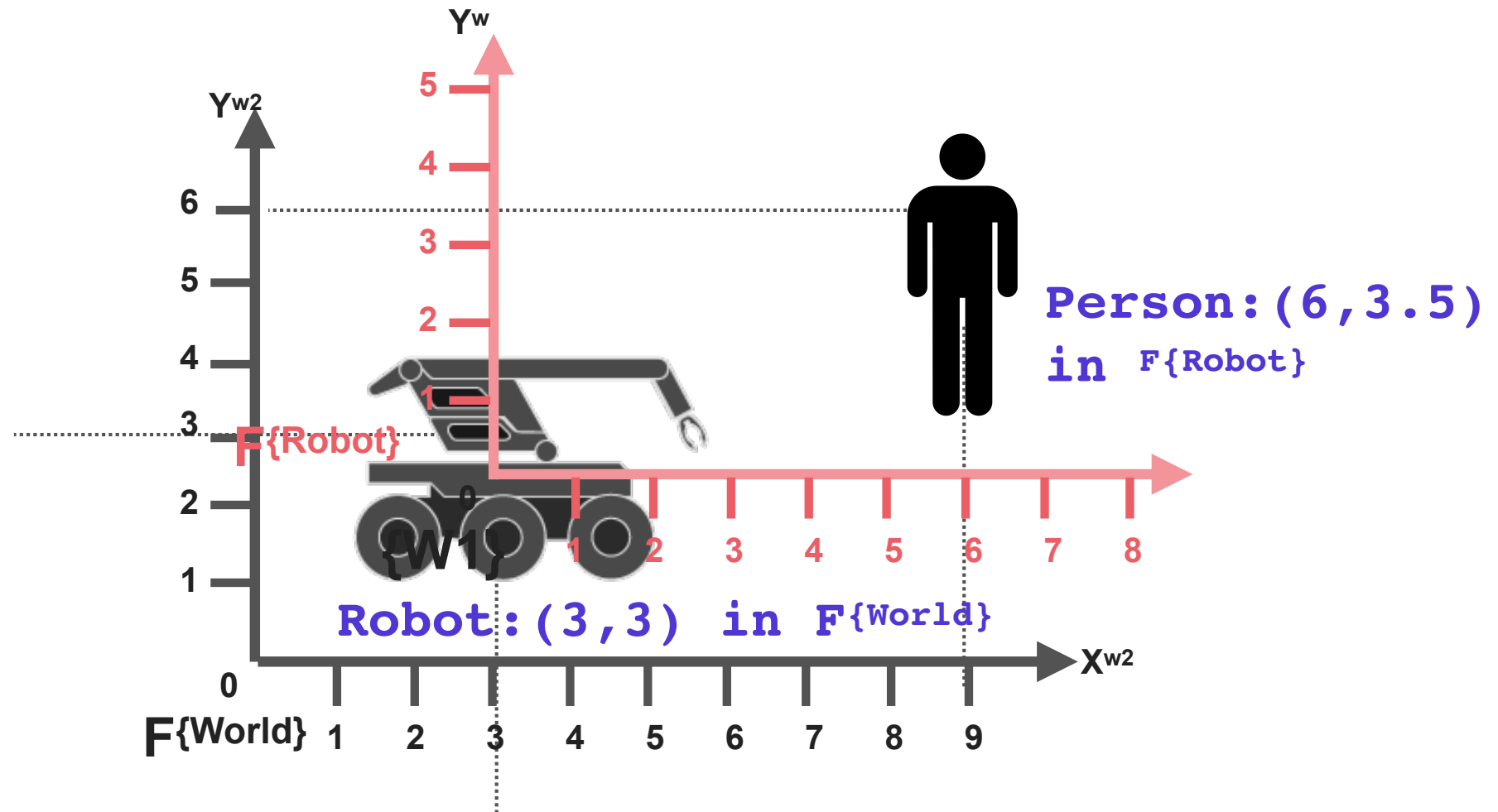
## LOCATION IN THE ROBOT COORDINATE FRAME



# LOCATION IN THE ROBOT AND WORLD COORDINATE FRAMES



## LOCATION IN THE ROBOT AND WORLD COORDINATE FRAMES



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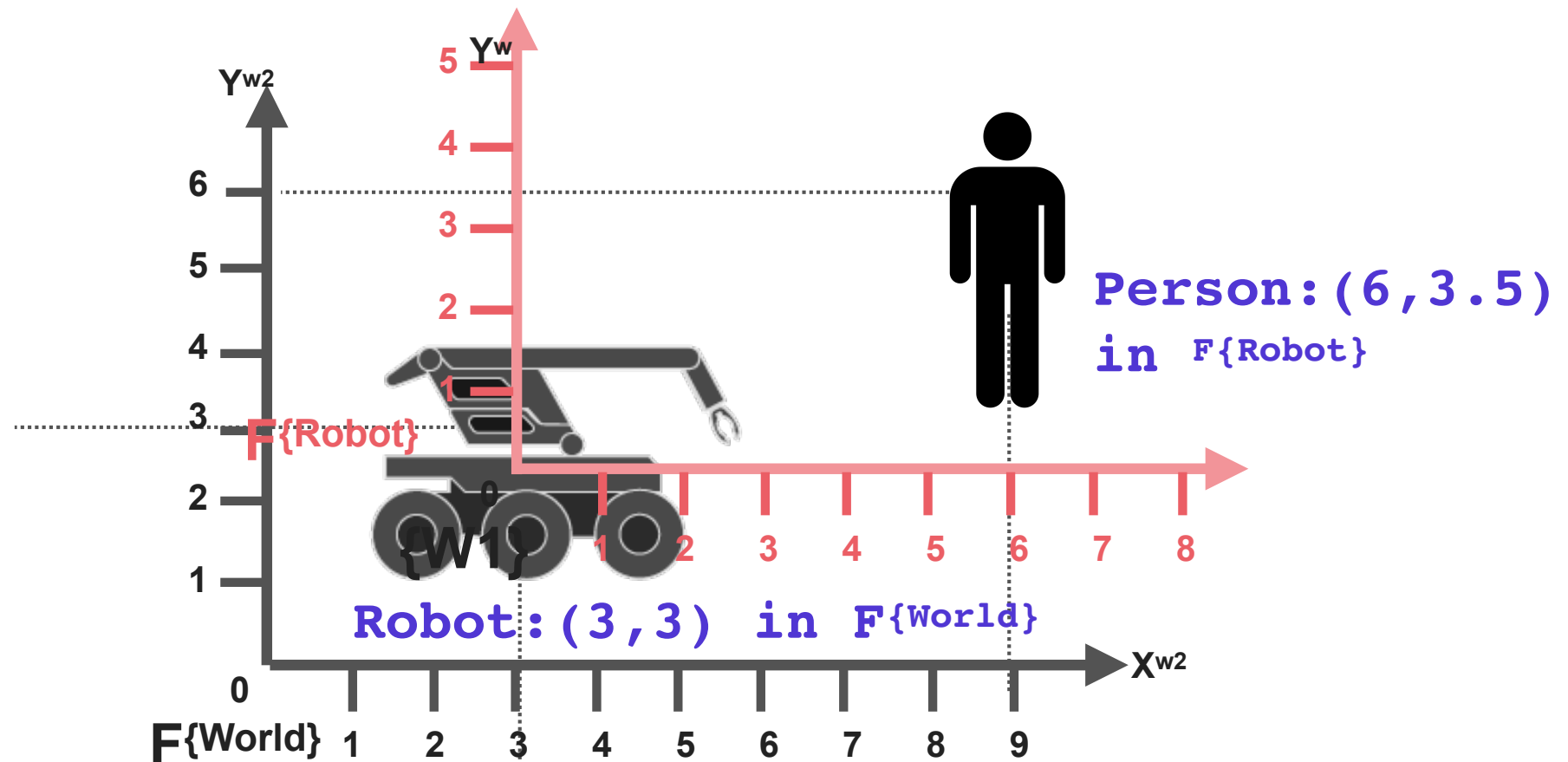
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# Coordinate Frame Transformation

