PHY321: Classical Mechanics 1

Homework 6, due Monday March 1

Feb 21, 2021

Practicalities about homeworks and projects.

1. You can work in groups (optimal groups are often 2-3 people) or by yourself. If you work as a group you can hand in one answer only if you wish. **Remember to write your name(s)**!
2. Homeworks are available ten days before the deadline.
3. How do I(we) hand in? You can hand in the paper and pencil exercises as a scanned document. For this homework this applies to exercises 1-5. Alternatively, you can hand in everyhting (if you are ok with typing mathematical formulae using say Latex) as a jupyter notebook at D2L. The numerical exercise(s) (exercise 6 here) should always be handed in as a jupyter notebook by the deadline at D2L.

Introduction to homework 6. This week's sets of classical pen and paper and computational exercises are again a continuation of the topics from the previous homework sets. We keep discussing conservation laws, conservative forces, energy, momentum and angular momentum. These conservation laws are central in Physics and understanding them properly lays the foundation for understanding and analyzing more complicated physics problems. The relevant reading background is

1. chapters 3 and 4 of Taylor (there are many good examples there) and
2. chapters 10-14 of Malthe-Sørenssen.
3. For exercise 4 will also need sections 5.1-5.3 of Taylor.

The numerical homework is based on what you did in homework 5 and you are going to test whether energy is conserved or not.

Exercise 1 (40pt), Summary of what we have done till now. Compile a summary of the material covered in this class so far thate covers the major topics (Newton's Laws, conservative and nonconservative forces, and conservation of energy, momentum, and angular momentum). This summary should not only contain a list of equations but should also include important concepts, numerical elements, and mathematical methods, and show the connections between different concepts. This assignment is meant to help you review to materials in the course in preparation for the first midterm project. Make this summary as long as you need to throughly review all of then covered topics.

Exercise 2 (10 pt), Conservative forces. Which of the following force are conservative?

- 2a (2pt) $\boldsymbol{F} = k(x\boldsymbol{i} + 2y\boldsymbol{j} + 3z\boldsymbol{k})$ where k is a consttant.
- 2b (2pt) $\boldsymbol{F} = y\boldsymbol{i} + x\boldsymbol{j} + 0\boldsymbol{k}$.
- 2c (2pt) $\boldsymbol{F} = k(-y\boldsymbol{i} + x\boldsymbol{j} + 0\boldsymbol{k})$ where k is a constant.
- 2d (4pt) For those which are conservative, find the corresponding potential energy V and verify that direct differentiation that $\boldsymbol{F} = -\boldsymbol{\nabla} V$.

Exercise 3 (10 pt), The Lennard-Jones potential. The Lennard-Jones potential is often used to describe the interaction between two atoms or ions or molecules. If you end up doing materals science and molecular dynamics calculations, it is very likely that you will encounter this potential model. The expression for the potential energy is of the molecule is:

$$V(r) = V_0 \left((\frac{a}{r})^{12} - (\frac{b}{r})^6 \right),$$

where V_0, a and b are constants and the potential depends only on the relative distance between two objects i and j, that is $r = |\boldsymbol{r}_i - \boldsymbol{r}_j| = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}$.

- 3a (3pt) Sketch/plot the potential (choose some values for the constants in doing so).
- 3b (3pt) Find and classify the equilibrium points.
- 3c (4pt) What is the force acting on one of the objects (an atom for example) from the other object? Is this a conservative force?

Exercise 4 (10 pt), particle in a new potential. Relevant reading here is Taylor chapter 5 and the lecture notes on oscillations. In particular, you will find useful sections 5.1, 5.2, and 5.4. They contain all material needed to solve this exercise.

Consider a particle of mass m moving in a one-dimensional potential,

$$V(x) = -\alpha\frac{x^2}{2} + \beta\frac{x^4}{4}.$$

- 4a (3pt) Plot the potential and discuss eventual equilibrium points. Is this a conservative force?
- 4b (3pt) Compute the second derivative of the potential and find its miminum position(s). Using the Taylor expansion of the potential around its minimum (see Taylor section 5.1) to define a spring constant k. Use the spring constant to find the natural (angular) frequency $\omega_0 = \sqrt{k/m}$. We call the new spring constant for an effective spring constant.
- 4c (4pt) We ignore the second term in the potential energy and keep only the term proportional to the effective spring constant, that is a force $F \propto kx$. Find the acceleration and set up the differential equation. Find the general analytical solution for these harmonic oscillations. You don't need to find the constants in the general solution.

