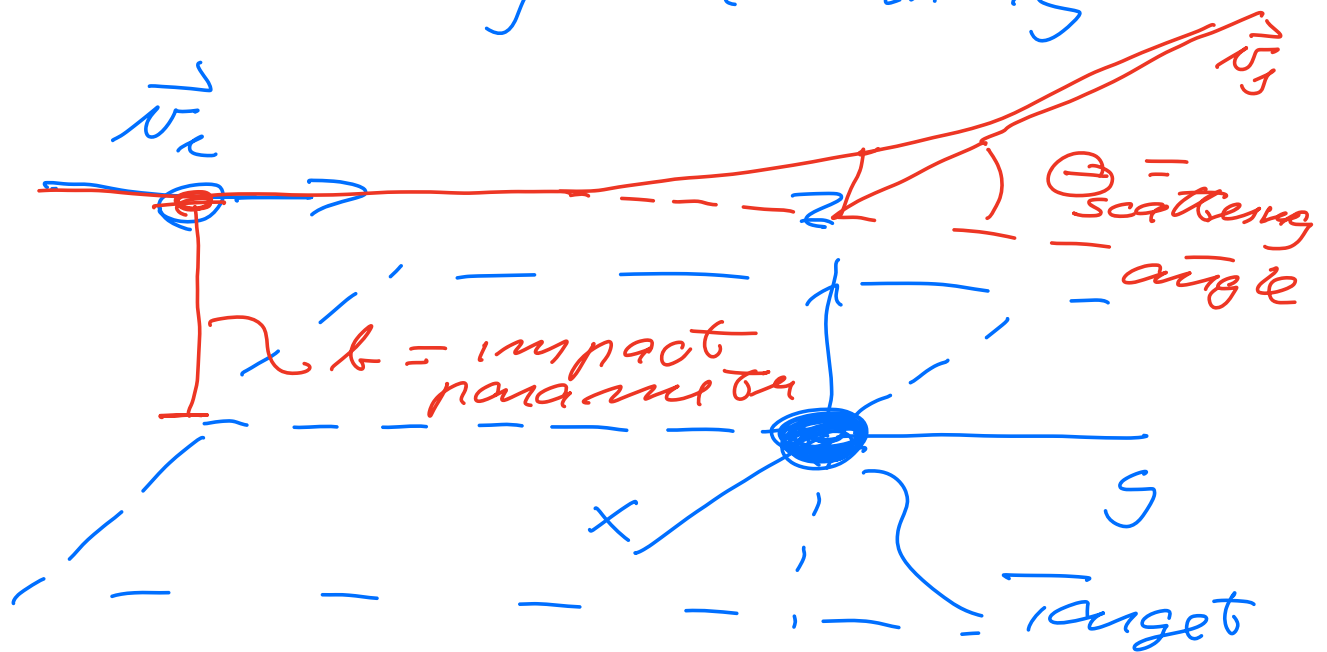


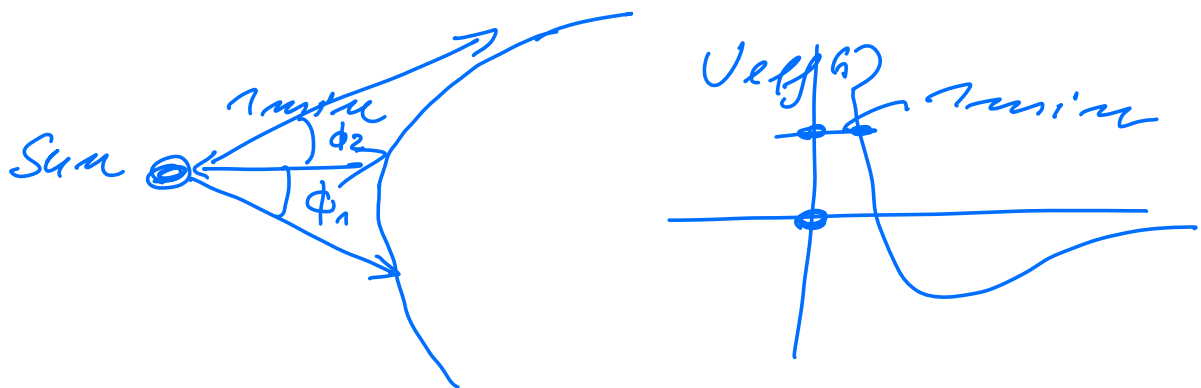
PHY 321, APRIL 11, 2022

## Two-body scattering



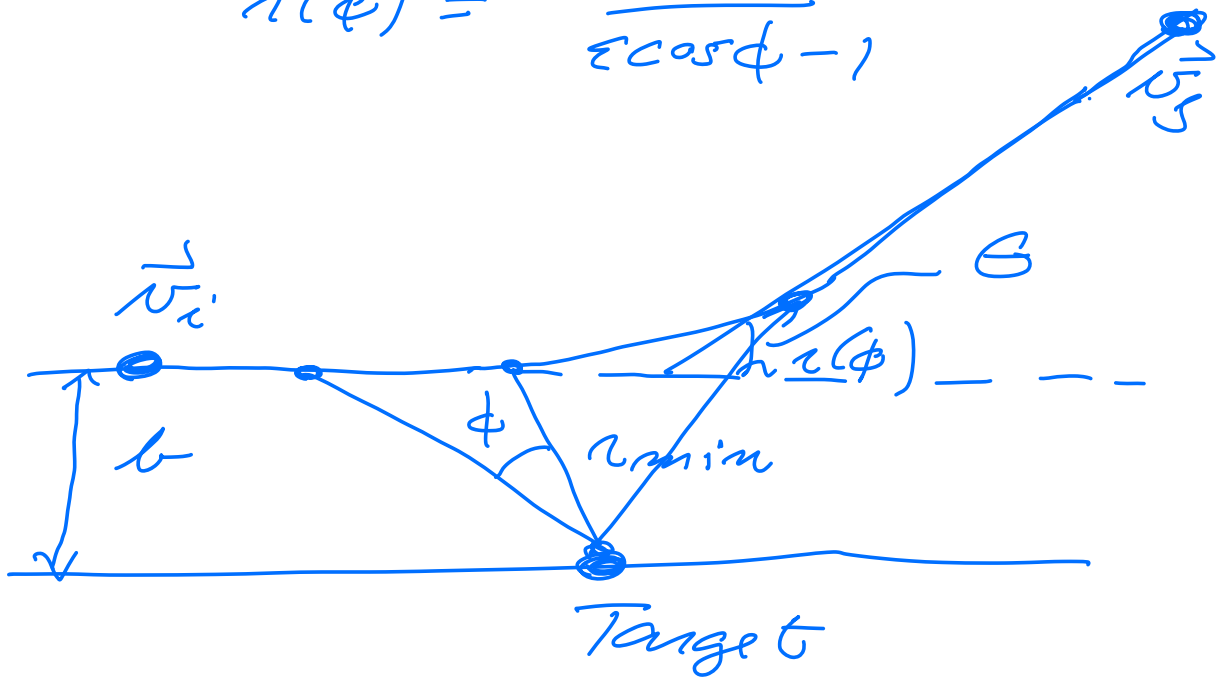
$$V(r) = -\alpha/r \quad \begin{cases} \alpha = Gm_1 m_2 \\ \alpha = \frac{1}{4\pi\epsilon_0} q_1 q_2 \end{cases}$$

$$r(\phi) = \frac{C}{1 + \epsilon \cos \phi}$$



$$V(r) = \alpha/r$$

$$r(\phi) = \frac{c}{\epsilon \cos \phi - 1}$$



when  $r = -\infty$  (far away)

$$E = \frac{1}{2} \mu v_i^2 \neq 0$$

only kinetic

$$r = +\infty$$

$$E = \frac{1}{2} v_f^2 \cdot \mu \text{ kinetic energy}$$

