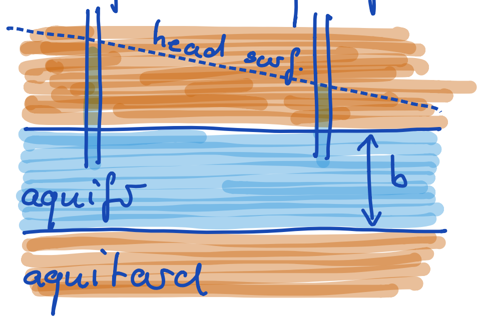
Flow in an unconfined aquifer

Confined aquifer

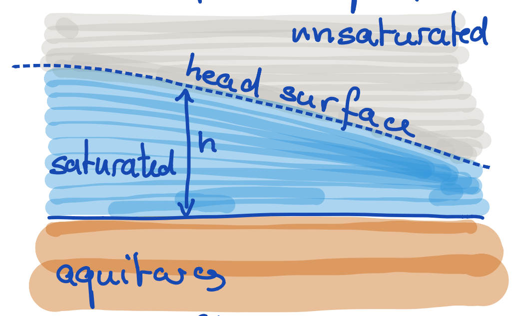


Steady flow:

$$-\nabla\cdot[b\,K\,\nabla h] = f_s$$

linear in h

Unconfined aquifer



Steady flow

$$-\nabla\cdot[h\,K\,\nabla h] = f_s$$

non-linear in h !

Non-linear model for unconfined aquifers

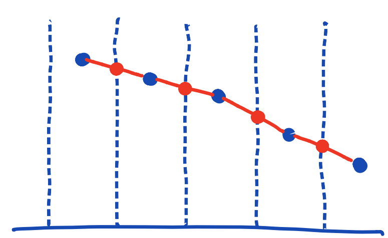
$$\boxed{\phi\frac{\partial h}{\partial t} - \nabla\cdot[K\,h\nabla h] = f_s}$$

Boussinesq eqn

Consider the discretization of steady problem:

continuous: $\quad -\nabla\cdot[K\,h\,\nabla h] = f_s$

discrete: $\quad -\underline{\underline{D}} * [\underline{\underline{Kd}} \;?\; \underline{\underline{G}} * \underline{h}] = \underline{f_s}$



We need to interpolate $\underline{h}$ to interfaces

$\Rightarrow$ arithmetic mean: $\quad \underline{H} = \underline{\underline{I}}_f(\underline{M} * \underline{h})$

$\underline{\underline{I}}_f$ is N_f by N_f diagonal matrix with vector $\underline{\underline{M}} * \underline{h}$ on diagonal

$$\Rightarrow \qquad \underbrace{-\underline{\underline{D}} * \underline{\underline{Kd}} * \underline{\underline{H}}(\underline{h}) * \underline{\underline{G}} * \underline{h}} = \underline{f}_s$$

$$\underline{N}(\underline{h}) = \underline{f}_s \qquad\qquad \underline{N}(\underline{h}) \neq \underline{\underline{L}} * \underline{h}$$

$\underline{N}(h)$ is a <u>non-linear system of algebraic equations</u>.

We cannot pull $\underline{h}$ out to form the system matrix $\underline{\underline{L}}$!

$\Rightarrow$ solve iteratively with Newton's method.