

Given a tool of radius r , determine the closest distance of approach d to the corner P_n , without the tool going outside the edges $P_n P_{n-1}$ and $P_n P_{n+1}$. If the distance d is greater than some threshold ϵ , output the polygon $V_1 P_n V_2$ for potential further processing with a smaller diameter tool.

First determine the angle α between one of the edges and the bisector of the angle between the edges. From the unit vectors \mathbf{u}_1 and \mathbf{u}_2 along the edges, the bisector is along the vector:

$$\mathbf{u} = \hat{\mathbf{u}}_1 + \hat{\mathbf{u}}_2 \quad (1)$$

$$\text{and } \alpha = \arccos(\hat{\mathbf{u}} \cdot \hat{\mathbf{u}}_1) \quad (2)$$

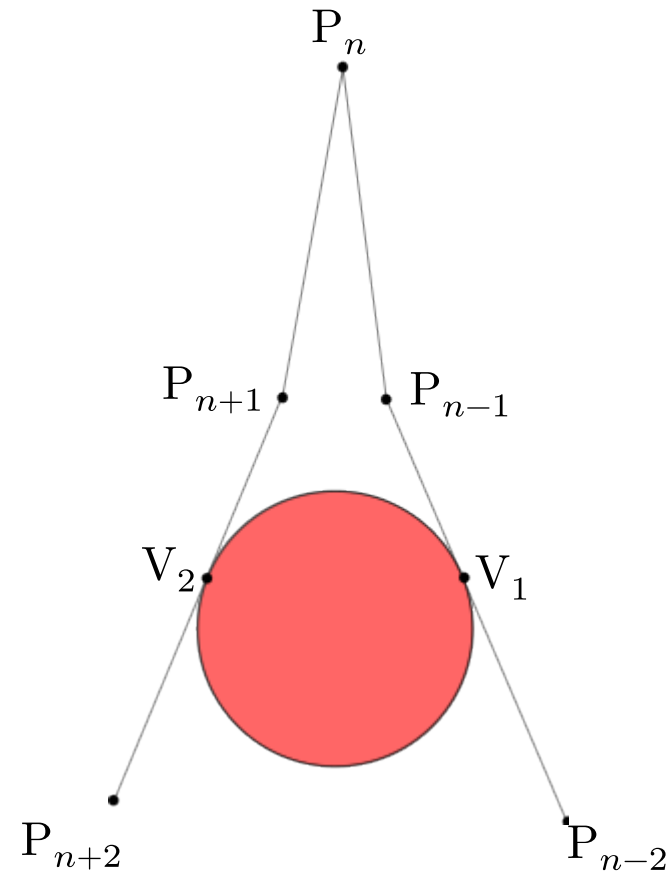
Then from the right triangle $cV_1 P_n$:

$$d = \frac{r}{\sin(\alpha)}. \quad (3)$$

The tangency points V_1 and V_2 can then be determined:

$$\mathbf{V}_1 = \mathbf{P}_n + d \cos(\alpha) \hat{\mathbf{u}}_1 \quad (4)$$

$$\mathbf{V}_2 = \mathbf{P}_n + d \cos(\alpha) \hat{\mathbf{u}}_2 \quad (5)$$



It might happen that the angle at P_n is too narrow or the tool diameter is too large for the tangency points to be on the corner edges. In that case the tangency points with the preceding and following edges will have to be found, and if those tangency points are also outside the edges, for the process to continue recursively.

If the point of intersection of the line segments $P_{n-1}P_{n-2}$ and $P_{n+1}P_{n+2}$ is found then the tangency points V_1 and V_2 can be found as before. The polygon $V_1, P_{n-2}, P_n, P_{n+1}, V_2$ is then output.

While obviously FreeCAD will have code to give the intersection of two line segments, it was not easy for me to locate the API, and I thought it was easy enough and faster to provide a small 2D geometry module to use in the computation. As a reminder, the parametric equation of a line segment from P_{n-1} to P_{n-2} and P_{n+1} to P_{n+2} are:

$$\mathbf{r}_1(t_1) = \mathbf{P}_{n-1} + t_1 (\mathbf{P}_{n-2} - \mathbf{P}_{n-1}) \quad 0 < t_1 < 1 \quad (1)$$

$$\mathbf{r}_2(t_2) = \mathbf{P}_{n+1} + t_2 (\mathbf{P}_{n+2} - \mathbf{P}_{n+1}) \quad 0 < t_2 < 1 \quad (2)$$

The intersection point is obtained by solving

$$\mathbf{r}_1(t_1) = \mathbf{r}_2(t_2) \quad (3)$$

for t_1 and t_2 .

Making use of the unit vectors $\hat{\mathbf{u}}_1$ along $P_{n-1}P_{n-2}$ and $\hat{\mathbf{u}}_2$ along $P_{n+1}P_{n+2}$ we get

$$t_1 = \frac{(\mathbf{P}_{n+1} - \mathbf{P}_{n-1}) \cdot \hat{\mathbf{u}}_{2\perp}}{(\mathbf{P}_{n-2} - \mathbf{P}_{n-1}) \cdot \hat{\mathbf{u}}_{2\perp}} \quad (1)$$

where $\hat{\mathbf{u}}_{2\perp}$ is a unit vector perpendicular to $\hat{\mathbf{u}}_2$. One can then find the intersection point from 2-1 and then proceed to find the tangency points V_1 and V_2 .