COMP20003
Algorithms and Data Structures
Dictionaries and Data Structures

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Semester 2

So far…

- We have:
  - Looked at algorithms, fast and slow.
  - Estimated computation time by counting operations.
  - Formalized a system for classifying algorithm efficiency.

Outline of the first few lectures

- Algorithms: general
- This subject: details
- Algorithm efficiency: intuitive
- Computational complexity
- Data structures
  - Basic data structures
  - Algorithms on basic data structures
  - Complexity analysis of algorithms on basic ds's

Textbook

- Skiena: Chapter 3, Data Structures

This section

- A lightning tour of fundamental data structures used for search:
  - Arrays
  - Linked Lists
  - Trees

Abstract Data Types *vs.* Data Structures

- *Abstract* data type: what it does
  - Stack, queue.
  - Dictionary: look up by key.
  - Does not specify an implementation
- *Concrete* data structure:
  - Array, linked list, tree
  - Implements an abstract data type.

Data structures

- Organizing data is important
- It is helpful to organize with the task in mind
- For searching, e.g.:
  - Some high-level languages have inbuilt "dictionaries", or associative arrays (Python, awk).
  - In lower-level languages, dictionaries are implemented directly using a fundamental data structure.

Searching

- Search Question:
  - Given a search key,
  - Find the record(s) that correspond to this key.
  - Typically we describe record simplistically with fields key and info (or just key).
- Examples:
  - Students and seat numbers, Telephone books

How you organize data is important for search!

Dictionaries

- Python built-in dictionary structure.

```
>>> tel = {'jack': 4098, 'sape': 4139}
>>> tel['guido'] = 4127
>>> tel
{'sape': 4139, 'guido': 4127, 'jack': 4098}
>>> tel['jack']
4098
```

- The dictionary is implemented using one of the underlying data structures.

Outline of the first few lectures

- Algorithms: general
- This subject: details
- Algorithm efficiency: intuitive
- Computational complexity
- Data structures
  - Basic data structures
  - Algorithms on basic data structures
  - Complexity analysis of algorithms on basic ds's

Array

- Def: given an index (location), we can retrieve any item in unit time.
- If items are in arbitrary order, finding a key in array of size *n* requires O(*n*) time.

Mimics the structure of random access memory (RAM), where the index is the memory address

Arrays in C (lightening review)

```
int  A[10];
```

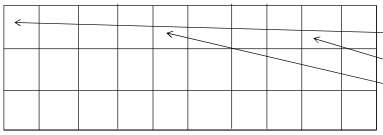
To access the value of the first element of the array:

```
A[0]
*A or *(A+0)
```

To access the value of the sixth element:

```
A[5]
*(A+5)
```

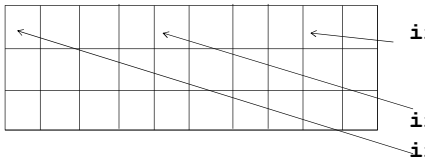
Random Access Memory



source: https://jdoodle.com/a/4eW

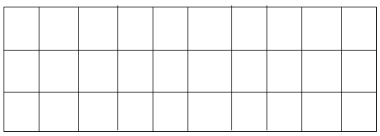
```
#include <stdio.h>
int main()
{
        int i,j,k;
        i = 5; j = 10; k = 15;
        printf("i: value = %d; address = %d\n", i, &i);
        printf("j: value = %d; address = %d\n", j, &j);
        printf("k: value = %d; address = %d\n", k, &k);
}
```


Random Access Memory




```
i: value =  5; address = 134509652
j: value = 10; address = 134509648
k: value = 15; address = 134509644
```

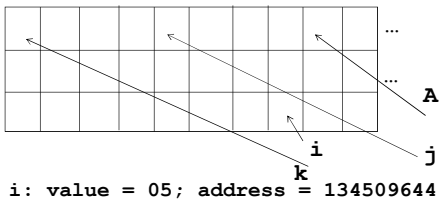

Random Access Memory



source: https://jdoodle.com/a/4eX

```
int i;
int A[100];
int j,k;
i = 5; j = 10; k = 15;
printf("i: value = %.2d; address = %d\n", i, &i);
printf("A:          ; address = %d\n", A );
printf("j: value = %.2d; address = %d\n", j, &j);
printf("k: value = %.2d; address = %d\n", k, &k);
```

