PHY 321, MARCH 27, 2023

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{M} \qquad M = m_1 + m_2$$

$$\vec{r} = \vec{r}_1 - \vec{r}_2$$

(i) CoM frame $\vec{R} = 0$

$$\frac{d\vec{P}}{dt} = 0 \qquad \vec{F}(r) = -\gamma \vec{r}/r^3$$

$\vec{P}$ is conserved

$$\vec{P} = M \cdot \frac{d\vec{R}}{dt}$$

(ii) $\vec{L} = \vec{r} \times \vec{p}$

$$\frac{d\vec{L}}{dt} = 0 \qquad \vec{L} \text{ is conserved}$$

$\Rightarrow$ two-dim

$$\vec{r} = x\vec{i} + y\vec{j}$$

$$r \in [0, \infty) \qquad r = \sqrt{x^2+y^2}$$

$$\phi \in [0, 2\pi]$$

$$\begin{cases} x \in (-\infty, +\infty) \\ y \in \text{—},\ \text{—} \end{cases}$$

$$\dot{r} = \frac{dr}{dt} \qquad \dot{\phi} = \frac{d\phi}{dt}$$

radial velocity $\qquad$ angular velocity

(iii)

in 2 dim (kinetic energy)

$$K = \frac{1}{2}(\dot{x}^2 + \dot{y}^2)\mu$$

$$= \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2)$$

$$\mu = \frac{m_1 m_2}{M}$$

(iv)

