PHY321, JANUARY 30, 2023

Work-Energy theorem

kinetic energy $K = \frac{1}{2} m \cdot v^2$

Force acting $\vec{F} = \vec{F}(\vec{r}, \vec{v}, t)$

$v^2 = \vec{v} \cdot \vec{v}$ $\qquad \vec{a} = \vec{F}/m$

assumption: no time dependence for m.

$$\frac{dK}{dt} = \frac{1}{2} m \frac{d}{dt}(\vec{v}\vec{v})$$

HW1, exercise 3

$$\frac{dK}{dt} = \frac{1}{2} m \left(\frac{d\vec{v}}{dt} \vec{v} + \vec{v} \frac{d\vec{v}}{dt} \right)$$

$$= \underbrace{m \frac{d\vec{v}}{dt}}_{\vec{F}} \cdot \vec{v}$$

$$\vec{F} \frac{d\vec{r}}{dt}$$

$$\frac{dK}{dt} = \lim_{\Delta t} \frac{K_2 - K_1}{t_2 - t_1} \qquad \Delta t = t_2 - t_1$$

Discretize

$dK \to \Delta K$

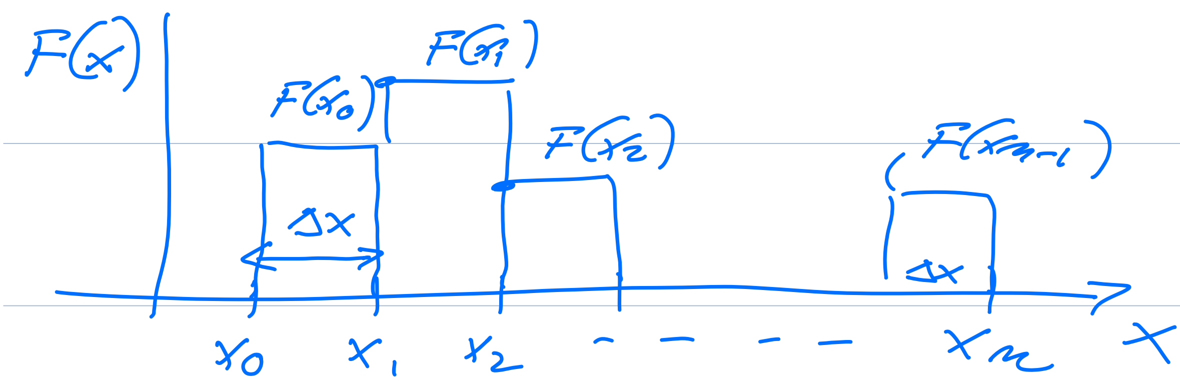
$$dt \Rightarrow \Delta t \qquad d\vec{r} = \Delta \vec{r}$$

$$\frac{dK}{dt} \Rightarrow \frac{\Delta K}{\Delta t} = \vec{F} \cdot \frac{\Delta \vec{r}}{\Delta t} = m\frac{\Delta \vec{v}}{\Delta t} \cdot \vec{v}$$

$$\Delta K = \underbrace{\vec{F} \Delta \vec{r}}$$

Def: work done by a force during a displacement $\Delta \vec{r}$

$$\Delta K = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$



$$X = \{x_0, x_1, x_2, \ldots x_n\}$$

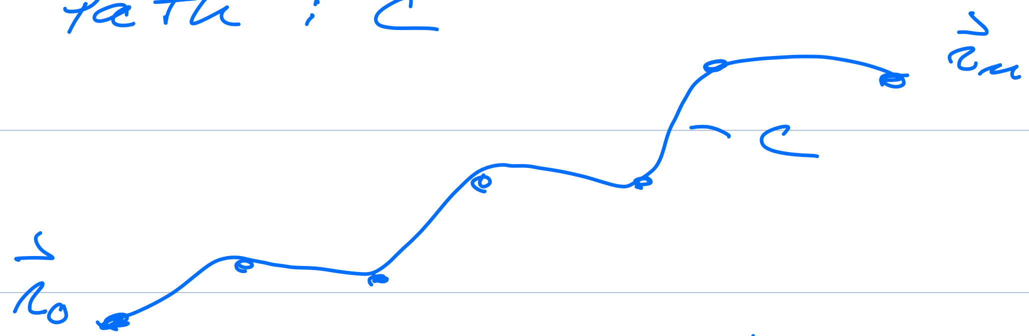
$$\Delta K = \frac{1}{2} m v_n^2 - \frac{1}{2} m v_0^2$$