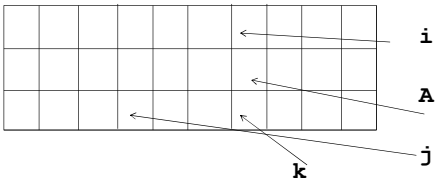

Random Access Memory




```
i: value = 05; address = 134509644
A:           ; address = 134509232
j: value = 10; address = 134509228
k: value = 15; address = 134509224

Element address =
  Addr Base + index * element_size
```

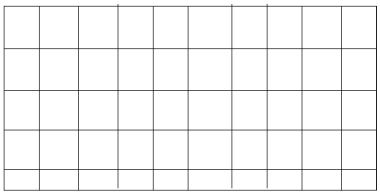
Random Access Memory



Although we often draw RAM to *look* like a 2-dimensional array,

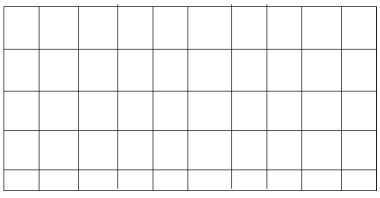
- *it is actually a linear (1-dimensional) array.*

A 2-Dimensional Array in C



- int A[rows][cols]
- int A[5][10]

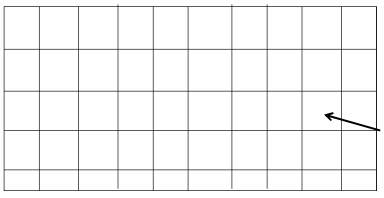
A 2-Dimensional Array in C



- int A[5][10]

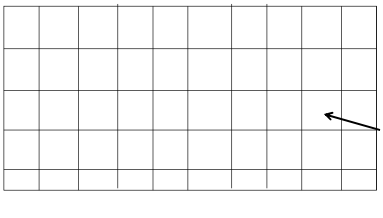
Actually, `A` is an array of size 5,
each element of `A` is an array of 10 ints.

A 2-Dimensional Array in C



- Address of element[thisrow][thiscolumn] =
  Address Base + thisrow * num_cols * sizeof(element)
  + thiscol * sizeof(element)

A 2-Dimensional Array in C



- Address of element[thisrow][thiscolumn] =
  Address Base + thisrow * num_cols * sizeof(element)
  + thiscol * sizeof(element)
- num_cols*sizeof(element) = size of one row

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Homework

- What is the difference between:
  - `int a[10][20];`
  - `int *b[10];`
- `? 2d in array vs 1d pointer to int array`
- Hint 1: Think memory allocation.
- Hint 2: See K&R section 5.9.
- Hint 3: How could you test your answer?

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Back to arrays as dictionaries

- To sort or not to sort?
- How to determine which is best?

  Use big-O analysis!

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Sorting: fine points

- Sorting assumes the keys are "sortable", *i.e.* comparable. E.g.:
  - *Comparable:* categories,
  - Not *Comparable*: colours, unless you associate an identifier.
- The CS definition of sorting means to put things into a *well-defined* order.
  - Other ways of sorting without comparing keys: counting sort. (next lectures)

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Sorting: fine points

- In CS applications of sorting, there is a *key*, and associated *information*.
- We sort by *key*, and the *information* comes along for the ride.
  - Student database, sorted on student ID.
  - Information: name, address, degree, *etc.*
- In our examples, we often do not show the *information* explicitly.