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## LOCATION IN THE ROBOT AND WORLD COORDINATE FRAMES

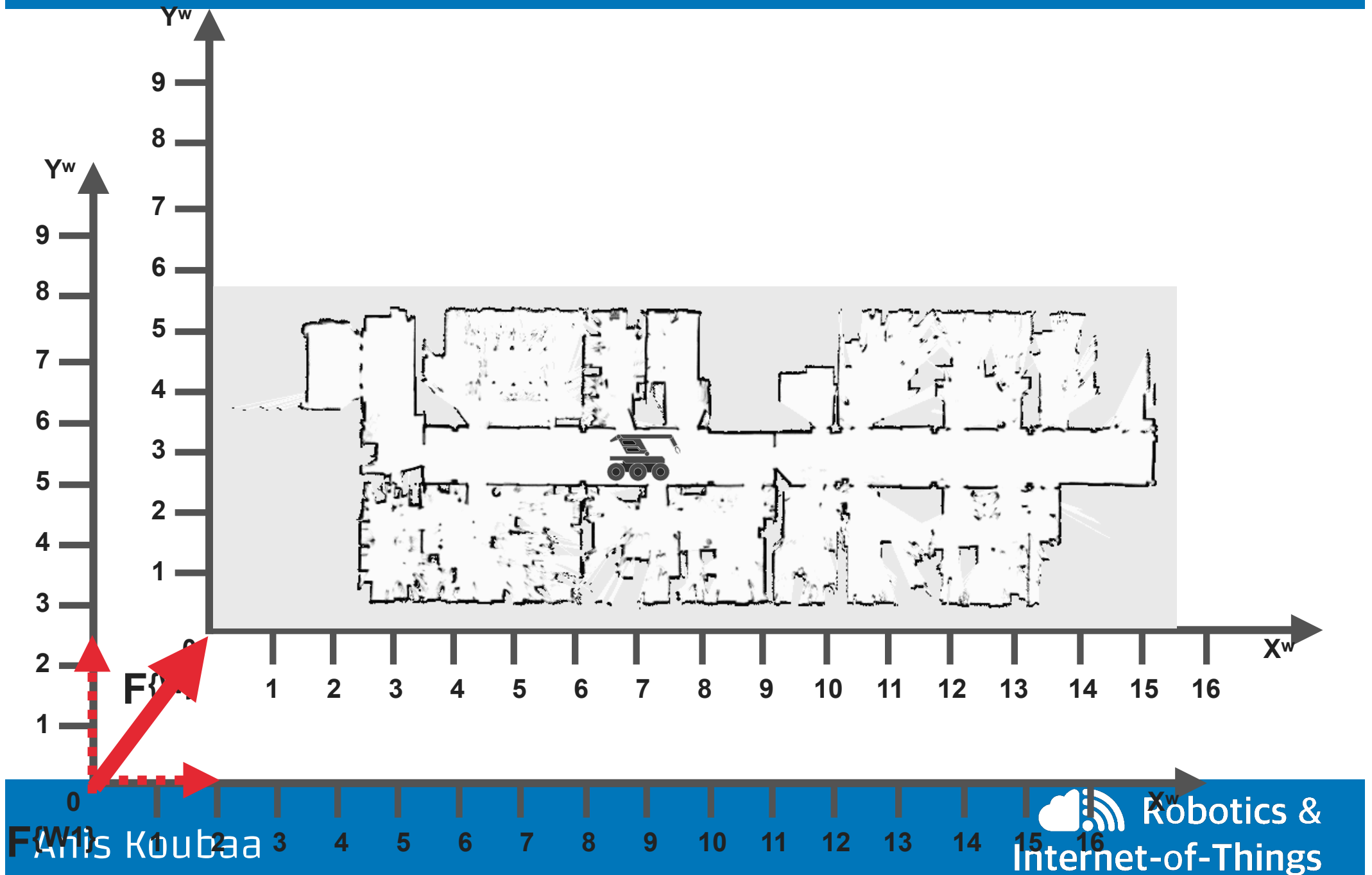


**Person in  $F\{world\}$ ?**



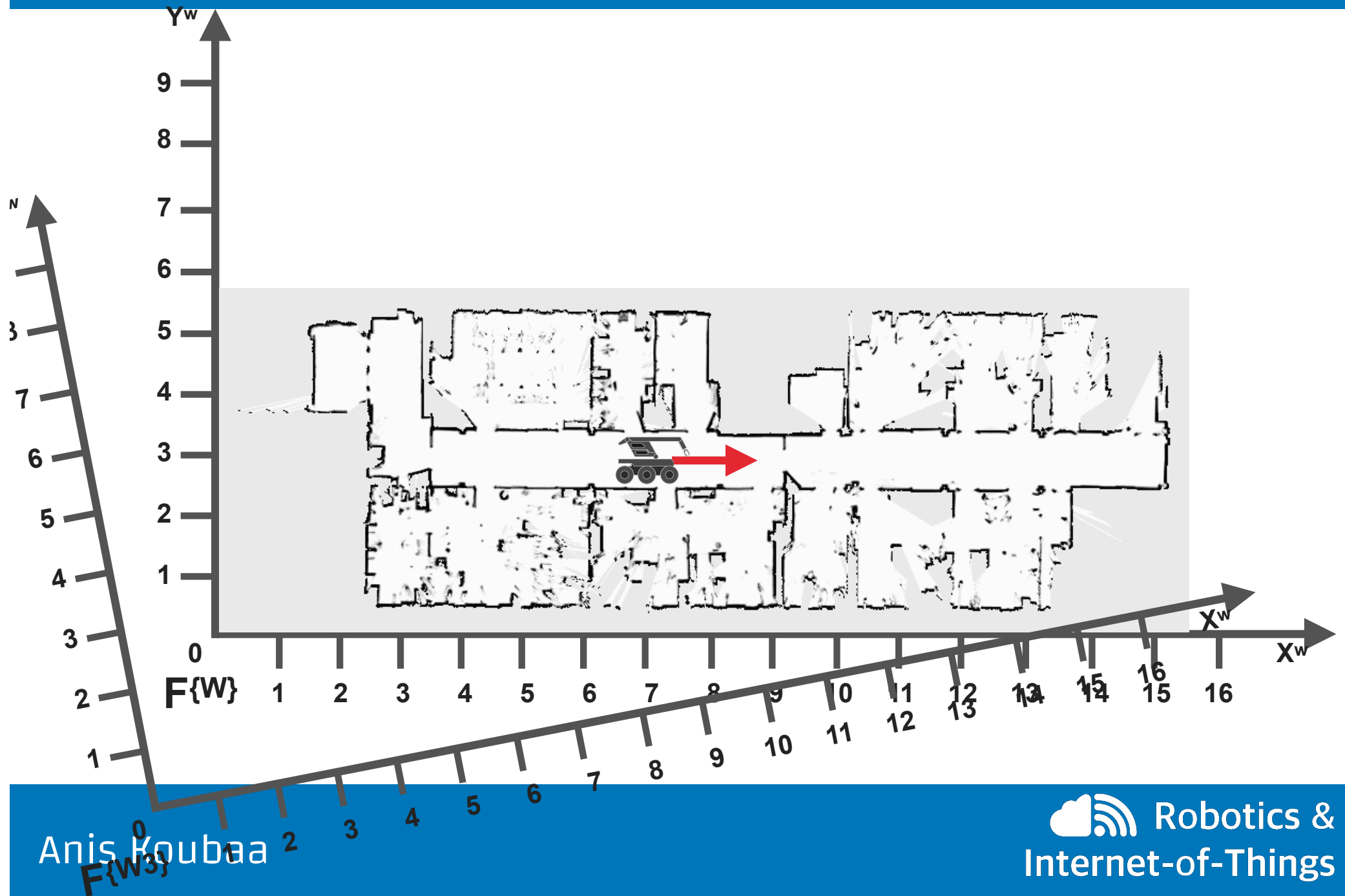
## TRANSFORMATION TYPES

- ▶ Translation
- ▶ Rotation





## ROS FOR BEGINNERS II: LOCALIZATION, NAVIGATION AND SLAM



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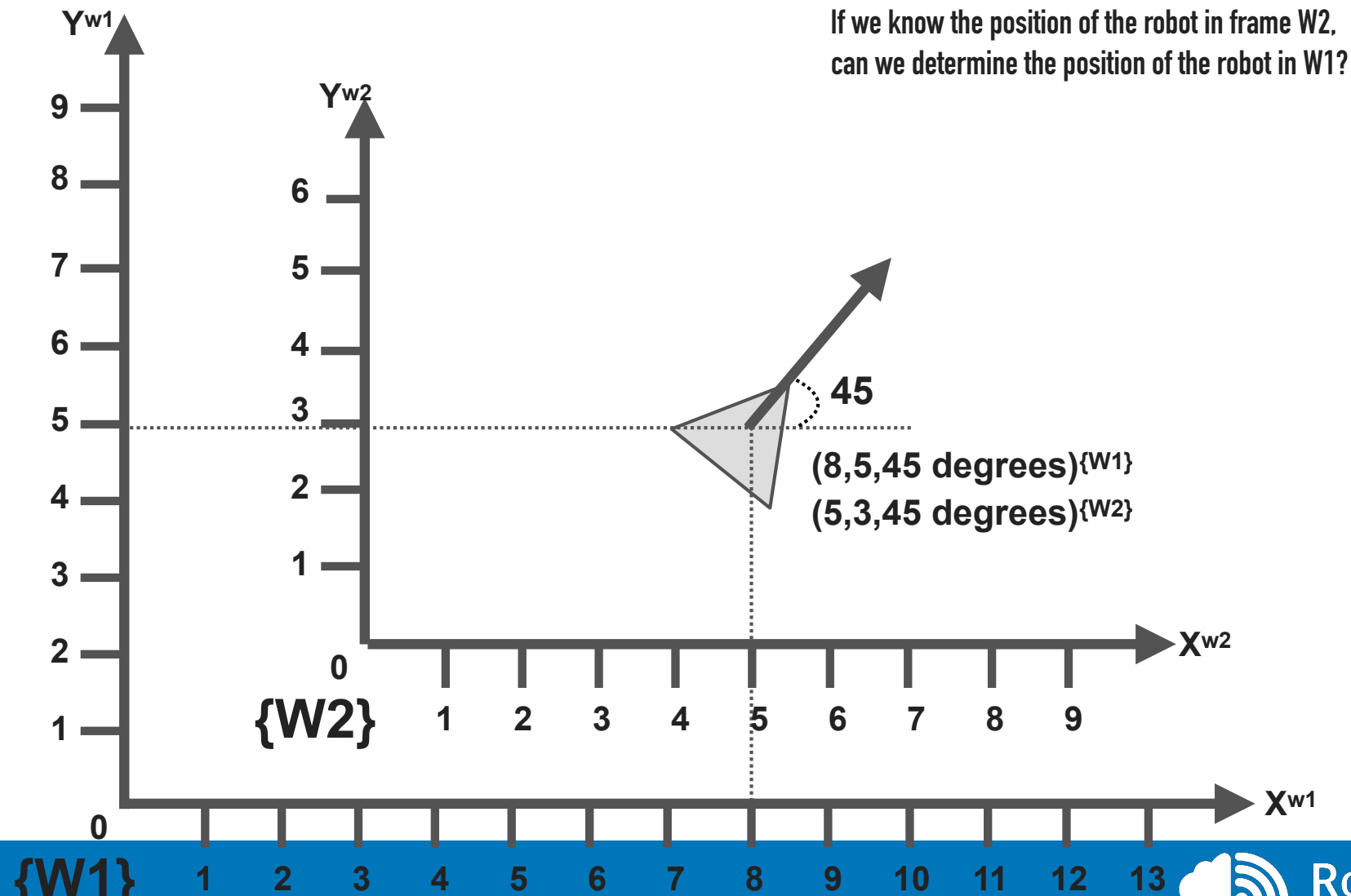
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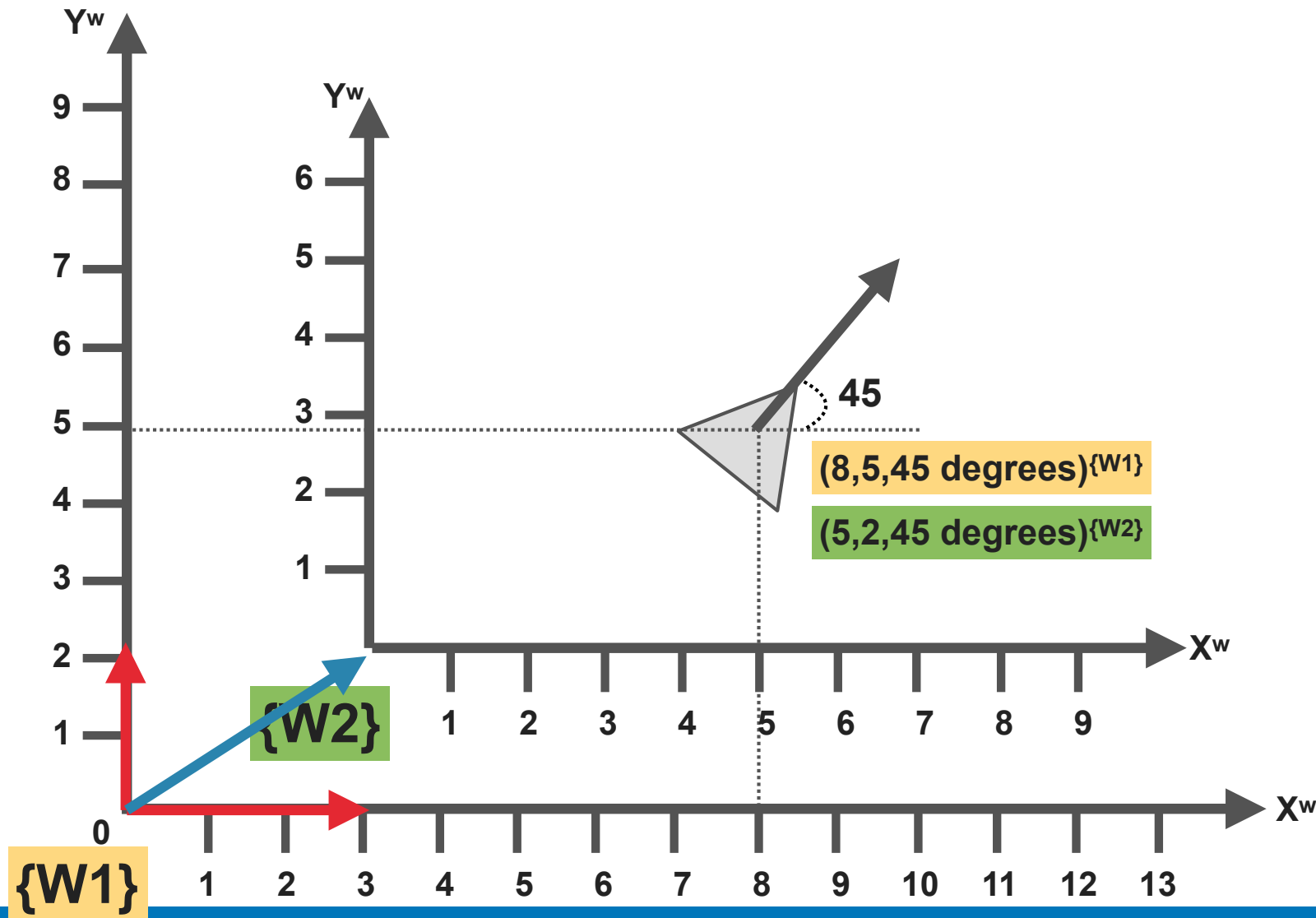
# 2D Translation

