Solution to homeworks

HW2

Exercise 1 (10pt)

1a (2pt) if the net forces on an object are zero, that is $\vec{F}^{net} = m\vec{a} = 0$, then assuming that $m \neq 0$, then $\vec{a} = 0$, However, the way the exercise is formulated, there is only a single force and we assume it is $\vec{F} \neq 0 \Rightarrow a \neq 0$, Answer = $\boxed{NO}$

1b (2pt) when a ball is thrown up vertically it reaches a maximum m point where it comes at rest since the net forces $\vec{F}^{net} = 0 \Rightarrow \boxed{\vec{a} = 0}$

1c (3pt)

Here we are measuring the acceleration of ourselves and since we are falling, it should match $g \Rightarrow$

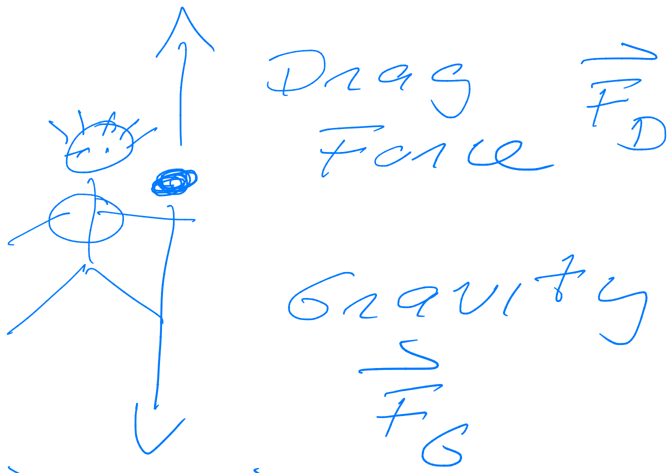
$$\boxed{\vec{a} = -g\,\vec{e}_g}$$

1d (3pt)

In vacuum there is no air resistance. This means that our velocity should be larger.

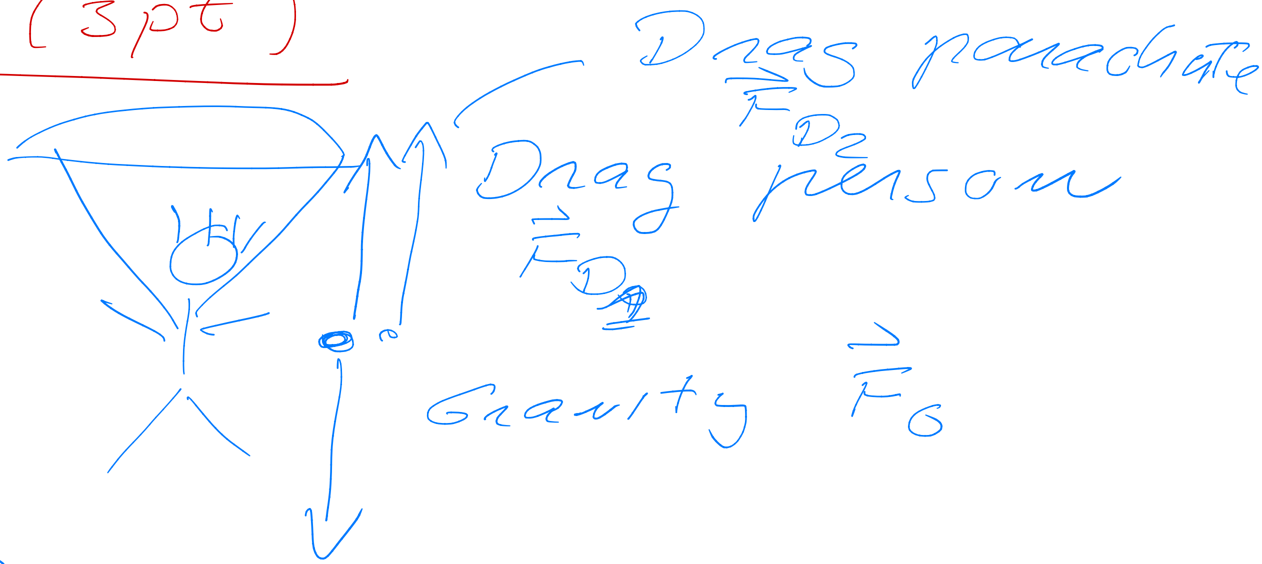
Exercise 2 (10pt)

