PHY 321, FEB 8, 2023

Conservative forces

(i) $\vec{F}(\vec{r})$

(ii) $\vec{\nabla}\times \vec{F}(\vec{r}) = 0$

(iii) $W_{12} = \int_{\vec{r}_1}^{\vec{r}_2} d\vec{r}\,\vec{F}(\vec{r})$

is independent of the path

(iv) $$\vec{F}(\vec{r}) = -\vec{\nabla} V(\vec{r})$$
$$= -\left(\frac{\partial}{\partial x} V(\vec{r})\,\vec{i} + \frac{\partial}{\partial y} V(\vec{r})\,\vec{j} + \frac{\partial}{\partial z} V(\vec{r})\,\vec{k}\right)$$

$\frac{dE}{dt} = 0$, Energy is conserved.

$$V = V(\vec{r})$$
$$E = \frac{1}{2} m v^2 + V(\vec{r})$$
$$= \frac{1}{2} m\, \vec{v}\cdot\vec{v} + V(\vec{r})$$
$$= \frac{1}{2} m \left[v_x^2 + v_y^2 + v_z^2\right] + V(\vec{r})$$

$$\frac{dE}{dt} = \frac{1}{2} m \left[2v_x \overset{a_x}{\frac{dv_x}{dt}} + 2v_y \overset{a_y}{\frac{dv_y}{dt}} + 2v_z \overset{a_z}{\frac{dv_z}{dt}} \right]$$

$$+ \underbrace{\frac{\partial V}{\partial x}}_{-F_x} \underbrace{\frac{dx}{dt}}_{v_x} + \underbrace{\frac{\partial V}{\partial y}}_{-F_y} \underbrace{\frac{dy}{dt}}_{v_y} + \underbrace{\frac{\partial V}{\partial z}}_{-F_z} \underbrace{\frac{dz}{dt}}_{v_z}$$

$$F_x v_x + F_y v_y + F_z v_z$$

$$- F_x v_x - F_y v_y - F_z v_z = 0!$$

Energy is conserved

in vector form

$$\frac{d}{dt}\left[\frac{m\vec{v}\cdot\vec{v}}{2} + V(\vec{r}) \right]$$

$$m \cdot \vec{v} \cdot \dot{\vec{v}} + \nabla(\vec{r})\, \dot{\vec{r}}$$

$$= \vec{v}\cdot\vec{F} - \vec{F}\cdot\vec{v} = 0$$

Ex3 hw4

$$V(x,y,z) = A \exp\left(\frac{-x^2-z^2}{2a^2}\right)$$

constant

$$\vec{\nabla} \times \vec{F} = \left(\frac{\partial}{\partial y} F_z - \frac{\partial}{\partial z} F_y \right) \vec{i}$$

$$+\left(\frac{\partial}{\partial z}F_x - \frac{\partial}{\partial x}F_z\right)\vec{j}$$

$$+\left(\frac{\partial}{\partial x}F_y - \frac{\partial}{\partial y}F_x\right)\vec{k}$$

