Lecture Jan 12 PHY 321

assume we know

$$y(t) = y(t_0) - \frac{1}{2} g \cdot t^2$$

$$g = 9.80665 \ m/s^2$$

initial time t_0 ($t_0 = 0s$)

$$y(t_0) = y_0$$

t_{final} when $y(t) = 0$

$$y(t_{final}) = 0 = y_0 - \frac{1}{2} g t_{final}^2$$

$$t_{final} = \sqrt{2 y_0 / g}$$

$$t \in [t_0, t_{final}]$$

$$t \to t_i$$

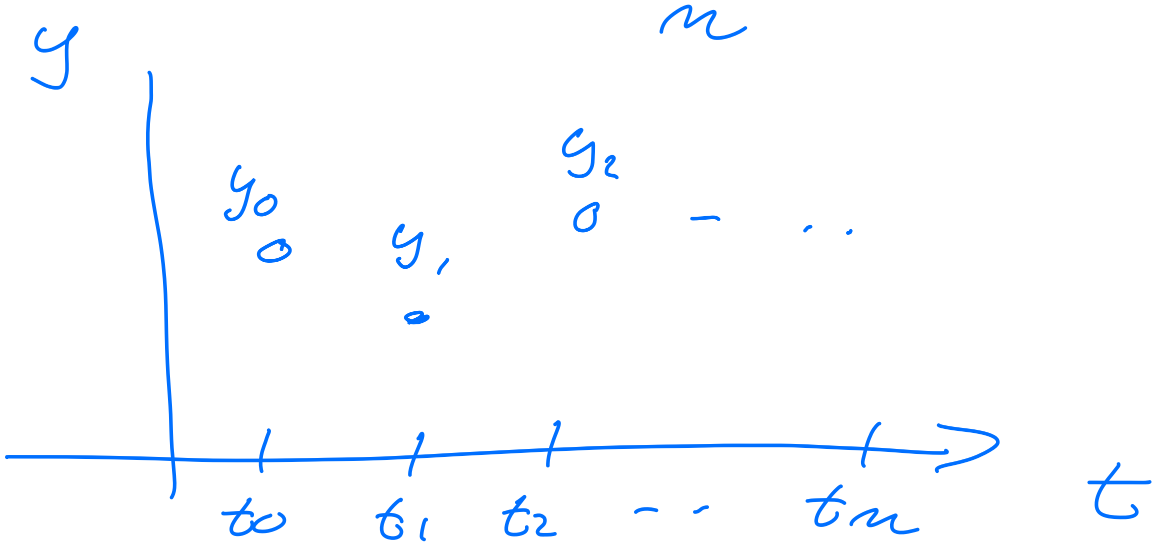
$$t_i = t_0 + i \Delta t$$

timestep

$$i = 0, 1, 2, \ldots n$$

$t_{final} = t_n$

$$\Delta t = \frac{t_n - t_0}{n}$$



y	y_0	y_1	y_2	- . . .	y_n
t	t_0	t_1	t_2		t_n

average velocity between $t_{i+1} = t_i + \Delta t$ and t_i

$$\bar{v}(t) = \frac{y(t+\Delta t) - y(t)}{\Delta t}$$

$$\Rightarrow \bar{v}_i = \frac{y_{i+1} - y_i}{\Delta t}$$

$$v(t) = \frac{dy}{} = \lim \frac{y(t+\Delta t) - y(t)}{}$$

dt $\Delta t \to 0$ Δt

$$a(t) = \frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{v(t+\Delta t) - v(t)}{\Delta t}$$

$$F = m\,a(t)$$

$$\Rightarrow$$

$$F/m = a(y, v, t)$$

$$= a\left(y, \frac{dy}{dt}, t\right)$$

$$a = \frac{dv}{dt}$$

initial conditions

$$v(t_0) = v_0 = 0\ m/s$$

$$t_0 = 0\ s$$

$$\int^{t} dt'\, a(t') = v(t) - v_0$$

$$\int_0 dt\, a(t) = v(t) - v_0$$

$$a(t) = -g$$

$$\int_0^t (-g)\, dt' = v(t) - \overset{0}{\overset{\shortparallel}{v_0}}$$

$$v(t) = -g \cdot t = \frac{dy}{dt}$$

$$v(t) = \frac{dy}{dt}$$

integrate again

$$\int_0^t dt' \underbrace{v(t')}_{(-g \cdot t')} = y(t) - y_0$$

$$\Rightarrow \quad y(t) = y_0 - \frac{1}{2} g t^2$$

approach $\underline{1}$

wisdom $\underline{1}$: if you can integrate, do it.

approach 2

solving differential equations

$$F = ma = m \cdot \frac{dv}{dt}$$

$$= m \frac{d^2y}{dt^2}$$

initial ~~con~~ conditions

$v_0, \ y_0, \ t_0$

two equations:

$$v(t) = \frac{dy}{dt}$$

$$a(t) = \frac{dv}{dt}$$

$$\int f(t)\,dt = \int dt\, f(t)$$

$$\approx \sum_i \Delta t\, f(t_i)$$