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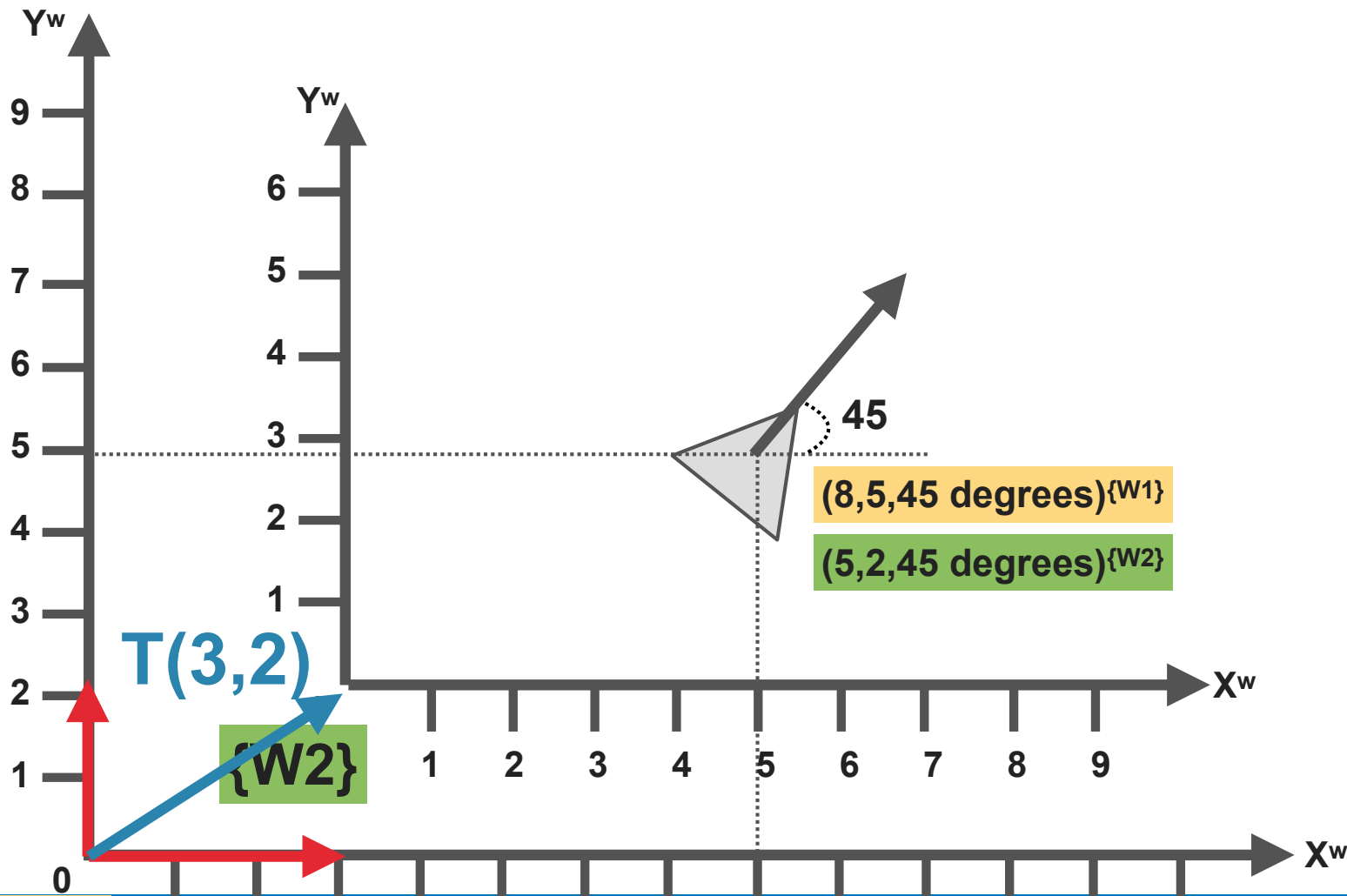
# COORDINATE FRAME: TRANSLATION



