PHY321, JANUARY 25, 2023

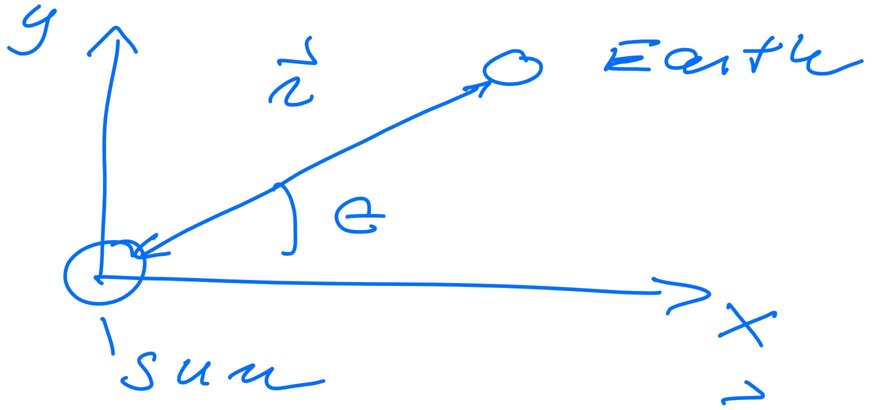
Example 3

$$\vec{F} = -\frac{G M_\odot M_E \vec{r}}{|\vec{r}|^3}$$

$M_E = 6\cdot 10^{24}\,\text{kg}$

$M_\odot = 2\cdot 10^{30}\,\text{kg}$

$$|\vec{r}| = r = \sqrt{x^2+y^2} = 1\,\text{AU} = 1{,}5\cdot 10^{11}\,\text{m}$$



$$x = \vec{r}\cdot\cos\theta$$

$$y = \vec{r}\cdot\sin\theta$$

$$F_x = -\frac{G M_\odot M_E\, x}{r^3}$$

$$F_y = -\frac{G M_\odot M_E\, y}{r^3}$$

$$\frac{F_x}{M_E} = a_x = -\frac{G M_B x}{r^3} = \frac{dv_x}{dt}$$

$$= -\frac{G M_B x}{\left(\sqrt{x^2+y^2}\right)^3}$$

$$\frac{F_y}{M_E} = a_y = -\frac{G M_B y}{\left(\sqrt{x^2+y^2}\right)^3} = \frac{dv_y}{dt}$$

$$v_x = \frac{dx}{dt} \quad \wedge \quad v_y = \frac{dy}{dt}$$

1. approach: integrate analytically

$$\frac{dv}{dt} = a(t)$$

$$dv = a(t)\,dt$$

$$\int_{v_0}^{v} dv' = \int_{t_0}^{t} a(t')\,dt'$$

2) solve coupled diff-eqs

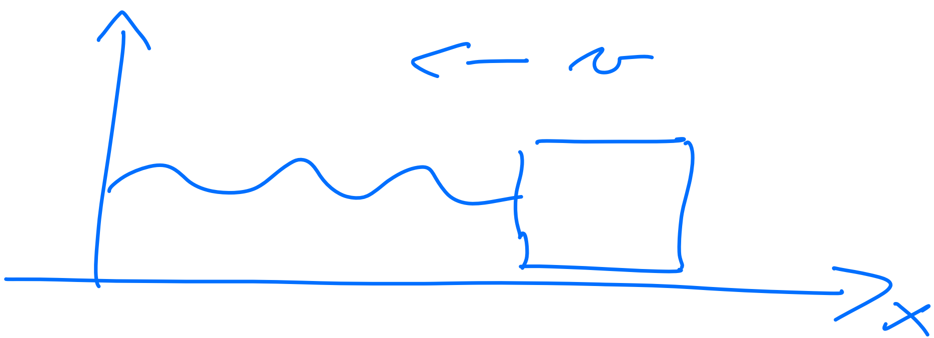
$$\frac{dv}{dt} = a$$

$$a = \frac{d^2x}{dt^2}$$

$$\frac{dx}{dt} = v$$

3) solve 2) numerically.

Example 4



$$F = F(x, v, t) = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

x_0, v_0

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

natural frequency $\omega_0^2 = \frac{k}{m}$

$$a = \frac{d^2x}{dt^2} = -\omega_0^2 x$$

$$a = \frac{dv}{dt} = -\omega_0 x$$

$$\frac{dx}{dt} = v$$

$$x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

HW 2 exercise 5 & 6

Exercise 5: Two forces

Normal force with magnitude N and perpendicular to to the board,

Then Gravity $\vec{G}$

perpendicular to the board

$-mg\cos\theta$

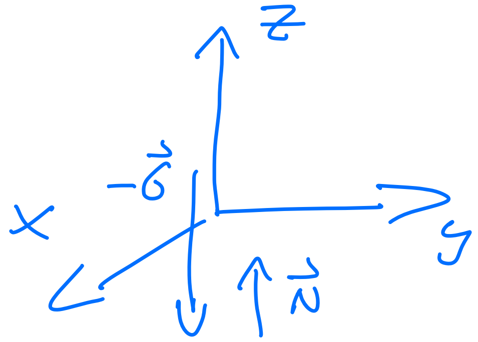
and the $-mg\sin\theta$

No force across the board.

$$\vec{r}_0 = (\overset{x}{0}, \overset{y}{0}, \overset{z}{0})$$

$$\vec{v}_0 = (v_{0x}, v_{0y}, 0)$$

$$\ddot{x} = \frac{d^2x}{dt^2}$$



$x: 0 = m\ddot{x}$

$y: -mg\sin\theta = m\ddot{y} = ma_y$

$z = N - mg\cos\theta = m\cdot\ddot{z} = 0$

no force in z direction

$$m\ddot{x} = m\frac{dv_x}{dt} = 0$$

$$\int_{v_{0x}}^{v_x} dv_x' = 0 \Rightarrow v_x = v_{0x} \neq 0$$

$$\frac{dx}{dt} = v_x \quad \Rightarrow$$

$$x(t) - x_0 = x(t) = \int_0^t v_{0x}\,dt$$

$$= v_{0x} \cdot t$$

y-direction

$$\cancel{m}\ddot{y} = \cancel{m}\frac{dv_y}{dt} = -\cancel{m} g \sin\theta$$

$$dv_y = -g \sin\theta \, dt$$

$$v_y(t) - v_{0y} = -g \sin\theta \cdot t$$

$$\frac{dy}{dt} = v_y = v_{0y} - g \sin\theta \cdot t$$

new integration:

$$y(t) = v_{0y} \cdot t - \frac{1}{2} g \sin\theta \cdot t^2$$

$$y_0 = 0$$

$$z(t) = 0$$

$$\vec{r}(t) = \left(\overset{x}{v_{0x} t},\ \overset{y}{v_{0y} t - g \sin\theta \frac{t^2}{2}},\ \overset{z}{0}\right)$$

$$y(t_f) = 0 = v_{0y} t_f - g \sin\theta \frac{t_f^2}{2}$$

$$t_f = \frac{2 v_{oy}}{g_{moe}}$$

it travels a distance

$$x(t_f) = v_{ox} \cdot t_f = \frac{2 v_{ox} v_{oy}}{g_{moe}}$$

python ways of defining

$\vec{r}(t)$ discretized time

$$t = \{\underbrace{t_0, t_1, \ldots, t_{99}}_{100 \text{ values}}\}$$

$$\vec{r}(t) = (x(t), y(t))$$

in python

$$\vec{r} \Rightarrow r = np.zeros((\overset{t}{100}, \overset{x,y}{2}))$$