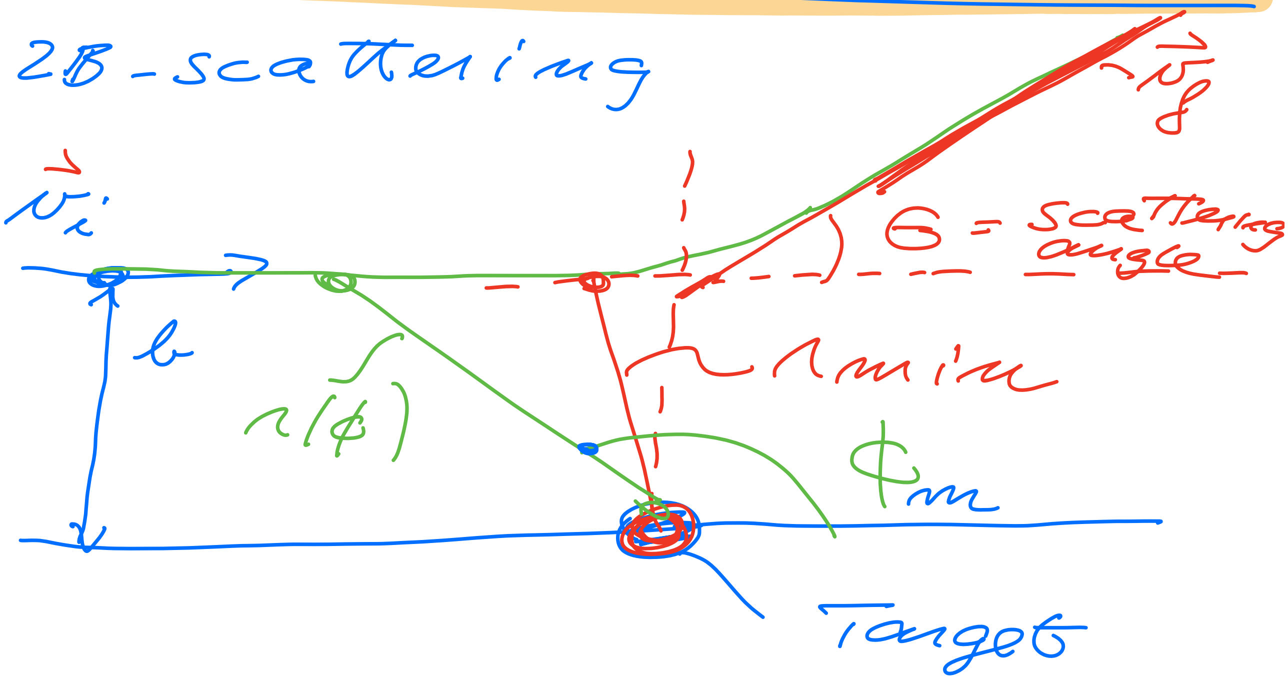
PHY 321, APRIL 13, 2022

2B-scattering



$$r(\phi) = \frac{c}{1+\varepsilon\cos\phi}$$ attractive force

$$= \frac{c}{\varepsilon\cos\phi - 1}$$ repulsive force

ϕ_m = projectile being infinitely far away from target

$$\Theta = |\pi - 2\phi_m|$$

(Taylor 14.6)

Cross section

$$\sigma = \int_0^{2\pi}\int_0^{\pi} \underbrace{\frac{d\sigma}{d\Omega}} d\phi \sin\theta \, d\theta$$

$$\frac{d\sigma}{d\Omega} = \frac{b \cdot db}{\sin\theta \, d\theta}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{a^2}{4\sin^4 \theta/2}}$$

$$a = \frac{\alpha}{2E} \qquad V(r) = -\alpha/r$$

$$\alpha = \frac{q_1 q_2}{4\pi\varepsilon_0}$$

E = incoming energy of projectile

$$\sigma = \int_0^{\pi} \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$$

$$= \int_0^{\pi} \frac{a^2}{4\sin^4 \theta/2} \sin\theta \, d\theta$$

Rutherford classical cross section

- **Lagrangian formalism**

can derive equations of motion from kinetic and potential energy

- Principle of least action
- Lagrangian (1-Dim)

$$\mathcal{L}(x, v, t) = K - V$$

- Euler-Lagrange eq

$$\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v} = 0$$

- variational calculus

Example 1

Harmonic oscillator

$$K = \frac{1}{2} m v^2$$

$$V = \frac{1}{2} k x^2$$

Euler-Lagrange Eqs

$$\frac{\partial \mathcal{L}}{\partial x} = -kx \qquad \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v} = m\frac{d}{dt}v = ma$$

$$\frac{\partial \mathcal{L}}{\partial v} = mv$$

$$\mathcal{L} = K - V = \frac{1}{2} m v^2 - \frac{1}{2} k x^2$$

$$\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v} = -kx - ma = 0$$

$$\Rightarrow \quad ma = \boxed{m\frac{d^2x}{dt^2} = -kx}$$

Example 2

$$x, y \Rightarrow r, \phi$$

$$K = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2}\mu (r\dot{\phi})^2$$

$$V = -\alpha/r$$

$$\mathcal{L} = \frac{1}{2}\mu \underbrace{\dot{r}}_{v_r}{}^2 + \frac{1}{2}\mu (r\underbrace{\dot{\phi}}_{v_\phi})^2 + \frac{\alpha}{r}$$

$$\mathcal{L} = \mathcal{L}(r, v_r, \phi, v_\phi, t)$$

$$\frac{\partial \mathcal{L}}{\partial r} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v_r} = 0$$

$$\underbrace{\frac{\partial \mathcal{L}}{\partial \phi}}_{=0} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v_\phi} = 0$$

$$\frac{\partial \mathcal{L}}{\partial v_\phi} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \mu \cdot r^2 \dot{\phi}$$

$$\dot{\phi} = \frac{L}{\mu r^2} \implies$$

$$\frac{\partial \mathcal{L}}{\cdots} = L \quad ?$$

$\dot{\phi}$

$$\frac{d}{dt} L = 0$$

zero torque.

$$\frac{\partial \mathcal{L}}{\partial r} = \mu r \dot{\phi}^2 - \alpha/r^2$$

$$\frac{\partial \mathcal{L}}{\partial \dot{r}} = \mu \dot{r} \qquad \frac{d}{dt} \mu \dot{r}$$

$$= \mu \ddot{r} \implies$$

$$\boxed{\mu \ddot{r} = -\alpha/r^2 + \mu r \dot{\phi}^2}$$