# COORDINATE FRAME: TRANSLATION

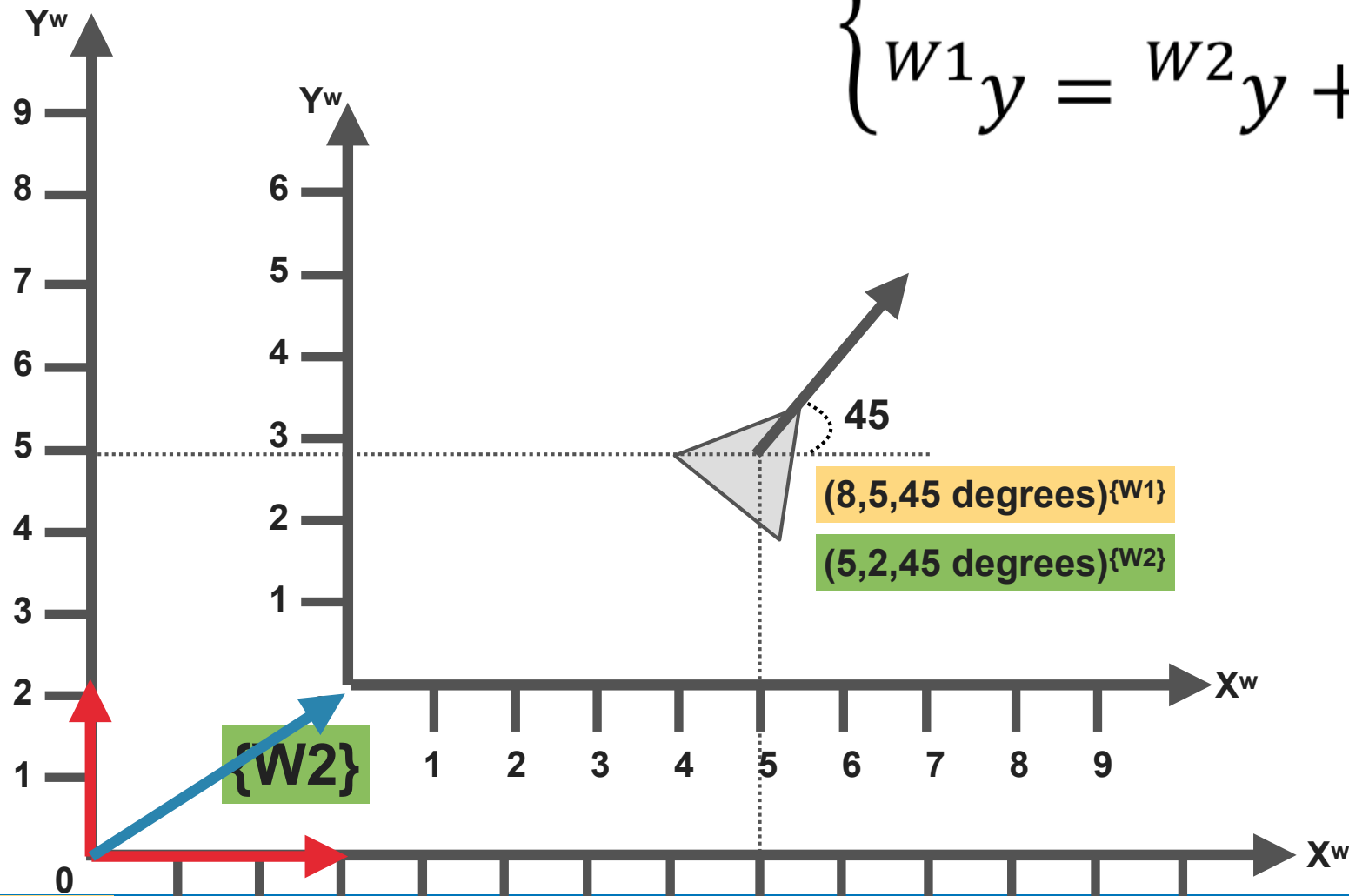


## COORDINATE FRAME: TRANSLATION





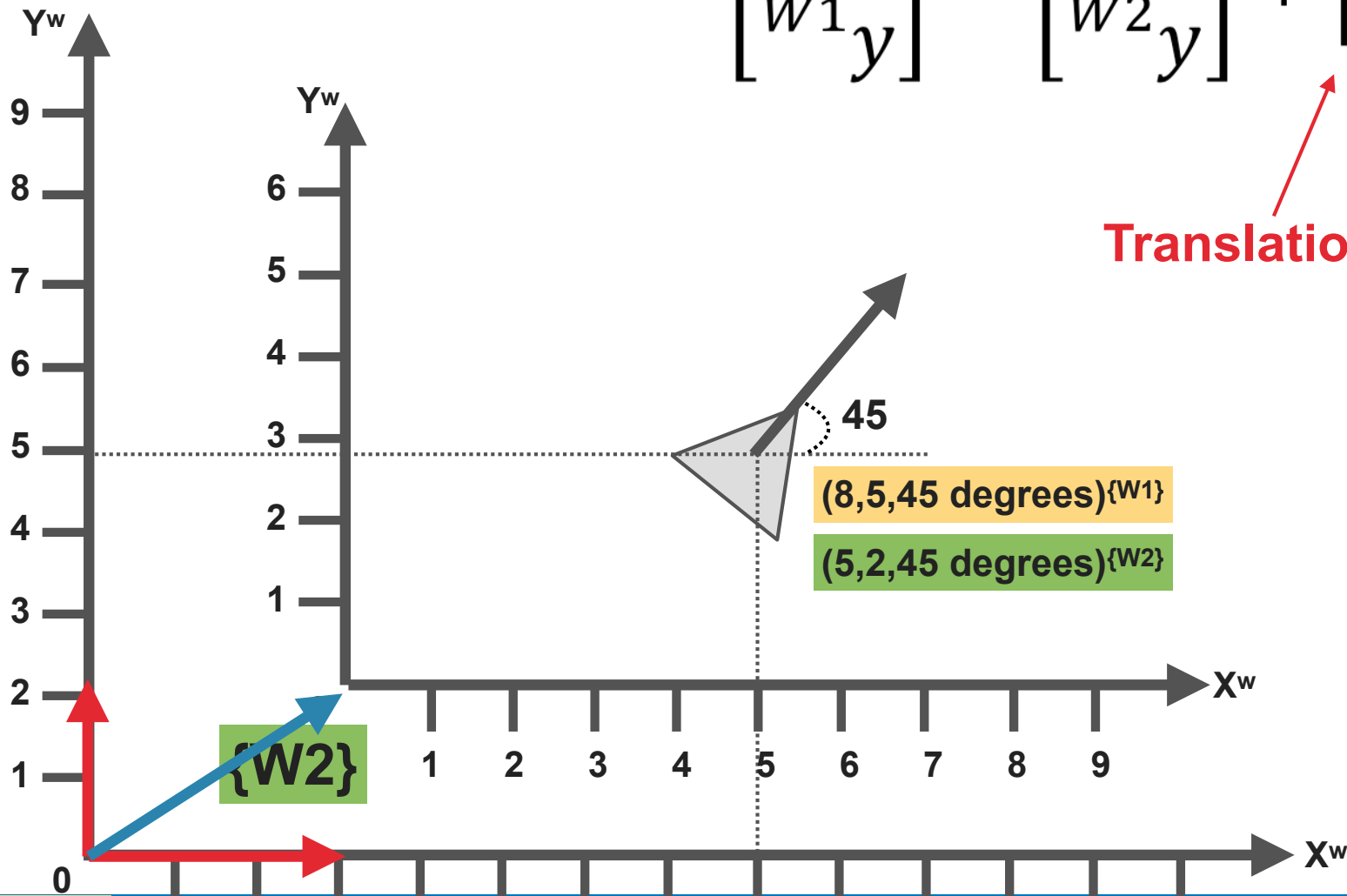
COORDINATE FRAME: TRANSL,  $\begin{cases} {}^{W1}x = {}^{W2}x + 3 \\ {}^{W1}y = {}^{W2}y + 2 \end{cases}$



COORDINATE FRAME: TRANSFORMATION

$$\begin{bmatrix} w^1 x \\ w^1 y \end{bmatrix} = \begin{bmatrix} w^2 x \\ w^2 y \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Translation vector



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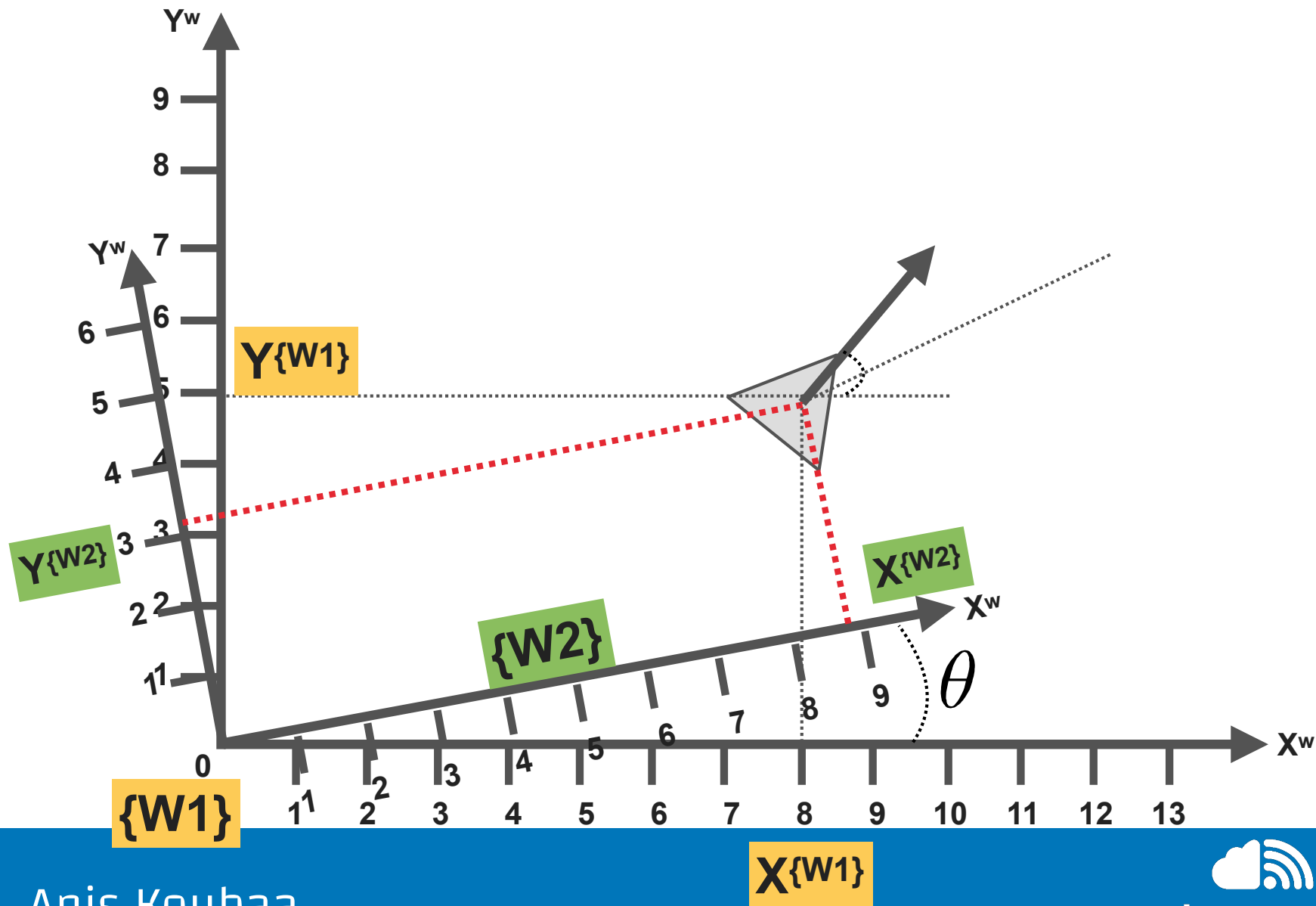
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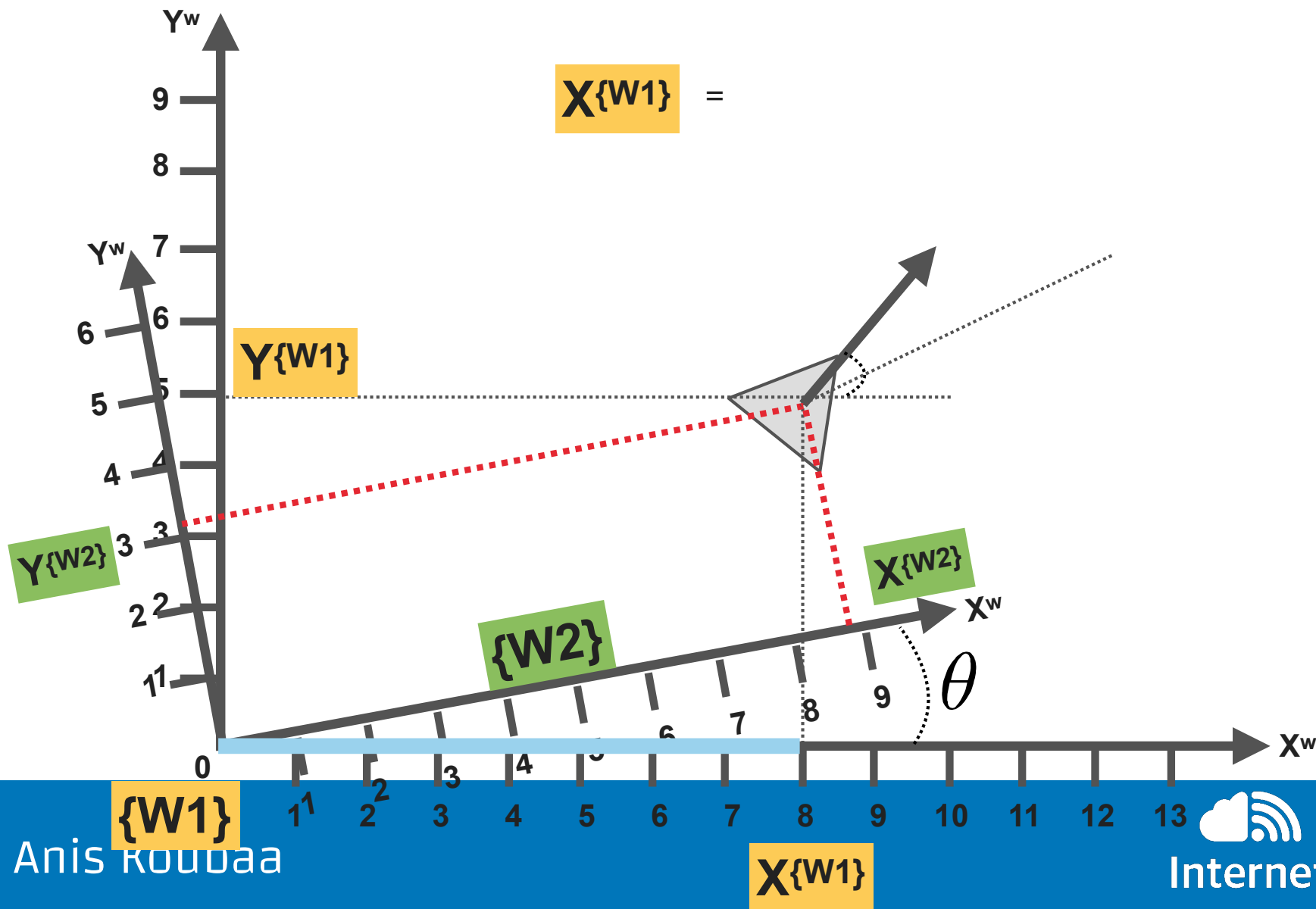
# 2D Rotation

