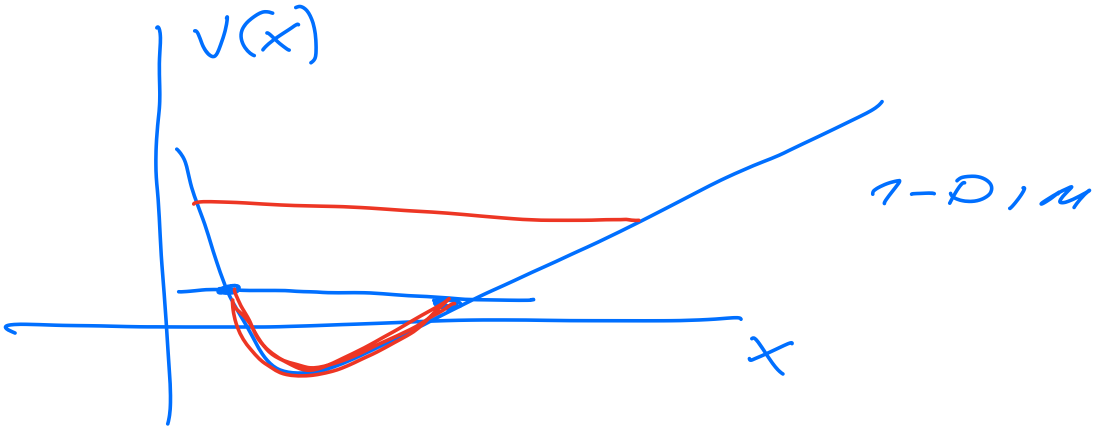
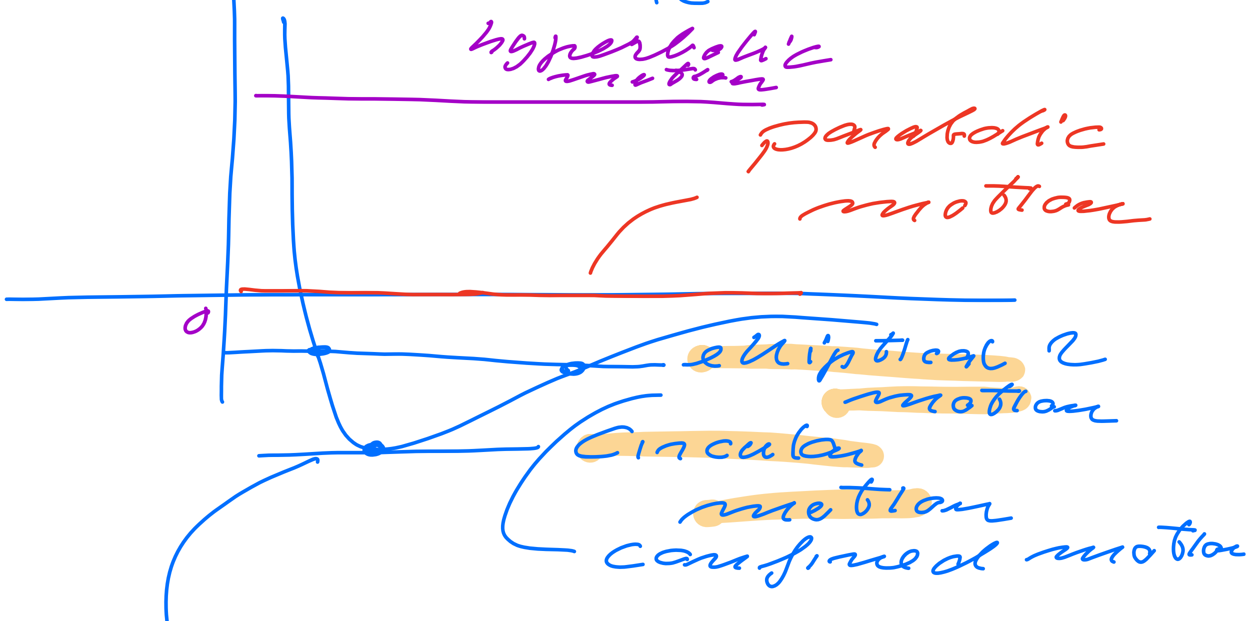
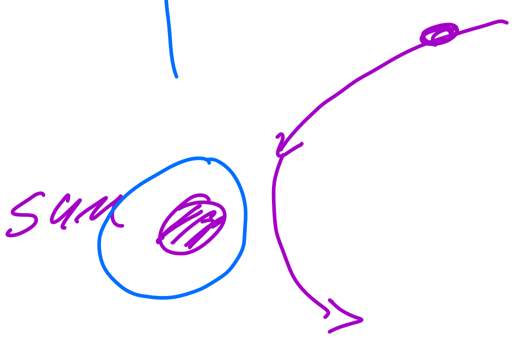
PHY 321, MARCH 21, 2022

