Differential Flatness of Two Quadrotors Carrying a Cable-suspended Payload

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According to the work in [1, 2] and other similar papers, two quadrotor and a cable-suspended payload is a differential flatness system. The detailed derivation and a matlab project will be summarized in this paper.

1 Dynamic Model

The basic nonlinear dynamic model is

$$m_{Qi}\ddot{\boldsymbol{\xi}}_{\boldsymbol{Qi}} = f_i \boldsymbol{R_i e_3} - m_{Qi} g \boldsymbol{e_3} - T_i \boldsymbol{\rho_i} \tag{1.1}$$

$$\boldsymbol{J_i \dot{\omega}_i} + \boldsymbol{\omega_i} \times \boldsymbol{J_i \omega_i} = \boldsymbol{M_i} \tag{1.2}$$

$$m_P \ddot{\boldsymbol{\xi}}_{\boldsymbol{P}} = T_1 \boldsymbol{\rho_1} + T_2 \boldsymbol{\rho_2} - m_P g \boldsymbol{e_3} \tag{1.3}$$

where, $i = 1, 2$ indicates the first and second quadrotor and $\boldsymbol{\rho_i}$ indicate unit direction vector from payload to quadrotors respectively.

2 Differential Flatness

Choosing $\{\boldsymbol{\xi_P}, T_2\boldsymbol{\rho_2}, \psi_1, \psi_2\}$ and their corresponding high order derivatives as flat output. As a result, we can describe both the states and control inputs with these outputs.

Step 1: Obtain T_1 and $\boldsymbol{\rho}_1$
Based on Eq(1.3), we have

$$\begin{aligned} T_1\boldsymbol{\rho_1} &= m_P \ddot{\boldsymbol{\xi}}_{\boldsymbol{P}} - T_2\boldsymbol{\rho_2} + m_P g \boldsymbol{e_3} \\ \Rightarrow \boldsymbol{\rho_1} &= \frac{T_1\boldsymbol{\rho_1}}{\left\| T_1\boldsymbol{\rho_1} \right\|} \\ T_1 &= T_1\boldsymbol{\rho_1} \cdot \boldsymbol{\rho_1} \end{aligned} \tag{2.1}$$

Step 2: $\dot{T}_1$, $\dot{\boldsymbol{\rho}}_{\boldsymbol{1}}$
Differentiating Eq(1.3), we have

$$m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(3)} = \dot{T}_1\boldsymbol{\rho_1} + T_1\dot{\boldsymbol{\rho}}_{\boldsymbol{1}} + \frac{d}{dt}\left(T_2\boldsymbol{\rho_2}\right) \tag{2.2}$$

According to $\dot{\boldsymbol{\rho}}_{\mathbf{1}} \cdot \boldsymbol{\rho}_{\mathbf{1}} = 0$, we have

$$\begin{aligned} m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{3})} \cdot \boldsymbol{\rho}_{\mathbf{1}} &= \dot{T}_1 + \frac{d}{dt}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \cdot \boldsymbol{\rho}_{\mathbf{1}} \\ \Rightarrow \dot{T}_1 &= m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{3})} \cdot \boldsymbol{\rho}_{\mathbf{1}} - \frac{d}{dt}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \cdot \boldsymbol{\rho}_{\mathbf{1}} \\ \dot{\boldsymbol{\rho}}_{\mathbf{1}} &= \frac{1}{T_1}\left(m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{3})} - \dot{T}_1 \boldsymbol{\rho}_{\mathbf{1}} - \frac{d}{dt}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right)\right) \end{aligned} \tag{2.3}$$

Step 3: $\ddot{T}_1$, $\ddot{\boldsymbol{\rho}}_{\mathbf{1}}$
Differentiating $\dot{T}_1$ obtained in former step yields,

$$\ddot{T}_1 = m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{4})} \cdot \boldsymbol{\rho}_{\mathbf{1}} + m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{3})} \cdot \dot{\boldsymbol{\rho}}_{\mathbf{1}} - \frac{d^2}{dt^2}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \cdot \boldsymbol{\rho}_{\mathbf{1}} - \frac{d}{dt}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \cdot \dot{\boldsymbol{\rho}}_{\mathbf{1}} \tag{2.4}$$

