

PHY 321, JANUARY 18, 2023

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analyzing a physics problem

- inertial masses determined by measuring the acceleration for a given applied force

$$\vec{F} = m \vec{a}$$

- inertial masses are additive (and time independent,  $\frac{dm}{dt} = 0$ )

$$M = m_1 + m_2 + \dots + m_N = \sum_{i=1}^N m_i$$

$$\vec{F} = M \cdot \vec{a}$$

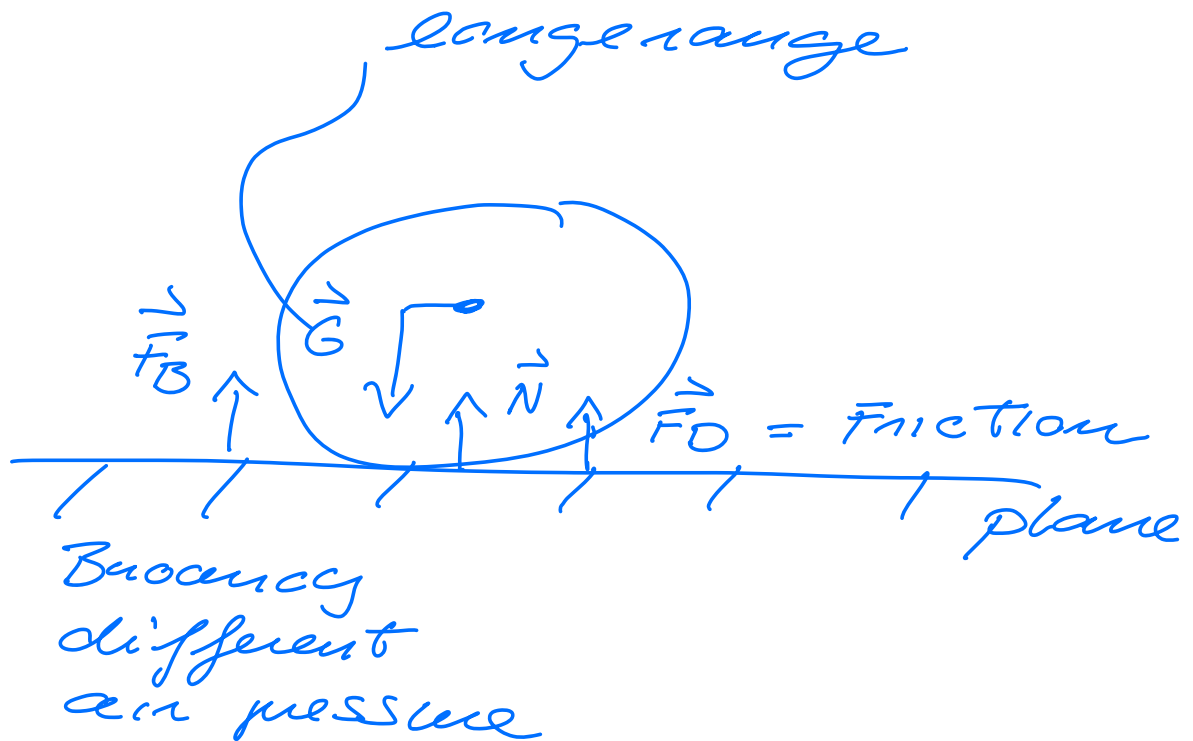
- only point particles

analyzing a problem:

- 1)
- Divide into system and environment
  - all forces must have sources when they act on the system
  - Forces are either long-range forces or contact forces

- 2)
- Draw a figure of the object and everything in contact with it
  - Make a sketch
  - Find contact points
  - name all forces
  - identify long-range and contact forces

- Decide upon choice of coordinate system



- Net external force

$$\vec{F}_{\text{net}} = m \cdot \vec{a} = \vec{G} + \vec{N} + \vec{F}_D + \vec{F}_B$$

Falling object  
(1-dim)

$$\vec{F} = -G = -m \cdot g$$

$$= m \cdot \frac{d^2 y}{dt^2}$$

$$\swarrow \quad \searrow$$

$$v = \frac{dy}{dt} \qquad a = \frac{dv}{dt}$$

Define initial conditions

$$v(t_0) = v_0 \quad \wedge \quad y(t_0) = y_0$$

$$\int_{t_0}^t a \, dt' = \int_{t_0}^t \frac{dv}{dt'} \, dt'$$

$$= v(t) - v(t_0)$$

$$= \int_{t_0}^t (-g) \, dt'$$

$$= -g(t - t_0)$$

$$t_0 = 0 \Rightarrow$$

$$v(t) = v_0 - g \cdot t$$

$$\int_{t_0}^t (v_0 - g \cdot t) dt = y(t) - y_0$$

$$v_0 = 0$$

$$= -\frac{1}{2} g t^2 = y(t) - y_0$$

$$y(t) = y_0 - \frac{1}{2} g t^2$$

approach 2      numerical approach

$$\frac{dv}{dt} = a \quad \wedge \quad \frac{dy}{dt} = v$$

Taylor expansion:

$$\begin{aligned} y(t + \Delta t) &= y(t) + \Delta t y'(t) \\ &\quad + \frac{(\Delta t)^2}{2!} y''(t) \\ &\quad + O(\Delta t^3) \end{aligned}$$

assume  $\Delta t$  small and  
truncate at  $\Delta t$

$$y(t + \Delta t) \approx y(t) + \Delta t y'(t)$$

$$t \rightarrow t_i = t_0 + i \Delta t$$

$$y(t + \Delta t) = y_{i+1}$$

$$y(t) = y_i$$

$$y_{i+1} = y_i + \Delta t v_i'$$

$$v(t + \Delta t) = v(t) + \Delta t v'(t)$$

$$\begin{aligned} y_{i+1} &= y_i + \Delta t v_i' \\ v_{i+1} &= v_i + \Delta t \cdot a_i' \end{aligned}$$

$$\Delta t = \frac{t_n - t_0}{n}$$

$y_0$  &  $v_0$  initial conditions