First midterm (part c)



$V(r) \propto -\frac{1}{r}\,\gamma \qquad r \in [0,\infty)$





Technicality

- center of mass motion
  and relative

CoM: $\vec{R} = \dfrac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$

$m_1 + m_2 = M$

relative $\vec{r} = \vec{r}_1 - \vec{r}_2$

$\vec{r}_1 = \vec{R} + \dfrac{m_2}{M} \vec{r}$

$\vec{r}_2 = \vec{R} - \dfrac{m_1 \vec{r}}{M}$

$V(r) = V(|\vec{r}|)$

$\vec{r} = \vec{r}_1 - \vec{r}_2$

$\vec{r}_i = x_i \vec{i} + y_i \vec{j} + z_i \vec{k}$

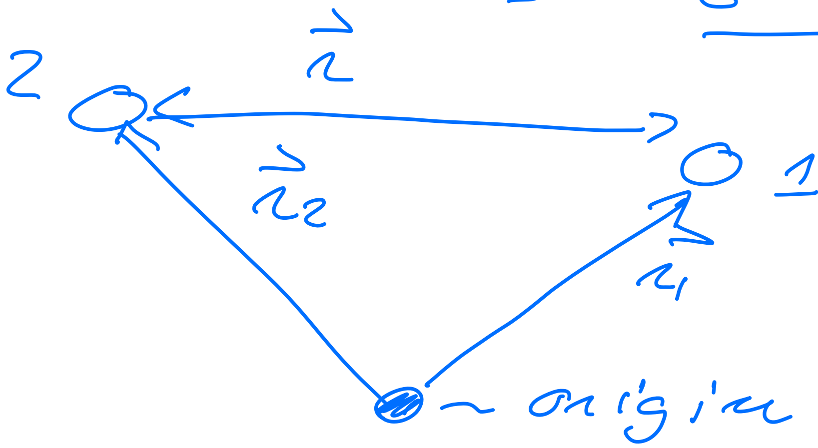
$|\vec{r}| = r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Gravitational potential

$V(r) = - \dfrac{G m_1 m_2}{|\vec{r}_1 - \vec{r}_2|}$

$|\vec{r}_1 - \vec{r}_2|$

$$= -\frac{G m_1 m_2}{r}$$



Total linear momentum

$$\vec{P} = \sum_{i=1}^{N} m_i \frac{d\vec{r}_i}{dt}$$

$$\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{M}$$

$$M \times \Big| \; \frac{d\vec{R}}{dt} = \frac{1}{M} \underbrace{\sum_{i=1}^{2} m_i \frac{d\vec{r}_i}{dt}}_{\vec{P}}$$

$$M \frac{d\vec{R}}{dt} = \vec{P}$$

$$\vec{F}^{net} = \frac{d\vec{P}}{dt} = M \frac{d^2\vec{R}}{dt^2}$$

$U(r)$ does not depend on

$V(r)$ does not

$\vec{R}$

$$-\vec{\nabla}_{\vec{R}} V(r) = 0$$

$$= F(\vec{R}) \Rightarrow$$

$$\frac{d\vec{P}}{dt} = 0$$

$$\vec{a}_R = \frac{d^2\vec{R}}{dt^2}$$

$M\vec{a}_R = 0$, can be trivially solved.