Meanwhile, differentiating Eq(2.2), we have

$$m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{4})} = \ddot{T}_1 \boldsymbol{\rho}_{\mathbf{1}} + 2\dot{T}_1 \dot{\boldsymbol{\rho}}_{\mathbf{1}} + T_1 \ddot{\boldsymbol{\rho}}_{\mathbf{1}} + \frac{d^2}{dt^2}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \tag{2.5}$$

As a result, we have

$$\ddot{\boldsymbol{\rho}}_{\mathbf{1}} = \frac{1}{T_1}\left(m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{4})} - \ddot{T}_1 \boldsymbol{\rho}_{\mathbf{1}} - 2\dot{T}_1 \dot{\boldsymbol{\rho}}_{\mathbf{1}} - \frac{d^2}{dt^2}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right)\right) \tag{2.6}$$

Step 4: $T_1^{(3)}$, $\boldsymbol{\rho}_{\mathbf{1}}^{(\mathbf{3})}$
Similarly, Differentiating $\ddot{T}_1$ yields,

$$\begin{aligned} T_1^{(3)} = {} & m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{5})} \cdot \boldsymbol{\rho}_{\mathbf{1}} + 2 m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{4})} \cdot \dot{\boldsymbol{\rho}}_{\mathbf{1}} + m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{3})} \cdot \ddot{\boldsymbol{\rho}}_{\mathbf{1}} \\ & - \frac{d^3}{dt^3}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \cdot \boldsymbol{\rho}_{\mathbf{1}} - 2\frac{d^2}{dt^2}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \cdot \dot{\boldsymbol{\rho}}_{\mathbf{1}} - \frac{d}{dt}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \cdot \ddot{\boldsymbol{\rho}}_{\mathbf{1}} \end{aligned} \tag{2.7}$$

Similarly, differentiating Eq(2.5), we have

$$m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{5})} = T_1^{(3)} \boldsymbol{\rho}_{\mathbf{1}} + 3\dot{T}_1 \ddot{\boldsymbol{\rho}}_{\mathbf{1}} + 3\ddot{T}_1 \dot{\boldsymbol{\rho}}_{\mathbf{1}} + T_1 \boldsymbol{\rho}_{\mathbf{1}}^{(\mathbf{3})} + \frac{d^3}{dt^3}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \tag{2.8}$$

Thus, we have

$$\boldsymbol{\rho}_{\mathbf{1}}^{(\mathbf{3})} = \frac{1}{T_1}\left(m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{5})} - T_1^{(3)} \boldsymbol{\rho}_{\mathbf{1}} - 3\dot{T}_1 \ddot{\boldsymbol{\rho}}_{\mathbf{1}} - 3\ddot{T}_1 \dot{\boldsymbol{\rho}}_{\mathbf{1}} - \frac{d^3}{dt^3}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right)\right) \tag{2.9}$$

Step 5: $T_1^{(4)}$, $\boldsymbol{\rho}_{\mathbf{1}}^{(\mathbf{4})}$
Similarly, Differentiating $T_1^{(3)}$ yields,

$$\begin{aligned} T_1^{(4)} = {} & m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{6})} \cdot \boldsymbol{\rho}_{\mathbf{1}} + 3 m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{5})} \cdot \dot{\boldsymbol{\rho}}_{\mathbf{1}} + 3 m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{4})} \cdot \ddot{\boldsymbol{\rho}}_{\mathbf{1}} + m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{3})} \cdot \boldsymbol{\rho}_{\mathbf{1}}^{(\mathbf{3})} \\ & - \frac{d^4}{dt^4}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \cdot \boldsymbol{\rho}_{\mathbf{1}} - 3\frac{d^3}{dt^3}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \cdot \dot{\boldsymbol{\rho}}_{\mathbf{1}} - 3\frac{d^2}{dt^2}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \cdot \ddot{\boldsymbol{\rho}}_{\mathbf{1}} - \frac{d}{dt}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \cdot \ddot{\boldsymbol{\rho}}_{\mathbf{1}} \end{aligned} \tag{2.10}$$

