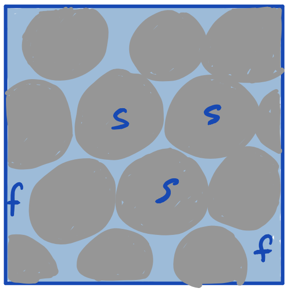
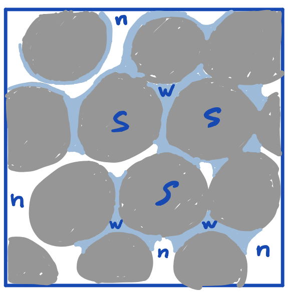
Introduction to porous media



A saturated porous medium comprises two phases:

1) solid (s)

2) pore-fluid (f)



An unsaturated porous medium comprises three phases:

1) solid (s)

2) wetting fluid (w) - water

3) non-wetting fluid (n) - air

Volume fractions: $\boxed{\phi_p = V_p / V_T}$

V_p = volume of phase p

$V_T = \sum_p V_p$ = total volume

$\Rightarrow \boxed{\sum_p \phi_p = 1}$ vol. frac. constraint

Porosity: $\phi = \phi_f$ (saturated)

$\phi = \phi_w + \phi_n$ (unsaturated)

ϕ is the fraction of space occupied by pores!

Fluid saturations: $\boxed{s_p = \phi_p / \phi}$ $p \in [w, n]$

$$\sum_p s_p = 1$$

s_p is the fraction of pore space occupied by fluid p.

$s_p = 0$ or $s_p = 1 \Rightarrow$ saturated medium

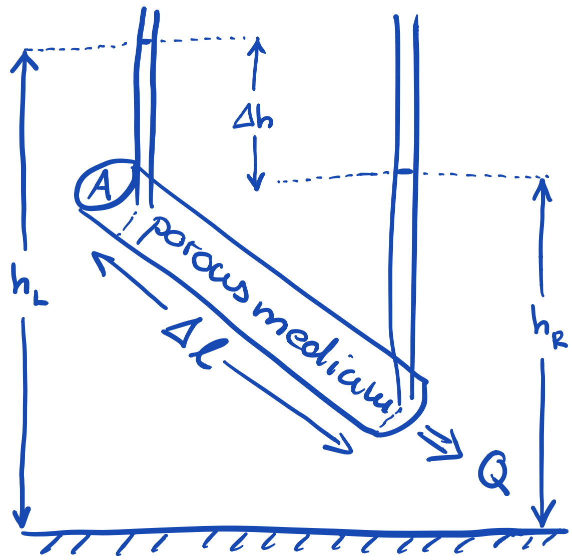
Darcy's law

Δl = sample length $[L]$

A = cross-section $[L^2]$

h_L, h_R = water elevations in left & right manometers $\Rightarrow$ hydraulic heads $[L]$

$\Delta h = h_R - h_L$ = head change $[L]$



Q = volumetric flow rate/discharge $[L^3/T]$

Experimental observations

$$\left.\begin{aligned} &1)\quad Q \sim -\Delta h \\ &2)\quad Q \sim 1/\Delta l \\ &3)\quad Q \sim A \end{aligned}\right\} \quad Q \sim -A\,\frac{\Delta h}{\Delta l}$$

$\Rightarrow$ Darcy's law $\boxed{Q = -K\,A\,\frac{\Delta h}{\Delta L}}$

Hydraulic conductivity: K $[L/T]$

(constant of proportionality)

Comments: 1) Empirical law (o.k.)

2) Macroscopic law (good)

3) Q is an integrated quanity it depends on A (bad)

For continuum theories we need fluxes not rates!

Rate: amount of something per time [#/T]

example: discharge Q $[L^3/T]$ → scalar

Flux: amount of something per area per time

[#/(L^2T)] → vector

example: specific discharge $q = \frac{Q}{A}\hat{n}_A$ $\left[\frac{L^3}{L^2T} = \frac{L}{T}\right]$

$\hat{n}_A$ = unit normal of A in dir. of flow

Note: q is not a velocity!

In 1D: $|q| = -K\frac{\Delta h}{\Delta \ell}$

Darcy's law for flux in 3D

$$\boxed{\underline{q} = -\underline{\underline{K}}\,\nabla h}$$

$$\underline{q} = \begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix}$$ flux vector

$$\nabla h = \begin{pmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \\ \frac{\partial h}{\partial z} \end{pmatrix}$$ gradient of the head

$$\underline{\underline{K}} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{xy} & K_{yy} & K_{yz} \\ K_{xz} & K_{yz} & K_{zz} \end{bmatrix}$$ hyd. conductivity tensor ($\underline{\underline{K}} = \underline{\underline{K}}^T$)