assume $N = 2$ and internal forces only

$$\vec{F}^{net} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

$$\vec{F}_{21} = -F_{12}$$

$$\frac{d^2x}{dt^2} = -kx$$

$$\boxed{\begin{aligned} \frac{dv}{dt} &= a = \frac{d^2x}{dt^2} = -kx \\ \frac{dx}{dt} &= v \end{aligned}}$$

<u>relative motion</u>

$$\vec{r} = \vec{r}_1 - \vec{r}_2 \qquad (\vec{r}_{ij} = \vec{r}_i - \vec{r}_j)$$

$$\frac{d^2\vec{r}}{dt^2} = \frac{d^2\vec{r}_1}{dt^2} - \frac{d^2\vec{r}_2}{dt^2}$$

$$= \vec{a}_1 - \vec{a}_2$$

$$= \frac{\vec{F}_{12}}{m_1} - \frac{\vec{F}_{21}}{m_2}$$

$$F_{21} = -F_{12}$$

$$= F_{12}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_1 m_2}{M}$$

$$\Rightarrow \boxed{\begin{aligned} \mu \frac{d^2 \vec{r}}{dt^2} &= \vec{F}_{12} \\ M \frac{d^2 \vec{R}}{dt^2} &= 0 \end{aligned}}$$

$$\mu \vec{a}_r = F_{12} = -G \frac{m_1 m_2 \vec{r}}{r^3}$$

$$\vec{a}_r = -\frac{G M \vec{r}}{r^3}$$

$$\vec{a}_R = 0$$

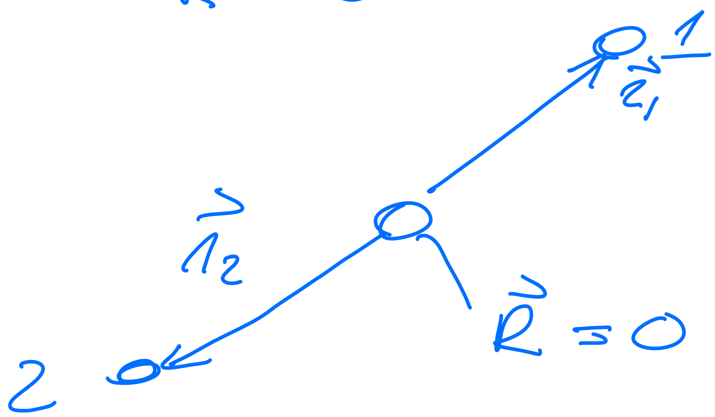
Angular Momentum

CoM-frame, center

of mass frame

of mass frame

$\vec{R} = 0$



$$\vec{r}_1 = \frac{m_2}{M}\vec{r}$$

$$\vec{r}_2 = -\frac{m_1}{M}\vec{r}$$

$$\vec{L} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$$

$$\vec{p}_i = m_i \frac{d\vec{r}_i}{dt}$$

$$\vec{L} = m_1\left(\vec{r}_1 \times \frac{d\vec{r}_1}{dt}\right) + m_2\left(\vec{r}_2 \times \frac{d\vec{r}_1}{dt}\right)$$

$$= m_1 \frac{m_2}{M}\vec{r} \times \frac{m_2}{M}\frac{d\vec{r}}{dt}$$

$$+m_2 \frac{m_1}{M} \vec{r} \times \frac{m_1}{M} \frac{d\vec{r}}{dt}$$

$$= \mu \left(\vec{r} \times \frac{d\vec{r}}{dt} \right)$$

$$\mu \vec{a}_r = \mu \frac{d^2\vec{r}}{dt^2} = \vec{F}(\vec{r})$$

$$\frac{d\vec{r}}{dt} = 0 \; ?$$