Differential Eq(2.8), we have

$$m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{6})} = T_1^{(4)} \boldsymbol{\rho}_{\mathbf{1}} + 4T_1^{(3)} \dot{\boldsymbol{\rho}}_{\mathbf{1}} + 6\ddot{T}_1 \ddot{\boldsymbol{\rho}}_{\mathbf{1}} + 4\dot{T}_1 \boldsymbol{\rho}_{\mathbf{1}}^{(\mathbf{3})} + T_1 \boldsymbol{\rho}_{\mathbf{1}}^{(\mathbf{4})} + \frac{d^4}{dt^4}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right) \tag{2.11}$$

As a result, we have

$$\boldsymbol{\rho}_{\mathbf{1}}^{(\mathbf{4})} = \frac{1}{T_1}\left(m_P \boldsymbol{\xi}_{\boldsymbol{P}}^{(\mathbf{6})} - T_1^{(4)} \boldsymbol{\rho}_{\mathbf{1}} - 4T_1^{(3)} \dot{\boldsymbol{\rho}}_{\mathbf{1}} - 6\ddot{T}_1 \ddot{\boldsymbol{\rho}}_{\mathbf{1}} - 4\dot{T}_1 \boldsymbol{\rho}_{\mathbf{1}}^{(\mathbf{3})} - \frac{d^4}{dt^4}\left(T_2 \boldsymbol{\rho}_{\mathbf{2}}\right)\right) \tag{2.12}$$

Step 6: $\boldsymbol{\xi}_{\boldsymbol{Q_1}}, \dot{\boldsymbol{\xi}}_{\boldsymbol{Q_1}}, \ddot{\boldsymbol{\xi}}_{\boldsymbol{Q_1}}, \boldsymbol{\xi}_{\boldsymbol{Q_1}}^{(\mathbf{3})}, \boldsymbol{\xi}_{\boldsymbol{Q_1}}^{(\mathbf{4})}, \boldsymbol{\xi}_{\boldsymbol{Q_2}}, \dot{\boldsymbol{\xi}}_{\boldsymbol{Q_2}}, \ddot{\boldsymbol{\xi}}_{\boldsymbol{Q_2}}, \boldsymbol{\xi}_{\boldsymbol{Q_2}}^{(\mathbf{3})}, \boldsymbol{\xi}_{\boldsymbol{Q_2}}^{(\mathbf{4})}$

According to the relationship of quadrotor and payload, we have

$$\begin{aligned}\boldsymbol{\xi}_{\boldsymbol{Q}_i} &= \boldsymbol{\xi}_{\boldsymbol{P}} + Lr\boldsymbol{\rho}_{\boldsymbol{i}} \\ \dot{\boldsymbol{\xi}}_{\boldsymbol{Q}_i} &= \dot{\boldsymbol{\xi}}_{\boldsymbol{P}} + Lr\dot{\boldsymbol{\rho}}_{\boldsymbol{i}} \\ \ddot{\boldsymbol{\xi}}_{\boldsymbol{Q}_i} &= \ddot{\boldsymbol{\xi}}_{\boldsymbol{P}} + Lr\ddot{\boldsymbol{\rho}}_{\boldsymbol{i}} \\ \boldsymbol{\xi}^{(3)}_{\boldsymbol{Q}_i} &= \boldsymbol{\xi}^{(3)}_{\boldsymbol{P}} + Lr\boldsymbol{\rho}^{(3)}_{\boldsymbol{i}} \\ \boldsymbol{\xi}^{(4)}_{\boldsymbol{Q}_i} &= \boldsymbol{\xi}^{(4)}_{\boldsymbol{P}} + Lr\boldsymbol{\rho}^{(4)}_{\boldsymbol{i}}\end{aligned} \tag{2.13}$$