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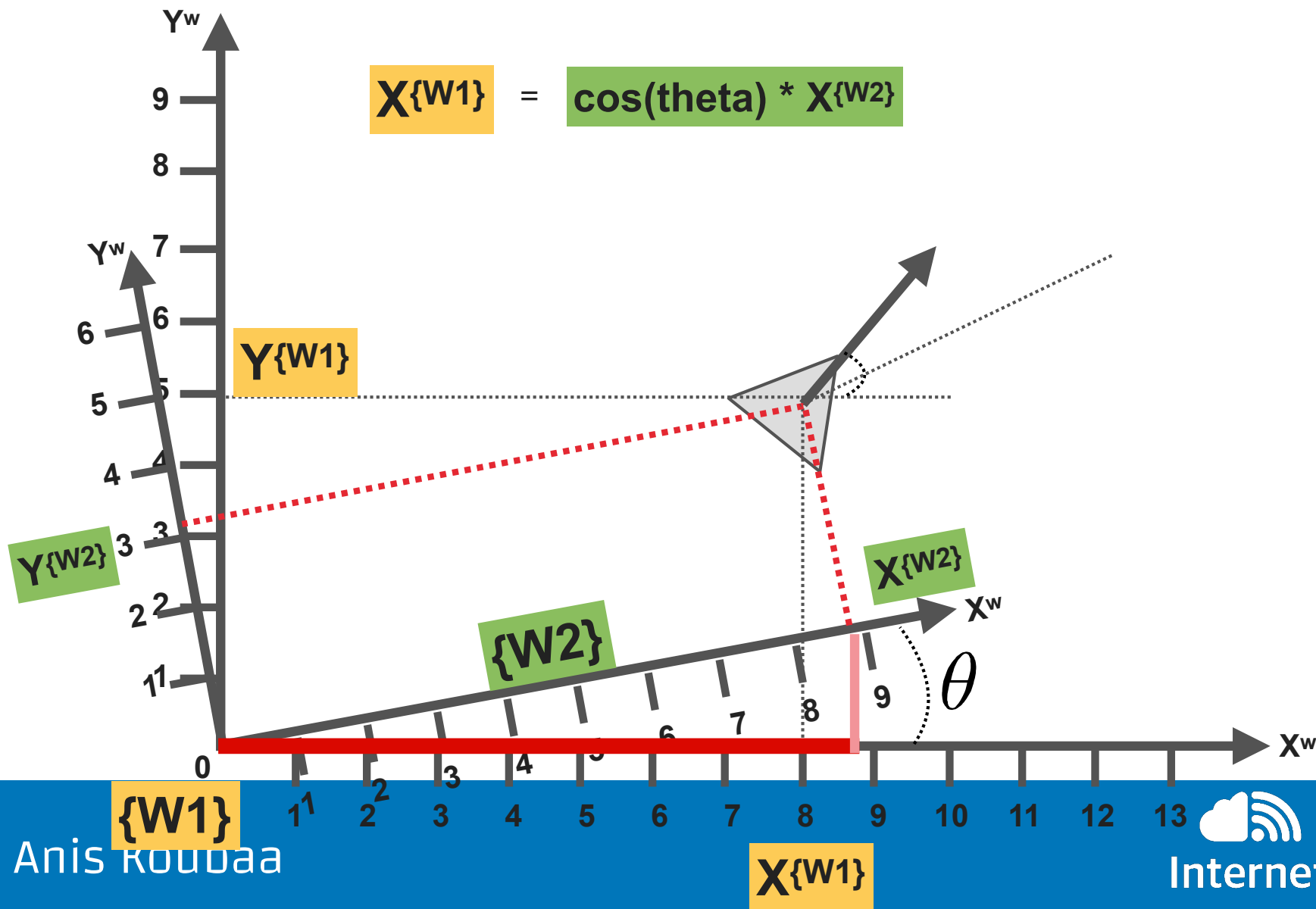
# COORDINATE FRAME: ROTATION



# COORDINATE FRAME: ROTATION

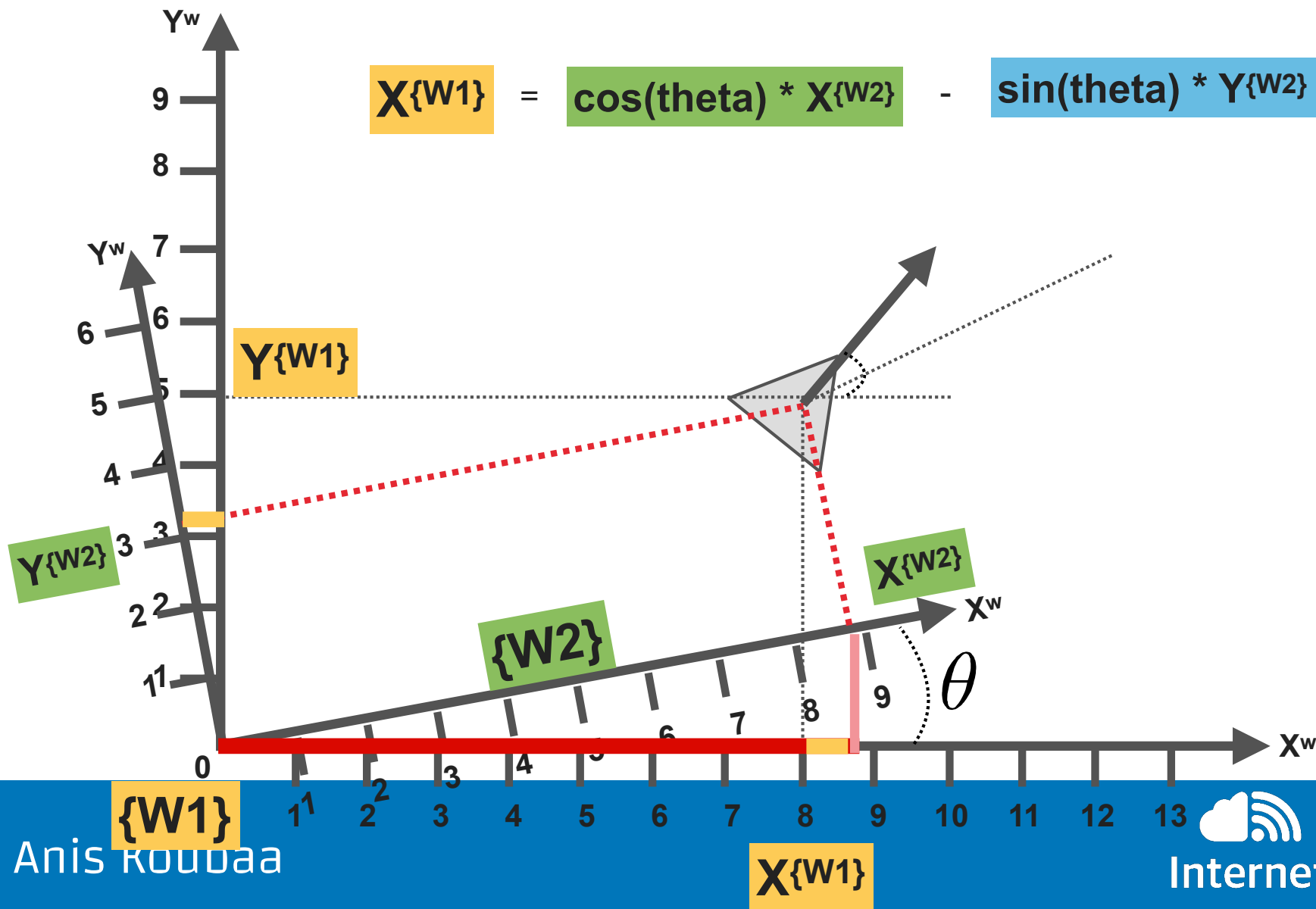


# COORDINATE FRAME: ROTATION



# COORDINATE FRAME: ROTATION

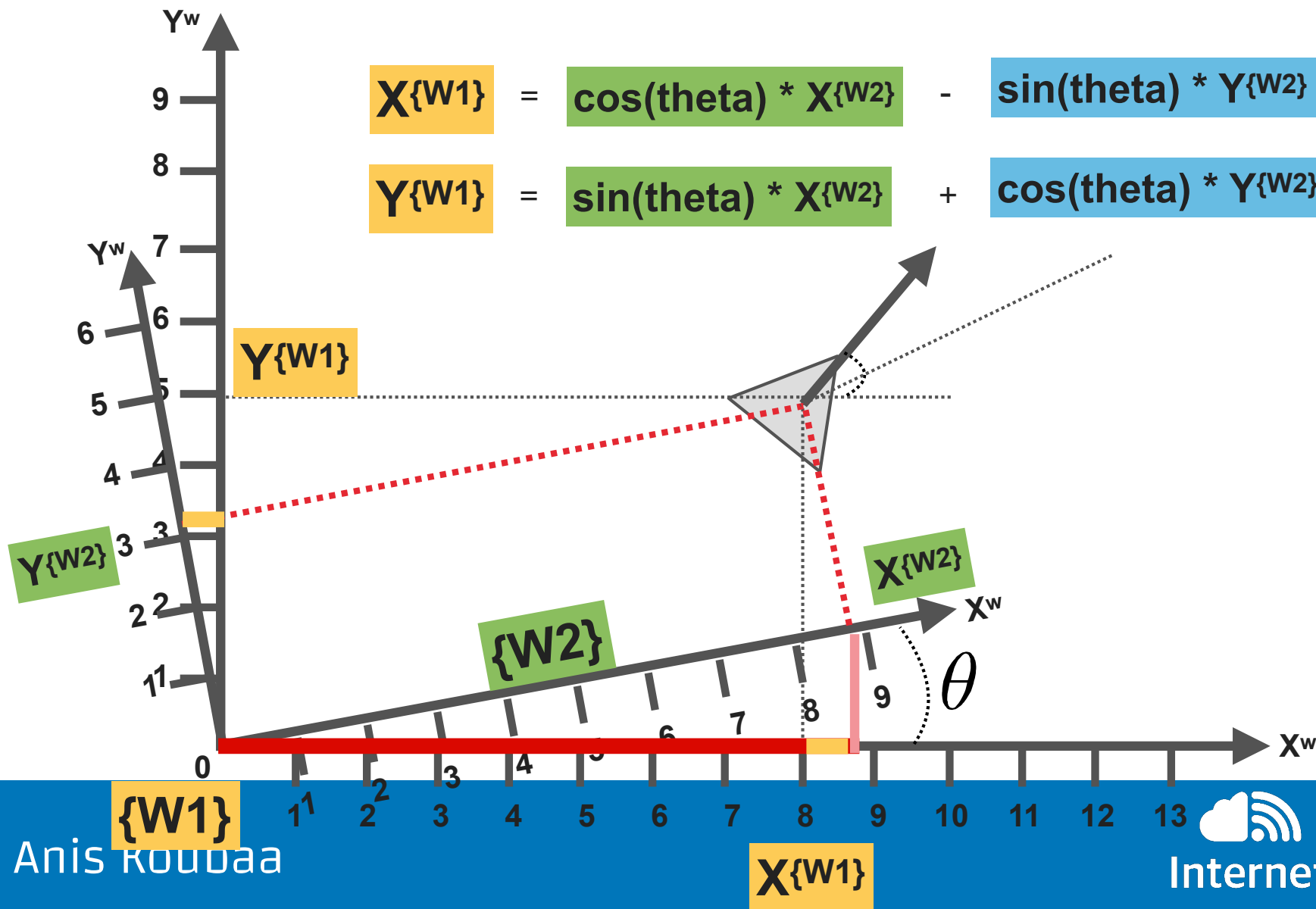
$$X^{W1} = \cos(\theta) * X^{W2} - \sin(\theta) * Y^{W2}$$