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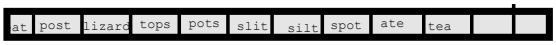
Unsorted arrays

- Just put the item at the end of the array:
  - Insertion is in $O(1)$



- How many comparisons you need to insert? NONE

Unsorted array: search

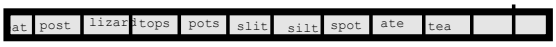
```
int i, cmps=0;    int EMPTY = 0;  int searchkey = 7;
int A[10] = {1,2,3,4,5,6,7,8,9,0};

for(i=0; A[i]!=EMPTY; i++)   /*A[] is an array of integers */
{
   cmps++;
   if (A[i] == searchkey)
   {
      printf("Found key %d at position %d\n", searchkey,i);
      printf("Key comparisons: %d\n", cmps);
      printf("FOUND");
      return 0;
   }
}
```

Source: https://jdoodle.com/a/4f1

Unsorted arrays

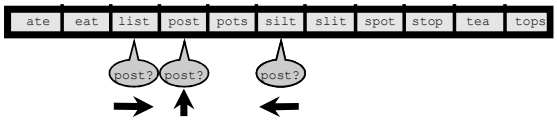
- Just put the item in at the end of the array:
  - Insertion is in $O(1)$



- What is the complexity of search?
  - O(n)

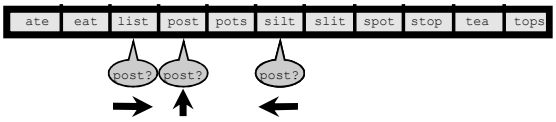
Sorted arrays

- How do you search for “post”?



Sorted arrays

- Binary search in sorted array:



Sorted array: binary search

```
int start = 0;
int end = ARRAYSIZE-1;
while(start <= end)
{
   cmps++;
   mid = (start + end ) / 2; //DIVIDE: Get the middle position
   if(A[mid]==searchkey)
   {
      printf("Found key %d at position %d, comparisons: %d \n",searchkey,start,cmps);
      printf("FOUND");
      return 0;
   }
   //If middle postion smaller than key, then update the end, discard A[mid,..,end]
   if (searchkey < A[mid])
      end = mid-1;
   else // otherwise, updat the beggining and discard A[start,..,mid]
      start = mid+1;
}
```

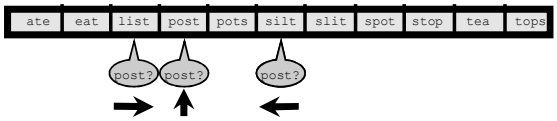
Source: https://jdoodle.com/a/4f3

Searching: analysis

- Unsorted array: O(n)
- Sorted array (binary search): O(logn)

Sorted arrays

- Binary search in sorted array:
  - Search is in O(logn)



  - What about insertion? O(n)

Sorted arrays: insertion

```
int i = ArrayFill-1;
// Shift values to the right until finding the correct position,
while(A[i]>INSERTNUM && i>=0)
{
   A[i+1] = A[i];
   i--;
}
/** only get here when A[i] <= INSERTNUM, i.e. have already moved
* previous contents
**/
A[i+1] = INSERTNUM;

/* to accomodate the new item */
ArrayFill++;

/* Any Assumptions */
```

Source: https://jdoodle.com/a/4fj

Sorted and unsorted arrays

Key comparisons:

- usually the most expensive operation in searching.

Analysis

- Search:
  - Unsorted array: O(n) per search
  - Sorted array (binary search): O(log n) per search
- Insert:
  - Unsorted array: O(1) per insertion
  - Sorted array: O(n) per insertion

Search: m lookups on n items

- Unsorted array, linear search:
  - n insertions @ 1 operation → n operations, O(n)
  - m lookups @ n operations → m*n (worst case)
  - O(n + m*n) = O(mn)
- Sorted array, binary search:
  - n insertions @ n comparisons and n data movements each → $O(n^2)$
  - m searches @ log n comps each -> $m*\log_2 n$
  - $O(n^2 + m \log n)$

Search: m lookups on n items

- Unsorted array, linear search:
  - n + m*n ~ m*n
- Sorted array, binary search:
  - $n^2 + m \log n$
- For m << n, unsorted arrays are better!
- But usually m > n, so use sorted array
  - Or even something better!?

Something better?

- What are the worst properties of a sorted array?

