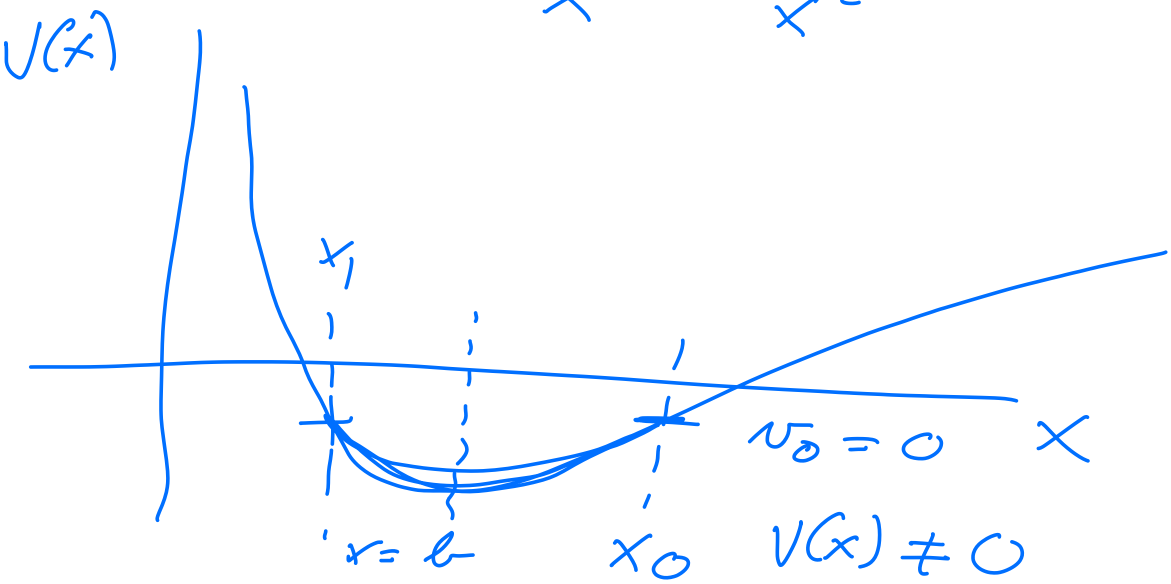
PHY 321 FEBRUARY 21, 2022

Midterm part 3

$$V(x) = -\frac{10}{x} + \frac{3}{x^2} + x$$



Harmonic oscillator:

$$V(x) = V(x=b) + (x-b)\left.\frac{\partial V}{\partial x}\right|_{x=b} + \frac{1}{2}(x-b)^2\left.\frac{\partial^2 V}{\partial x^2}\right|_{x=b} + O\left((x-b)^3\right)$$

$V(x=b)$ is the minimum of the potential.

$$\left.\frac{\partial V}{\partial x}\right|_{x=b} = 0$$

Drop higher-order terms

$$V(x) = V(b) + \frac{1}{2}(x-b)^2 \frac{\partial^2 V}{\partial x^2}$$

$$\left.\frac{\partial^2 V}{\partial x^2}\right|_{x=b} = k$$

$$k \geq 0$$

$$V(x) = V(b) + \frac{1}{2}(x-b)^2 k$$

$$\vec{F}(\vec{r}) = F(x) = -\frac{dV}{dx}$$

$$= -(x-b)k$$

$$b = 0$$

$$F(x) = -kx = m\frac{d^2x}{dt^2}$$

$$\omega_0 = \sqrt{k/m}$$

$$[K] = \text{Energy}/\text{length}^2$$
$$= \text{mass} \cdot \text{length}^2/\text{time}^2 / \text{length}^2$$
$$= \text{mass}/\text{time}^2$$
$$[\omega_0] = \sqrt{\text{mass}/\text{time}^2/\text{mass}}$$
$$= \frac{1}{\text{time}}$$

natural frequency.

