PHY 321, MARCH 20, 2023

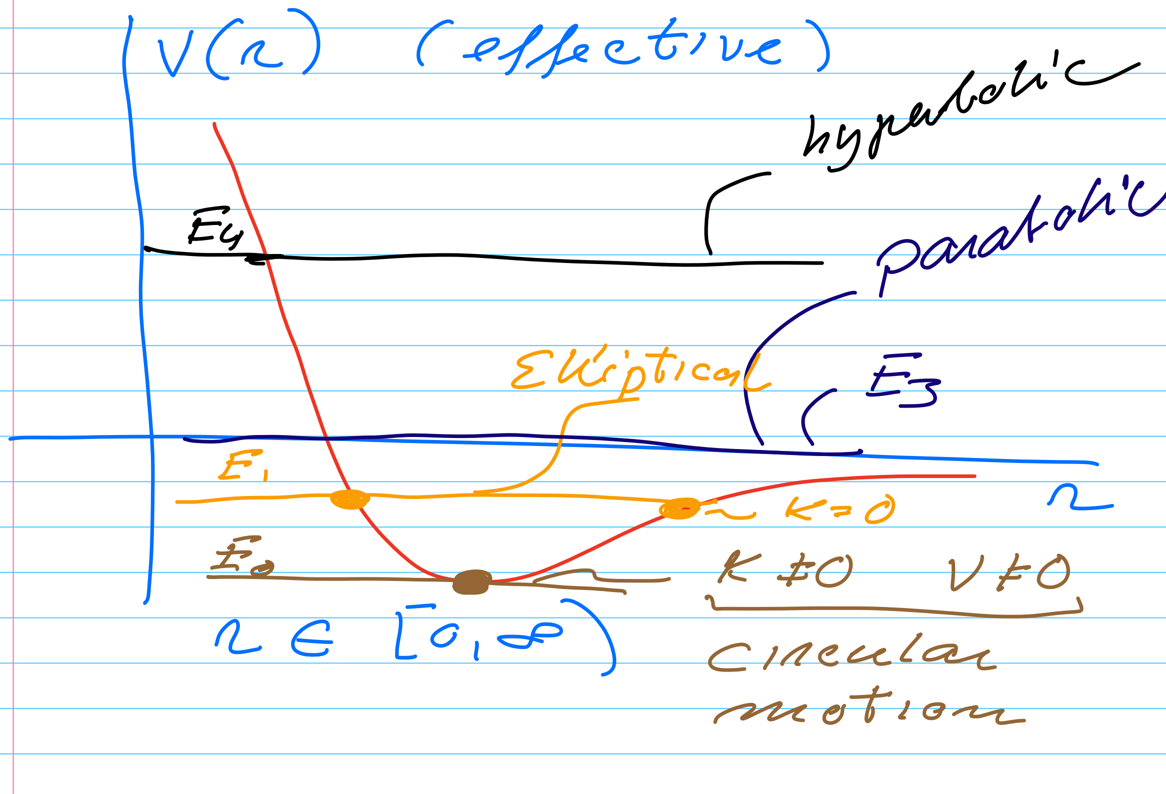
$$\vec{F}(\vec{r}) = -\vec{\nabla} V(r)$$

