PHY 321, MARCH 31, 2023

$$\mu \ddot{r} = F(r) + \frac{\ell^2}{\mu r^3}$$

$$r(\phi) = \frac{c}{1+\varepsilon\cos\phi}$$

$\varepsilon = 0 \Rightarrow$ circular motion

$x = r\cos\phi \wedge y = r \cdot \sin\phi$

$$(1+\varepsilon\cos\phi)\cdot r = c$$

$$r + x\varepsilon = c$$

squared

$$r^2 = x^2+y^2 = c^2+\varepsilon^2x^2 - 2x\varepsilon c$$

$$x^2(1-\varepsilon^2) + y^2 + 2x\varepsilon c = c^2$$

Divide by $1-\varepsilon^2$ $\quad d = \frac{c\varepsilon}{1-\varepsilon^2}$

$$x^2 + 2dx + \frac{y^2}{1-\varepsilon^2} = \frac{c^2}{1-\varepsilon^2}$$

$$+d^2 \qquad\qquad + d^2$$

$$\left(x+d\right)^2+\frac{y^2}{1-\varepsilon^2}=\frac{c^2}{1-\varepsilon^2}+d^2$$

$$a^2=\frac{c^2}{(1-\varepsilon^2)^2}$$

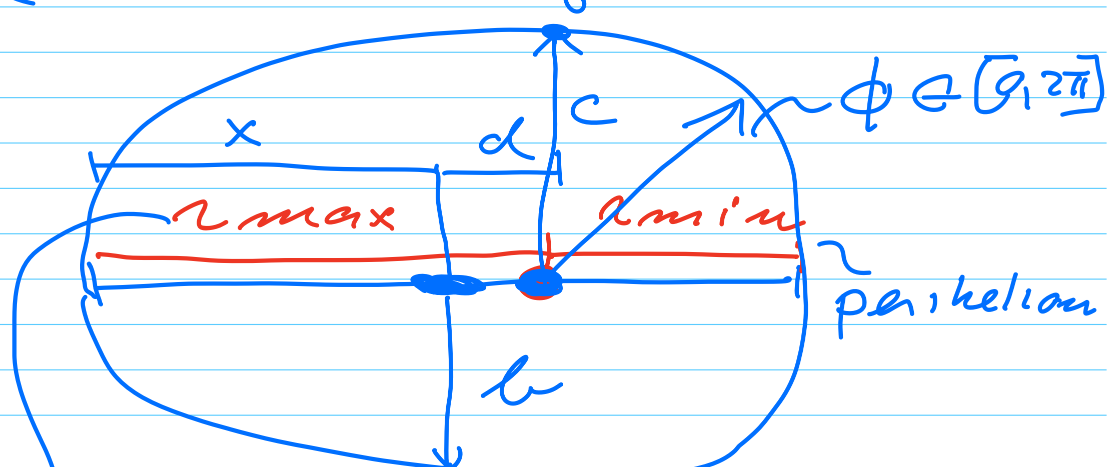
$$(x+d)^2+\frac{y^2}{1-\varepsilon^2}=\frac{c^2}{1-\varepsilon^2}+\frac{c^2\varepsilon^2}{(1-\varepsilon^2)^2}$$

$$=a^2$$

$$\left(\frac{x+d}{a}\right)^2+\frac{y^2}{(1-\varepsilon^2)a^2}=1$$

$$b=a\sqrt{1-\varepsilon^2}$$

$$\left(\frac{x+d}{a}\right)^2+\frac{y^2}{b^2}=1$$



aphelion.

$\varepsilon = 0$ circle $E < 0$
bounded orbit

$0 < \varepsilon < 1$ ellipse $E < 0$
bounded orbit

$\varepsilon = 1$ parabola $E = 0$
unbounded
orbit

$\varepsilon > 1$ hyperbolic $E > 0$
unbounded
orbit.