Limited size

- We can overcome the limited size problem using dynamic memory allocation
- C library functions:
  - void *calloc(size_t nobj, size_t size)
  - void *realloc(void *p, size_t size)
  - also, of course void *malloc(size_t size)
  - All defined in `stdlib.h`

malloc(): size_t

- malloc(size_t size)
- size_t is:
  - an unsigned integer type
  - the type returned by the **sizeof** operator
  - widely used in the standard library (**stdlib**) to represent sizes
- *e.g.* malloc(**sizeof**(int))

malloc() example (part 1)

```
#define NUMBER 5
int main (argc, argv)
{
        int   var;
        var = NUMBER;
        printf("%d - %d\n", &var, var);
        return 0;
}

>a.out
134509940 - 5
```


malloc() example: (part 2)

```
#include<stdlib.h>
#include<stdio.h>
#define NUMBER 5
int main ()
{
   int* ptr;
   ptr =  (int *)malloc(sizeof(int));
   *ptr = NUMBER; /* note '*' */
   printf("%d - %d\n", ptr, *ptr);
   return 0;
}
```

```
>a.out
134613280 - 5
```

Source: https://jdoodle.com/a/4fjm

malloc():check return value

- Be aware: malloc() can fail!
  - If malloc() fails, it returns NULL
  - Never use a pointer to something where the memory allocation has failed!

```
int* B;
B = (int*) malloc( NUMBER * sizeof(int));
/* always check return value of malloc()*/
If( B == NULL )
{
     printf("malloc() error\n");
     exit(1);
}
```

→ write a function safemalloc() that does this

Getting memory for an array using malloc()

```
int A[NUMBER];
/**
 * while insertions < NUMBER array is OK
 * BUT… has a limit
 */
int* B;
/* always check return value of malloc()*/
if( (B = (int *) malloc( NUMBER * sizeof(int) )) == NULL )
{
        printf("malloc() error\n");
        exit(1);
}
/**
 * B can now be used like A
 * better to use calloc(NUMBER,sizeof(int))
 */
```

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Getting memory for an array using calloc()

```
int* B;
/* always check return value of malloc()*/
if( (B = (int*) calloc( NUMBER, sizeof(int) )) == NULL )
{
        printf("malloc() error\n");
        exit(1);
}

/* B now comes with each slot initialized to 0 */
```

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realloc()

```
/**
 * as previously, used malloc(),calloc()
 * RESIZE when insertions == NUMBER
 */
B = realloc( B, ( NUMBER * 2 ) * sizeof(int) );
/* should also check realloc() for NULL*/

/* now initialize new part of array */
for( i = NUMBER; i < NUMBER * 2; i++ )
        B[ i ] = NULL;
/* now we have a bigger array, first half copied from the old B  */
```

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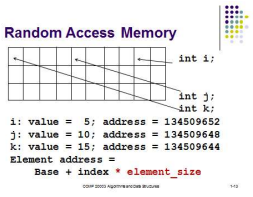
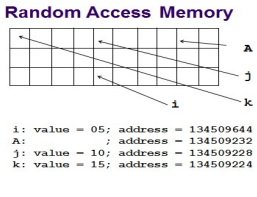
Details about malloc() and friends

- malloc() returns a pointer to a place in memory.
- Argument to malloc() specifies how much space to reserve in memory





What's a pointer?

- A pointer is an address in memory
- What is the output of this code?

```
int* ptr;
ptr = (int *)malloc(sizeof(int));
*ptr = 5;
printf("%d, %d", ptr, *ptr);
```

- Now what is the output of *this* code? Try it:https://jdoodle.com/a/4fP

```
int* ptr;
ptr = (int*) malloc(sizeof(int));
ptr = 5;
printf("%d, %d", ptr, *ptr);
```

- Int* p; or int *p; ? What the inventor of C++ thinks:
  - http://www.stroustrup.com/bs_faq2.html#whitespace

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Details about malloc() and friends

- #include<stdlib.h>
- Read the documentation for fine points:
  - `malloc()` returns uninitialized space
  - `calloc()` returns space initialized to 0
  - `realloc(void *p, size_t size)` returns space where the start is copied from p and the rest is unintialized.
- Check return value of all memory alloc functions!

malloc() and free()

- `malloc()` allocates memory.
- `free()` use to deallocate memory

```
void* ptr;
 ptr = malloc(NUMBER_OF_BYTES);
 /* do things until finished with the
    contents pointed to by ptr */
free(ptr);
```


Back to sorted arrays…

- Space limitations:
  - Can use `realloc()`.
  - Or can use linked list (sorted linked list).