2a (3pt)



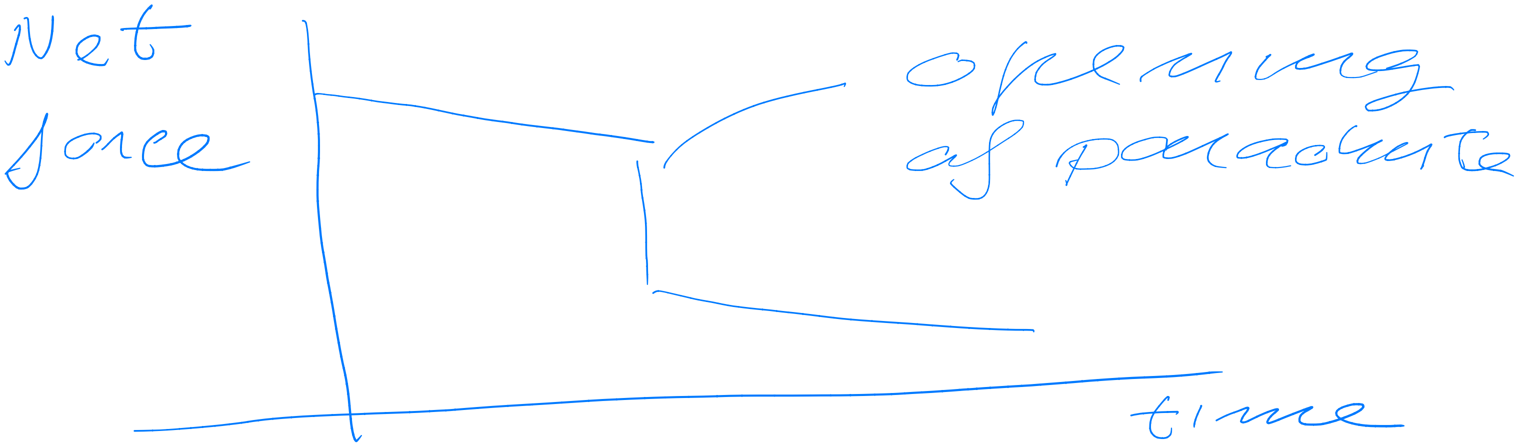
$$\vec{F}_{net} = \vec{F}_G + \vec{F}_D$$

2b (3pt)



$$\vec F_{net} = \vec F_{D1} + \vec F_{D2} + \vec F_G$$

2c (4pt)



We assume here that

$$\vec F_D = -D\vec v|\vec v|$$

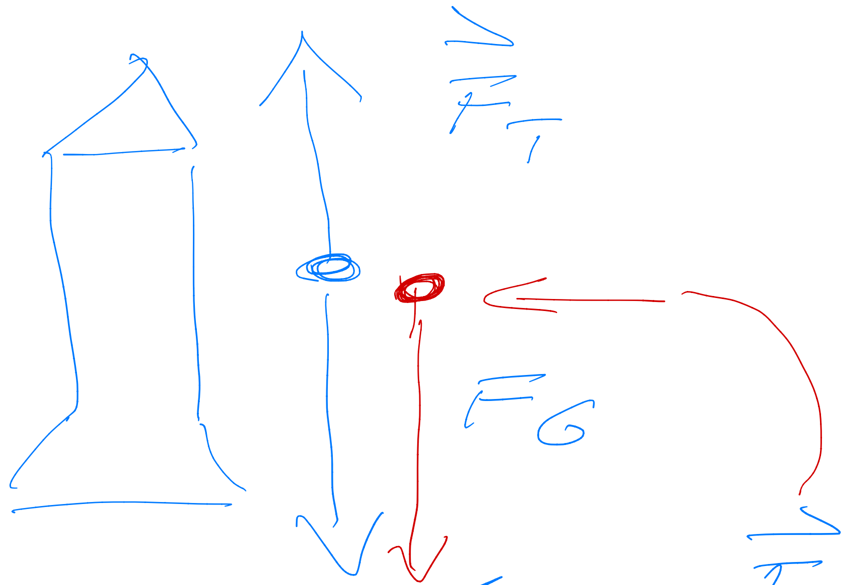
Since $|\vec v|$ increases with time, the net force is assumed to decrease. When the parachute is opened, there is a sudden change,