# COORDINATE FRAME: ROTATION

$$X^{W1} = \cos(\theta) * X^{W2} - \sin(\theta) * Y^{W2}$$

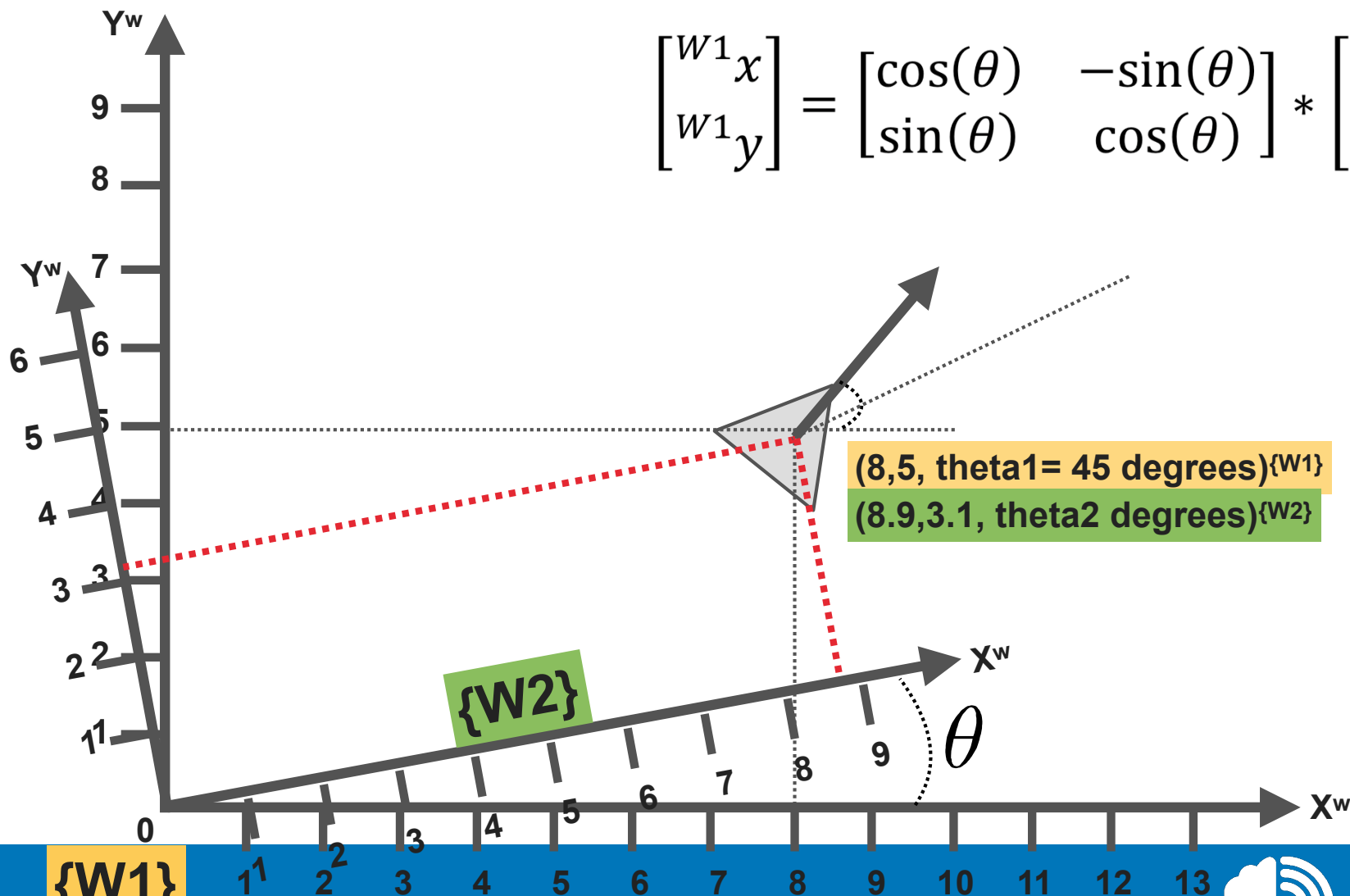
$$Y^{W1} = \sin(\theta) * X^{W2} + \cos(\theta) * Y^{W2}$$





# COORDINATE FRAME: ROTATION

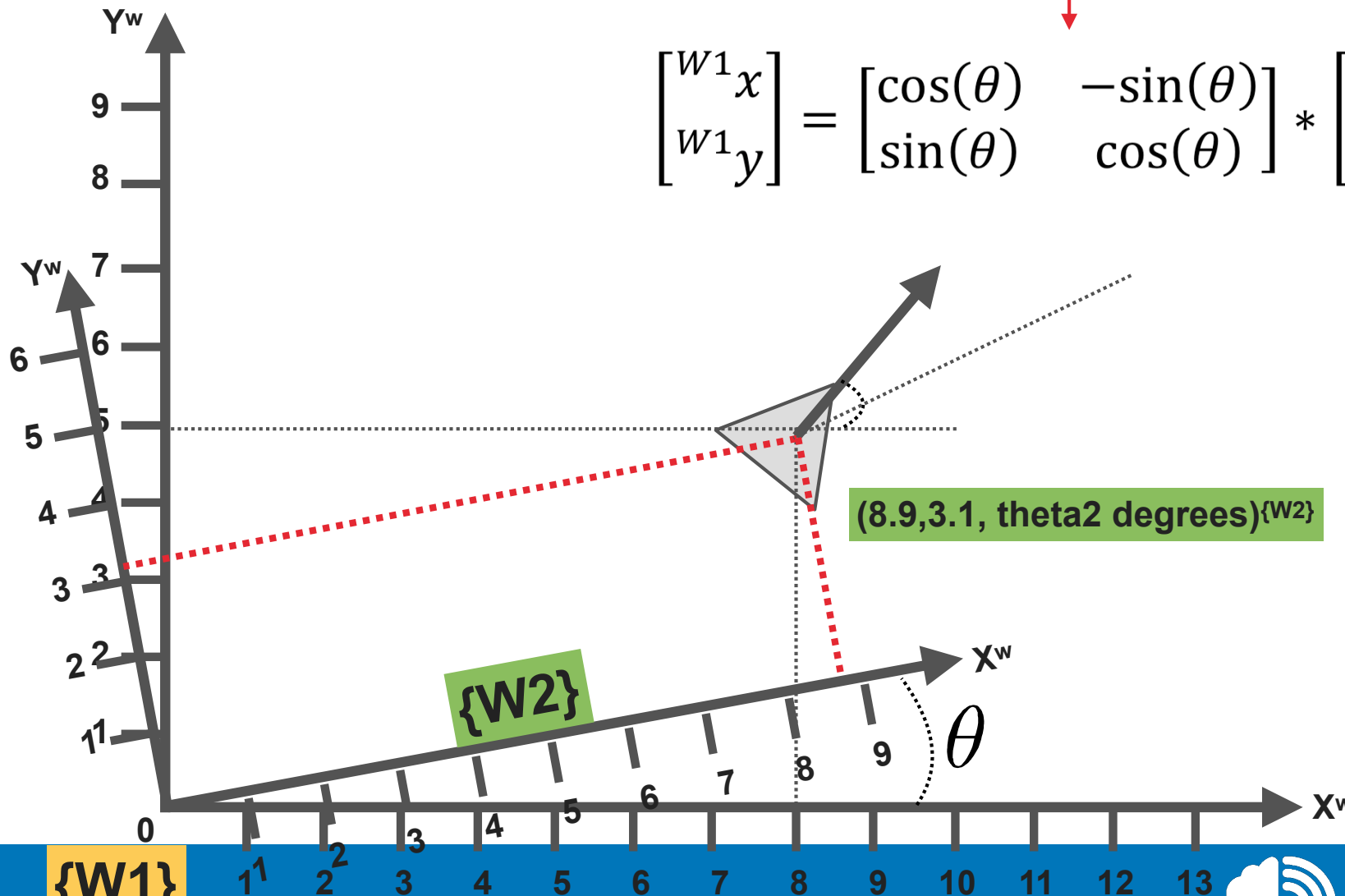
$$\begin{bmatrix} {}^{W1}x \\ {}^{W1}y \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} * \begin{bmatrix} {}^{W2}x \\ {}^{W2}y \end{bmatrix}$$



Rotation Matrix

## COORDINATE FRAME: ROTATION

$$\begin{bmatrix} {}^{W1}x \\ {}^{W1}y \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} * \begin{bmatrix} {}^{W2}x \\ {}^{W2}y \end{bmatrix}$$



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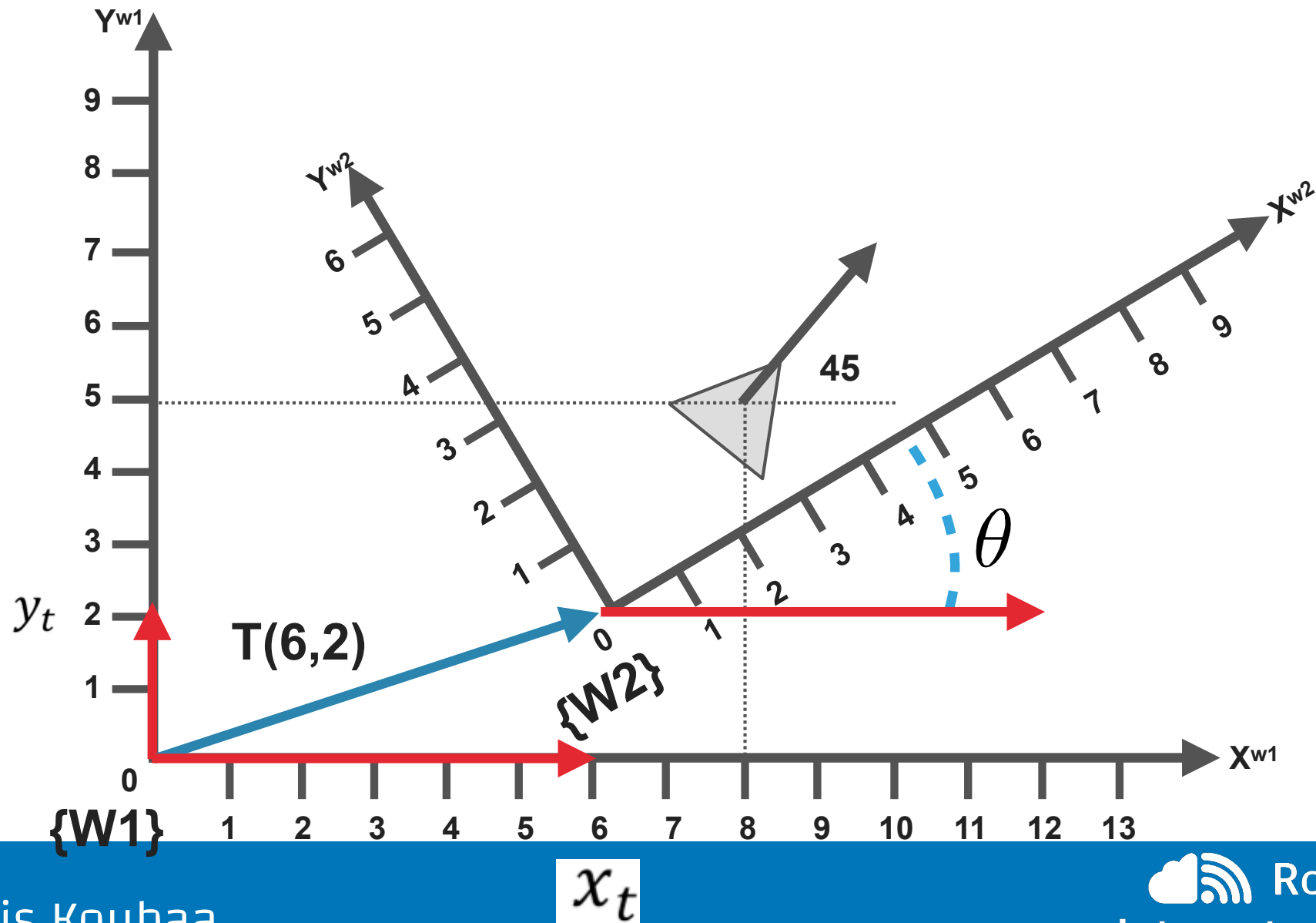
2D Transformation:  
Translation + Rotation

