PHY 321, FEB 18, 2022

Velocity Verlet

$$x(t+\Delta t) = x(t) + \Delta t \frac{dx}{dt} + \frac{(\Delta t)^2}{2}\frac{d^2x}{dt^2} + O(\Delta t^3)$$

$$\simeq x(t) + \Delta t\, v(t) + \frac{(\Delta t)^2}{2} a(t)$$

Discretize

$$x_{i+1} = x_i + \Delta t\, v_i + \frac{(\Delta t)^2}{2} a_i$$

$$v(t+\Delta t) = v(t) + \Delta t \frac{dv}{dt} + \frac{(\Delta t)^2}{2}\frac{d^2v}{dt^2} + O(\Delta t^3)$$

$$\simeq v(t) + \Delta t\, a(t) + \frac{(\Delta t)^2}{2}\frac{d^2v}{dt^2}$$

$$\frac{da}{dt} \simeq \frac{a_{i+1} - a_i}{\Delta t}$$

$$v_{i+1} = v_i + \Delta t\, a_i + \frac{(\Delta t)^2}{2} \times \left(\frac{a_{i+1} - a_i}{\Delta t} \right)$$

$$= v_i + \frac{\Delta t}{2} \left(a_{i+1} + a_i \right)$$

$$x_{i+1} = x_i + \Delta t\, v_i + \frac{\Delta t^2}{2} a_i$$

$$a_{i+1} = f(x_{i+1})$$

$$v_{i+1} = v_i + \frac{\Delta t}{2} \left(a_{i+1} + a_i \right)$$