Lamé's stress ellipsoid

is a graphical representation of the state of stress at a point.

In the frame of the principal axes $\{\underline{n}_i\}$ the traction

$$\underline{t} = \underline{\underline{\sigma}}\,\underline{n} = \sigma_1 (\underline{n}_1 \otimes \underline{n}_1)\,\underline{n} + \sigma_2 (\underline{n}_2 \otimes \underline{n}_2)\,\underline{n} + \sigma_3 (\underline{n}_3 \otimes \underline{n}_3)\,\underline{n}$$

$$\underline{t} = \sigma_1 (\underline{n}_1 \cdot \underline{n})\,\underline{n}_1 + \sigma_2 (\underline{n}_2 \cdot \underline{n})\,\underline{n}_1 + \sigma_3 (\underline{n}_3 \cdot \underline{n})\,\underline{n}_3$$

$$= \sigma_1 n_1 \underline{n}_1 + \sigma_2 n_2 \underline{n}_2 + \sigma_3 n_3 \underline{n}_3$$

Components of the traction are : $\underline{t} = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix}$

$$t_1 = \sigma_1 n_1 \qquad t_2 = \sigma_2 n_2 \qquad t_3 = \sigma_3 n_3$$

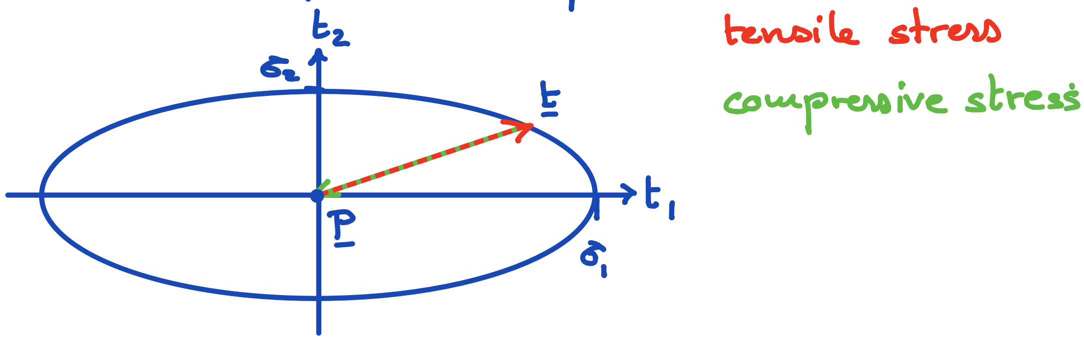
where $n_1^2 + n_2^2 + n_3^2 = 1$ because $\underline{n}$ is a unit vector.

substituting the tractions we have

$$\boxed{\left(\frac{t_1}{\sigma_1}\right)^2 + \left(\frac{t_2}{\sigma_2}\right)^2 + \left(\frac{t_3}{\sigma_3}\right)^2 = 1}$$

the equation for an ellipsoid with the σ_i's as principal semi-major axes.

In 2D for graphical simplicity:



Ellipse is the locus of the end/start points of all traction vectors on all planes at point $\underline{P}$ (center).

However, the stress ellipsoid does <u>not</u> indicate the plane on which the traction vector acts, except for the principal directions.

In 2D the plane a given traction is acting on can be constructed grapically with help of exterior circle as shown. At this moment it is not clear to me why this works.