Exercise 3 (10pt)

3a (3pt)

if we assume no drag force we have that the thrust force is F_T, Gravity is F_G

$$\vec{F}_{net} = \vec{F}_T + \vec{F}_G$$



if we add drag $\vec{F}_D$ we have

$$\vec{F}_{net} = \vec{F}_T + \vec{F}_D + \vec{F}_G$$

3b (3pt)

$$\vec{a} = \frac{\vec{F}_T + \vec{F}_G}{m}$$

$$= \frac{\vec{F}_T - mg\vec{e}_y}{m}$$

$$= \frac{F_T\vec{e}_y - mg e_y}{m}$$

$$|\vec{a}| = \frac{35\times10^6 N - 2\times10^6 kg\ 9.8 m/s^2}{2\times10^6 kg}$$

$$= \boxed{7.7\ m/s^2}$$

3c (4pt)

$$\vec{v}(t) = \vec{v}_0 + \int_{t_0}^{t} \vec{a}(t)\,dt$$

$$|\vec{v}_0| = 0\ m/s$$

$$\vec{v}(t) = v(t)\vec{e}_y = \int_{t_0}^{t} a\vec{e}_y\,dt$$

inserting for $a = 7.7\, m/s^2$
and setting $t = 20s$
we get

$$v(t) = a.t = 7.7 \times 20\, m/s^2$$

$$= \underline{\underline{154\, m/s}}$$

Exercise 4 (15pt) Taylor 1.35

4a (4pt) initial velocity

$$\vec{v}_0 = v_{0x}\vec{e}_x + v_{0y}\vec{e}_y + v_{0z}\vec{e}_z$$

$$= v_0 \cos\theta\, \vec{e}_x + 0\, \vec{e}_y + v_0 \sin\theta\, \vec{e}_z = (v_0\cos\theta, 0, v_0 \sin\theta)$$

integrating gives

$$\vec{r}(t) = v_0 \cos\theta . t\, \vec{e}_x + 0\, \vec{e}_y + (v_0 \sin\theta t - g t^2/2)\vec{e}_z$$

4b (3pt)

Returns to ground when

$$z(t_f) = 0 = v_0 \sin\theta \cdot t_f - g t_f^2/2$$

$$\Rightarrow \quad t_f = \frac{2 v_0 \sin\theta}{g}$$

4c (3pt)

it travels a distance

$$x(t_f) = v_0 \cos\theta \, t_f =$$

$$\frac{2 v_0^2 \sin\theta \cos\theta}{g}$$

$$= \frac{v_0^2}{g} \sin 2\theta$$

Exercise 5 (15pt) Taylor 1.38

5a (3pt)

There are two forces, the normal force with magnitude N and perpendicular to the board and gravity with magnitude mg pointing vertically downward.

We can split the latter into two components, one perpendicular to the board ($-mg\cos\theta$) and parallel down the board ($-mg\sin\theta$). There is no force across the board.

Newton's 2nd law gives (assuming the puck remains on the board)

$$x: \quad 0 = m\ddot{x}$$

$$y: \quad -mg\sin\theta = m\ddot{y}$$

$$z: \quad N - mg\cos\theta = m\ddot{z}$$

$\Rightarrow$ gives 0 force in z-direction

5b (3pt) initial conditions

$\vec{r}_0 = (0, 0, 0)$

$\vec{v}_0 = (v_{0x}, v_{0y}, 0)$

gives when integrating

$$\boxed{\vec{r}(t) = (v_{0x} t, v_{0y} t - g_{nme} t^2/2, 0)}$$

5c (2~~pt~~) Returns to floor level when

$$y(t_f) = 0 = v_{0y} t_f - g_{nme} t_f^2/2$$

$$\Rightarrow \quad t_f = \boxed{\frac{2 v_{0y}}{g_{nme}}}$$

5d (2pt) It travels a distance

$$x(t_f) = v_{0x} t_f =$$

$$\boxed{\frac{2 v_{0x} v_{0y}}{g_{nme}}}$$