Category Theory Anki Study Document

Denis Erfurt

4. Januar 2018

Def. Category

- Objects: A, B, C, ...
- Arrows: f, g, h, ...
- for each $f : A \to B$
  domain: A = dom(f)
  codomain: B = cod(f)
- for $f : A \to B$, $g : B \to C$ with $cod(f) = dom(g)$
  composite of f and g: $g \circ f : A \to C$
- for each object A:
  identity arrow: $1_A : A \to A$
- Associativity: $h \circ (g \circ f) = (h \circ g) \circ f$
- Unit: $f \circ 1_A = f = 1_B \circ f$

Def. concrete Categories

Categories in which Objects are Sets, possibly equipped with some structure, and arrows are certain, possibly structure-perserving, functions.

Def. functor

Let **C**, **D** be categories, then $F : \mathbf{C} \to \mathbf{D}$ is a functor with:

1. $F(f : A \to B) = F(f) : F(A) \to F(B)$
2. $F(g \circ f) = F(g) \circ F(f)$
3. $F(1_A) = 1_{F(A)}$

Def. discrete categories

---

categories with only the identity arrows

Def. monoid

A set M with an associative binary operation $\cdot : M \times M \to M$ and unit element $u \in M$.

Def. isomorphism

In any category **C**, an arrof $f : A \to B$ is an isomorphism, if there is an arrow $g : B \to A$ in **C** such that.

$$g \circ f = 1_A \text{ and } f \circ g = 1_B$$

we write $f^{-1} = g$. A is isomorphic to B ($A \cong B$) if there exists an isomorphism between them.

Def. group

A group G is a monoid with an inverse g^{-1} for every element g.

Def. Free Monoid

A monoid M is **freely generated** by a subset A of M with:

1. **no junk**: every element $m \in M$ can be written as a product of elemens of A

   $$m = a_1 \cdot_M \dots \cdot_M a_n, \quad a_i \in M$$

2. **no noise**: No "nontrivial"relations hold in M: if $a_1 \dots a_n = a'_1 \dots a'_n$ then this is required by the axioms for monoids.

Def. Universal Mapping Propperty M(A)

---

given $i : A \to |M(A)|$, Monoid N and $f : A \to |N|$ there is a unique monoid homomorphism $\bar{f} : M(A) \to N$ s.t. $|\bar{f}| \circ i = f$