COMP20003
Algorithms and Data Structures
Distribution Counting

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Semester 2

Why Sorting?

Census 1890 led to IBM
- 1960: ¼ CPU resources worldwide spent on sorting

- Pervarsive for human experience:
  - Digital: FB, Tweet, Email, Reedit, google search, etc.
  - Physical: Bills, papers, books, socks, etc.
  - Societal: Sports - grades - college ranking, credit rating, etc.
  - Animal Behaviour: group sorting, bigger groups -> confrontations, fade-out with time

Sort Is Prophylaxis for Search:
- CS: should you sort at all? All about trade-offs

Sort is a preemptive strike for future search
- Mess can be a bliss

Estimate ahead of time the future usage

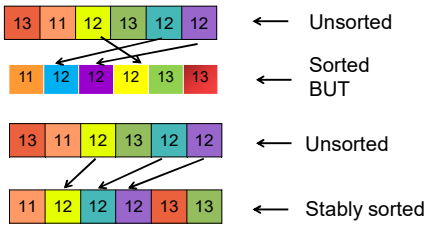
Sorting by Counting

- Distribution counting:
  - unusual approach to sorting
- Later we will look at more common approaches
- Distribution counting requires:
  - Key values to be within a certain range, *lower* to *upper*.

Sorting by Counting: Approach

- Steps in distribution counting:
  - Start with array of:
    - Records, or
    - Keys + pointers to records
  - Count number of records associated with each key value (*lower* to *upper*)
  - Redistribute array elements
- Net result:
  - Sorted array
  - Stable sort

Segue: What is a stable sort?

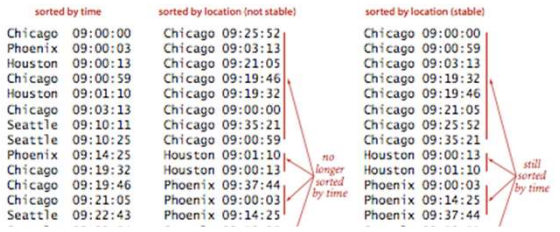


Stable sorting: definition

Stable sorting algorithms maintain relative order of records with equal key values.

Stable sorting: Applications

- Want file sorted on one key, and within each group, sorted on another key:



Example from Sedgewick and Wayne, Algorithms, 4th Edition, 2011

Back to Distribution Counting: Approach

- Steps in distribution counting:
  - Input: array of:
    - records, or
    - keys + pointers to records
  - Count number of records associated with each key value (*lower* to *upper*).
  - Redistribute array elements.
  - Output: stably sorted array.

Distribution Counting Example:

- Sort [4,4,2,2,0,2,1,3,2,4,3,1,4,3,1,4]
- Count records for each key [, , , ,]

  Pos 0 1 2 3 4
  - CumulativeCount = [, , , ,] (#items < pos_key)

    Pos 0 1 2 3 4
- Redistribute
  - Create auxiliary array
  - [, , , , , , , , , , , , , , ,]

    Pos 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
  - traverse original array copying each item to position:
    - aux_array[cumulativeCount[item.key]] = item
    - Increase cumulativeCount[itemkey] + 1

Back to Distribution Counting: Example:

- Sort [4,4,2,2,0,2,1,3,2,4,3,1,4,3,1,4]
- Count records for each key [1,3,4,3,5]
  - CumulativeCount = [0,1,4,8,11]
- Redistribute
  - Create auxiliary array
  - traverse original array copying each item to position:
    - aux_array[cumulativeCount[item.key]] = item
    - Increase cumulativeCount[itemkey] + 1

Distribution Counting: Analysis

- Time:
  - Worst-case:
  - Average-case:
- Space:

Does the key range influence the complexity?

- *O(n)* if range *r* of keys is in *O(n)*
  - `count[]` array size is *r*
  - Initialization and shuffling are *O(r)*
  - So if *r* > *n*…

But what about theory?

- we said weeks ago:
  - Comparison-based sorting is $\Omega(n \log n)$
- Does distribution counting contradict that statement?

Sorting without comparing

- Other non-comparison-based sorting algorithms include:
  - LSD Radix sort
  - MSD Radix sort
  - Several others
- Drawbacks:
  - Take extra space
  - Generally less flexible than comparison-based
  - Can be fiddly if keys are not the same length, *e.g.* variable length strings in MSD radix