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## OUTLINE

- ▶ 2D Transformation involves
  - ▶ one translation
  - ▶ one rotation

# TRANSFORMATION: TRANSLATION + ROTATION



## COORDINATE FRAME 2D TRANSFORMATION

for counter clock wise rotation angle theta

$$\begin{bmatrix} {}^{W1}x \\ {}^{W1}y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & x_t \\ \sin(\theta) & \cos(\theta) & y_t \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} {}^{W2}x \\ {}^{W2}y \\ 1 \end{bmatrix}$$

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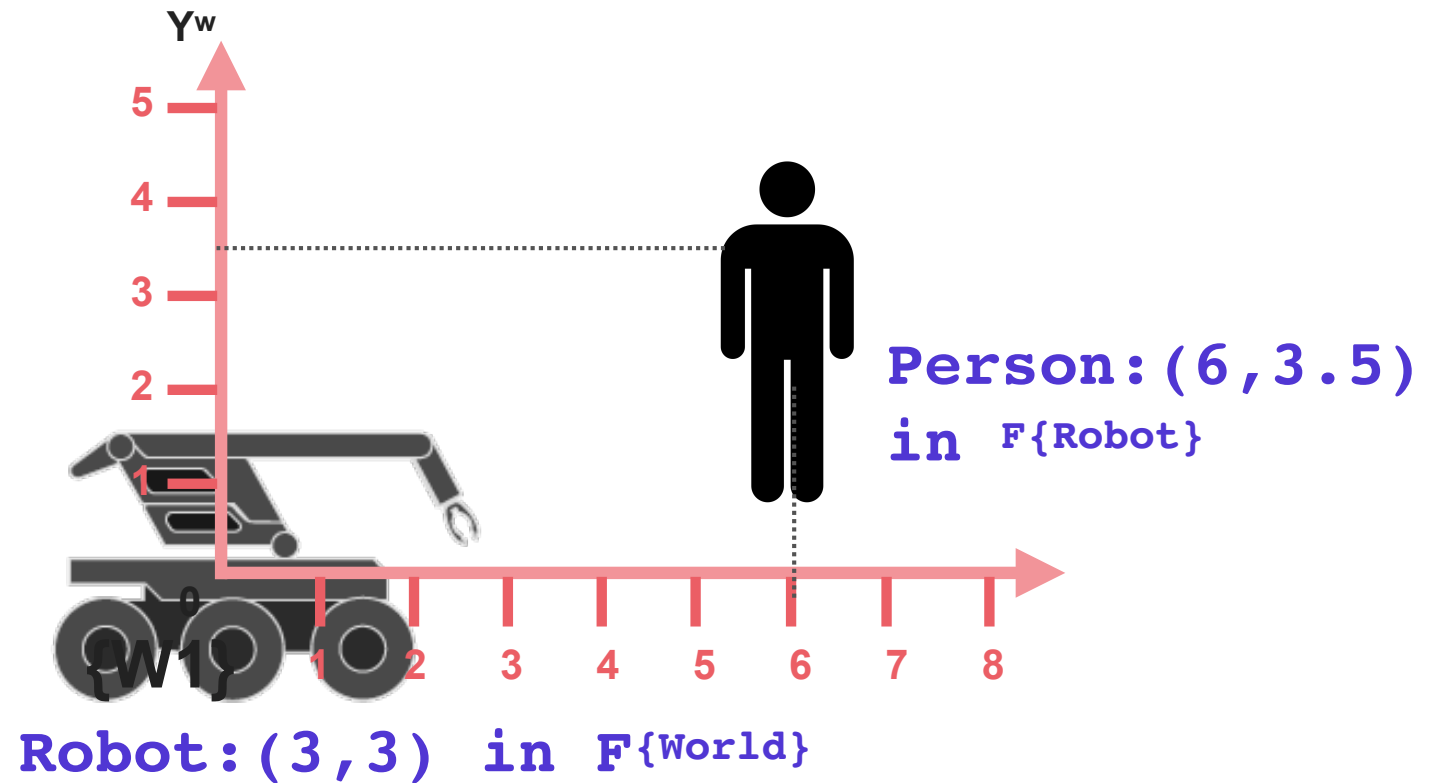
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# Transformation Example

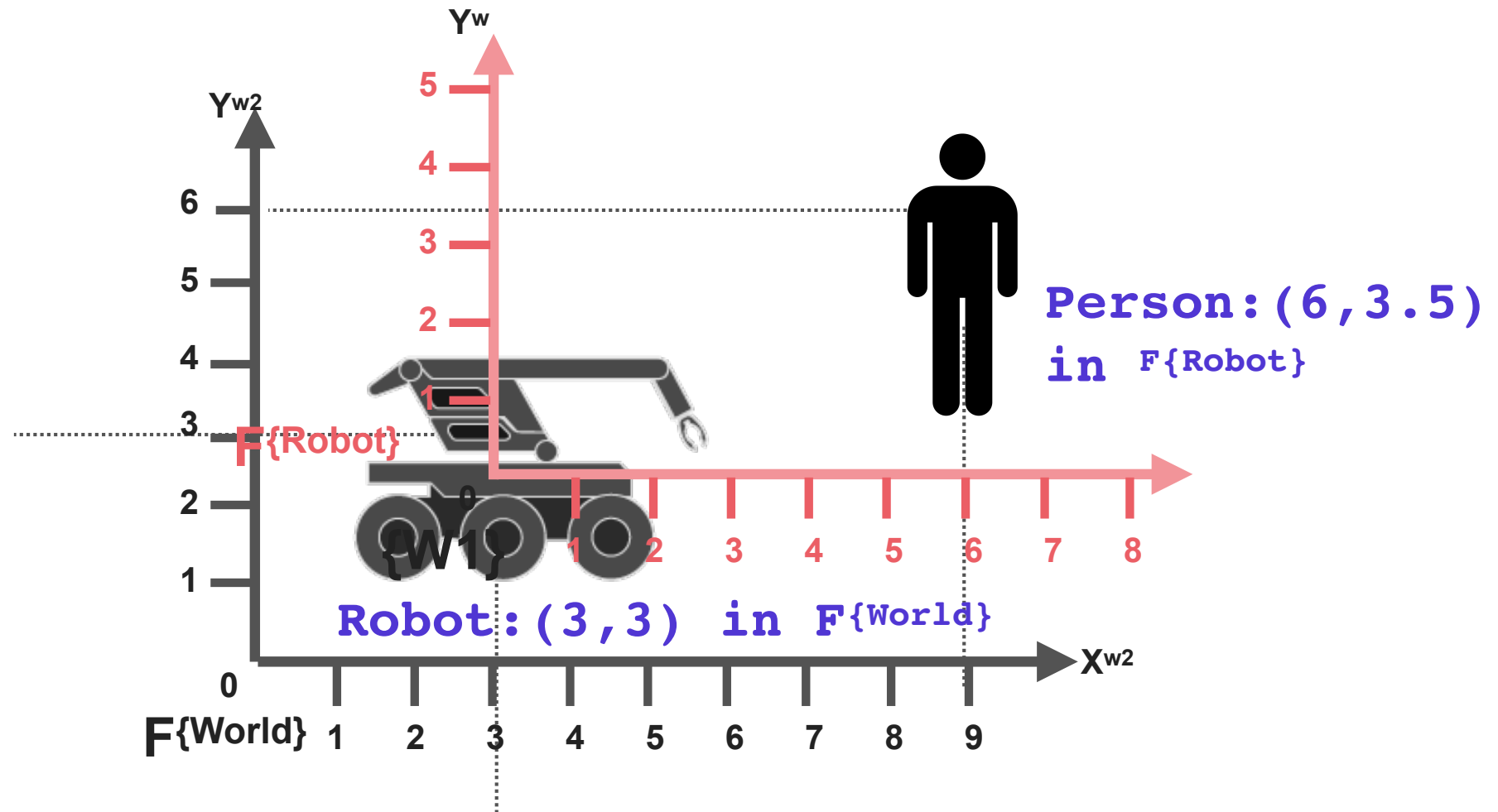
<https://www.udemy.com/user/anis-koubaa/>

