PHY321, FEB 22, 2023

HOs

Harmonic oscillations (HOs)

$$F(x) = -kx = m\frac{d^2x}{dt^2}$$

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\omega_0 = \sqrt{k/m}$$

$$[k] = \text{mass}/\text{time}^2$$

$$[\omega_0] = \left[\sqrt{\frac{\text{mass}}{\text{mass time}^2}}\right]$$

$$= \frac{1}{\text{time}}$$

ω_0 = natural frequency

$$\frac{d^2x}{dt^2} = -\omega_0^2 x$$

ω_0 is real

$$x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

$$v_0 = v(t_0) = 0 \qquad t_0 = 0$$

$$x(t_0) = x_0$$

$$x(t_0) = x(0) = A \overbrace{\cos(\omega_0 \cdot 0)}^{=1} + B \underbrace{\sin(\omega_0 0)}_{=0}$$

$$= A = x_0$$

$$B = ?$$

$$\left.\frac{dx}{dt}\right|_{t=0} = v(t_0) = v_0 = 0$$

$$= -A\omega_0 \overbrace{\sin(\omega_0 \cdot 0)}^{=0} + B\omega_0 \underbrace{\cos(\omega_0 \cdot 0)}_{=1}$$

$$v_0 = 0 = B \cdot \omega_0$$

$$\omega_0 \neq 0$$

$$\Rightarrow \quad B = 0 \quad \Rightarrow$$

$$x(t) = x_0 \cos(\omega_0 t)$$

Period τ

$$x(\tau + t) = x(t)$$

$$\boxed{\tau = \frac{2\pi}{\omega_0}}$$

Energy Conservation

$v_0 = 0 \qquad x(t_0) = x_0$

$$E(t_0) = E_0 = \frac{1}{2} m v_0^2 + \frac{1}{2} k x_0^2 = \frac{1}{2} k x_0^2$$

$$= E(t) = \frac{1}{2} m v^2(t) + \frac{1}{2} k x^2(t)$$

$$x(t) = x_0 \cos(\omega_0 t)$$

$$\frac{dx}{dt} = v(t) = -\omega_0 x_0 \sin(\omega_0 t)$$

$$E(t) = \frac{1}{2} m \omega_0^2 x_0^2 \sin^2(\omega_0 t) + \frac{1}{2} k x_0^2 \cos^2(\omega_0 t)$$

$$\omega_0^2 = k/m$$

$$= \frac{1}{2} k x_0^2 \left[\cos^2(\omega_0 t) + \sin^2(\omega_0 t)\right]$$

$$= \boxed{\frac{1}{2} k x_0^2 = E_0}$$

energy is conserved,

Damping

$$m \frac{d^2x}{dt^2} = -k x(t)$$

velocity dependent damping (Friction)

$$b \cdot \frac{dx}{dt} = b \cdot v$$

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

$$\left[m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F(t) \right]$$

$$a = \frac{d^2x}{dt^2} \qquad k/m = w_0^2$$

Divide by m

$$\frac{d^2x}{dt^2} + \frac{b}{m} \cdot \frac{dx}{dt} + w_0^2 x = 0$$

Introduce $\hat{t} = w_0 \cdot t$

Dimless

$$t = \hat{t}/w_0$$

$$w_0^2 \frac{d^2x}{d\hat{t}^2} + \frac{b\,w_0}{m}\frac{dx}{d\hat{t}} + w_0^2 x = 0$$

$$w_0 \neq 0$$

Divide by w_0^2

$$\frac{d^2x}{d\hat{t}^2} + 2\gamma \frac{dx}{d\hat{t}} + x = 0$$

$$\gamma = \frac{b}{2m w_0}$$

$$a(\hat{t}) + 2\gamma v(\hat{t}) + x(\hat{t}) = 0$$

$$a(\hat{t}) = -x(\hat{t}) - 2\gamma v(\hat{t})$$