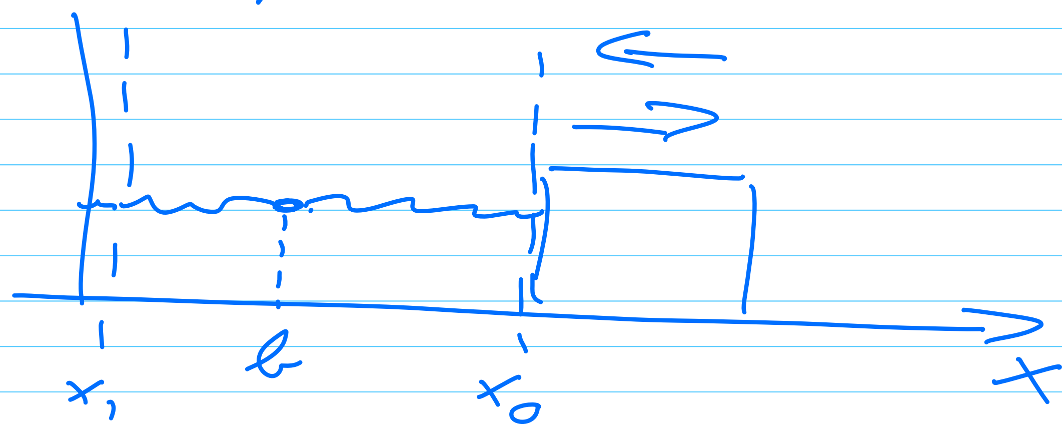
PHY321, FEB 13, 2023

Potential and Energy landscape.

- Limits of possible motion
- stable and unstable points



$$E = V(x) + \frac{1}{2} m v^2$$

WE - theorem

$$V(x) - V(x_0) = - \int_{x_0}^{x} F(x') dx'$$

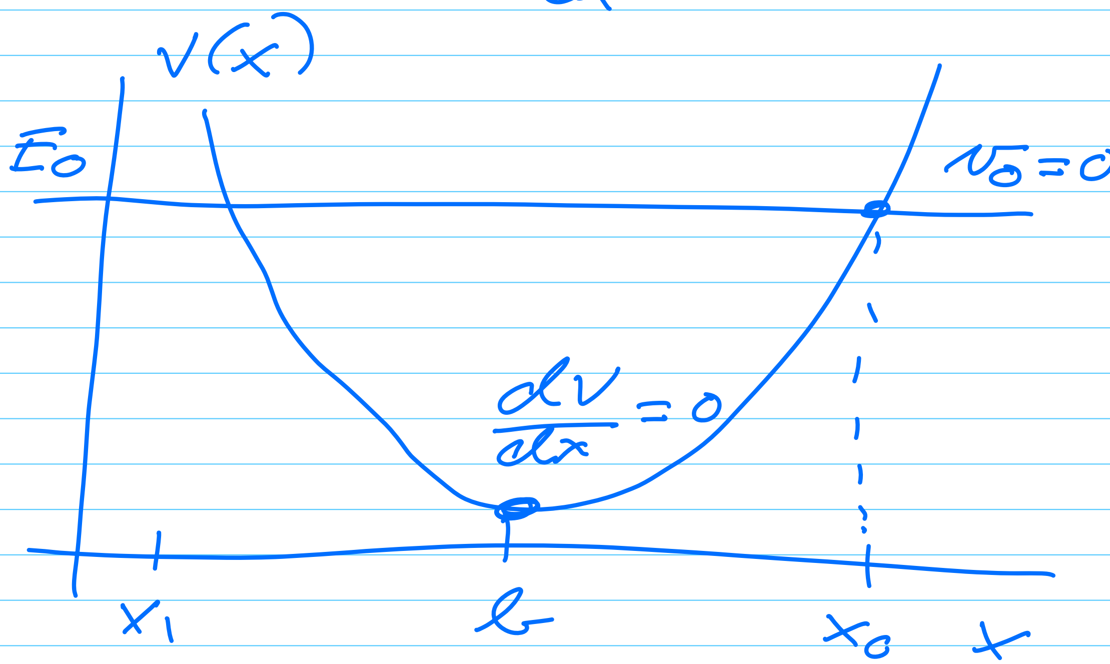
$$F(x) = -k(x-b)$$

$$V(x) - V(x_0) = \frac{1}{2} k (x-b)^2 - \frac{1}{2} k (x_0 - b)^2$$

$x_0 = b \Rightarrow$

$$V(x) = \frac{1}{2} k (x-b)^2$$

$$F(x) = -\frac{dV}{dx} = -k(x-b)$$



b = equilibrium point

V(x) is convex

$$\frac{d^2V}{dx^2} > 0$$

Assume small pertur-
bations around $x = b$

Taylor expand

$$V(x) = V(x-b) + (x-b)\frac{dV}{dx}\Big|_{x=b} + \frac{(x-b)^2}{2}\frac{d^2V}{dx^2}\Big|_{x=b} + O\left((x-b)^3\right)$$

