PHY 321, MARCH 16, 2022

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos(\omega t)$$

$$x_p(\tilde{t}) = D\cos(\tilde{\omega}\tilde{t} - \delta)$$

$$D = \frac{\tilde{F}_0}{\sqrt{(1-\tilde{\omega}^2)^2 + 4\tilde{\omega}^2\gamma^2}}$$

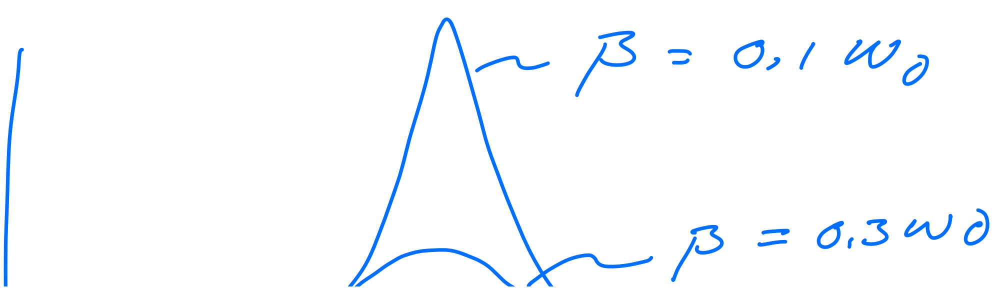
$$\gamma = \frac{b}{2m\omega_0} \qquad \omega_0 = \sqrt{k/m}$$

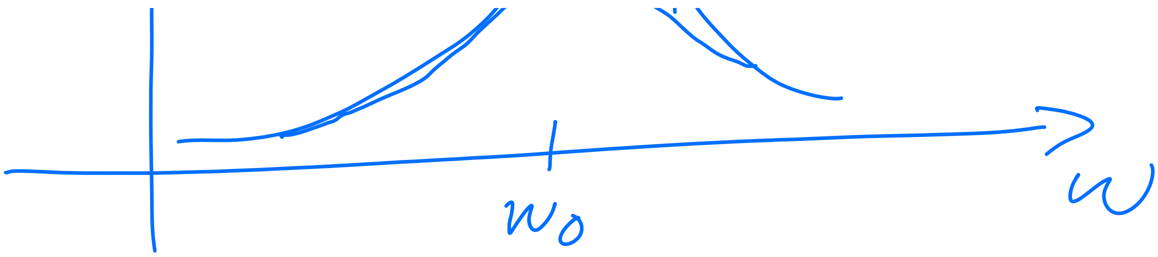
$$\beta = \frac{b}{2m} \qquad \gamma = \frac{\beta}{\omega_0}$$