Exercise 5 (30pt), Testing Energy Conservation. This week we will reuse our code from homework 4 (exercises 6) and replace the air resistance and force from the ground with the gravitational force. Then we will study the stability of the system as function of initial conditions and the time step length.

We start by importing some standard packages

```
# let's start by importing useful packages we are familiar with
import numpy as np
from math import *
import matplotlib.pyplot as plt
%matplotlib inline
```

- 5a (10 pt) Show that the **curl** of the gravitational force is zero and discuss whether the gravitational force is a conservative force or not.
- 5b (20pt) Using the code from homework 5 (see also the solution), compute the kinetic and potential energies as functions of time. You will need the velocties and positions to compute these quantities. Remember to define initial positions and velocities, as well as the initial kinetic and potential energies. Use Euler's methods, the Euler-Cromer method or the Velocity-Verlet

method from the code you developed in homework 5 and study if energy is conserved over time. For a circular motion, are the potential and kinetic energies conserved separately? Comment your results.

Classical Mechanics Extra Credit Assignment: Scientific Writing and attending Talks. The following gives you an opportunity to earn **five extra credit points** on each of the remaining homeworks and **ten extra credit points** on the midterms and finals. This assignment also covers an aspect of the scientific process that is not taught in most undergraduate programs: scientific writing. Writing scientific reports is how scientist communicate their results to the rest of the field. Knowing how to assemble a well written scientific report will greatly benefit you in you upper level classes, in graduate school, and in the work place.

The full information on extra credits is found at `https://github.com/mhjensen/Physics321/blob/master/doc/Homeworks/ExtraCredits/`. There you will also find examples on how to write a scientific article. Below you can also find a description on how to gain extra credits by attending scientific talks.

This assignment allows you to gain extra credit points by practicing your scientific writing. For each of the remaining homeworks you can submit the specified section of a scientific report (written about the numerical aspect of the homework) for five extra credit points on the assignment. For the two midterms and the final, submitting a full scientific report covering the numerical analysis problem will be worth ten extra points. For credit the grader must be able to tell that you put effort into the assignment (i.e. well written, well formatted, etc.). If you are unfamiliar with writing scientific reports, see the information here

The following table explains what aspect of a scientific report is due with which homework. You can submit the assignment in any format you like, in the same document as your homework, or in a different one. Remember to cite any external references you use and include a reference list. There are no length requirements, but make sure what you turn in is complete and through. If you have any questions, please contact Julie Butler at butler@frib.msu.edu.

HW/Project	Due Date	Extra Credit Assignment
HW 3	2-8	Abstract
HW 4	2-15	Introduction
HW 5	2-22	Methods
HW 6	3-1	Results and Discussion
Midterm 1	**3-12**	*Full Written Report*
HW 7	3-22	Abstract
HW 8	3-29	Introduction
HW 9	4-5	Results and Discussion
Midterm 2$\vert\,_4 - 16$	*Full Written Report*	
HW 10	4-26	Abstract
Final	**4-30**	*Full Written Report*

You can also gain extra credits if you attend scientific talks. This is described here.

Integrating Classwork With Research. This opportunity will allow you to earn up to 5 extra credit points on a Homework per week. These points can push you above 100% or help make up for missed exercises. In order to earn all points you must:

1. Attend an MSU research talk (recommended research oriented Clubs is provided below)
2. Summarize the talk using at least 150 words
3. Turn in the summary along with your Homework.

Approved talks: Talks given by researchers through the following clubs:

- Research and Idea Sharing Enterprise (RAISE): Meets Wednesday Nights Society for Physics Students (SPS): Meets Monday Nights
- Astronomy Club: Meets Monday Nights
- Facility For Rare Isotope Beam (FRIB) Seminars: Occur multiple times a week

If you have any questions please consult Jeremy Rebenstock, rebensto@msu.edu.

All the material on extra credits is at `https://github.com/mhjensen/Physics321/blob/master/doc/Homeworks/ExtraCredits/`.