$$r = |\vec{r}_1 - \vec{r}_2| =$$

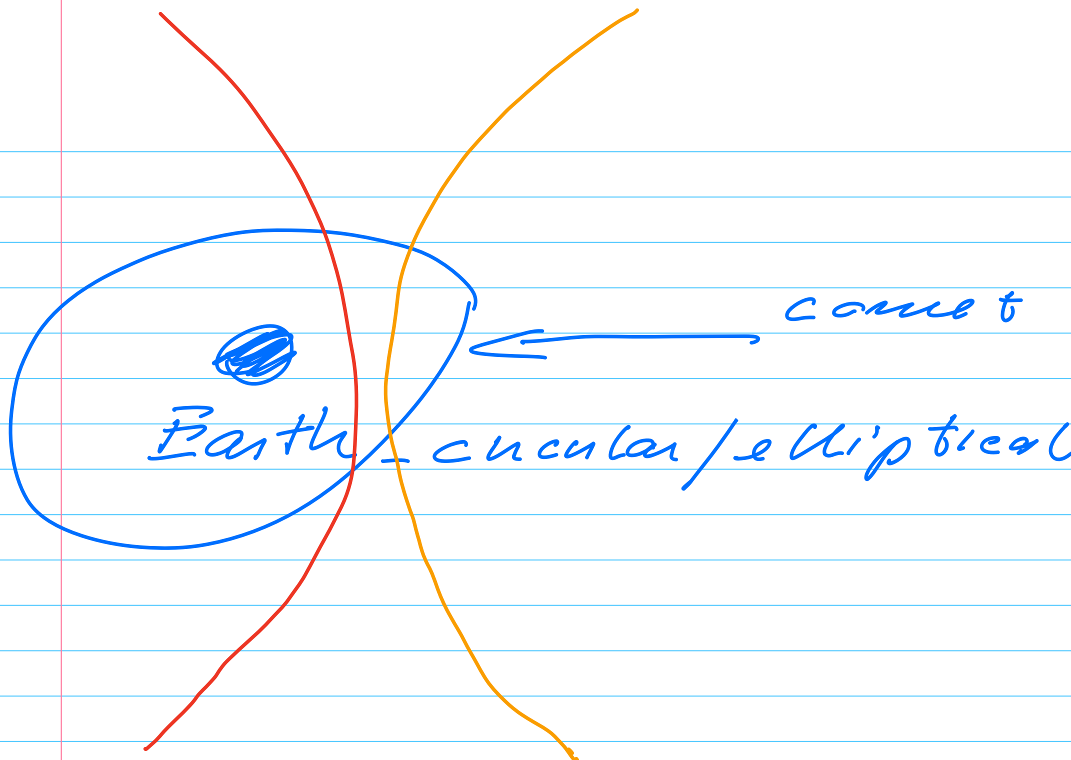
$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$V(r) = -\frac{G m_1 m_2}{r} = -\frac{\gamma}{r}$$

$$\gamma = G m_1 m_2$$



$r \in [0, \infty)$



Technicalities (hw 7, ex 2)

$N = 2$

Center-of-mass position (COM)

$$\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{M}$$

$$M = m_1 + m_2$$

Relative distance

$$\vec{r} = \vec{r}_1 - \vec{r}_2$$

$$\vec{r}_2 = \vec{R} - \frac{m_1}{M}\vec{r}$$

$$\vec{r}_1 = \vec{R} + \frac{m_2}{M}\vec{r}$$

Total Linear Momentum

$$\vec{P} = \sum_{i=1}^{N} \vec{P}_i = \sum_{i=1}^{N} m_i \frac{d\vec{r}_i}{dt}$$

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{M}$$

$$M\frac{d\vec{R}}{dt} = M\dot{\vec{R}} = \sum m_i \frac{d\vec{r}_i}{dt}$$

$$= \sum \vec{P}_i = M \cdot \dot{\vec{R}}$$

$$\vec{F}_{net} = \dot{\vec{P}} = M \cdot \ddot{\vec{R}}$$

(internal forces only)

$$\vec{F}_{net} = \vec{0} = \sum_{i \neq j}^{N} \vec{F}_{ij}$$

$N = 2$

$$\vec{F}_{net} = \vec{F}_{12} + \vec{F}_{21} =$$

$$\vec{F}_{12} - \vec{F}_{12} = 0 \qquad \vec{F}_{21} = -\vec{F}_{12}$$

$$M \cdot \ddot{\vec{R}} = 0 \implies$$

linear total momentum

$$\vec{P} = M \cdot \dot{\vec{R}} = \text{constant}$$

<u>Relative motion</u>

acceleration $\dfrac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}}$

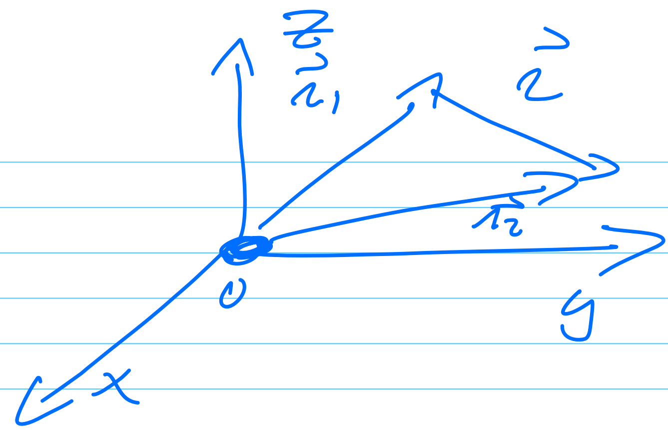
$$= \ddot{\vec{r}}_1 - \ddot{\vec{r}}_2$$

