PHY 321, FEBRUARY 28, 2022

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x(t) = -\omega_0^2 x$$

$$\omega_0 = \sqrt{k/m}$$

$$x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

$$e^{\pm i\omega_0 t} = \cos(\omega_0 t) \pm i\sin(\omega_0 t)$$

$$x(t) = C e^{i\omega_0 t}$$

$$\frac{d^2x}{dt^2} = -\omega_0^2 C e^{i\omega_0 t}$$

$$= -\omega_0^2 x(t)$$

$$x(t) = C e^{i\omega_0 t} + D e^{-i\omega_0 t}$$

$$= (C+D)\cos(\omega_0 t) + i(C-D)\sin(\omega_0 t)$$

$$A = C+D$$

$$B = i(C-D)$$

Another alternative: new

$$A = \sqrt{B_1^2 + B_2^2}$$

$$B_2 = A \sin\delta$$

$$B_1 = A\cos\delta$$

$$x(t) = B_1 \cos(\omega_0 t) + B_2 \sin(\omega_0 t)$$

$$= A\left[\frac{B_1}{A}\cos(\omega_0 t) + \frac{B_2}{A}\sin(\omega_0 t)\right]$$

$$= A\left[\cos(\omega_0 t)\cos\delta + \sin(\omega_0 t)\sin\delta\right]$$

$$\cos(\omega_0 t - \delta)$$

$$= A\cos(\omega_0 t - \delta)$$

Damping

add $b \cdot v = b \frac{dx}{dt}$

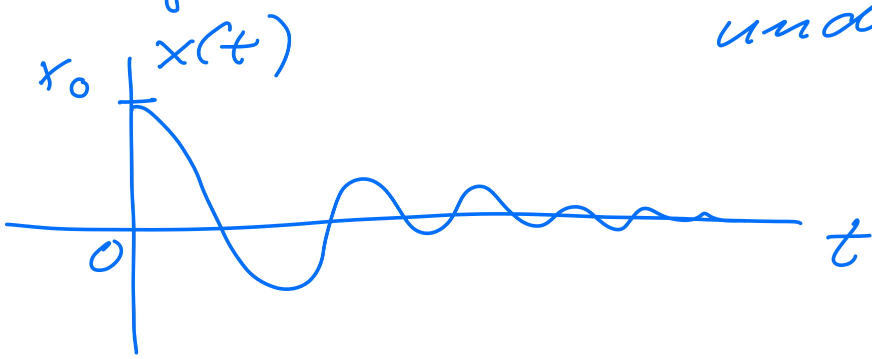
$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

