PHY 321, MARCH 15, 2023

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos(\omega t)$$

$$\omega_0 = \sqrt{k/m} \qquad \gamma = \frac{b}{2m\omega_0}$$

$$= \frac{\beta}{\omega_0} \qquad \beta = \frac{b}{2m}$$

$$\tilde{\omega} = \frac{\omega}{\omega_0}$$

$$\tilde{t} = \omega_0 \cdot t$$

HW6 parameters

$$\omega = 2\pi$$

$$\omega_0 = 5\omega \qquad F_0 = 1000$$

$$T = \frac{2\pi}{\omega} \quad \text{(period)}$$

$$= \underline{1}$$

$$m = \underline{1} \qquad b = \underline{1}$$

$$\tilde{F}_0 = \frac{F_0}{m\omega_0^2} = \frac{1000}{(5\cdot 2\pi)^2}$$

$\approx 1.01 \quad (= \underline{1})$

$$\gamma = \frac{b}{2m\omega_0} = \frac{1}{20\pi}$$

$$t = \tau / \omega_0 = \frac{\tau}{2\pi \cdot 5}$$

$$\tau = 10\pi \cdot t$$

Particular solution

$$x_p(\tau) = D \cdot \cos(\tilde{\omega}\tau - \delta)$$

$$D = \frac{\tilde{F}_0}{\sqrt{(1-\tilde{\omega}^2)^2 + 4\tilde{\omega}^2\gamma^2}}$$

$\beta = \frac{b}{2m}$ damping constant

$$\frac{d^2x}{d\tau^2} + 2\gamma\frac{dx}{d\tau} + x = \tilde{F}_0 \cos(\tilde{\omega}\tau)$$

$$D_{max} = \frac{F_0}{2\beta\omega_0}$$

$$\omega = \sqrt{\omega_0^2 - 2\beta^2}$$

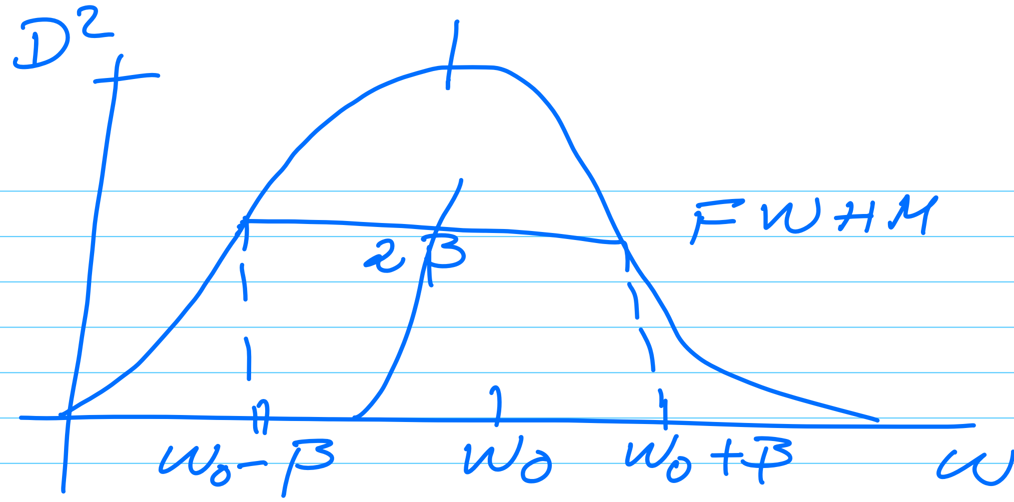
$\beta \ll \omega_0 \rightarrow \omega \simeq \omega_0$

as β decreases the peak (resonance) gets sharper and sharper.

Common to define a quantity called Full Width at Half Maximum (FWHM)

$$\frac{D^2_{max}}{2} = \frac{F_0^2}{4\beta^2\omega_0^2}\,\frac{1}{2}$$

$$\omega \simeq \omega_0 \pm \beta$$



if we want a sharp resonance, we need to tune $\beta \ll \omega_0$

common to introduce the quality factor

$$Q = \frac{\omega_0}{2\beta}$$

Large $Q \Rightarrow$ narrow resonance
Small $Q \Rightarrow$ wide resonance

well-defined resonance requires a small β

<u>ODE</u>

$$\frac{d^2x}{d\tilde{\tau}^2} + \frac{1}{Q}\frac{dx}{d\tilde{\tau}} + x = \tilde{F}_0 \cos(\tilde{\omega}\tilde{\tau})$$