Step 7: $\boldsymbol{R_1}$, f_1, $\boldsymbol{R_2}$ and f_2
According to Eq(1.1), we have

$$m_{Q1}\ddot{\boldsymbol{\xi}}_{\boldsymbol{Q1}} = f_1\boldsymbol{z_{B1}} - m_{Q1}g\boldsymbol{e_3} - T_1\boldsymbol{\rho_1} \tag{2.14}$$

where, $\boldsymbol{z_{B1}} = \boldsymbol{R_1}\boldsymbol{e_3}$ defines the body frame z axis of the first quadrotor.
Furthermore, we have

$$\begin{aligned} f_1\boldsymbol{z_{B1}} &= m_{Q1}\ddot{\boldsymbol{\xi}}_{\boldsymbol{Q1}} + m_{Q1}g\boldsymbol{e_3} + T_1\boldsymbol{\rho_1} \\ \boldsymbol{z_{B1}} &= \frac{f_1\boldsymbol{z_{B1}}}{\|f_1\boldsymbol{z_{B1}}\|} \\ f_1 &= f_1\boldsymbol{z_{B1}}\cdot\boldsymbol{z_{B1}}\end{aligned} \tag{2.15}$$

Given the yaw angle ψ_1, we can write the unit vector

$$\boldsymbol{x_{C1}} = \left[\cos(\psi_1), \sin(\psi_1), 0\right]^T \tag{2.16}$$

Then we can determine $\boldsymbol{x_{B1}}$ and $\boldsymbol{y_{B1}}$ as follows:

$$\begin{aligned}\boldsymbol{y_{B1}} &= \frac{\boldsymbol{z_{B1}}\times\boldsymbol{x_{C1}}}{\|\boldsymbol{z_{B1}}\times\boldsymbol{x_{C1}}\|} \\ \boldsymbol{x_{B1}} &= \boldsymbol{y_{B1}}\times\boldsymbol{z_{B1}}\end{aligned} \tag{2.17}$$

Provided that $\boldsymbol{y_{B1}}\times\boldsymbol{x_{C1}} \neq 0$, in other words, provided that we never encounter the singularity where $\boldsymbol{z_{B1}}$ is parallel to $\boldsymbol{x_C}$.
Thus, we can uniquely determine

$$\boldsymbol{R_1} = \left[\boldsymbol{x_{B1}}, \boldsymbol{y_{B1}}, \boldsymbol{z_{B1}}\right] \tag{2.18}$$

The similar result for the second quadrotor is available by changing subscript 1 into 2.
Step 8: $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_2}$
Take the first derivative of Eq(2.14), we have

$$m_{Q1}\boldsymbol{\xi}^{(3)}_{\boldsymbol{Q1}} = \dot{f}_1\boldsymbol{z_{B1}} + \boldsymbol{\omega_1}\times f_1\boldsymbol{z_{B1}} - \frac{d}{dt}\left(T_1\boldsymbol{\rho_1}\right) \tag{2.19}$$

Projecting this expression along $\boldsymbol{z_{B1}}$ yields $\dot{f}_1 = m_{Q1}\boldsymbol{\xi}^{(3)}_{\boldsymbol{Q1}}\cdot\boldsymbol{z_{B1}} + \frac{d}{dt}\left(T_1\boldsymbol{\rho_1}\right)\cdot\boldsymbol{z_{B1}}$, then we can substitute $\dot{f}_1$ into Eq(2.19) to define the vector $\boldsymbol{h_{\omega 1}}$ as

