PHY 321 FOURIER ANALYSIS

$$F(t) = F(t+T)$$

$$\text{Period} = 2\pi/\omega = T$$

$$F(t) = \sum_{n=0}^{\infty} \left[a_n \cos(n\omega t) + b_n \sin(n\omega t) \right]$$

$x \in [-1, 1]$

$P_n(x)$ orthogonal polynomials

$$\int_{-1}^{1} P_m(x) P_n(x)\, dx = 0 \text{ if } m \neq n$$

$$\int_{-1}^{1} a_1 x\, a_2 x^2\, dx = a_1 a_2 \frac{1}{4} x^4 \Big|_{-1}^{1} = 0$$

$x \in [-\pi, \pi]$

$$\int_{-\pi}^{\pi} \sin(mx) \sin(nx)\, dx = 0 \text{ unless } m = n$$

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,dx$$

$$= 0 \quad \text{unless } m = n$$

$$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx$$

$$m = n$$

$$= \left. \frac{1 - \cos(2mx)}{4} \right|_{-\pi}^{\pi} = 0$$

$$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx$$

$$m \neq n$$

$$= -\frac{1}{2}\left\{ \frac{\cos(m-n)x}{m-n} + \frac{\cos(m+n)x}{m+n} \right\}\Bigg|_{-\pi}^{\pi}$$

$$= 0$$

$$F(t) = \sum_{n=0}^{\infty} \left[a_n \cos(n\omega t) + b_n \sin(n\omega t) \right]$$

$$x \in [-\pi, \pi]$$

$$T = 2\pi /$$

$T = 2\pi/\omega$

$$x \in \left[-\frac{T}{2}, T/2\right] = \left[-\frac{\pi}{\omega}, \frac{\pi}{\omega}\right]$$

$$t \in \left[-\frac{T}{2}, T/2\right]$$

How do we find b_n and a_n?

multiply with $\cos(m\omega t)$ on both sides and integrate from $-T/2$ to $T/2$

$m > 0$

$$\int_{-T/2}^{T/2} F(t) \cos(m \omega t)\, dt$$

$$= \sum_{n>0} \int_{-T/2}^{T/2} \cos(m\omega t) \cos(n\omega t) \times a_n \cdot dt$$

$$\boxed{m = n}$$

$$= \frac{T}{2} a_n \Rightarrow$$

$T/2$

$$a_n = \frac{2}{T} \int_{-T/2}^{} F(t) \cos(n\omega t)\, dt$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} F(t) \sin(n\omega t)\, dt$$

$n = m = 0$

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} F(t)\, dt$$

$$\underline{F(t) = t}$$

$$a_n = a_0 = 0$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} t \sin(n\omega t)\, dt$$

$$b_n = \frac{2(-1)^{n+1}}{\pi \cdot n}$$

$$t = \sum_{n=1}^{\infty} \frac{2}{\pi n} (-1)^{n+1} \sin(\cancel{n}\omega t)$$

$\omega = 1$

$$= \sum_{n=1}^{\infty} \frac{2}{\pi n} (-1)^{n+1} \sin(n\pi t)$$

Harmonic oscillations

$$\frac{d^2y}{d\tau^2} + 2\gamma \frac{dy}{d\tau^2} + y = F(\tau)$$

$$\tau \Rightarrow t \qquad \gamma = 0$$

$$y'' + y = F(t) = t$$

$$= \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{\pi n} \sin(n\pi t)$$

$$y(t) = \sum_{n=1}^{\infty} c_n \sin(n\pi t)$$

$$y'' = \sum_{n=1}^{\infty} c_n(-n^2\pi^2) \sin(n\pi t)$$

n - specific value

$$(-n^2\pi^2 + 1) c_n = \frac{2}{\pi n}(-1)^{n+1}$$

$$c_n = \frac{2}{\pi n(1-n^2\pi^2)}(-1)^{n+1}$$

$$y(t) = \sum_{n=1}^{\infty} \frac{2}{\pi n(1-n^2\pi^2)}(-1)^{n+1} \times \sin(n\pi t)$$

$$\stackrel{\circ}{\wedge}$$