$$\frac{d^2x}{dt^2} = -\omega_0^2 x$$

$$x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

$$v(t) = \frac{dx}{dt} = -A\omega_0 \sin(\omega_0 t) + B\omega_0 \cos(\omega_0 t)$$

$$\frac{d^2x}{dt^2} = -A\omega_0^2 \cos(\omega_0 t) - B\omega_0^2 \sin(\omega_0 t)$$

$$= -\omega_0^2 \left(A\cos(\omega_0 t) + B\sin(\omega_0 t) \right)$$

$$= -\omega_0^2 \, x(t)$$

specify $x(t_0) = x_0$

$v(t_0) = v_0 = 0$

$$x(t_0) = x_0 = A\underbrace{\cos(0)}_{=1} + B\underbrace{\sin(0)}_{=0}$$

$t_0 = 0$

$$\Rightarrow x_0 = A$$

$$\left.\frac{dx}{dt}\right|_{t_0} = v_0 = 0$$

$$= -A\omega_0 \sin(\omega_0 t_0) + B\omega_0\cos(\omega_0 t_0)$$

$$\Rightarrow B = 0 \quad \Rightarrow$$

$$x(t) = x_0 \cos(\omega_0 t)$$

$$t = \frac{2\pi}{\omega_0} = \text{Period} = T$$

Energy conservation

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$v_0 = 0 \qquad x(t_0) = x_0$$

$$x(t) = x_0 \cos(\omega_0 t)$$

$$t \neq 0$$

$$E(t) = \frac{1}{2} m v^2(t) + \frac{1}{2} k x^2(t)$$

at t_0 $\quad E(t_0) = E_0 = \frac{1}{2} k x_0^2$

$$E(t) = E_0$$

$$E(t) = \frac{1}{2} k x_0^2 \cos^2(\omega_0 t)$$

$$\left(v(t) = \frac{dx}{dt} = -x_0 \omega_0 \sin(\omega_0 t) \right)$$

$$+ \frac{1}{2} m x_0^2 \omega_0^2 \sin^2(\omega_0 t)$$

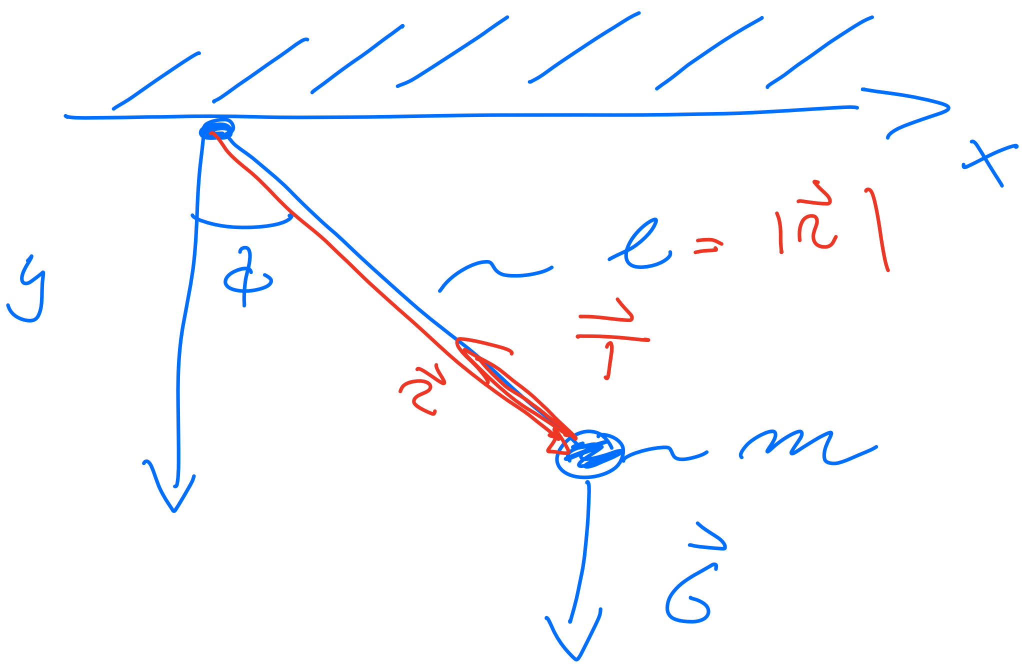
$$\omega_0^2 = k/m$$

$$= \frac{1}{2} k x_0^2 \left(\cos^2(\omega_0 t) + \sin^2(\omega_0 t) \right)$$

$$= \frac{1}{2} k x_0^2 = E_0$$

Energy is conserved.

Example: Math pendulum



$$\frac{d^2\phi}{dt^2} = -g/l \sin\phi$$

$$\phi \ll 1$$

$$\frac{d^2\phi}{dt^2} = -g/l\,\phi$$

$$\omega_0 = \sqrt{g/l}$$