PHY 321, MARCH 2, 2022

$\omega_0 = \sqrt{k/m}$ $\quad \tau = \omega_0 t$

$\gamma = \dfrac{b}{2m\omega_0}$

$$\frac{d^2x}{d\tau^2} + 2\gamma\frac{dx}{d\tau} + x = \tilde{F}_0 \cos(\tilde{\omega}\tau)$$

$\tilde{\omega} = \dfrac{\omega}{\omega_0}$

$\tilde{F}_0 = 0$

Homogeneous solution

$$x(\tau) = A_1 e^{r_1\tau} + A_2 e^{r_2\tau}$$

$r_1 = -\gamma + \sqrt{\gamma^2 - 1}$

$r_2 = -\gamma - \sqrt{\gamma^2 - 1}$

$\gamma < 1$ underdamping

$\gamma = 1$ critical damping

$\gamma > 1$ over critical damping

$\tilde{F}_0 \neq 0$ $\quad x_p(\tau) =$ particular solution.

$$x_p(\tau) = D\cos(\tilde{\omega}\tau - \delta)$$

plug into the ODE

$$D\left\{-\tilde{\omega}^2\cos(\tilde{\omega}\tau-\delta) - 2\gamma\tilde{\omega}\sin(\tilde{\omega}\tau-\delta) + \cos(\tilde{\omega}\tau-\delta)\right\} = \tilde{F}_0\cos(\tilde{\omega}\tau)$$

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$$

$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$D\left\{\left(-\tilde{\omega}^2\cos\delta + 2\gamma\tilde{\omega}\sin\delta + \cos\delta\right)\cos(\tilde{\omega}\tau) + \underbrace{\left(-\tilde{\omega}^2\sin\delta - 2\gamma\tilde{\omega}\cos\delta + \sin\delta\right)}_{=0}\sin(\tilde{\omega}\tau)\right\} = \tilde{F}_0\cos(\tilde{\omega}\tau)$$

$$-\tilde{\omega}^2\sin\delta - 2\gamma\tilde{\omega}\cos\delta$$

$+ \sin\delta = 0$

$\to$ Divide by $\cos\delta$ $\Rightarrow$

$$\tan\delta = \frac{2\gamma\tilde{\omega}}{1-\tilde{\omega}^2}$$

$$\sin\delta = \frac{\tan\delta}{\sqrt{\tan^2\delta - 1}}$$

$$\cos^2\delta = 1 - \sin^2\delta$$

$$\sin\delta = \frac{2\gamma\tilde{\omega}^2}{\sqrt{(1-\tilde{\omega})^2 + 4\gamma^2\tilde{\omega}^2}}$$

$$\cos\delta = \frac{(1-\tilde{\omega}^2)^2}{\sqrt{(1-\tilde{\omega})^2 + 4\gamma^2\tilde{\omega}^2}}$$

insert back into ODE

$$D = \frac{\tilde{F}_0}{\sqrt{(1-\tilde{\omega}^2)^2 + 4\tilde{\omega}^2\gamma^2}}$$

$$\delta = \tan^{-1}\left(\frac{2\gamma\tilde{\omega}}{1-\tilde{\omega}^2}\right)$$

$$\omega = \omega_0 \Rightarrow \tilde{\omega} = 1$$

$$D = \frac{\tilde{F}_0}{2\gamma}$$

$$\gamma = \frac{b}{2m\omega_0}$$

$$x_p(\tau) = D\cos(\tilde{\omega}\tau - \delta)$$

$$V(r) = r^2 \cdot V_0 \qquad r = \sqrt{x^2+y^2}$$

$$F_x = -\frac{dV}{dx} = -V_0\left(\frac{d}{dx}(x^2+y^2)\right) = -2V_0 x$$

$$F_y = -\frac{dV}{dy} = -2V_0 y$$

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

$$= -2V_0(x\vec{i} + y\vec{j})$$

$$= -2v_0 \vec{r} \cdot \frac{r}{r}$$

$$= -2v_0 r \cdot \hat{r}$$

Circular motion:

$$r_0 = ?$$

$$|\vec{a}| = v^2/r_0$$