More about Pointers

- For an excellent exposition of pointers in C, see the excellent tutorial by Ted Jensen:
  - LMS Resources → Pointers and Arrays in C

101 on Pointers

- Declaration: pointer * in C is an address
- Operators:

*	Value at Operator (dereferencing)	Gives Value stored at Particular address
&	Address of Operator	Gives Address of Variable

```
int k;
int* ptr;

k=5;
ptr = &k;
printf("%d", *ptr);
```

<5>

101 on Pointers

- A pointer in C is an address.
- When A is the name of an array, A is a pointer to the array, and
  - A[0] is equivalent to *(A+0),
  - A[5] is equivalent to *(A+5)…

Memory Allocation: Summary

- `malloc(),calloc(),and realloc()` return:
  - The (untyped) address of allocated memory;
  - *i.e.* a pointer to allocated memory.

```
/*allocates just enough room for an address */
struct node* ptr;
/* allocates enough room for the node */
ptr = (struct node*) malloc(sizeof(struct node));
```

We have discussed the strong and weak points of sorted arrays as data strucutres for search

Linked lists: flexibility, but overheads

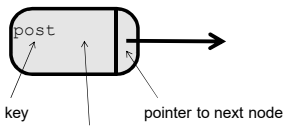
- In a linked list:
  - each item (or key) is located in an arbitrary place in memory,
  - with a link (pointer) to the next item
- Search Operations:
  - If unsorted, finding item is still $\Theta(n)$-time.
  - Once insertion point has been determined, easy to insert (or delete) a new item, by rearranging links.

Linked lists: flexibility, but overheads

- Takes extra space for each item in the list.
- Takes extra time to allocate the memory for the node for each item.

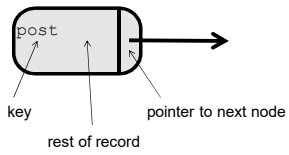
The node

```
struct node{
    record r;
    struct node *next;
    };
```



The node

```
typedef struct node{
        record r;
        struct node *next;
        } node_t;
```



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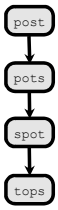
A linked list of nodes




```
struct node
{
   char* key;
   char* info;
   struct node* next;
};

struct node* newnode;

newnode = /*malloc space and put in the key and info */

/* suggested declaration style for beginner and intermediate */
```

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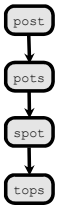
A (sorted) linked list of nodes




```
/* record struct contains key and other info */

typedef struct node{
        record r;
        struct node* next;
        } node_t;

typedef node_t* node_ptr;

node_ptr   newnode;

/* more advanced style */
```

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Traverse the list

```
p = listhead;
if(p!=NULL){ /* empty list */
   while( p->next !=NULL)
   {
      printf("%d\n", p->key);
      p = p->next;
   }
   printf("%d\n",p->key);
}
```

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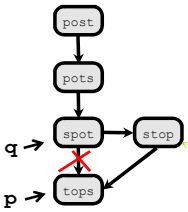
Traverse the list

```
p = listhead;
if(p==NULL)
{
   printf("List empty\n");
   return;
}

/* traverse and print key */
while( p->next !=NULL)
{
   printf("%d\n", p->key);
   p = p->next;
}
printf("%d\n",p->key);
```


Inserting a new node into a sorted linked list



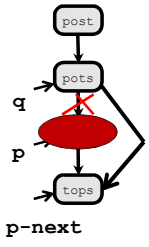

```
/* find correct place in list to insert */

/* how? */

/* then adjust pointers*/
q->next = newnode;
newnode->next=p;
```

newnode

Deleting a node




```
/* p points to the node we want to delete */
q->next = p->next;
free(p);
```

