Hardware simulation of the Deutsch algorithm

Problem statement [edit]

In the Deutsch–Jozsa problem, we are given a black box quantum computer known as an oracle that implements some function:

$$f: \{0,1\}^n \rightarrow \{0,1\}$$

The function takes n-bit binary values as input and produces either a 0 or a 1 as output for each such value. We are promised that the function is either constant (0 on all inputs or 1 on all inputs) or *balanced* (1 for exactly half of the input domain and 0 for the other half).[1] The task then is to determine if f is constant or balanced by using the oracle.

For the remainder, we consider **the case n=1.**

The idea is that the 'oracle' gets fed with only one input (either '0' or '1') and decides if the (embedded) function f is or 'balanced' (output '0') or 'constant' (output '1').

This problem is defined such that a classical computer cannot solve it by using just a single input, as it needs to evaluate **both** '0' **and** '1' to determine if the function f is 'constant' or 'balanced'.

In **[1],** a simulation is described using classic (optical) hardware to mimic (emulate) the Deutsch experiment on a quantum computer. This memo describes the experimental setup of the hardware simulation.

Short Description of the experiment and setup

1. Basis states and superposition

To mimic basis states [0> or [1>, horizontal (0°) or vertical polarization (90°) of a laser pulse is used. Again, it is simulation, not a single quantum particle.

By using a Hadamard gate, a superposition state can be obtained by $1/\sqrt{2}$ ([0> + [1>).
This represents a quantum state, specifically the equal superposition of the basis states |0⟩ and |1⟩. Created by applying a Hadamard gate (H) to the |0⟩ state.
We can do a similar transform on [1>, with result $1/\sqrt{2}$ ([0> - [1>).

Additional notation: a negative expression, like -[0>, represents a horizontal polarization with an 180-degree phase shift, compared to [0>. Similar for –[1>.

1.1 Implementation of superposition in our experiment
A small laser (< 5mW) is used to generate a short (~20 ms) laser pulse.
Choosing input '0' or '1' is done by choosing an orientation of the polarization filter that is installed directly after the laser output.
For input '0', the filter **(I0)** is oriented at 45°. This simulates a superposition state.
For input '1', the filter **(I1)** is oriented at -45°.
Without loss of generality, we continue with input '0', as input '1' is a symmetric case.

2. Function ‘f’

Possible functions f0, f1, f2, f3 can be implemented in the oracle.

✅ All Possible $f(x) : \{0, 1\} \rightarrow \{0, 1\}$

Function	$f(0)$	$f(1)$	Type
f_0	0	0	Constant
f_1	1	1	Constant
f_2	0	1	Balanced
f_3	1	0	Balanced

In our experiment, it would be convenient for implementing ‘f’ if we could transform a horizontal polarity into a vertical one (and vice versa), independent of input ‘0’ or ‘1’. Unfortunately, this is not implementable.

To overcome this, a trick is done. For the output value of ‘f’ to become a ‘1’, a 180° phase shift is applied to the input. For the output value of ‘f’ to become ‘0’, no phase shift is applied to the input.

2.1 Implementation for the balanced functions

$\mathbf{f_2}(1) = 1$ is implemented by taking the vertical polarization component and apply a 180° phase shift on it, while simultaneously, $f_2(0) = 0$ is implemented by keeping (the phase of the) horizontal polarization component unchanged. For this, we can use a special optical filter: a **half-wave plate** is an optical device that shifts the phase of a wave by half a wavelength (180°).
The net result of $\mathbf{f_2}$ is that the superpositioned state is rotated -90°.

$\mathbf{f_3}(0) = 1$ is implemented by taking the horizontal polarization component and apply a 180° phase shift on it, while simultaneously, $f_3(0) = 0$ is implemented by keeping (the phase of the) vertical polarization component unchanged. For this, we can use the same special optical filter: a **half-wave plate** is an optical device that shifts the phase of a wave by half a wavelength (180°).
The net result of $\mathbf{f_3}$ is that the superpositioned state is rotated +90°.

2.2 Implementation for the constant functions

$\mathbf{f_0}(0) = 0$ and $\mathbf{f_0}(1) = 0$ is implemented by keeping the phase(s) of both polarization components unchanged. This can be implemented by using no filter at all.
The net result of $\mathbf{f_0}$ is that the superpositioned state is rotated 0°.

$\mathbf{f_1}(0) = 1$ and $\mathbf{f_1}(1) = 1$ is implemented by taking both the horizontal and vertical polarization components and apply a 180° phase shift on them. For this, we can use two consecutive **half-wave plates** (90° rotated with respect to each other).
The net result of $\mathbf{f_1}$ is that the superpositioned state is rotated 180°.

3. Measuring

So, the simulated system can determine which function (f0, f1, f2, f3) was used, based on the change in orientation (of the polarization) **and** phase shift (of the polarized laser pulse), of the resulting laser pulse (after the function ‘f’ is applied).

