PHY 321 FEBRUARY 14, 2022

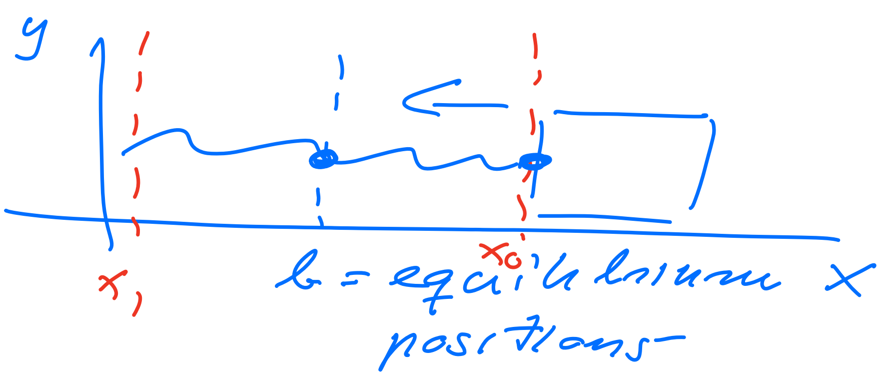
Prelude to HOs (harmonic oscillations)

- Energy conservation

  $E = K + V$

- potential energy landscape
  - limits of possible motion
  - Equilibrium points
  - stable and unstable points

HO in 1-Dim



$$\vec{F} = F(x) = -k(x-b)$$

$$E = V(x) + K$$

$$V(x) - V(x_0) = -\int^{x} F(x)\,dx$$

$$V(x) - V(x_0) = -\int_{x_0}^{x} F\,dx$$

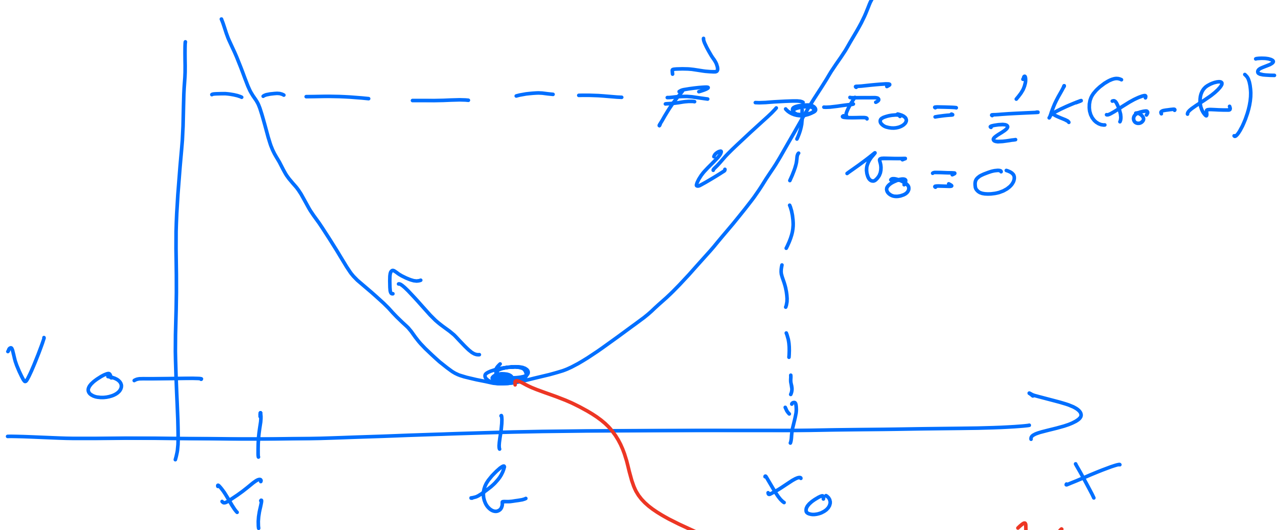
$$= \frac{1}{2}k(x-b)^2 - \frac{1}{2}k(x_0-b)^2$$

We could define

$V(x=b) = 0 \Rightarrow$

$$V(x) = \frac{1}{2}k(x-b)^2$$

$$F(x) = -\frac{d}{dx}V(x) = -k(x-b)$$



equilibrium
local/global
min

x_0 : $F(x) = -\frac{dV}{dx}$

$x_0 > b$ $\quad \frac{dV}{dx} > 0$

$$\left.\frac{dV}{dx}\right| = k(x_0-b) \Rightarrow$$

$\Big|_{x=x_0}$

F is negative

at $x = b$ $\quad V(x=b) = 0$

$$E(x=b) = \frac{1}{2} m v_b^2 + 0$$

at $x = b$ $\quad \left.\frac{dV}{dx}\right|_{x=b} = 0$

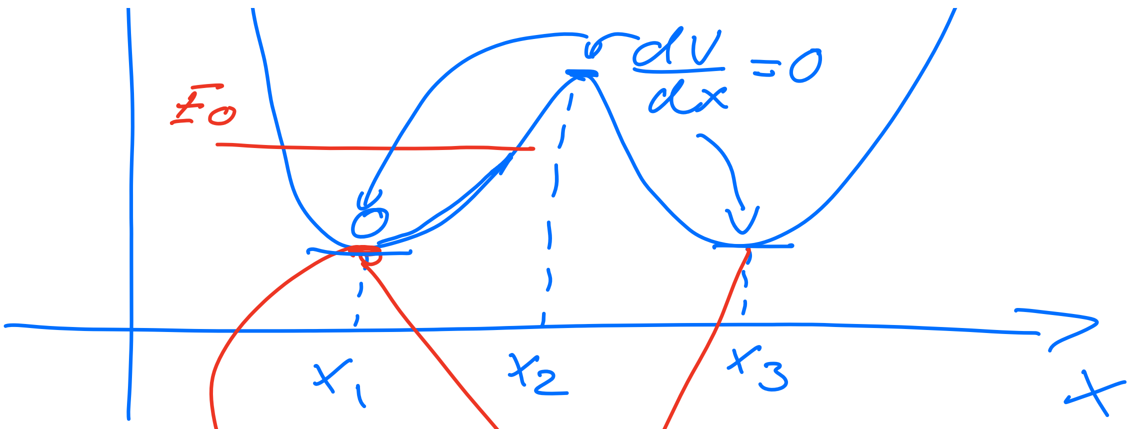
$$\frac{dV}{dx} = k(x-b)$$

$$\frac{d^2V}{dx^2} = k > 0$$

$x < b$ moving towards x, on the graph

$$F = -\frac{dV}{dx} > 0$$

Example 2



$V(x_1) = V(x_3) = 0$

$\boxed{x_1 \text{ and } x_3}$ $\quad \frac{d^2V}{dx^2} > 0$

local minima

$x_2 \quad \frac{d^2V}{dx^2} < 0 \Rightarrow$

$$F = -\frac{dV}{dx} \Rightarrow$$

$$\frac{dF}{dx} = -\frac{d^2V}{dx^2} > 0$$

at x_2 $\quad \frac{dV}{dx} = 0 = F$

but $\frac{dF}{dx} > 0 \implies$ unstable maximum

(unstable point)