Fill the missing code: https://jdoodle.com/a/4fT

Linked lists: sorted *vs.* unsorted

- What are the advantages of keeping a linked list in sorted order?
  - Print in sorted order is trivial
  - Avg search n/2
- What are the disadvantages?
  - Insertion more costly

Search: Arrays vs. Linked Lists

- Sorted arrays:
  - Fast search (binary search), but
  - Slow insertion (keeping sorted order)
- Sorted array:
  - Fixed size, but
  - Can grow with realloc()
- Array needs (in general) only 1 memory allocation
  - Linked list needs many

Table of "running times"

	One Search	One Insert
Unsorted array	*n*	1
Sorted array	log *n*	*n*
Unsorted linked list	*n*	1
Sorted linked list	*n*	*n*

Exercize

How many operations are needed for *m* searches in a dictionary of *n* items?

	Each Insertion	Each Search	Build + Search
Unsorted array	O(1)	O(*n*)	***O(mn)***
Sorted array	O(*n*) comps + O(*n*) data movements	O(log *n*)	
Unsorted linked list	O(1)	O(*n*)	
Sorted linked list	O(n) comps	O(*n*)	

Practical complexity and algorithms

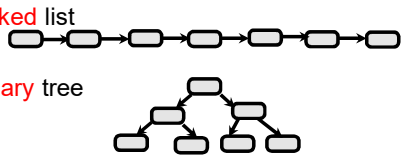
- O(1): Execute instructions once (or a few times), independent of input
  - Example: pick a lottery winner
- O(log n): keep splitting the input, and only operate on *one* section of the input.
  - Example: Binary Search
- O(n): Execute instruction(s) once for each item:
  - Example: search in sorted or unsorted array

Practical complexity and algorithms

- O(n log n): split the input repeatedly, and do something to *all* the segments
  - Example: Many sorting algorithms
- $O(n^2)$: For each item, do something to all the others. (Nested loops.)
  - Example:
    - Note: getting slow for large data…
- $O(n^3)$: matrix multiplication
- $O(2^n)$: combinatorial problems

Breaking out of linearity

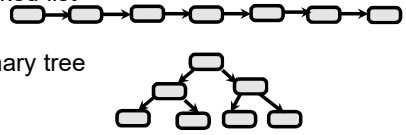
- Compare:
  - Linked list
  - Binary tree



- If we reliably know whether the desired item is in the left subtree or the right subtree, we could find it more quickly!

Breaking out of linearity

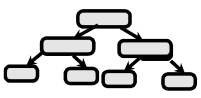
- Compare:
  - Linked list
  - Binary tree



- Note: for a complete binary tree, half the nodes are at the bottom level…

What is a complete binary tree?

- A binary tree is **complete** if **every level**, **except** possibly the **last**, is:
  - completely filled, and
  - **all nodes are as far left** as possible.



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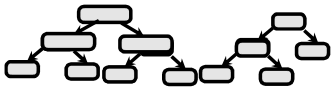

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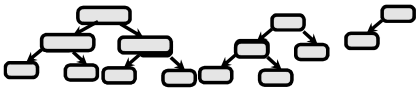

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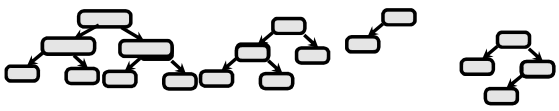
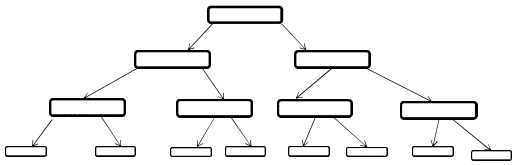


