$$V(r) = \frac{1}{2} kr^2$$
$$= \frac{1}{2} k(x^2 + y^2)$$

↳ 2D

Week 12 notes 2

lectures

$$v^2 = \dot{x}^2 + \dot{y}^2 = \dot{r}^2 + r^2\dot{\theta}^2$$

$$E = KE + PE$$
$$= \frac{1}{2}mv^2 + \frac{1}{2}kr^2$$
$$= \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}kr^2$$

centrifugal term

effective potential

$$V_{eff} = \frac{1}{2} k r^2 + \frac{1}{2} m r^2 \dot{\theta}^2$$

$$= \frac{1}{2} k r^2 + \frac{l^2}{2 m r^2}$$

$r_{min} \rightarrow$ value of $r$ that minimizes $V_{eff}$

$$\frac{\partial V_{eff}}{\partial r} = 0 \Rightarrow$$

$$k r_{min} - \frac{l^2}{m r_{min}^3} = 0$$

$$k r_{min} = \frac{l^2}{m r_{min}^3}$$

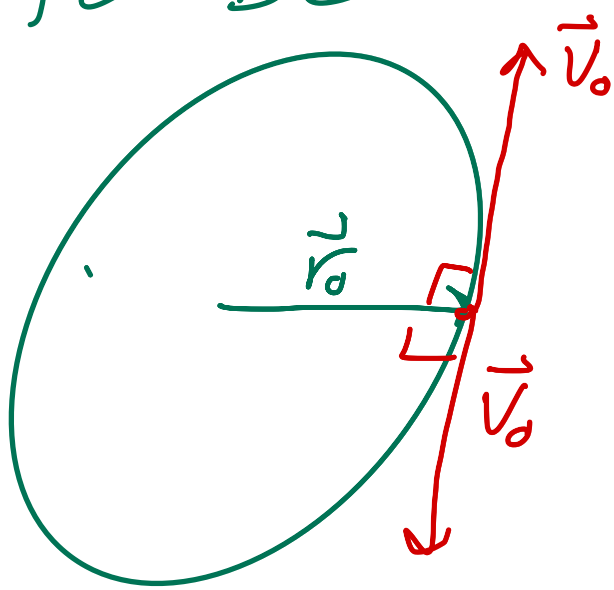
$$k r_{min}^4 = \frac{l^2}{m} \Rightarrow r_{min} = \left(\frac{l^2}{mk}\right)^{1/4}$$

Note: $\vec{r} = r_{min}$ , $\dot{r} = 0 \Rightarrow$ circular orbit, change L to change the radius

$$w_0 = \sqrt{\frac{k}{m}}$$

if $r = r_{min}$ then we have a circular orbit

1) initial position & the initial velocity have to be perpendicular



2) $r_0$ must be equal to $r_{min}$

$$\vec{r_0} = (r_{min}, 0)$$

3) $V(r) = \frac{1}{2} k r^2$

$$F(\vec{r}) = -\nabla V = -k\vec{r}$$

$$\vec{a}(\vec{r}) = -\frac{k}{m}\vec{r}$$

$$a(r) = \frac{k}{m} r$$

centripetal acceleration

$$a_c = \frac{V_0^2}{r_{min}} = \frac{k}{m} r_{min}$$

$$V_0^2 = \frac{k}{m} r_{min}^2$$

$$V_0 = \sqrt{\frac{k}{m}}\, r_{min}$$

$V_0 = \omega_0 r_{min} \rightarrow$ magnitude

$\vec{r}_0 = (r_{min}, 0)$

$v_0 = (0, \omega_0 r_{min})$

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$$\vec{a}(\vec{r}) = -\frac{k}{m}\vec{r}$$

$$= -\omega_0^2 \vec{r}$$

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Analytical Solutions for Position

Because $\ddot{x} = -\frac{k}{m} x$

$$\ddot{y} = -\frac{k}{m} y$$

$x$ & $y$ equations of motion are separate

$$x = A\cos(w_0 t) + B\sin(w_0 t)$$
$$y = C\cos(w_0 t) + D\sin(w_0 t)$$

<u>initial conditions</u>

$\vec{r}_0 = (r_{min}, 0)$ $\quad A = r_{min}$, $C = 0$

$\vec{V}_0 = (0, w_0 r_{min})$ $\quad B = 0$, $D = r_{min}$

$x(0) = A = r_{min} \Rightarrow A = r_{min}$

$y(0) = C = 0 \Rightarrow C = 0$

$x = r_{min}\cos(w_0 t) + B\sin(w_0 t)$

$\dot{x} = -r_{min} w_0 \sin(w_0 t) +$
$\quad w_0 B\cos(w_0 t)$

$\dot{x}(0) = w_0 B = 0 \Rightarrow B = 0$

$y = D\sin(w_0 t)$ $\quad \dot{y}(0) = w_0 D =$
$\quad w_0 r_{min}$

$\dot{y} = w_0 D\cos(w_0 t)$ $\quad \Rightarrow D = r_{min}$

$$x = r_{min} \cos(w_0 t)$$

$$y = r_{min} \sin(w_0 t)$$

$$\vec{a} = -w_0^2 \, \vec{r}$$

$$\vec{r}_0 = (r_{min}, 0)$$

$$\vec{v}_0 = (0, \; w_0 r_{min})$$