Category Theory Anki Study Document

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3. Januar 2018

Def. Category

- Objects: A, B, C, ...
- Arrows: f, g, h, ...
- for each $f : A \to B$
  domain: A = dom(f)
  codomain: B = cod(f)
- for $f : A \to B, g : B \to C$ with $cod(f) = dom(g)$
  composite of f and g: $g \circ f : A \to C$
- for each object A:
  identity arrow: $1_A : A \to A$
- Associativity: $h \circ (g \circ f) = (h \circ g) \circ f$
- Unit: $f \circ 1_A = f = 1_B \circ f$

Def. concrete Categories

Categories in which Objects are Sets, possibly equipped with some structure, and arrows are certain, possibly structure-perserving, functions.

Def. functor

Let $\mathbf{C}$, $\mathbf{D}$ be categories, then $F : \mathbf{C} \to \mathbf{D}$ is a functor with:

1. $F(f : A \to B) = F(f) : F(A) \to F(B)$
2. $F(g \circ f) = F(g) \circ F(f)$
3. $F(1_A) = 1_{F(A)}$

Def. discrete categories

categories with only the identity arrows

Def. monoid

A set M with an associative binary operation $\cdot : M \times M \to M$ and unit element $u \in M$.

Def. isomorphism

In any category $\mathbf{C}$, an arrof $f : A \to B$ is an isomorphism, if there is an arrow $g : B \to A$ in $\mathbf{C}$ such that.

$$g \circ f = 1_A \text{ and } f \circ g = 1_B$$

we write $f^{-1} = g$. A is isomorphic to B ($A \cong B$) if there exists an isomorphism between them.