Category Theory Anki Study Document

Denis Erfurt

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Def. Category

- Objects: A, B, C, ...
- Arrows: f, g, h, ...
- for each $f : A \to B$
  domain: A = dom(f)
  codomain: B = cod(f)
- for $f : A \to B$, $g : B \to C$ with $cod(f) = dom(g)$
  composite of f and g: $g \circ f : A \to C$
- for each object A:
  identity arrow: $1_A : A \to A$
- Associativity: $h \circ (g \circ f) = (h \circ g) \circ f$
- Unit: $f \circ 1_A = f = 1_B \circ f$

Def. concrete Categories

Categories in which Objects are Sets, possibly equipped with some structure, and arrows are certain, possibly structure-perserving, functions.

Def. functor

A "homomorphism of categories" is called a functor. Let **C**, **D** be categories, then $F : \mathbf{C} \to \mathbf{D}$ is a functor with:

1. $F(f : A \to B) = F(f) : F(A) \to F(B)$
2. $F(g \circ f) = F(g) \circ F(f)$
3. $F(1_A) = 1_{F(A)}$

Def. discrete categories

categories with only the identity arrows

Def. monoid

A set M with an associative binary operation $\cdot : M \times M \to M$ and unit element $u \in M$.

Def. isomorphism

An abstract categroy In any category **C**, an arrow $f : A \to B$ is an isomorphism, if there is an arrow $g : B \to A$ in **C** such that.

$$g \circ f = 1_A \text{ and } f \circ g = 1_B$$

we write $f^{-1} = g$. A is isomorphic to B ($A \cong B$) if there exists an isomorphism between them.

Def. group

A group G is a monoid with an inverse g^{-1} for every element g.

Def. Free Monoid

A monoid M is **freely generated** by a subset A of M with:

1. **no junk**: every element $m \in M$ can be written as a product of elemens of A

$$m = a_1 \cdot_M \dots \cdot_M a_n, \quad a_i \in M$$

2. **no noise**: No "nontrivial"relations hold in M: if $a_1 \dots a_n = a'_1 \dots a'_n$ then this is required by the axioms for monoids.

Def. Universal Mapping Propperty M(A)

given $i : A \to |M(A)|$, Monoid N and $f : A \to |N|$ there is a unique monoid homomorphism $\bar{f} : M(A) \to N$ s.t. $|\bar{f}| \circ i = f$

Def. Monoid In Terms Of A Category

A one object category, where the composition of morphisms satisfy the exact same associative and untial laws that the monoid operation needs to.

Cayley's theorem regarding abstract groups

Any abstract group can be represetned as a "concrete" one, that is, a group of permutations of a set. Gerneralized to any category that is not "too big" can be represetned as one that is "concrete," that is, a category of sets and functions.
while not every category has special sets and functions as its objects and arrows, every category is isoomorphic to such a one.

The opposite (or "dual") category C^{op} of a category C

Has the same objects as **C**, and an arrow $f : C \to D$ in C^{op} is an arrow $f : D \to C$ in **C**. C^{op} is **C** with all the arrows turned around.

Def Monomorphism $f : A \rightarrowtail B$

If given any $g, h : C \to A$, $fg = fh$ implies $g = h$

$$C \underset{h}{\overset{g}{\rightrightarrows}} A \xrightarrow{f} B$$

*related to injective (1-to-1)

Def Epimorphism $f : A \twoheadrightarrow B$

If given any $i, j : B \to D$, $if = jf$ implies $i = j$

$$f : A \xrightarrow{f} B \underset{j}{\overset{i}{\rightrightarrows}} D$$

*related to surjective (onto)

Def. initial object 0 in a category **C**

there exists a unique morphism: $0 \to C$

Def. terminal object 1 in a category **C**

there exists a unique morphism: $C \to 1$