PHY 321 February 7, 2022

Center of mass is the average position of a set of masses weighted by the total mass

$$M = \sum_{i=1}^{N} m_i$$

$$\vec{R} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i} = \frac{1}{M} \sum_{i=1}^{N} m_i \vec{r}_i$$

rate of change

$$\frac{d\vec{R}}{dt} = \frac{1}{M} \sum_{i=1}^{N} m_i \underbrace{\frac{d\vec{r}_i}{dt}}_{\vec{v}_i}$$

$$= \frac{1}{M} \underbrace{\sum_{i=1}^{N} \vec{p}_i}_{\vec{P}}$$

$$= \underline{\vec{P}}$$

$\overline{M}$

if $\frac{d\vec{p}}{dt} = 0$, the rate of change $\vec{R}$ $\left(\frac{d\vec{R}}{dt}\right)$ is constant.

Example: Torque (hw4, ex4)

$t_0 = 0 \qquad \vec{r}_0 = x_0 \vec{i} + y_0 \vec{j}$

Add a force in the x-direction

$$\vec{F} = F_x \vec{i} = \frac{d\vec{p}}{dt}$$

$$\vec{v}_0 = 0 \text{ m/s}$$

$$\vec{a} = \vec{F}/m = F_x \vec{i}$$

$$\vec{v}(t) = \vec{v}_0 + \int_{t_0}^{t} \vec{a}\, dt'$$

$$\vec{v}(t) = \left(\int_{t_0}^{t} \frac{F_x}{m}\, dt' \right) \vec{i}$$

$$= \frac{F_x}{m}\, t \cdot \vec{i}$$

$$p = m \cdot \vec{v}(t)$$

$$= F_x \cdot t \cdot \vec{i}$$

$$\vec{r}(t) = \vec{r}_0 + \int_{t_0}^{t} \vec{v}(t)\, dt)$$

$$\vec{r}_0 = x_0 \vec{i} + y_0 \vec{j}$$

$$\vec{r}(t) = \left(x_0 + \frac{1}{2}\frac{F_x}{m} t^2\right)\vec{i} + y_0 \vec{j}$$

angular momentum

$$\vec{\ell} = \vec{r} \times \vec{p}$$

$$\vec{\ell} = -y_0 F_x \cdot t \cdot \vec{k}$$

$$\frac{d\vec{\ell}}{dt} = \tau = -y_0 F_x \vec{k}$$

Conservative forces

(i) $\vec{F} = \vec{F}(\vec{r})$

(ii) $W = \int_C \vec{F}(\vec{r})\, d\vec{r}$

= independent of C

is independent of ...

(iii) $\vec{\nabla} \times \vec{F} = 0$

We will show that

$$\vec{F}(\vec{r}) = -\vec{\nabla} \underbrace{V(\vec{r})}_{\text{number}}$$

work-energy theorem

work done by displacing an object from $\vec{r}$ to $\vec{r} + d\vec{r}$

$$W(\vec{r} \to \vec{r} + d\vec{r}) = \vec{F}(\vec{r}) d\vec{r}$$

$$= -\left[V(\vec{r} + d\vec{r}) - V(\vec{r})\right]$$

Back to last week

$$\frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 = W_{01}$$

$$= \cancel{V}(x_0) - V(x_1)$$

$$F(x) = -kx$$

$$\frac{1}{2} k x_0^2 - \frac{1}{2} k x_1^2$$

$$\frac{1}{2} m v_1^2 + \frac{1}{2} k x_1^2 = \frac{1}{2} m v_0^2 + \frac{k}{2} x_0^2$$

$$W(\vec{r} \to \vec{r} + d\vec{r}) = \vec{F}(\vec{r})\, d\vec{r}$$

$$= -\left[V(\vec{r} + d\vec{r}) - V(\vec{r})\right]$$

$$= F_x\, dx + F_y\, dy + F_z\, dz$$

$$V(\vec{r} + d\vec{r}) - V(\vec{r}) =$$

$$V(x+dx, y+dy, z+dz) - V(x, y, z)$$

$$f(x, y, z)$$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

$$W(\vec{r} \to \vec{r} + d\vec{r}) = -dV$$

$$= -\left[\frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz\right]$$

$$= F_x\, dx + F_y\, dy + F_z \cdot dz$$

$$F_x = -\frac{\partial V(x, y, z)}{\partial x}$$

$$F_y = -\frac{\partial V}{\partial y}$$

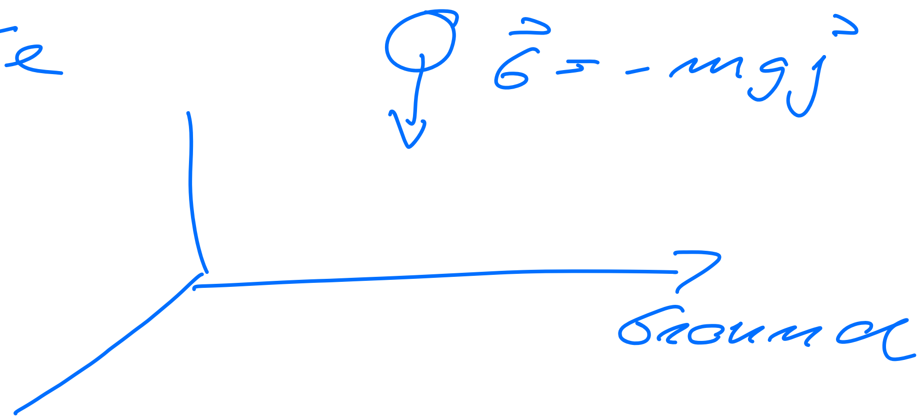
$$F_z = -\frac{\partial V}{\partial z}$$

$$\Rightarrow \vec{F} = -\vec{\nabla} V$$

Example (hw4, exercises)

Falling object

phase 1



phase 2