COMP20003
Algorithms and Data Structures
Introduction to Graphs

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Semester 2

Application: Image synthesis

A. A. Efros & W. T Freeman. Image quilting for texture synthesis. SIGGRAPH 2002

The key to solving many algorithmic problems is to think of them in terms of graphs. Graph theory provides a language for talking about the properties of relationships, and it is amazing how often messy applied problems have a simple description and solution in terms of classical graph properties.

Designing truly novel graph algorithms is a very difficult task. The key to using graph algorithms effectively in applications lies in correctly modeling your problem so you can take advantage of existing algorithms. Becoming familiar with many different algorithmic graph *problems* is more important than understanding the details of particular graph algorithms, particularly since Part II of this book will point you to an implementation as soon as you know the name of your problem.

Here we present basic data structures and traversal operations for graphs, which will enable you to cobble together solutions for basic graph problems. Chapter 6 will present more advanced graph algorithms that find minimum spanning trees, shortest paths, and network flows, but we stress the primary importance of correctly modeling your problem. Time spent browsing through the catalog now will leave you better informed of your options when a real job arises.

S. S. Skiena. The Algorithm Design Manual.

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Graph definition

Graph:

- a representation of a set of objects
- and the relationships between them

Graph definition

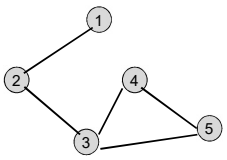
Graph G = {V,E} Set of:

- Vertices V: can contain information
- Edges E (links between vertices): can have direction and/or weight

Compared to trees and linked lists:

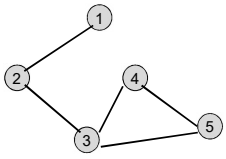
- vertices = nodes
- edges = links

Undirected graph



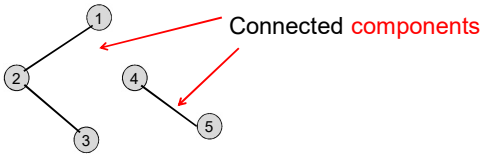
Edges have no direction specified

Connected Undirected graph

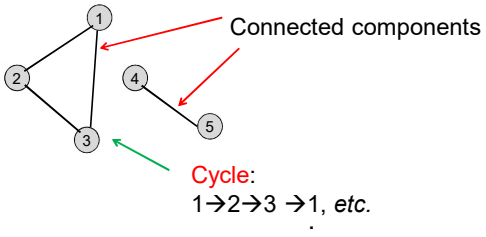


Every pair of vertices is connected (possibly indirectly)

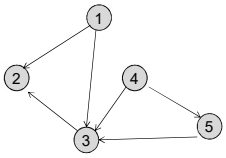
Unconnected Undirected graph



Unconnected Undirected graph with cycle

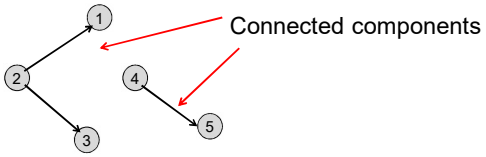


Directed graph



- Edge direction is specified
- Links are not symmetrical

Acyclic, unconnected directed graph



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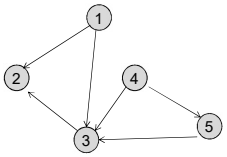
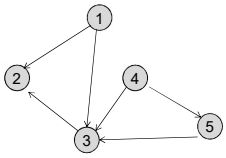
Directed graph



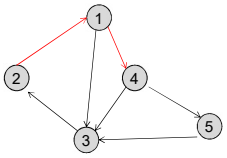
Reachability:
Can you get from Vertex 1 to Vertex 5?

Weakly connected Directed graph



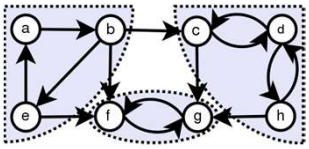
Replace all directed edges with undirected edges, to obtain a connected (undirected) graph

Strongly connected Directed graph



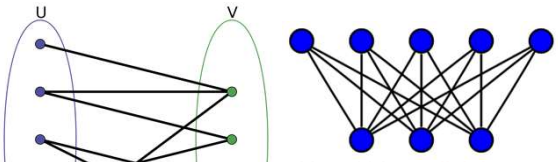
If every vertex is reachable from every other vertex

Strongly connected Components in a Digraph



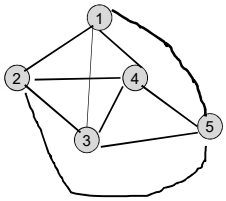
If every vertex is reachable from every other vertex within the same component

Bipartite Graph



- U and V are disjoint sets of vertexes
- Every vertex in U connects to a vertex in V, and viceversa

Complete graph



V(V-1)/2 edges

Trees and Graphs

Tree, undirected graph that is:

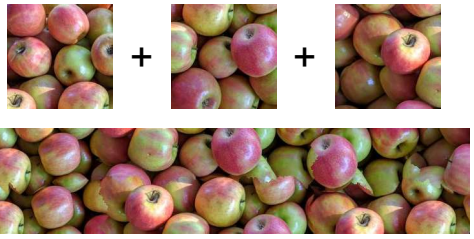
- Connected
- Acyclic

- *n.b.* Any two vertices are connected by exactly one simple path
- *n.b.* All vertices are connected

Demos…

Application: image synthesis

- How does it use graphs?



Application: image synthesis

- To find the best splice, represent overlapping images as a graph:
  - Edge weight = difference in colour
  - Problem = find minimum path