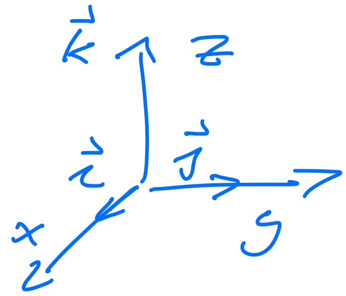
PHY 321, JANUARY 23, 2023

Falling object

$$\vec{a}(t) = -g\vec{j}$$



1-Dim (y-direction)

$$\vec{F} = m\vec{a} = m\frac{d^2\vec{r}}{dt^2}$$

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

1-Dim

$$F = G = -mg \qquad a = -g$$

$$v = \frac{dy}{dt} \quad \wedge \quad a = \frac{dv}{dt}$$

$$v(t) - v_0 = -g\int_{t_0}^{t} dt' = -g(t-t_0)$$

Add air resistance in 1 Direction

$$\vec{F}_D \rightarrow F_D = \begin{cases} Dv^2 & (\text{if } v \text{ large}) \\ \Gamma \cdot v & (\text{if } v \text{ small}) \end{cases}$$

$$F_{net} = G + F_D$$

$$= -mg + Dv^2$$

$$a = F_{net}/m = -g + \frac{D}{m} v^2$$

$$= -g + \tilde{D} v^2$$

$$= \frac{dv}{dt} = -g + \tilde{D} v^2$$

$$\frac{dv}{-g + \tilde{D} v^2} = dt$$

$$t_0 = 0\,s \qquad v_T = \sqrt{g/\tilde{D}}$$

$$\int_0^v \frac{dv'}{-g + \tilde{D} v'^2} = \int_{t_0}^{t} dt' = t$$

$$v_0 = 0\ m/s$$

Look up table of integrals

$$= \frac{v_T}{g} \tanh^{-1}\left(\frac{v}{v_T}\right) = -t$$

$$\Rightarrow v(t) = v_T \tanh\left(-\frac{gt}{v_T}\right)$$

$$\frac{dy}{dt} = v(t) \Rightarrow$$

$$y(t) - y_0 = \int_{t_0=0}^{t} dt' \, v(t')$$

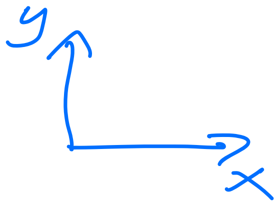
$$\Rightarrow y(t) = y_0 + \int_0^t dt' \, v_T \tanh\left(\frac{-gt'}{v_T}\right)$$

$$y(t) = y_0 - \frac{v_T^2}{g} \log\left[\cosh\left(\frac{gt}{v_T}\right)\right]$$

Example : 2Dim with

$$\vec{F}_D = D\vec{v}\,|\vec{v}(t)|$$

- choose coordinate system



$$\vec{v} = [v_x, v_y]$$

$$\vec{v} = v_x \vec{i} + v_y \vec{j}$$

$$\vec{F}_D = D\vec{v}\,|\vec{v}|$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

$$\vec{F}_{net} = \vec{G} + \vec{F}_D$$

$$F_x^D = D v_x \vec{i} \, |\vec{v}| = m a_x$$

$$F_y^D = D v_y \vec{j} \, |\vec{v}| = m a_y$$

$$a_x = \frac{dv_x}{dt} \wedge v_x = \frac{dx}{dt}$$

$$a_y = \frac{dv_y}{dt} \wedge v_y = \frac{dy}{dt}$$

$$F_y^{net} = -mg + D v_y \, |\vec{v}|$$

$$a_y^{net} = -g + \frac{D}{m} v_y \, |\vec{v}|$$

$$a_x^{net} = \frac{D}{m} v_x \, |\vec{v}|$$

$$\frac{d v_x^{tot}}{dt} = \frac{D}{m} v_x \, |\vec{v}|$$

$$\frac{d v_y^{tot}}{dt} = \frac{D}{m} v_y \, |\vec{v}| - g$$

$$\frac{dv_x^{tot}}{dt} = \frac{D}{m} v_x \sqrt{v_x^2 + v_y^2}$$

coupled diff eq

$$\frac{dv_y^{tot}}{dt} = \frac{D}{m} v_y \sqrt{v_x^2 + v_y^2} - g$$

cannot separate the equations wrt v_x and v_y? No analytical solutions or we may have to change to other coordinate systems?

<u>Example</u>

$$\vec{F}_D = -m\gamma \vec{v}(t)$$

$$\vec{G} = -mg\vec{j}$$

$$\vec{F}_{net} = \vec{F}_D + \vec{G}$$

$$a_x = \frac{dv_x}{dt} = -\gamma v_x(t) = a_x(t)$$

$$a_y = \frac{dv_y}{dt} = -\gamma v_y(t) - g$$

$$\frac{dx}{dt} = v_x \quad \wedge \quad \frac{dy}{dt} = v_y$$

$$\frac{dv_x}{dt} = -\gamma v_x$$

$$\frac{dv_x}{v_x} = -\gamma dt$$

$$\int_{v_{0x}}^{v_x} \frac{dv_x'}{v_x'} = -\gamma \int_{t_0=0}^{t} dt'$$

$$\ln \frac{v_x}{v_{0x}} = -\gamma t$$

$$v_x(t) = v_{0x} \exp(-\gamma t)$$

$$v_x = \frac{dx}{dt}$$

$$\int_{x_0}^{x} dx' = \int_{t_0=0}^{t} dt' v_x(t')$$

$$x(t) - x_0 = \int_0^t dt' \, v_{0x} \, e^{-\gamma t'}$$

$$x(t) = x_0 + \frac{v_{0x}}{\gamma}\left(1 - e^{-\gamma t}\right)$$

$$\frac{dv_y}{dt} = -\gamma v_y - g$$

$$\frac{dv_y}{v_y + g/\gamma} = -\gamma \, dt$$

$$\int_{v_{0y}}^{v_y} \frac{dv_y'}{v_y' + g/\gamma} = -\gamma \int_0^t dt' = -\gamma t$$

$$= \ln\left[\frac{v_y + g/\gamma}{v_{0y} + g/\gamma}\right] = -\gamma t$$

$$v_y(t) = \left(-g/\gamma + e^{-\gamma t}\right) \times$$

$$\left(v_{0y} + g/\gamma\right)$$

$$v_y(t) = \frac{dy}{dt}$$

$$y(t) = y_0 - gt/\gamma + (v_{0y} + g/\gamma)(1 - e^{-\gamma t})\frac{1}{\gamma}$$