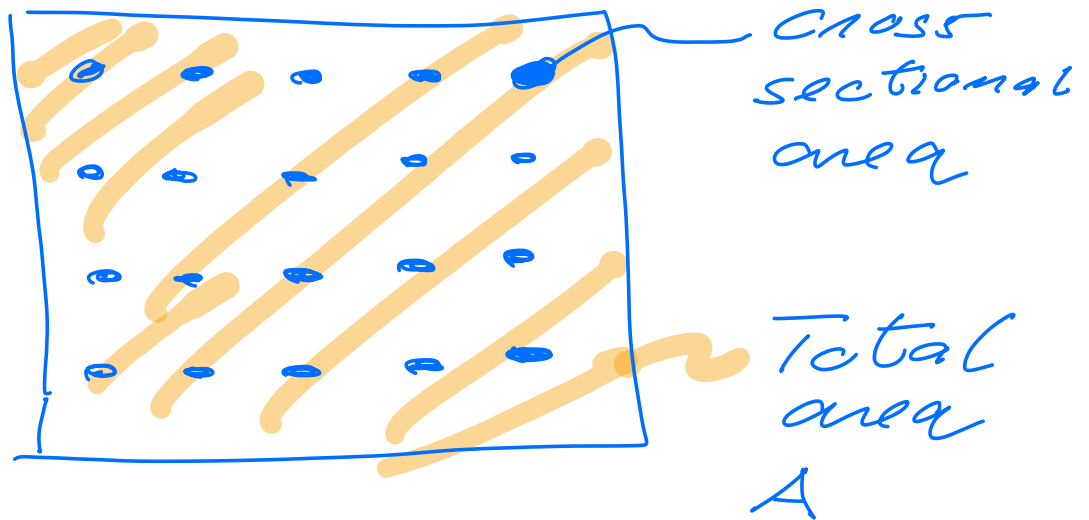
$r(\phi)$  (theoretical result)

we want to relate this to  $\theta$  (expt), the scattering angle (expt)

cross-section  $\lambda$  impact  
and the impact parameter  
-  $b$  -

### More definitions

- cross section
- Target assembly



$\rho_t$  = number of targets  
per area

$$\begin{aligned}\# \text{ targets} &= \text{number of} \\ &\quad \text{targets} \\ &= \rho_t \cdot A\end{aligned}$$

- Likelihood of making

... ..  
a hit;

- cross sectional area

$$\sigma = \pi R^2$$

 - target with  
radius  $R$ .