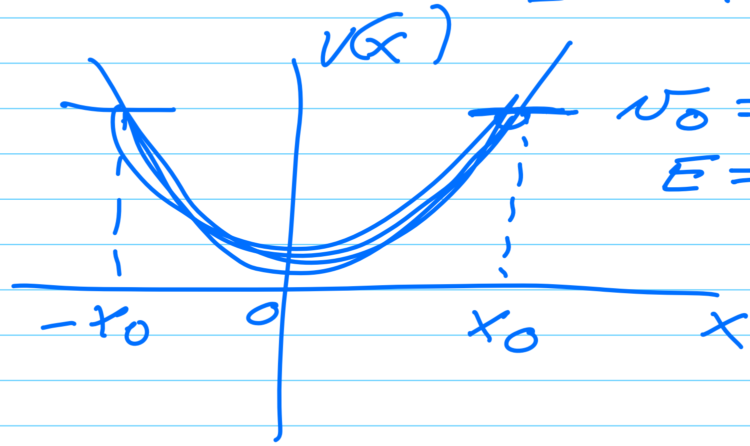
Ex 5 HW 4

$$V(x) = \frac{1}{2}kx^2$$

$$\vec{F} = -\vec{\nabla} V(\vec{r})$$

$$\vec{F} \Rightarrow F_x = -\frac{d}{dx}V(x)$$

$$= -kx$$

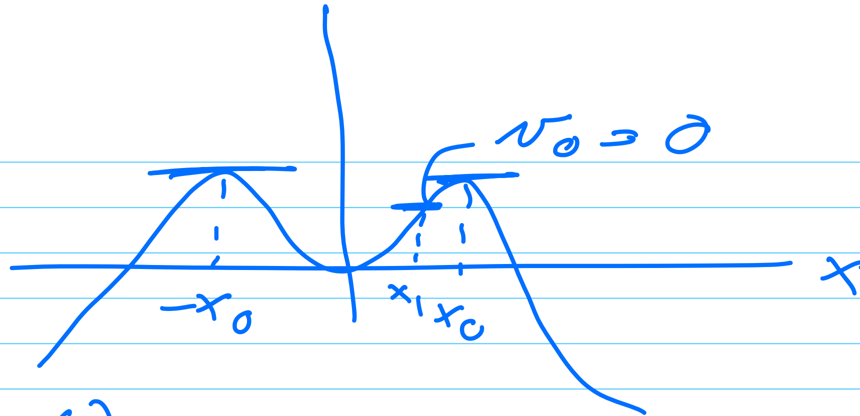


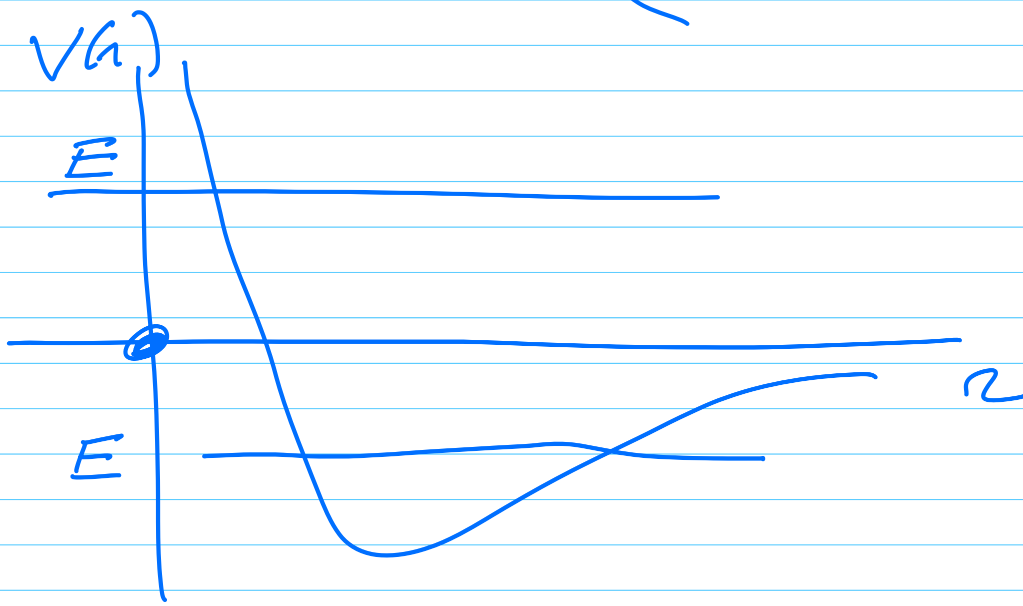
$$v_0 = 0$$

$$E = \frac{1}{2}kx_0^2$$

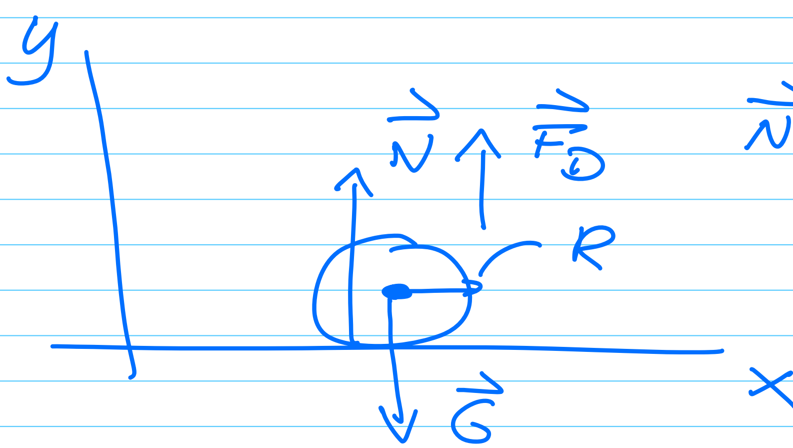
$$V(x) = \frac{kx^2}{2} - \frac{kx^4}{4\alpha^2}$$

$\alpha = 1.0$ $\quad k = 1.0$





Ex 6



$$\vec{N} = -k(R - y(t)) \times \vec{j}$$

$$\vec{N} = \begin{cases} -k\left(R - y(t)\right)\vec{j} & y(t) \leq R \\ 0 & y(t) > R \end{cases}$$