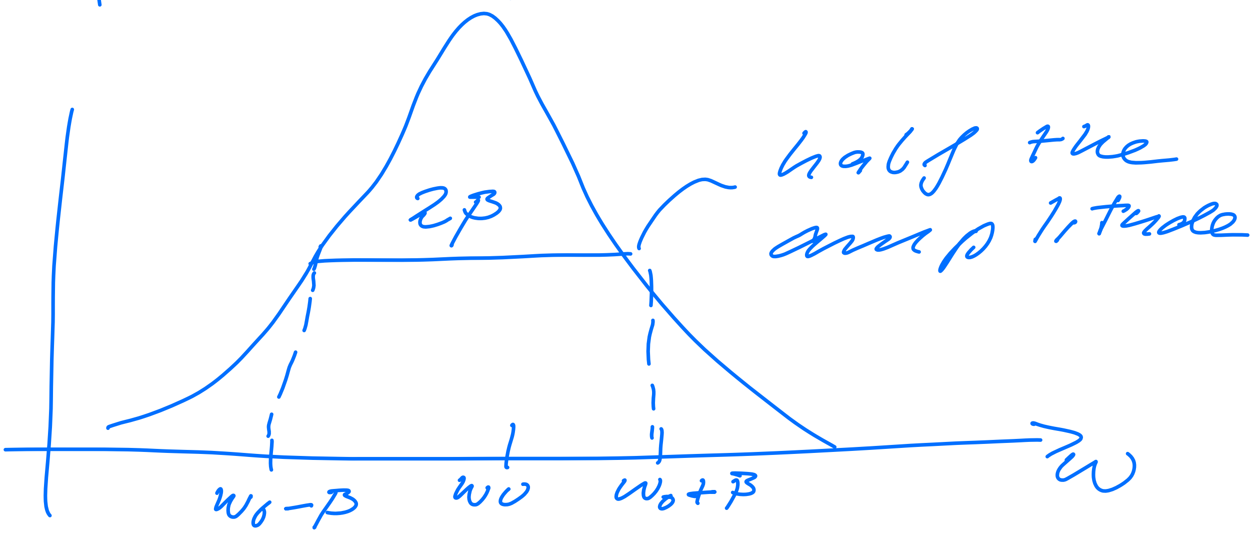
$$\beta \ll \omega_0$$

Max amplitude $\omega = \omega_0$

$$D_{max} = \frac{F_0}{2\beta\omega_0}$$







Quality factor

$$Q = \frac{\omega_0}{2\beta}$$

$Q \sim 100$ Grandfather clock

$Q \sim 10^4$ quartz crystal

$$\tau = \omega_0 t$$

$$\frac{d^2x}{d\tau^2} + 2\gamma \frac{dx}{d\tau} + x = \tilde{F}_0 \cos(\tilde{\omega}\tau)$$

$$\frac{1}{Q} = 2\gamma$$

$$\frac{d^2x}{d\tau^2} + \frac{1}{Q}\frac{dx}{d\tau} + x = \tilde{F}_0 \cos(\tilde{\omega}\tau)$$
$$= f(\tau)$$

Damping is given by

$$e^{-\gamma\tau} \propto e^{-\frac{1}{Q}\tau}$$

Driving period (without $F(\tau)$)

$$T = \frac{2\pi}{\omega_0}$$

$$\sqrt{k/m}$$

$$\beta \ll \omega_0 \qquad Q = \frac{\omega_0}{2\beta}$$

$$\frac{b}{2m}$$

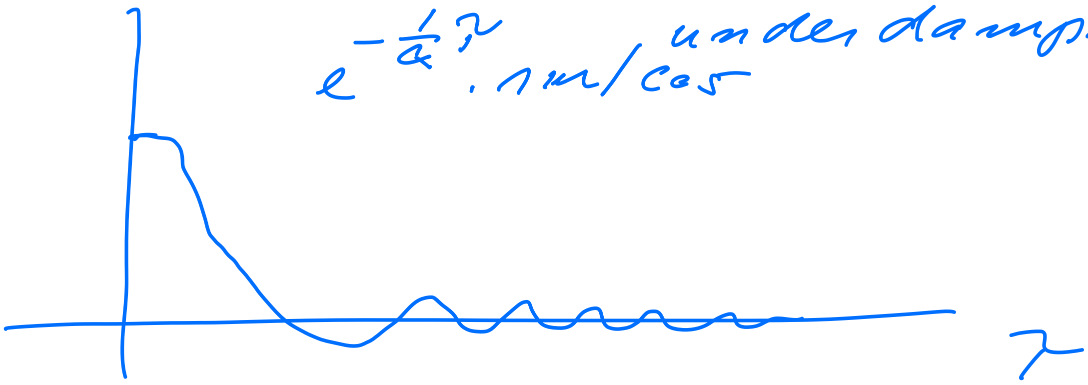
$$Q = \frac{\pi\, 1/\beta}{2\pi/\omega_0}$$

$$= \pi \cdot \frac{\text{decay time}}{\text{period}}$$

= number of periods in the decay time

$= \pi \times$ number of cycles or oscillations makes in one decay time

$e^{-\frac{1}{2}\gamma} \cdot \sin/\cos$ underdamping



Fourier analysis

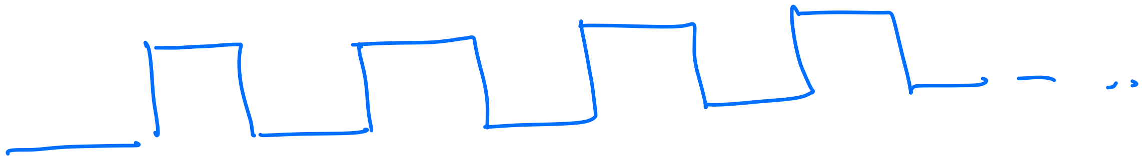
washboard case

$$f(t) = f(t + \underset{\text{period}}{T})$$

$$= \sum_{n=0}^{\infty} \left[a_n \cos(n\omega t) + b_n \sin(n\omega t) \right]$$

Fourier theorem

Fourier [illegible] ;



$x \in [-\pi, \pi]$

$$\int_{-\pi}^{\pi} \cos(nx) \sin(mx)\, dx$$

$$= 0$$