Total area of all  
targets

$$= \sigma \cdot p_t \cdot A$$

probability of hit

$$= \frac{\text{area occupied by targets}}{\text{total area}}$$

$$= \frac{p_t \cdot A \sigma}{A} = p_t \cdot \sigma$$

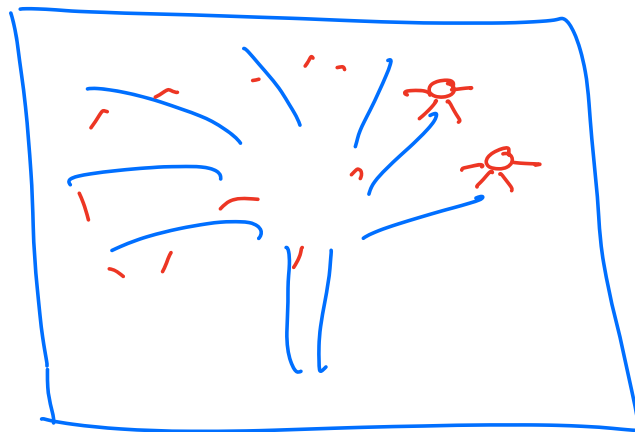
↙  
want this as  
function of  $G, b$

- Number of scattered:

$$= N_{\text{scatter}} = N_{\text{incoming}} \times \rho_t \cdot \nabla$$

$\nabla$  = effective area of the target interacting with a beam of particles

Example (Taglar)



oak  
tree

$$A = 150 \text{ ft}^2$$

50 pigeons each with

$$\nabla = 1 \text{ ft}^2$$

Fire 60 bullets randomly

$$N_{hits} = N_{incoming} \times \sigma_{pigeons} \quad \nabla$$

$$\sigma_{pigeons} = \frac{50}{150} = \frac{1}{3} \text{ ft}^{-2}$$

$$N_{hits} = 60 \times \left(\frac{1}{3} \text{ ft}^{-2}\right) \left(\frac{1}{2} \text{ ft}^2\right) \\ \approx 10 \text{ pigeons}$$

In nuclear physics

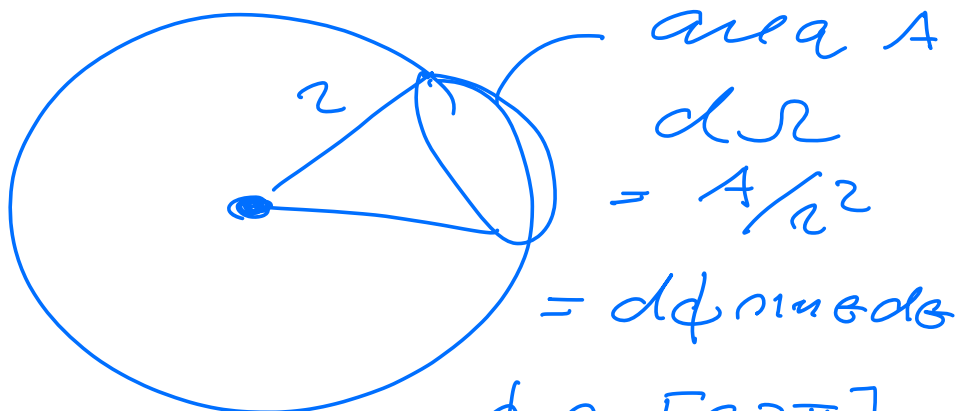
$$R \sim 10^{-14} \text{ m}$$

$$\sigma \sim 10^{-28} \text{ m}^2 = 1 \text{ barn}$$

Expt :  $\sigma$ , cross section  
 $\nabla$

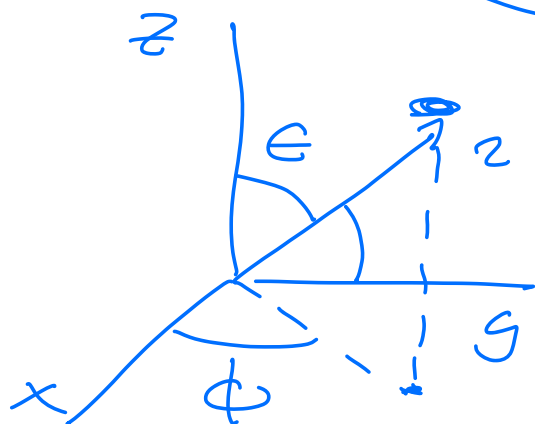
$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$

# Sphere and solid angle



$$\phi \in [0, 2\pi]$$

$$\theta \in [0, \pi]$$



$$\frac{dA}{d\Omega} = \frac{r dr}{\sin\theta d\theta}$$