$$\frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + w_0^2 x = 0$$

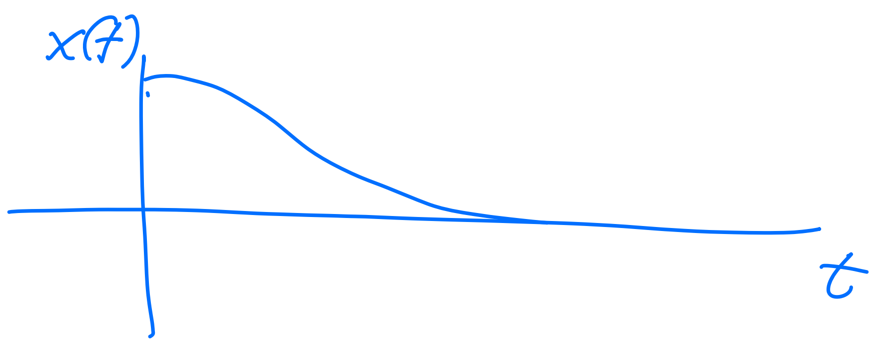
$\tau = w_0 \cdot t$ Dimless

$\gamma = \frac{b}{m w_0 \cdot 2}$ Damping coefficient

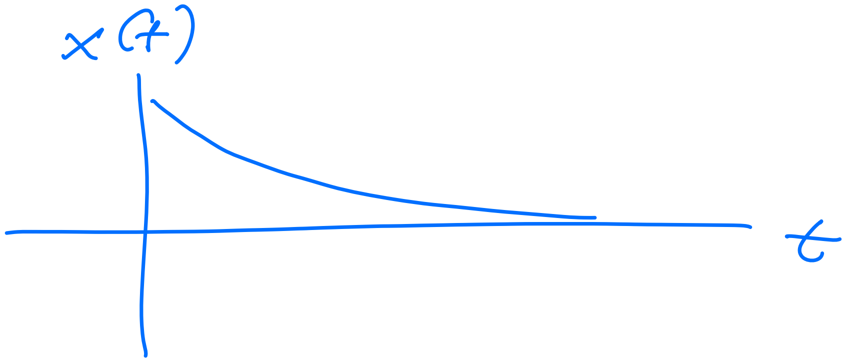
$\gamma < 1$ underdamping



$\gamma = 1$ critical damping



$\gamma > 1$ overdamping



$$m \frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \cos(\omega t)$$

with $\tau = \omega_0 \cdot t$

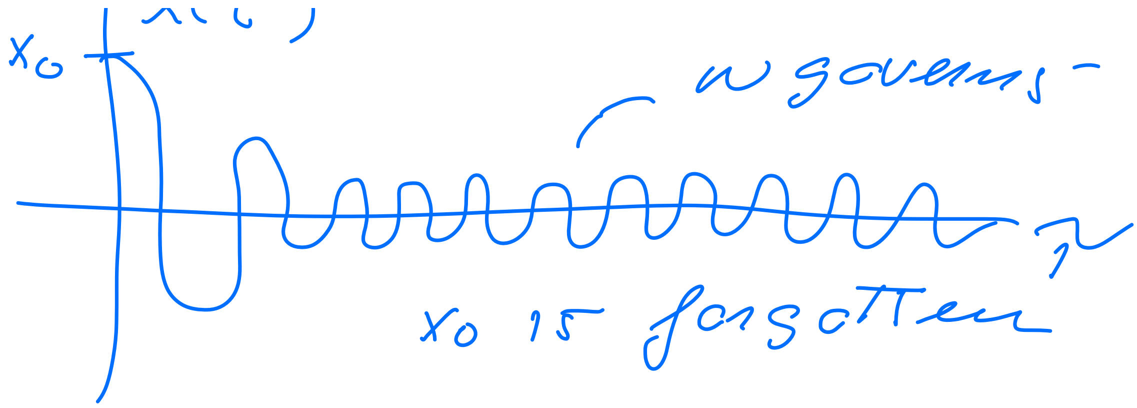
$$\frac{d^2x}{d\tau^2} + 2\gamma \frac{dx}{d\tau} + x = ?$$

$$\underbrace{\frac{F_0}{m\,\omega_0^2}}_{\tilde{F}_0} \cos\Big(\underbrace{\frac{\omega}{\omega_0}}_{\tilde{\omega}}\tau\Big)$$

$$\frac{d^2x}{d\tau^2} + 2\gamma \frac{dx}{d\tau} + x = \tilde{F}_0 \cos(\tilde{\omega}\tau)$$

$\tilde{F}_0 \neq 0$

$x(\tau)$



$$\tilde{T}_0 = 0$$

$$\frac{d^2x}{d\tau^2} + 2\gamma\frac{dx}{d\tau} + x = 0$$

$$x(\tau) = Ae^{r\tau}$$

not yet specified

$$Ae^{r\tau}\left(r^2 + 2\gamma r + 1\right) = 0$$

$$A \neq 0 \qquad e^{r\tau} \neq 0$$

$$r^2 + 2\gamma r + 1 = 0$$

$$r_1 = -\gamma + \sqrt{\gamma^2 - 1}$$

$$r_2 = -\gamma - \sqrt{\gamma^2 - 1}$$

$$x(\tau) = A_1 e^{r_1\tau} + A_2 e^{r_2\tau}$$

(i) underdamping $\gamma < 1$

$$\sqrt{\gamma^2-1} = i\gamma' \qquad \gamma' \in \mathbb{R}$$

$$\gamma \in \mathbb{R}$$

$$x(\tau) = A_1 e^{-\gamma\tau} e^{+i\gamma'\tau} + A_2 e^{-\gamma\tau} e^{-i\gamma'\tau}$$

Damping

cos & sin oscillations

$$= (A_1+A_2) e^{-\gamma\tau} \cos(\gamma'\tau)$$

$$+ i(A_1-A_2) e^{-\gamma\tau} \sin(\gamma'\tau)$$

(ii) $\gamma = 1$ critical damping

$$\sqrt{\gamma^2-1} = 0$$

$$-\gamma\tau \qquad -\gamma\tau$$

$$x(\tau) = A_1 e + A_2 e$$

$$x(\tau) = A_1 e^{-\gamma\tau} + A_2 \tau e^{-\gamma\tau}$$

Damping

$$\gamma = \frac{b}{m \cdot \omega_0 \cdot 2} = \underline{1}$$

(iii) Strong damping

$$\gamma > \underline{1}$$

$\sqrt{\gamma^2 - 1}$ always real

$$x(\tau) = A_1 e^{-(\gamma - \sqrt{\gamma^2-1})\tau} + A_2 e^{-(\gamma + \sqrt{\gamma^2-1})\tau}$$

Damping only

$$\gamma \gg \underline{1}$$

$$\sqrt{\gamma^2 - 1} \simeq \gamma$$

$$x(\tau) \simeq A_1 + A_2 e^{-2\gamma\tau}$$