PHY 321, FEB 6, 2023

$$\vec{F}^{net} = \vec{F}^{ext} + \vec{F}^{int}$$

$$\sum_{i=1}^{N} \vec{F}_i^{int} = \vec{F}^{int} =$$

$$\sum_{i=1}^{N} \sum_{i \neq j}^{N} \vec{F}_{ij} = 0$$

if no $\vec{F}^{ext}$ $\Rightarrow$

$$\vec{F}^{int} + \vec{F}^{ext} = \frac{d\vec{P}}{dt} = 0$$

$\vec{P}$ is a constant of motion

Center of mass

$$\vec{R} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i} =$$

$$\frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M} \qquad \vec{p}_i$$

$$\frac{d\vec{R}}{dt} = \frac{\sum_{i=1}^{N} m_i \frac{d\vec{r}_i}{dt}}{M}$$

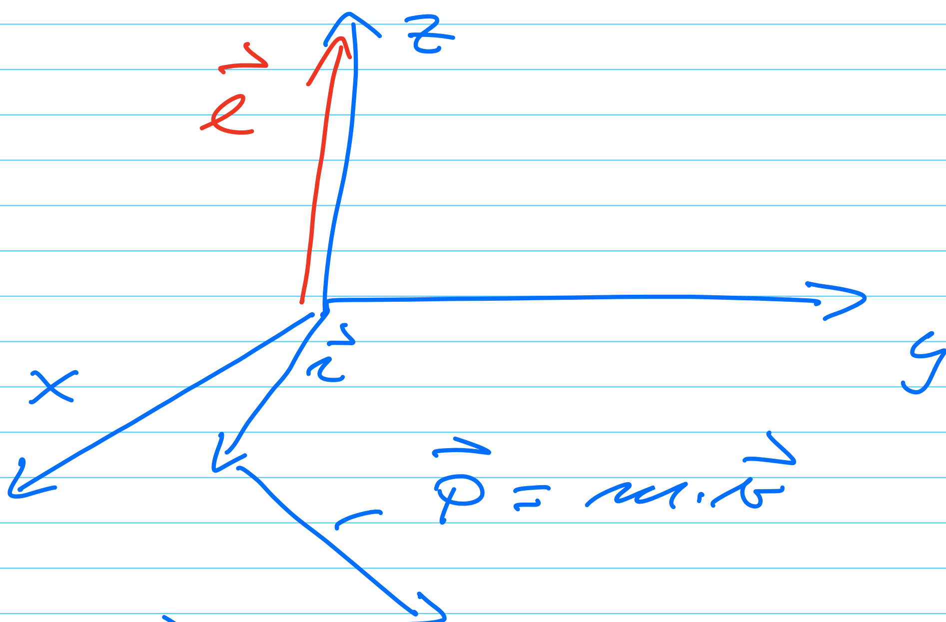
$$= \frac{\vec{P}}{M} = \sum_{i=1}^{N} \vec{p}_i / M$$

$$\frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} = \text{constant}$$

$$\Rightarrow \frac{d\vec{R}}{dt} = \text{constant}$$

$\vec{R}$ moves with constant

Angular Momentum



$$\vec{\ell} = \vec{r} \times m \cdot \vec{v} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{\ell}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F}$$

(Ex4 from hw 1)

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{N} \frac{d}{dt}\vec{l_i}$$

if conservative and internal force

$$\frac{d\vec{L}}{dt} = \tau = 0$$

Torque τ (hw ex 4)

initial $\vec{r}_0 = x_0\vec{i} + y_0\vec{j}$

$t_0 = 0$

$$\vec{F} = F_x\vec{i} = \frac{dP_x}{dt}\vec{i} = m\frac{dv_x}{dt}\vec{i}$$

$$\vec{v}_0 = 0$$

$$\vec{v}(t) = \vec{v}_0 + \int_0^t \frac{\vec{F}}{m}\,dt'$$

$$= \left(\int_0^t \frac{F_x}{m}\,dt'\right)\vec{i}$$

$$= \frac{F_x}{m}\,t\,\vec{i}$$

no change in y-direction

$$\vec{r}(t) = \left(x_0 + \frac{1}{2}\frac{F_x}{m}t^2\right)\vec{i}$$

$$+ y_0 \vec{j}$$

$$\vec{l} = \vec{r} \times \vec{p} = \vec{r}(t) \times \vec{p}(t)$$

$$= \left[\left(x_0 + \frac{F_x}{m} \frac{1}{2} t^2 \right) \vec{i} + y_0 \vec{j} \right] \times$$

$$F_x t \vec{i}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= (a_y b_z - a_z b_y) \vec{i}$$

$$+ (b_z a_x - a_z b_x) \vec{j}$$

$$+ (a_x b_y - b_x a_y) \vec{k}$$

no z-direction

$$\vec{l} = - F_x y_0 \cdot t \vec{k}$$

$$\frac{d\vec{l}}{dt} = \vec{\tau} = -y_0 F_x \cdot \vec{k}$$

$$\neq 0$$

$\vec{l}$ is not conserved

Conservation of Energy

(i) $\vec{F} = \vec{F}(\vec{r})$

(ii) $\vec{\nabla} \times \vec{F}(\vec{r}) = 0$

$$= \left(\frac{\partial}{\partial y}F_z - \frac{\partial}{\partial z}F_y\right)\vec{i}$$

$$+ \left(\frac{\partial}{\partial z}F_x - \frac{\partial}{\partial x}F_z\right)\vec{j}$$

$$+ \left(\frac{\partial}{\partial x}F_y - \frac{\partial}{\partial y}F_x\right)\vec{k}$$

$\Rightarrow$

(iii) Path independence

$$W_{12} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r})\, d\vec{r}$$

$$\vec{r} \rightarrow \vec{r} + d\vec{r}$$

$$\Delta W = W(\vec{r} \rightarrow \vec{r} + d\vec{r})$$

$$= \vec{F}(r)\, d\vec{r}$$

$$F_x\, dx + F_y\, dy + F_z\, dz$$

work-energy theorem

$$\frac{1}{2} m \underset{\vec{r}+d\vec{r}}{\vec{v}}^2 + V(\vec{r}+d\vec{r}) = \frac{1}{2} m \underset{\vec{r}}{\vec{v}_0}^2 + V(\vec{r})$$

$$V(\vec{r}+d\vec{r}) - V(\vec{r}) =$$

$$V(x+dx, y+dy, z+dz) - V(x, y, z) = dV$$

$$f(x, y, z)$$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

$$W(\vec{r} \to \vec{r}+d\vec{r}) = -dV$$

$$= -\left[\frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz\right]$$

$$= F_x dx + F_y dy + F_z dz$$

$$\vec{F} = -\frac{\partial V}{\partial x}\vec{i} - \frac{\partial V}{\partial y}\vec{j} - \frac{\partial V}{\partial z}\vec{k}$$

$$= -\vec{\nabla} V(\vec{r})$$