$$\begin{aligned}\boldsymbol{h_{\omega 1}} &= \boldsymbol{\omega_1}\times\boldsymbol{z_{B1}} \\ &= \frac{1}{f_1}\left(m_{Q1}\boldsymbol{\xi}^{(3)}_{\boldsymbol{Q1}} - \dot{f}_1\boldsymbol{z_{B1}} + \frac{d}{dt}\left(T_1\boldsymbol{\rho_1}\right)\right) \\ &= \frac{1}{f_1}\left[\left(m_{Q1}\boldsymbol{\xi}^{(3)}_{\boldsymbol{Q1}} + \frac{d}{dt}\left(T_1\boldsymbol{\rho_1}\right)\right) - \left(m_{Q1}\boldsymbol{\xi}^{(3)}_{\boldsymbol{Q1}}\cdot\boldsymbol{z_{B1}} + \frac{d}{dt}\left(T_1\boldsymbol{\rho_1}\right)\cdot\boldsymbol{z_{B1}}\right)\boldsymbol{z_{B1}}\right]\end{aligned} \tag{2.20}$$

Provided that $\boldsymbol{\omega_1} = p_1\boldsymbol{x_{B1}} + q_1\boldsymbol{y_{B1}} + r_1\boldsymbol{z_{B1}}$, we have

$$\begin{aligned} p_1 &= -\boldsymbol{h_{\omega 1}}\cdot\boldsymbol{y_{B1}} \\ q_1 &= \boldsymbol{h_{\omega 1}}\cdot\boldsymbol{x_{B1}} \\ r_1 &= \dot{\psi}_1\boldsymbol{e_3}\cdot\boldsymbol{z_{B1}}\end{aligned} \tag{2.21}$$

The similar result for the second quadrotor is available by changing subscript 1 into 2.

Step 9: $\dot{\boldsymbol{\omega}}_{\mathbf{1}}, \dot{\boldsymbol{\omega}}_{\mathbf{2}}$
Take the second derivative of Eq(2.14), we have

$$\begin{aligned} m_{Q1}\boldsymbol{\xi}_{\mathbf{Q1}}^{(\mathbf{4})} &= \left(\ddot{f}_1\boldsymbol{z}_{\boldsymbol{B1}} + \boldsymbol{\omega}_{\mathbf{1}} \times \dot{f}_1\boldsymbol{z}_{\boldsymbol{B1}}\right) \\ &+ \left(\dot{\boldsymbol{\omega}}_{\mathbf{1}} \times f_1\boldsymbol{z}_{\boldsymbol{B1}} + \boldsymbol{\omega}_{\mathbf{1}} \times (\dot{f}_1\boldsymbol{z}_{\boldsymbol{B1}} + \boldsymbol{\omega}_{\mathbf{1}} \times f_1\boldsymbol{z}_{\boldsymbol{B1}})\right) \\ &- \frac{d^2}{dt^2}\left(T_1\boldsymbol{\rho}_{\mathbf{1}}\right) \end{aligned} \tag{2.22}$$

Projecting this expression along $\boldsymbol{z}_{\boldsymbol{B1}}$ yields

$$\ddot{f}_1 = m_{Q1}\boldsymbol{\xi}_{\mathbf{Q1}}^{(\mathbf{4})} \cdot \boldsymbol{z}_{\boldsymbol{B1}} - \boldsymbol{\omega}_{\mathbf{1}} \times (\boldsymbol{\omega}_{\mathbf{1}} \times f_1\boldsymbol{z}_{\boldsymbol{B1}}) \cdot \boldsymbol{z}_{\boldsymbol{B1}} + \frac{d^2}{dt^2}\left(T_1\boldsymbol{\rho}_{\mathbf{1}}\right) \cdot \boldsymbol{z}_{\boldsymbol{B1}}$$