Damping factor

$$e^{-\gamma\tilde{\tau}} \propto e^{-1/Q\,\tilde{\tau}} \propto e^{-\beta\tilde{\tau}}$$

MORE ODE Solvers

- Euler's method (no energy conserv.)
- Euler-Cromer, conserves energy ($O(\Delta t^2)$ for x, $O(\Delta t)$ for v)
- Velocity-Verlet ($O(\Delta t^3)$ for x and v) conserves energy
- Family of RK-methods

RK = Runge-Kutta,

$$\frac{dy}{dt} = f(t, y)$$

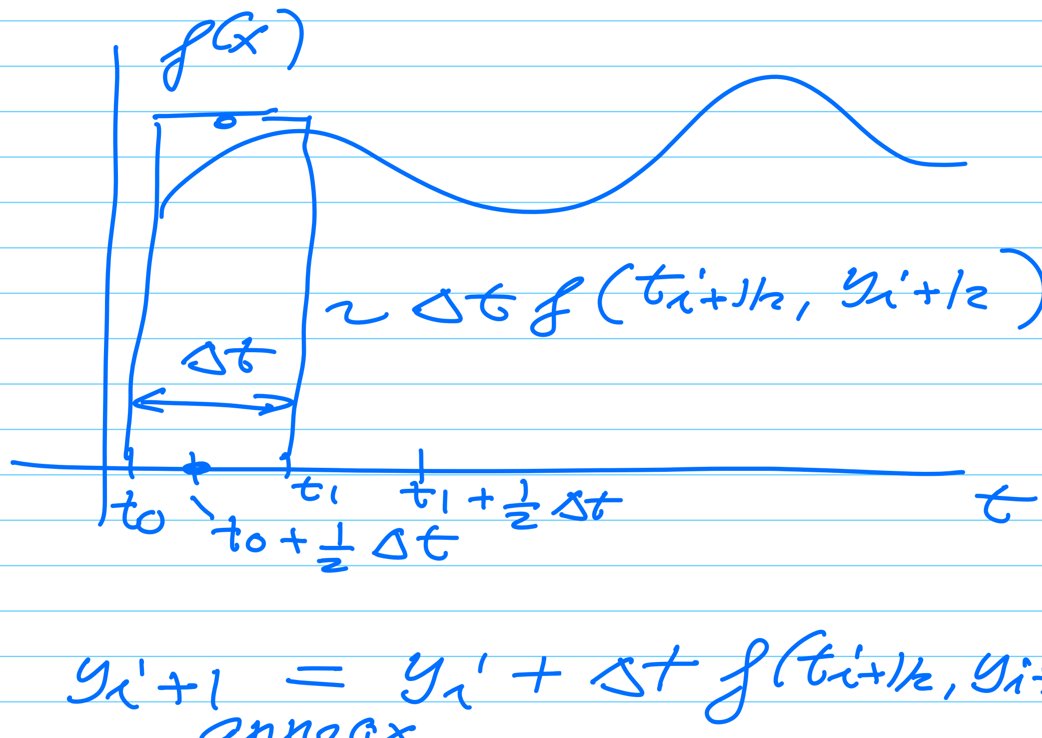
$$t_{i+1} = t_i + \Delta t \qquad y_i = y(t_i)$$

$$y_{i+1} = y_i + \int_{t_i}^{t_{i+1}} f(t, y)\,dt$$

RK2

$$\int_{t_i}^{t_{i+1}} f(t, y)\,dt = \Delta t \cdot f(t_{i+1/2},\, y_{i+1/2}) + O(\Delta t^3)$$

Midpoint rule



$$\underset{\text{approx}}{y_{i+1}} = y_i + \Delta t\, f(t_{i+1/2}, y_{i+1/2})$$

$y_{i+1/2} = ?$

Euler !

$$y_{i+1/2} = y_i + \frac{\Delta t}{2} \cdot f(t_i, y_i)$$

$$k_1 = \Delta t \cdot f(t_i, y_i)$$

$$k_2 = \Delta t \, f\left(t_{i+1/2}, y_i + \frac{k_1}{2}\right)$$

$$y_i + k_1/2 = y_{i+1/2}$$

$$y_{i+1} = y_i + k_2 \quad \boxed{\left(O(\Delta t^3)\right)}$$