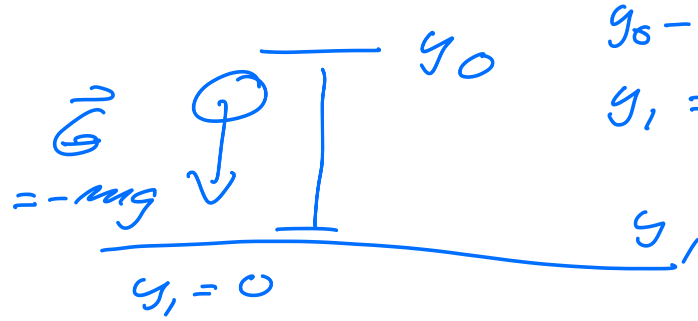
PHY 321 February 9

Falling ball

phase 1



$y_0 - y_1 = h$

$y_1 = 0$

$$W_{01} = \int_{y_0 = h}^{y_1 = 0} (-mg)\,dy$$

$$= m \cdot g h = K_1 - K_0$$

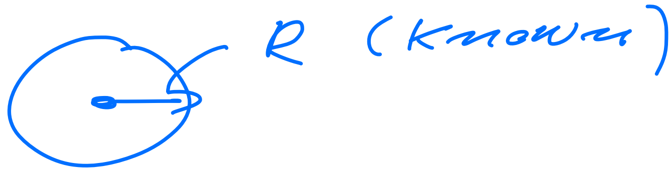
$K_0 = \frac{1}{2} m v_0^2 = 0$

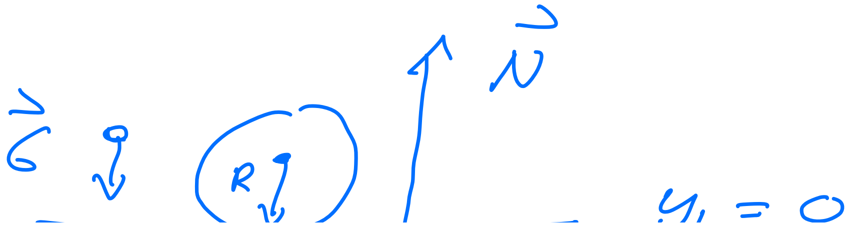
$v_0 = 0 \qquad v_1 = ?$

$$mgh = \frac{1}{2} m v_1^2 - 0$$

$$\Rightarrow \quad v_1 = \pm \sqrt{2gh}$$

phase 2

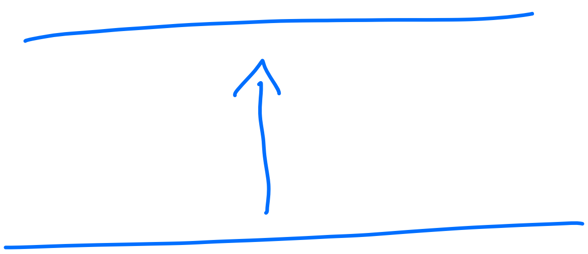




$$\vec{F}^{net} = \vec{G} + \vec{N}$$

$$\vec{N} = -k(R-y)\vec{j}$$

$$= \begin{cases} -k(R-y)\vec{j} & y \le R \\ 0 & else \end{cases}$$



$y_2 = ?$ $v_2 = 0$

$y_1 = 0$

$v_1 = \pm\sqrt{2gh}$ $y_1 = 0$

h is known

K is known

$$W_{12} = \int_{y_1=0}^{y_2} (\vec{N} - mg)\,dy$$

$$= \int_{y_1=0}^{R} -(k(R-y)+mg)\,dy + \int_{R}^{y_2} (-mg)\,dy$$

$$= -k\int_0^{R}(R-y)\,dy - mg\int_0 dy$$

$$y_2 > R$$

$$= -kR^2 + \frac{kR^2}{2} - mg\cdot y_2$$

$$= \frac{1}{2}m\underset{\overset{\shortparallel}{0}}{v_2^2} - \frac{1}{2}m\underset{\overset{\shortparallel}{\left(\pm\sqrt{2gh}\right)^2}}{v_1^2}$$

$$\Rightarrow$$

$$-\frac{1}{2}kR^2 - mg\,y_2 = -mg h$$

$$y_2 = h - \frac{1}{2}\frac{kR^2}{mg}$$

Example 2

$$\vec{F} = -\gamma\frac{\vec{r}}{|\vec{r}|^3}$$

$$|\vec{r}| = r = \sqrt{x^2+y^2+z^2}$$

$$W_{01} = -\gamma\int_{r_0}^{r_1}\frac{\vec{r}\,d\vec{r}}{r^3}$$

$$\vec{r}\,d\vec{r} = d\left(\tfrac{1}{2}\vec{r}\vec{r}\right)$$

$$= d\left(\tfrac{1}{2}r^2\right) = r\,dr$$

$$W_{01} = -\gamma \int_{r_0}^{r_1} \frac{r\,dr}{r^3} = -\gamma \int_{r_0}^{r_1} \frac{dr}{r^2}$$

$$= \frac{\gamma}{r}\Bigg|_{r_0}^{r_1} = \underbrace{-\gamma\frac{1}{r_0}}_{V(r_0)} + \gamma\frac{1}{r_1}$$

$r_0 =$ origin $\qquad \overset{\parallel}{V_0}$

$$\gamma\frac{1}{r_1} = -V(r_1) \longrightarrow$$

$$r_1 \to r \Rightarrow V(r) = -\gamma\frac{1}{r}$$

$$\vec{F}(r) = -\vec{\nabla}\,V(r)$$

$$= -\gamma\left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)\left(\frac{1}{r}\right)$$

$$\frac{1}{r} = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial}{\ } \frac{1}{\ } = \frac{\cancel{2}x}{\cancel{2}(x^2+y^2+z^2)^{3/2}}$$

$\partial x \quad \sqrt{x^2+y^2+z^2} \qquad (x^2+y^2+z^2)$

$$\frac{\partial}{\partial y}\left(\text{—} , \text{—} \right) = \frac{y}{(x^2+y^2+z^2)^{3/2}}$$

$$\frac{\partial}{\partial z}\left(\text{—} , \text{—} \right) = \frac{z}{(x^2+y^2+z^2)^{3/2}}$$

$\Rightarrow$

$$\vec{F} = - \nabla V(\vec{r}) =$$

$$-\gamma \left(\frac{x\,\vec{i}}{(x^2+y^2+z^2)^{3/2}} + \frac{y\,\vec{j}}{(\ldots)^{3/2}} + \frac{z\,\vec{k}}{(\ldots)^{3/2}} \right)$$

$$= - \gamma \frac{\vec{r}}{r^3}$$

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$$\vec{F} = - \vec{\nabla} V(\vec{r})$$

$$\vec{\nabla} \times \vec{F} = 0$$

$$(\vec{\nabla} \times \vec{F})_x = \left(\frac{\partial}{\partial y} F_z - \frac{\partial}{\partial z} F_y\right)\hat{x}$$