$$\dot{\phi} = \frac{d\phi}{dt} = \frac{L}{\mu r^2}$$

$$\mu \ddot{r} = \mu a_r = F(r) + \frac{L^2}{\mu r^3}$$

$$= F_{eff}(r)$$

(v) $E = K + V$

$$= \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2) + V(r)$$

$$\frac{1}{2}\mu \dot{r}^2 + V_{eff}(r)$$

$$V_{eff}(r) = V(r) + \frac{L^2}{2\mu r^2}$$

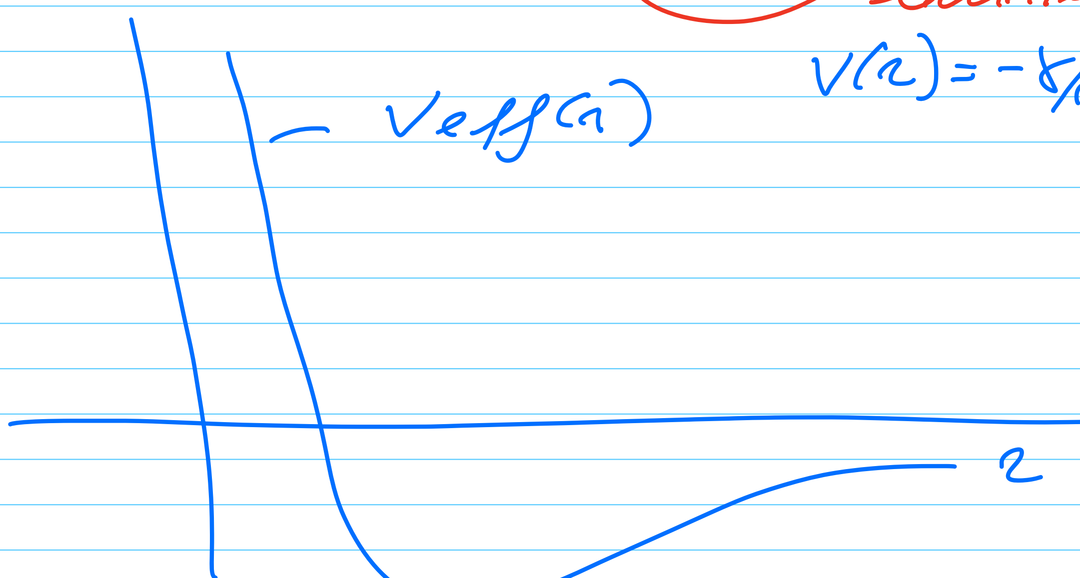
$$F_{eff}(r) = -\frac{dV_{eff}}{dr} =$$

$$= -\frac{dV}{dr} + \frac{L^2}{\mu r^3}$$

$$= F(r) + \frac{L^2}{m r^3}$$

centri'
fugal
barrier

$V(r) = -k/r$



2-Dim Harmonic oscillator

$$V(r) = \frac{1}{2} k r^2 =$$

$$V(x,y) = \frac{1}{2} k (x^2+y^2)$$

$x, y \Rightarrow r, \phi$

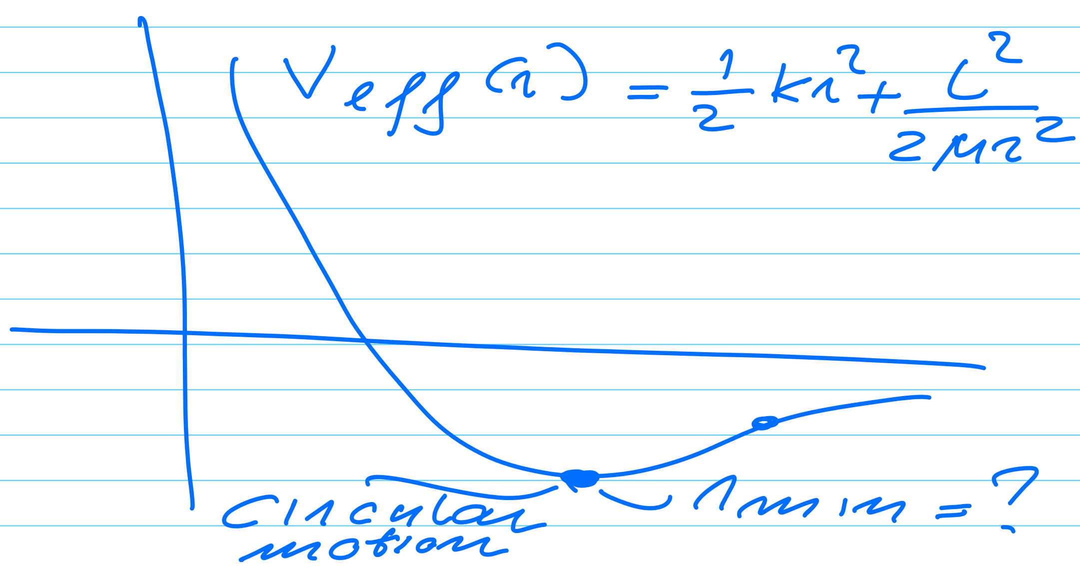
$$E = \frac{1}{2}\mu(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}k(x^2+y^2)$$

$$= \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{1}{2}kr^2$$

$$V_{eff}(r) = \frac{1}{2}\mu r^2\dot{\phi}^2 + \frac{1}{2}kr^2$$

$$= \frac{1}{2}\frac{L^2}{\mu r^2} + \frac{1}{2}kr^2$$

$$\mu\ddot{r} = \mu a_r = -kr + \frac{L^2}{\mu r^3}$$



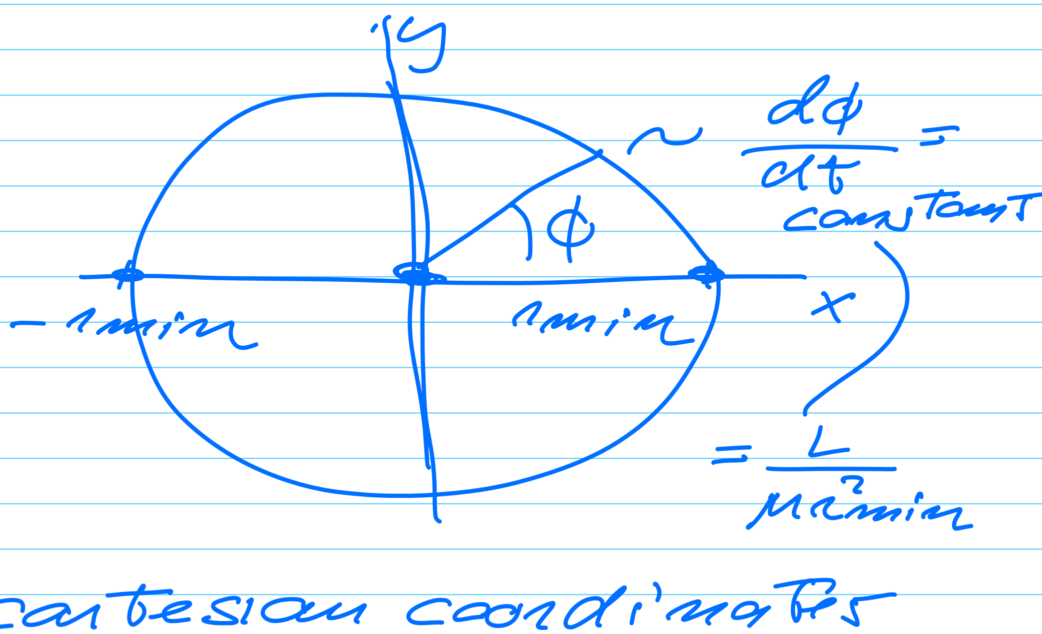
$$\left.\frac{dV_{eff}(r)}{dr}\right|_{r=r_{min}} = 0$$

