COMP20003
Algorithms and Data Structures
Mergesort

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Mergesort

- Skiena Chapter 4.5

Merging

- We have two lists (stored as linked lists or arrays), each already in sorted order
- We would like to merge them into one sorted list that includes every element

3	5	6	8

1	2	4	7	9

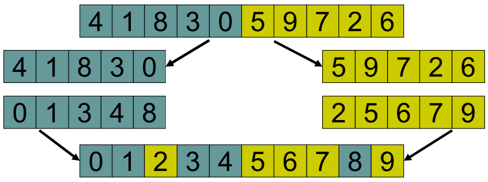
1	2	3	4	5	6	7	8	9

How do you merge?

- 2 linked lists or arrays
  - Two pointers (or indices): to smallest element
  - Compare elements pointed to
  - Output smallest and move pointer
- How many comparisons?
- Code…

Sorting using the merge operation

1. If list has one element, return
2. Split list into two equal-sized pieces (recursively, until singleton)
3. Sort each half
4. Merge two sorted halves



Merge Code

```
merge(item C[], item A[], item B[],
        int n, int m) /* n is size of A, m size of B */
{




}
```


Merge Code

```
merge(item C[], item A[], item B[],
        int n, int m) /* n is size of A, m size of B */
{
    int i,j,k;
    for( i=0,j=0,k=0; k < n+m; k++ )
    {
      /* shortcut at the end of A or B*/
      if(i == n) { C[k] = B[ j++ ]; continue; }
      if(j == m) { C[k] = A[ i++ ]; continue; }
      if(A[i] <= B[j]) C[k] = A[ i++ ];
      else C[k] = B[ j++ ];
     }
}
```


Mergesort: topdown (recursive)

```
main() {/* code */ mergesort(A,0,n-1); /* more code */

mergesort(A,first,last)
{
        if( first < last){
                int i;
                item B[], item C[];
                mid = (int)(last-first+1)/2;
                for(i=0;i<mid;i++) B[i] = A[i];
                for(i=mid;i<=last;i++) C[i-mid] = A[i];
                B = mergesort(B,0,mid-1);
                C = mergesort(C,0,mid-1);
                A = merge(B,C);
        }
}
```

Analyzing mergesort

- We are concerned with:
  - Accuracy
    - Does mergesort work?
    - Is it stable?
  - Efficiency
    - Does it take extra space? How much?
    - Analyze time efficiency using recurrences

Recurrences

- Recurrence relation "mathematical def'n":
  - an equation that recursively defines a sequence
  - each further term of the sequence is defined as a function of the preceding terms

Remember Fibonacci numbers (week 1)

Mergesort recurrences

Recurrence for number of comparisons:

- Cost of sorting n items =
  - 2*Cost of sorting n/2 items + merge n items
  - $C(n) = 2C(n/2) + n-1$ (worst case)
  - $C(1) = 0$

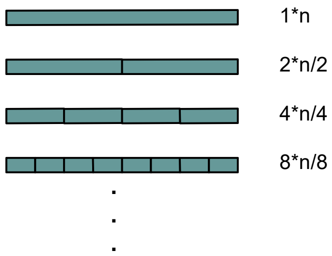
Solving recurrence

- Approximate n as power of 2:
  - $C(n) = 2C(n/2) + n-1$
  - $= 2[2C(n/4) + (n/2-1)] + (n-1)$
  - $= 4C(n/4) + (n-2) + (n-1)$
  - $= 8C(n/8) + (n-4) + (n-2) + (n-1)$
  - $= 16C(n/16) + (n-8) + (n-4) + (n-2) + (n-1)$
  - $2^k C(n / 2^k) + kn - (2^k - 1)$

Solving recurrence

- Approximate *n* as power of 2:
  - $2^k C(n / 2^k) + kn - (2^k - 1)$
  - What is k?
  - Base case: $(n / 2^k) = 1$
    - $n = 2^k$
    - $k = \log_2 n$
  - $n C(1) + n \log_2 n - (n - 1)$

Intuition



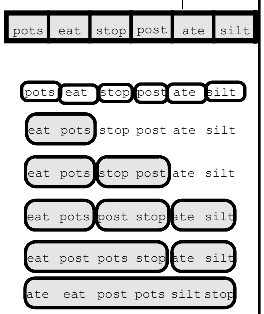
Mergesort: Top-down (recursive)

- Top-down mergesort works well with arrays
  - Also with linked lists (pointer to find midpoint of list)
- Worst case O(n log n)
- Average case O(n log n)
- Stable?
- Requires O(n) extra space

Bottom-up mergesort

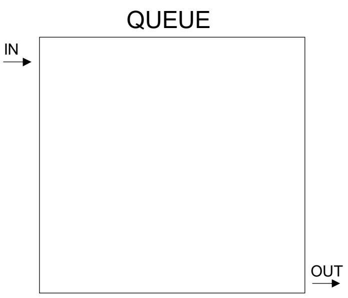
- Break list into *n* singleton lists
- Insert single lists into a queue
- deQueue the first two items, merge them, and enQueue them

Merged Items go at the end



Bottom-up mergesort Example

[4, 2, 5, 3, 0, 1]



Mergesort: Implementation

- Top-down mergesort (recursive)
- Bottom-up mergesort (iterative)
- Demos:
  - https://www.cs.usfca.edu/~galles/visualization/ComparisonSort.html
  - https://www.toptal.com/developers/sorting-algorithms
  - http://www.youtube.com/watch?v=XaqR3G_NVoo

Quicksort vs. Mergesort

	Quicksort	Mergesort
Compare during?	split	merge
Average O()	n log n	n log n
Worst O()	n^2	n log n
In-place sort?	yes	no
Stable sort?	no	yes

Mergesort: Summary

- Analysis similar for recursive and non.
  - Θ(n log n)
  - Stable
  - Reliable, and work with both arrays and lists
  - Can sort huge files on disk
    - Use disk fetching just the portions of data you need
- Would be the perfect sort, except that:
  - Arrays require O(n) extra space
  - Slower than quicksort in practical cases