$$V(x) \simeq V(x-b) + V(x-b)\frac{dV}{dx}\Big|_{x=b} + \frac{(x-b)^2}{2}\frac{d^2V}{dx^2}\Big|_{x=b}$$

$$V(x) = \underbrace{V(x-b)}_{\text{constant}} = V_0 + \frac{(x-b)^2}{2}\underbrace{\frac{d^2V}{dx^2}\Big|_{x=b}}_{>0} = k$$

$$V(x) = V_0 + \frac{(x-b)^2}{2}k$$

$$-\frac{dV}{dx} = F(x) = -k(x-b)$$

$$= m\frac{d^2x}{dt^2}$$

<u>Analysis</u>

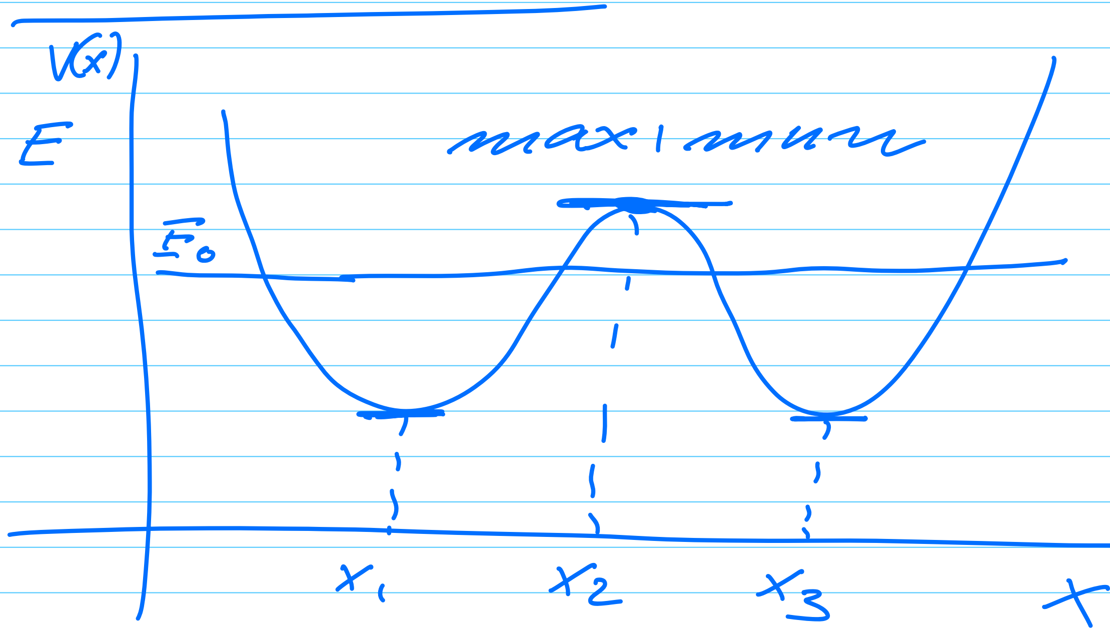
$x = x_0 > \ell \qquad \frac{dV}{dx} = k(x-\ell)$

> 0

$\Rightarrow \quad F < 0$

$F = m \cdot a \qquad a < 0$

<u>Example 2</u>



$V(x_1) = V(x_3) = 0$

at x_1 and x_3

$$\left.\frac{dV}{dx}\right|_{x=x_1} = \left.\frac{dV}{dx}\right|_{x=x_3}$$

$$= 0$$

$$\frac{dF}{dx} < 0 \, ,$$

local or global minima.

$$\left.\frac{dV}{dx}\right|_{x=x_2} = 0$$

$$\frac{dF}{dx} = -\frac{d^2V}{dx^2} > 0$$