Then we can substitute $\dot{f}_1$ and $\ddot{f}_1$ into Eq(2.22) to define $\boldsymbol{h}_{\dot{\omega}\mathbf{1}}$ as

$$\begin{aligned} \boldsymbol{h}_{\dot{\omega}\mathbf{1}} &= \dot{\boldsymbol{\omega}}_{\mathbf{1}} \times \boldsymbol{z}_{\boldsymbol{B1}} \\ &= \frac{1}{f_1}\left[m_{Q1}\boldsymbol{\xi}_{\mathbf{Q1}}^{(\mathbf{4})} - \left(\ddot{f}_1\boldsymbol{z}_{\boldsymbol{B1}} + \boldsymbol{\omega}_{\mathbf{1}} \times \dot{f}_1\boldsymbol{z}_{\boldsymbol{B1}}\right) - \boldsymbol{\omega}_{\mathbf{1}} \times (\dot{f}_1\boldsymbol{z}_{\boldsymbol{B1}} + \boldsymbol{\omega}_{\mathbf{1}} \times f_1\boldsymbol{z}_{\boldsymbol{B1}}) + \frac{d^2}{dt^2}\left(T_1\boldsymbol{\rho}_{\mathbf{1}}\right)\right] \end{aligned} \tag{2.23}$$

Provided that $\dot{\boldsymbol{\omega}}_{\mathbf{1}} = p_1\dot{\boldsymbol{x}}_{\boldsymbol{B1}} + q_1\dot{\boldsymbol{y}}_{\boldsymbol{B1}} + r_1\dot{\boldsymbol{z}}_{\boldsymbol{B1}}$, we have

$$\begin{aligned} \dot{p}_1 &= -\boldsymbol{h}_{\dot{\omega}\mathbf{1}} \cdot \boldsymbol{y}_{\boldsymbol{B1}} \\ \dot{q}_1 &= \boldsymbol{h}_{\dot{\omega}\mathbf{1}} \cdot \boldsymbol{x}_{\boldsymbol{B1}} \\ \dot{r}_1 &= \ddot{\psi}_1\boldsymbol{e}_{\mathbf{3}} \cdot \boldsymbol{z}_{\boldsymbol{B1}} \end{aligned} \tag{2.24}$$

The similar result for the second quadrotor is available by changing subscript 1 into 2.
Step 10: $\boldsymbol{M}_{\mathbf{1}}$ and $\boldsymbol{M}_{\mathbf{2}}$

$$\boldsymbol{M}_{\mathbf{1}} = \begin{bmatrix} M_{x1} \\ M_{y1} \\ M_{z1} \end{bmatrix} = \boldsymbol{\omega}_{\mathbf{1}} \times \boldsymbol{I}_{\boldsymbol{B}}\boldsymbol{\omega}_{\mathbf{1}} + \boldsymbol{I}_{\boldsymbol{B}}\dot{\boldsymbol{\omega}}_{\mathbf{1}} \tag{2.25}$$

Similarly,

$$\boldsymbol{M}_{\mathbf{2}} = \begin{bmatrix} M_{x2} \\ M_{y2} \\ M_{z2} \end{bmatrix} = \boldsymbol{\omega}_{\mathbf{2}} \times \boldsymbol{I}_{\boldsymbol{B}}\boldsymbol{\omega}_{\mathbf{2}} + \boldsymbol{I}_{\boldsymbol{B}}\dot{\boldsymbol{\omega}}_{\mathbf{2}} \tag{2.26}$$

3 Matlab Implementation

Based on these expressions, once given flat outputs and their higher derivatives, the states and controls are determined.
A matlab project is given based on these derivations and it is available on `https://github.com/mhguo321/differential-flatness.git`.

References

[1] Daniel Mellinger and Vijay Kumar. Minimum snap trajectory generation and control for quadrotors. In *Robotics and Automation (ICRA), 2011 IEEE International Conference on*, pages 2520–2525. IEEE, 2011.

[2] Koushil Sreenath and Vijay Kumar. Dynamics control and planning for cooperative manipulation of payloads suspended by cables from multiple quadrotor robots. *rn*, 1(r2):r3, 2013.