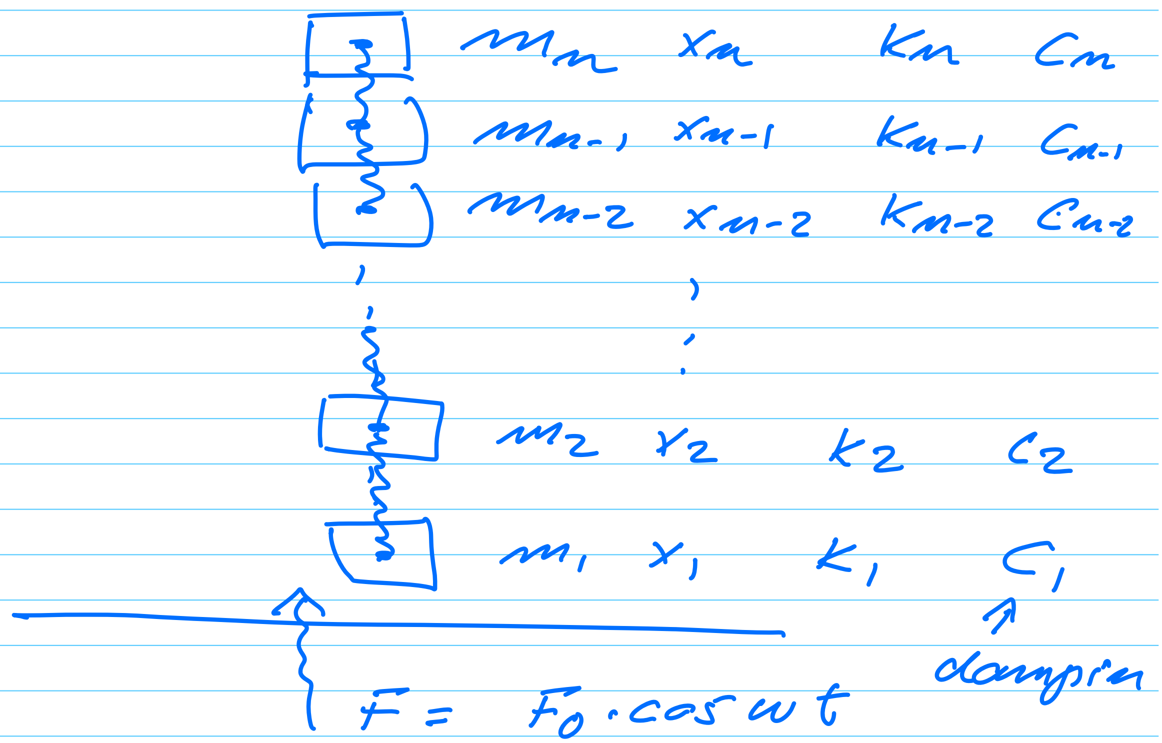
PHY 321, APRIL 24, 2023

Harmonic oscillations model for earthquakes



$$m_1 x_1'' = -k_1 x_1 + k_2 (x_2 - x_1) - c_1 (x_1' - x_2') + F_0 \cos \omega t$$

$$m_2 x_2'' = -k_2 (x_2 - x_1) + k_3 (x_3 - x_2) - c_2 (x_2' - x_1') - c_3 (x_2' - x_3')$$

$$- F_0 \cos \omega t$$

$$\vdots$$

$$m_n \ddot{x}_n = -k_n (x_n - x_{n-1}) - c_{n-1} (\dot{x}_n - \dot{x}_{n-1}) + F_0 \cos \omega t$$

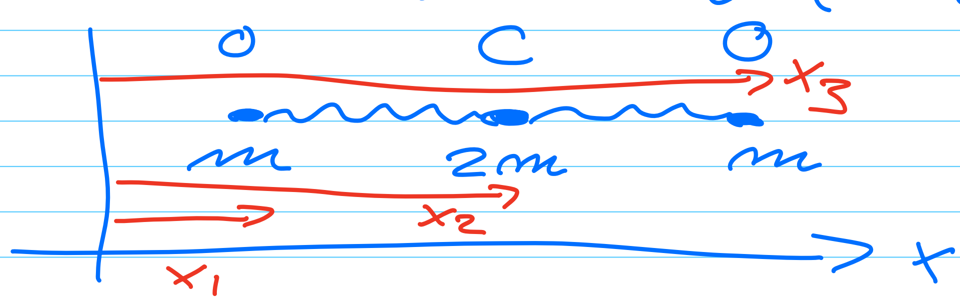
$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad M = \begin{bmatrix} m_1 & & 0 \\ & m_2 & \\ 0 & & \ddots \\ & & & m_n \end{bmatrix}$$

$$K = \begin{bmatrix} -(k_1+k_2) & k_2 & 0 & \cdots & 0 \\ k_2 & -(k_2+k_3) & k_3 & 0 & \cdots \\ 0 & k_3 & \ddots & & \\ \vdots & & & \ddots & \\ \vdots & & & & \ddots \end{bmatrix}$$

$$M\ddot{X} = KX + C\dot{X} + \bar{F}$$

Example

linear chain (CO_2)



Harmonic oscillator potential

$$\mathcal{L} = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} 2m \dot{x}_2^2$$
$$+ \frac{1}{2} m \dot{x}_3^2$$
$$- \frac{k}{2} (x_2 - x_1)^2 - \frac{1}{2} k (x_3 - x_2)^2$$

$$\frac{\partial \mathcal{L}}{\partial x_1} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}_1} = 0$$

$$m \ddot{x}_1 = -k (x_1 - x_2)$$
$$2m \ddot{x}_2 = -k (x_2 - x_1) - k (x_2 - x_3)$$
$$= -k [2x_2 - x_1 - x_3]$$
$$m \ddot{x}_3 = -k [x_3 - x_2]$$

$$q_1 = x_1 - x_2$$

$$q_3 = x_3 - x_2$$

$$X = \frac{x_1 m + 2m x_2 + x_3 m}{4m}$$

$$X = \frac{x_1 + 2x_2 + x_3}{4}$$

$$x_2 = (4X - x_1 - x_3)/2$$

$$x_1 = (3q_1 - q_3 + 4X)/2$$

$$x_3 = (3q_3 - q_1 + 4X)/2$$

$$\mathcal{L} = K - V$$

$$V = \frac{k}{2}\left(q_1^2 + q_3^2\right)$$

$$K = \frac{3m}{8}\left[\dot{q}_1^2 + \dot{q}_3^2\right] - \frac{m\,\dot{q}_1\dot{q}_3}{4} + 2m\dot{X}^2$$