$$= \sum_{i=0}^{n-1} F(x_i) \Delta x$$

$$\lim_{\Delta x \to 0} \sum_{i=0}^{n-1} F(x_i) \Delta x = \int_{x_0}^{x_n} dx\, F(x)$$

$$= W = \frac{1}{2}mv_n^2 - \frac{1}{2}mv_0^2$$

Path : C



$$W = \int_C F(\vec{r})\, d\vec{r}$$

$$= \frac{1}{2}mv_n^2 - \frac{1}{2}mv_0^2$$

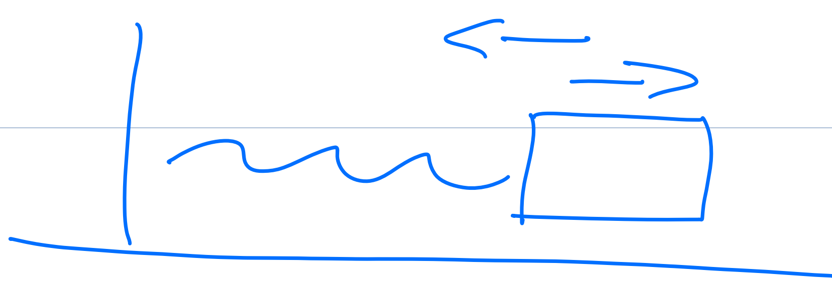
1) $\vec{F} \cdot \Delta\vec{r}$, can it be negative?

2) $\vec{F} \perp \Delta\vec{r} \Rightarrow \vec{F} \cdot \Delta\vec{r} = 0$

what happens to $\vec{v}$?

$\Delta K = 0 \Rightarrow$ velocity is constant.

Example 1



$$F = -kx$$

we move from x_0 to x_1

$$\frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 = \int_{x_0}^{x_1} (-kx)\,dx$$

$$= -\frac{1}{2} k x_1^2 + \frac{1}{2} k x_0^2$$

$$\frac{1}{2} m v_1^2 + \frac{1}{2} k x_1^2 = \frac{1}{2} m v_0^2 + \frac{1}{2} k x_0^2$$

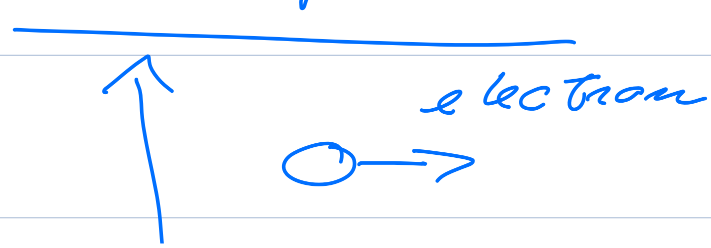
what does it mean?
what is $\frac{1}{2} k x_1^2$, $\frac{1}{2} k x_0^2$?

1) Energy (total energy) is conserved

2) Total energy = kinetic + potential energy.

3) $F(x) = -kx$ has a potential $V(x) = \frac{1}{2} k x^2$

Example 2



$x_0 = 0\,m \qquad v_0 = 0\,m/s$

$$F(x) = -F_0 \sin\left(\frac{2\pi x}{b}\right)$$

WE – theorem

$$\frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 = \frac{1}{2} m v_1^2$$

$$= -\int_{x_0=0}^{x_1} F_0 \sin\left(\frac{2\pi x}{b}\right) dx$$

$$\left[\begin{array}{ll} u = \dfrac{2\pi x}{b} & du = \dfrac{2\pi\, dx}{b} \\ dx = \dfrac{b\, du}{2\pi} & \end{array}\right]$$

$$\frac{F_0 b}{2\pi}\left[\cos\frac{2\pi x_1}{b} - \cos\frac{2\pi x_0}{b}\right]$$

$$\frac{1}{2} m v_1^2 = \frac{F_0 b}{2\pi}\left[\cos\frac{2\pi x_1}{b} - 1\right] \qquad x_0 = 0$$

$$\Rightarrow v_1 = \pm\sqrt{\frac{F_0 b}{m\pi}\left[\cos\frac{2\pi x_1}{b} - 1\right]}$$

Conservation laws

$\frac{dE}{dt} = 0$; energy is conserved

linear momentum $\vec{p}$ $\frac{d\vec{p}}{dt} = 0$;
momentum is conserved.

Two important conditions

(i) $\vec{F} = \vec{F}(\vec{r})$
only dependence on $\vec{r}$
The path chosen in the integral for $W = \int_{x_0}^{x_n} F(x)dx$

leads to a result which is independent of path
$\Rightarrow$ energy conservation

(ii) To have a path independent work
$\vec{\nabla} \times \vec{F} = 0$

These are called conservative forces ?

Conservation of linear momentum