$$= \frac{\vec{F}_{12}}{m_1} - \frac{\vec{F}_{21}}{m_2} = \vec{F}_{12} \times \left(\frac{1}{m_1} + \frac{1}{m_2} \right)$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \implies$$

$$\ddot{\vec{r}} = \vec{F}_{12} / \mu \implies$$

$$\boxed{\begin{aligned} \mu \ddot{\vec{r}} &= \vec{F}_{12} \\ M \cdot \ddot{\vec{R}} &= 0 \end{aligned}}$$



$$\vec{F}_{12} = ? \qquad \vec{F}_{12} \Rightarrow \vec{F}(\vec{r})$$

$$\vec{F}(\vec{r}) = -\vec{\nabla} V(r)$$

$$= -\left[\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right]$$

$$\times \left(-\gamma / \sqrt{x^2+y^2+z^2}\right)$$

$$\frac{\partial}{\partial x}\frac{1}{\sqrt{x^2+y^2+z^2}} = -\frac{x}{(x^2+y^2+z^2)^{3/2}}$$

$$\vec{F} = -\gamma \frac{x\vec{i} + y\vec{j} + z\vec{k}}{r^3}$$

$$= -\gamma \frac{\vec{r}}{r^3} = -\gamma \frac{1}{r^2}\hat{r}$$

$$\hat{r} = \frac{\vec{r}}{r}$$

$$\mu \ddot{\vec{r}} = -\frac{\gamma}{r^3}\vec{r}$$

$$\gamma = G m_1 m_2$$

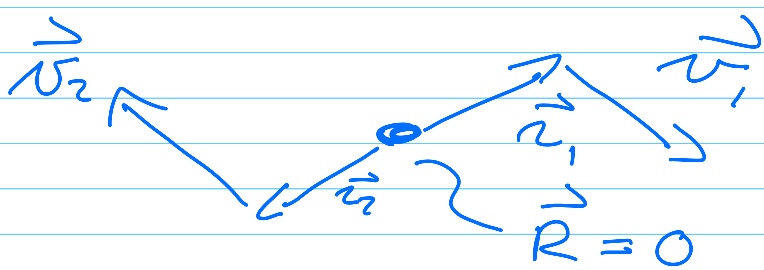
$$\mu = \frac{m_1 m_2}{M} \Rightarrow$$

$$\ddot{\vec{r}} = \vec{a}_r = -\frac{GM\vec{r}}{r^3}$$

COM frame

$$\vec{R} = 0$$

$$\vec{r}_1 = \frac{m_2 \vec{r}}{M} \wedge \vec{r}_2 = -\frac{m_1 \vec{r}}{M}$$



$$\vec{p}_i = m_i \dot{\vec{r}}_i$$

Total angular momentum

$$\vec{L} = (\vec{r}_1 \times \vec{p}_1) + (\vec{r}_2 \times \vec{p}_2)$$

$$= m_1 (\vec{r}_1 \times \dot{\vec{r}}_1) + m_2 (\vec{r}_2 \times \dot{\vec{r}}_2)$$

$$\vec{r}_1 = \frac{m_2 \vec{r}}{M} \qquad \vec{r}_2 = \frac{m_1 \vec{r}}{M}$$

$$= \mu (\vec{r} \times \dot{\vec{r}})$$

$$\frac{d\vec{L}}{dt} = 0 = \underbrace{\mu \left(\frac{d\vec{r}}{dt} \times \frac{d\vec{r}}{dt} \right)}_{=0} + \mu \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2} \right)$$

$$= \mu (\vec{r} \times \ddot{\vec{r}})$$

$$= \vec{r} \times \vec{F}(\vec{r}) = 0$$

$$\vec{F}(\vec{r}) = -\frac{\gamma}{r^3}\,\vec{r}$$

$$\Rightarrow \quad \frac{d\vec{L}}{dt} = 0$$

$\vec{L}$ is a constant