$$r_{min} = \left[\frac{L^2}{k\mu}\right]^{1/4}$$

$$-\left.\frac{dV_{eff}}{dr}\right|_{r=r_{min}} = 0 = F_{eff}(r)$$

$$= \mu a_r = \mu \ddot{r} = 0$$

$$\frac{d\phi}{dt} = \frac{L}{\mu r^2}$$



cartesian coordinates

$$\mu \frac{d^2x}{dt^2} = -kx$$

$$\mu \frac{d^2y}{dt^2} = -ky$$

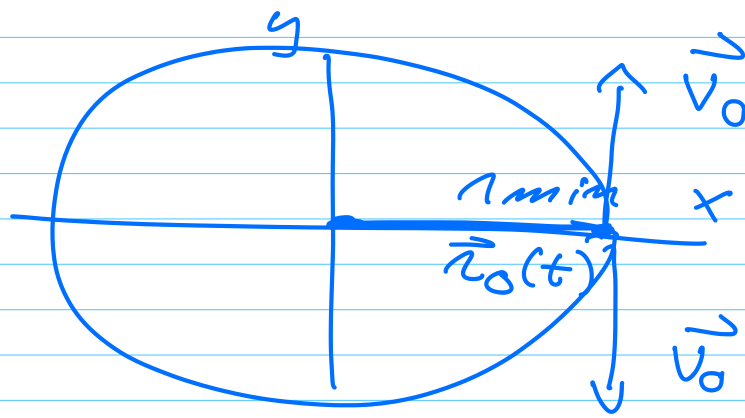
$$x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

$$\omega_0 = \sqrt{k/m}$$

$$y(t) = C\cos(\omega_0 t) + D\sin(\omega_0 t)$$

at r_{min} we have $\ddot{r} = a_r = 0$

initial conditions



$$\vec{r}_0(t_0) = \vec{r}_0 = x_0\vec{i} + y_0\vec{j}$$

$$y_0 = 0 \qquad x_0 = r_{min}$$

Centripetal acceleration

$$a_c = a_r = \frac{v_0^2}{r_{min}}$$

$$= \frac{k r_{min}}{\mu} \Rightarrow$$

$$(\omega_0^2 = k/\mu)$$

$$v_0^2 = \omega_0^2 r_{min}^2$$

$$v_0 = \omega_0 r_{min}$$

$$\vec{v}_0 = \vec{v}(t_0) = v_{0x}\vec{i} + v_{0y}\vec{j}$$

$$\vec{v}_0 = 0 \cdot \vec{i} + \underbrace{\omega_0 r_{min}}_{v_{0y}}\vec{j}$$

$$x_0 = A \cdot \cos(0) + B \sin(0)$$

$$= r_{min} = A$$

$$v_{0x} = 0 = -A\omega_0 \cdot \sin(\omega_0 t_0)$$

$$+ B\omega_0 \cos(\omega_0 t_0)$$

$$\Rightarrow B = 0$$

$$x(t) = r_{min} \cos(\omega_0 t)$$

$$y(t) = r_{min} \sin(\omega_0 t)$$