## LOCATION IN THE ROBOT AND WORLD COORDINATE FRAMES

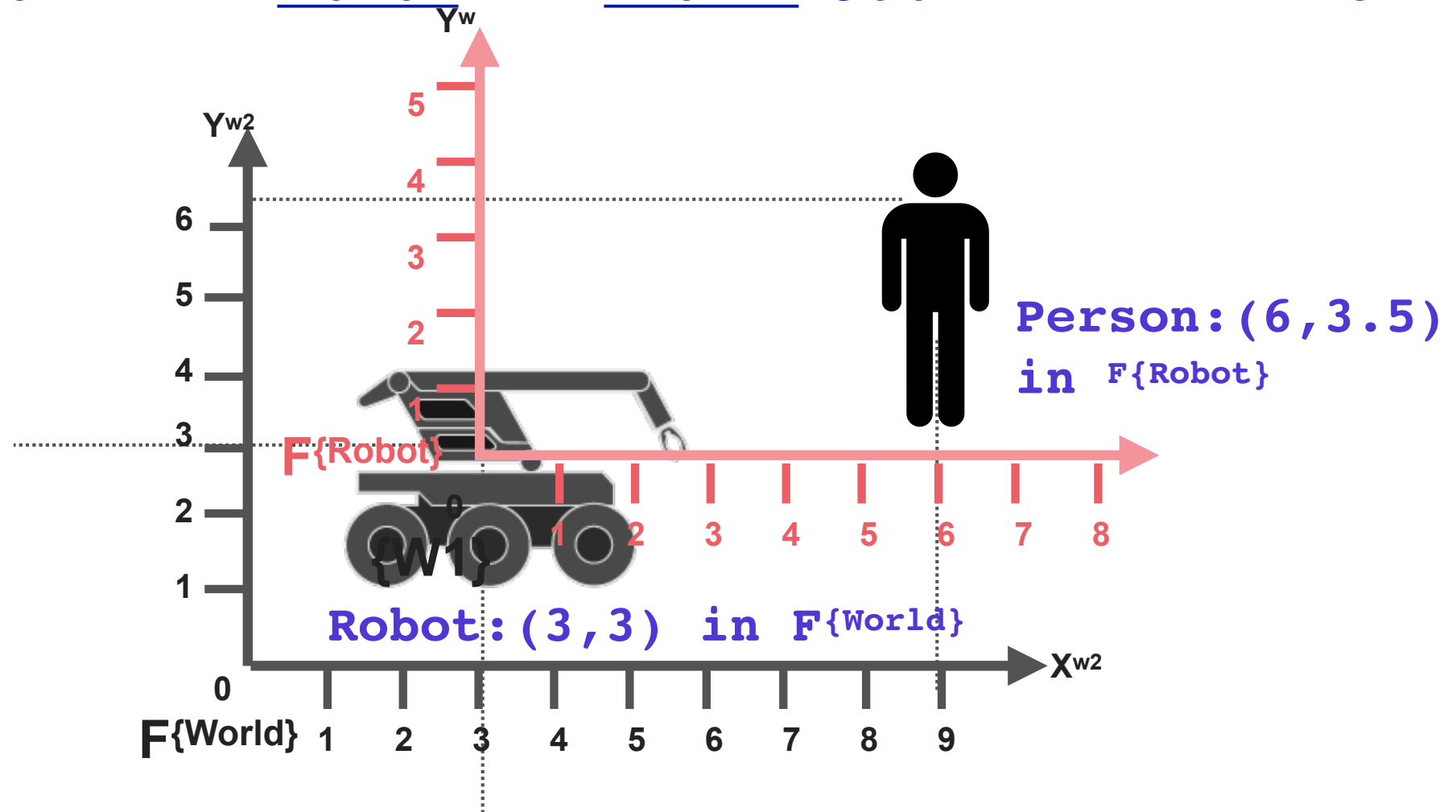




## LOCATION IN THE ROBOT AND WORLD COORDINATE FRAMES



## LOCATION IN THE ROBOT AND WORLD COORDINATE FRAMES



## TRANSFORMATION MATRIX

$$\begin{bmatrix} {}^{w^1}x \\ {}^{w^1}y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & x_t \\ \sin(\theta) & \cos(\theta) & y_t \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} {}^{w^2}x \\ {}^{w^2}y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_{\text{world}} \\ Y_{\text{world}} \\ 1 \end{bmatrix} = \begin{bmatrix} \text{Cos}(0) & -\text{Sin}(0) & 3 \\ \text{Sin}(0) & \text{Cos}(0) & 3 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X_{\text{robot}} \\ Y_{\text{robot}} \\ 1 \end{bmatrix}$$

# TRANSFORMATION MATRIX

$$\begin{bmatrix} X_{\text{world}} \\ Y_{\text{world}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X_{\text{robot}} \\ Y_{\text{robot}} \\ 1 \end{bmatrix}$$

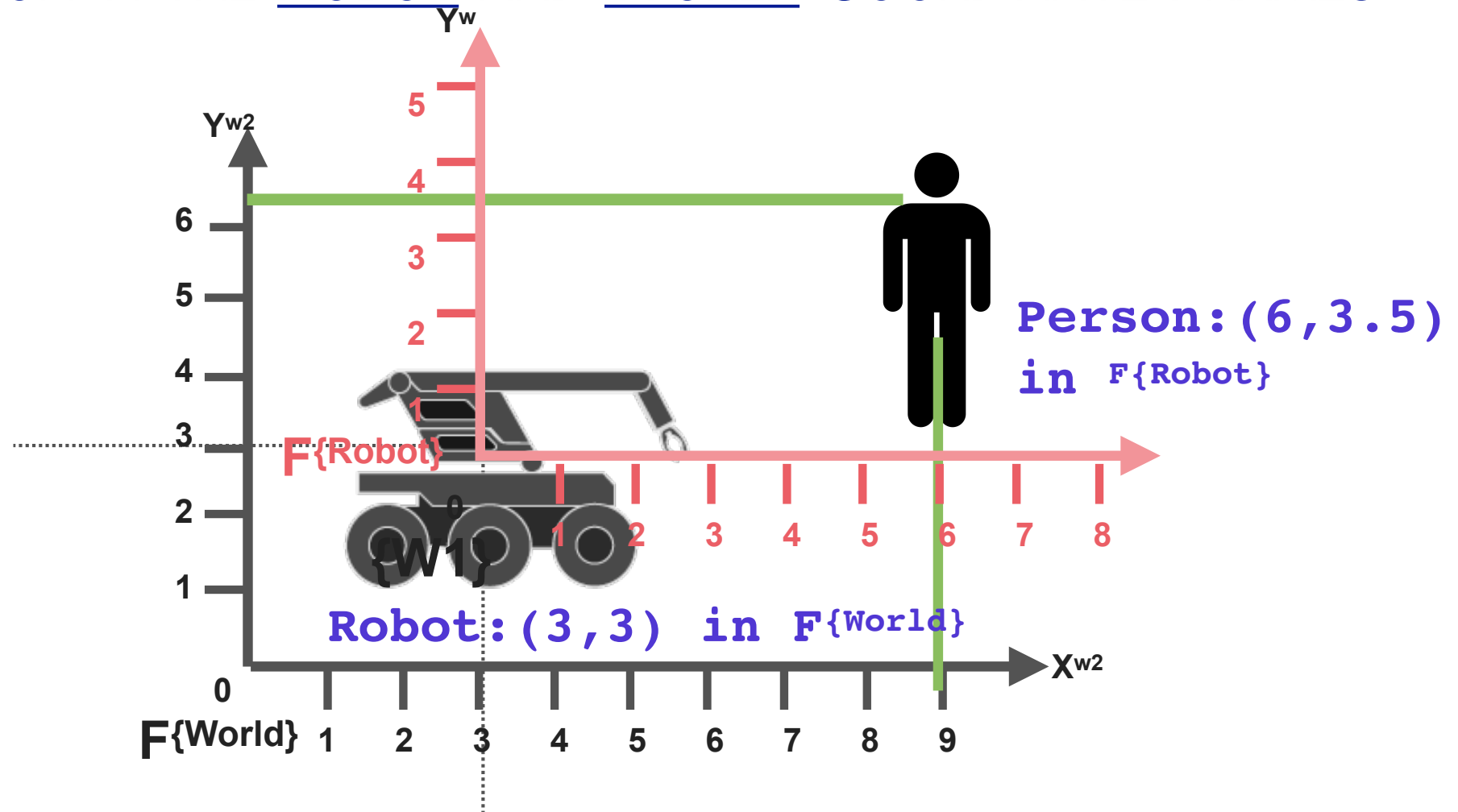
# TRANSFORMATION MATRIX

$$\begin{bmatrix} X^{\{world\}} \\ Y^{\{world\}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 6 \\ 3.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ Y^{\{world\}} \\ 1 \end{bmatrix}$$

# TRANSFORMATION MATRIX

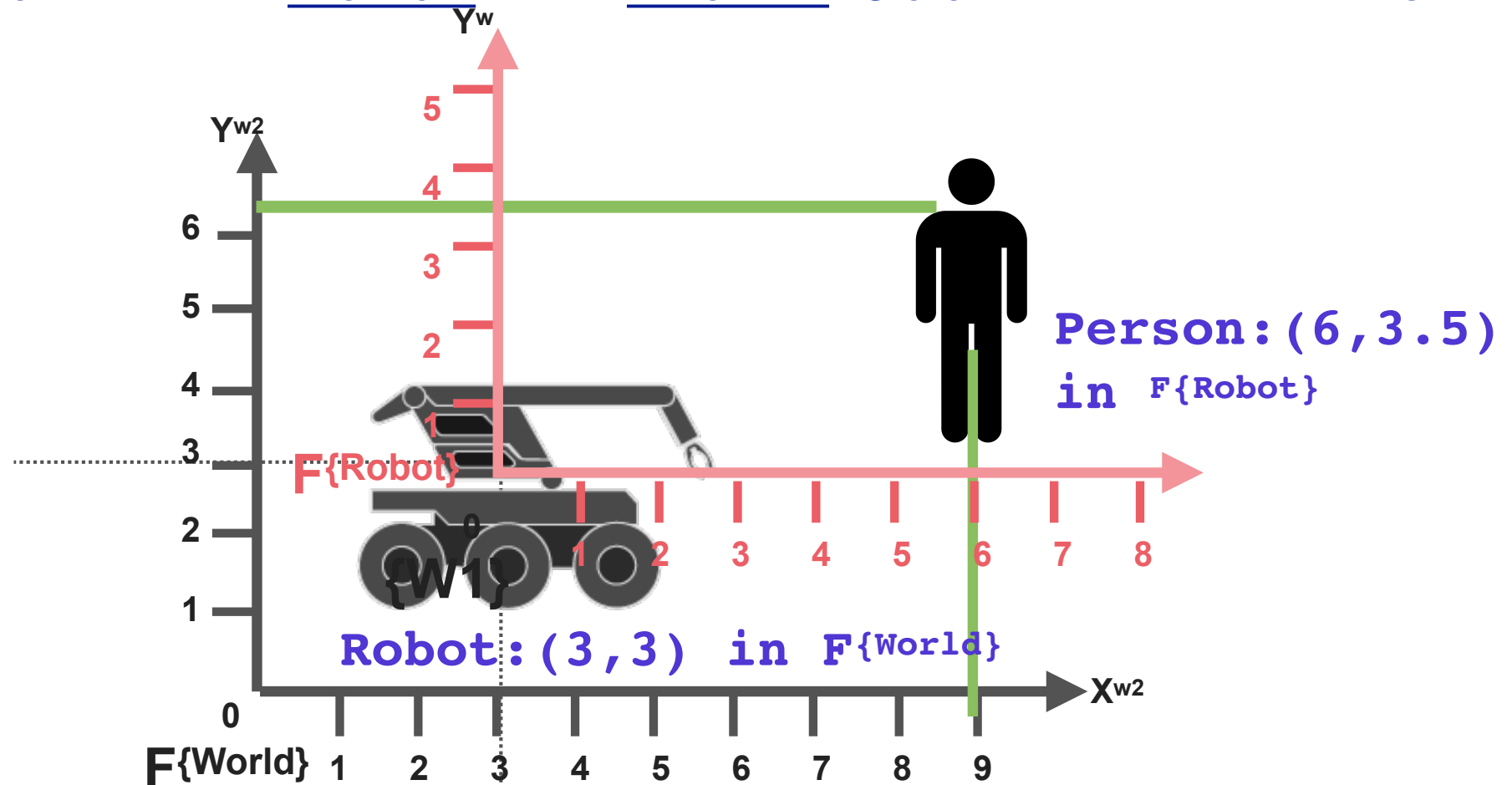
$$\begin{vmatrix} X^{\{world\}} \\ Y^{\{world\}} \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} 6 \\ 3.5 \\ 1 \end{vmatrix} = \begin{vmatrix} 9 \\ 6.5 \\ 1 \end{vmatrix}$$

## LOCATION IN THE ROBOT AND WORLD COORDINATE FRAMES



**Person in  $F\{world\}$ ?**

## LOCATION IN THE ROBOT AND WORLD COORDINATE FRAMES



**Person in  $F\{world\}$ ?**