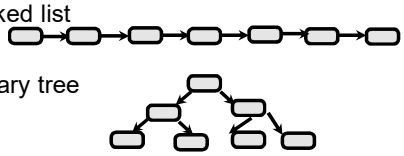
complete binary tree



A complete binary tree of n nodes has depth approximately $\log_2 n$

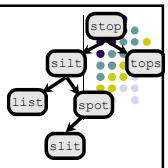
Breaking out of linearity

- Compare:
  - Linked list
  - Binary tree



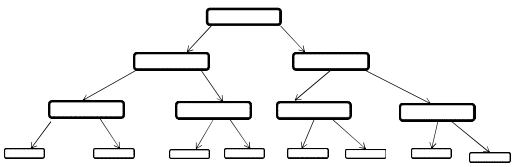
- As we will see, both insertion and search are O(log *n*) operations.

How a binary search tree work?



- In a sorted linked list:
  - `next` links to a record with a key ≥ this one.
- In a BST
  - `left` links to items with key < current key
  - `right` links to items with key ≥ current key.
- Insert node with key `slit` in this tree.

Looking at a complete binary tree



Nodes

2^0

2^1

2^2

2^3

….

$2^{\log n-1}$

For a complete binary tree of n nodes, to get to the bottom takes not more than $\log_2 n$ key comparisons

linked list, to get to the end, we need how many comparisons? n

Binary tree exercizes

- Put the following numeric keys into a bst:
  - 45, 37, 86, 90, 50, 16, 37
  - How long (how many key comparisons) does it take to search for key=5? 3
- Put the following numeric keys into a bst:
  - 90, 86, 50, 45, 37, 32, 16
  - How long does it take to search for key=5? 7

Best case run time in bst: Perfectly balanced tree

- Best case for BST: perfectly balanced
- Height of tree with n items: $\log_2 n$
- Path from root to any node:
  - Maximum length: $\log_2 n$
  - Average length: $\log_2 n$
- Insertion/search/deletion are all O(logn) for a well-balanced tree

Worst case run time in bst: Stick

- Worst case for BST: a stick.
  - e.g. when items are inserted in sorted order.
  - The BST degenerates to a linked list!
- Height of tree with n items: n
- Path from root to any node:
  - maximum length: n
  - average length: n/2
- Insertion/search/deletion are O(n)!

Binary search trees

- Deletion? Next lectures

Something to think about

- Why don't we just randomize the order of the items we insert into the binary search tree, to prevent worst case behavior?

If you've got the data before do it.

Binary search trees

- Good average case behavior – logn
- Bad worst case behavior – n
- So overall BST O(n).
  - Actual behavior usually not linear
  - But potentional linearity
- Balanced trees: AVL, red-black; 2,3,4; B+tree.

Dictionaries: Summary

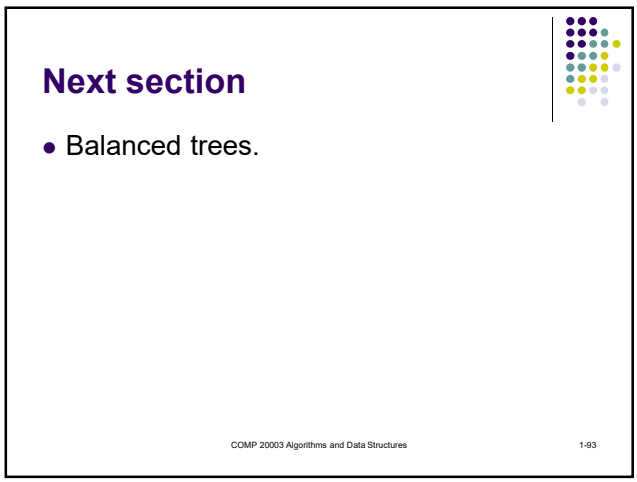
- We have looked at various underlying data structures for implementing dictionaries:
  -
  -
  -
  -
  -

Dictionaries: Summary

- We have analyzed the computational complexity for these data structures:
  -
  -
  -
  -
  -

Dictionaries: Summary

- So far the best we have done is log n search, where either:
  - Insertion is O(n); or
  - O(log n) average case but O(n) worst case.
- We can do better…


Next section
Balanced trees.
COMP 20003 Algorithms and Data Structures
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