However, in our simple experiment, we can **only measure** the change in **orientation** (of the polarization), not the phase shift (of the polarized laser pulse). In fact, we can detect the two following changes:

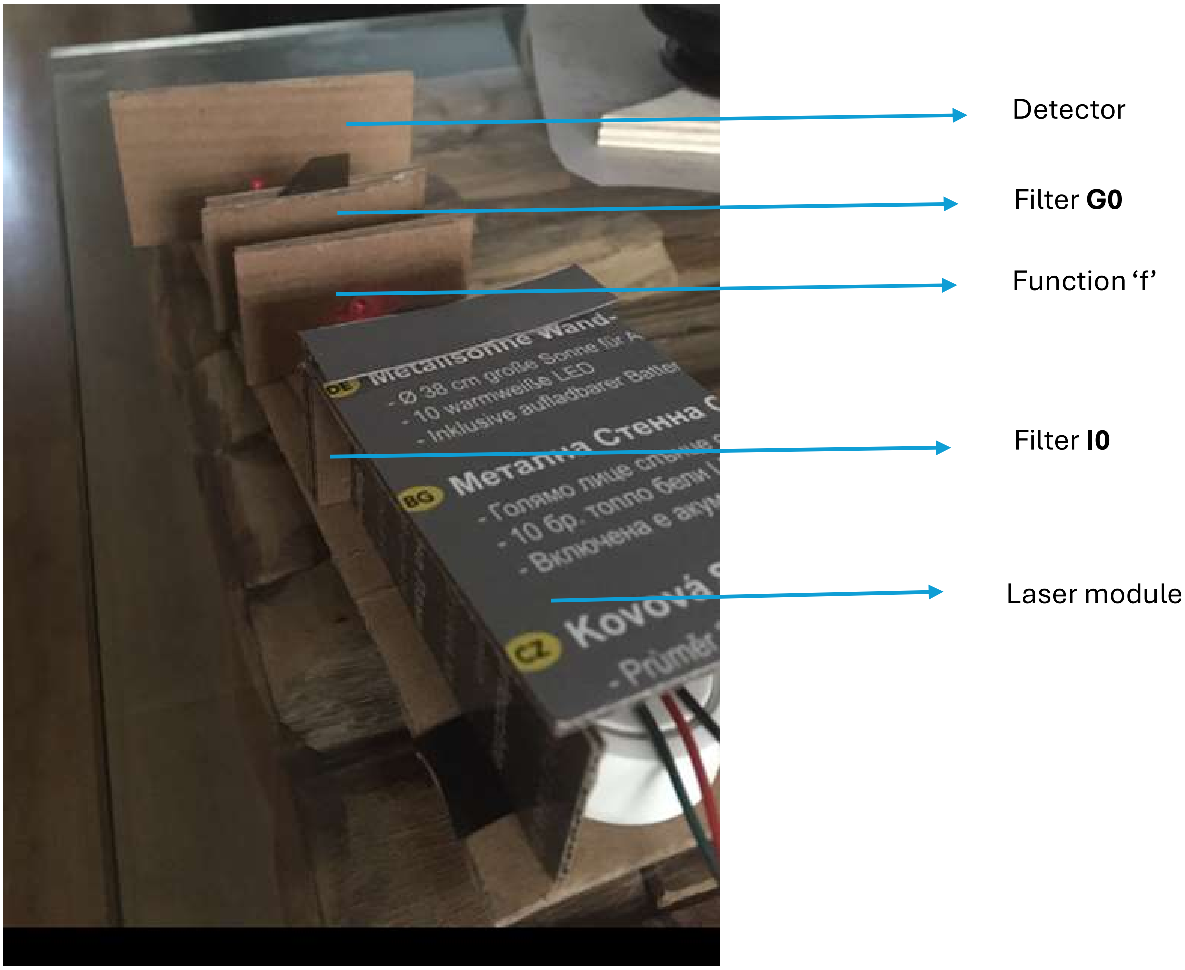
Orientation change, 0° / 180°	‘f’ is constant	Apply filter **G0** (*)
Orientation change, -90° / 90°	‘f’ is balanced	Apply filter **G0** (**)

(*)
Filter **G0** is a polarization filter that is applied to the result of function ‘f’, and the filter’s orientation is equal to 45° (in fact, it is <u>chosen</u> to be the same polarization orientation of filter **I0** (see section 1.1)). Since the orientation of polarization is (visibly) <u>not</u> changed, the result is that the filter G0 passes the result of I0 (and ‘f’); and produces a resulting laser pulse (a ‘1’ state)
If we do see a laser pulse at the end, the function is constant

(**), similar as for (*):
Since the orientation of polarization is <u>changed</u> by +/- 90°, the result is that the filter G0, as define above, blocks the resulting laser pulse; and produces <u>no</u> laser pulse (‘0’ state)
If we do <u>not</u> see a laser pulse at the end, the function is balanced

4. Hardware implementation

Experimental setup (breadboard) with laser compartment and filters I0 (directly after the laser), function ‘f’, and filter G0, as well as a detector (card at the end).



5. References

[0] Youtube movie: https://www.youtube.com/watch?v=muoIG732fQA

[1] Youtube movie: https://www.youtube.com/watch?v=tHfGucHtLqo