PHY 321, MARCH 14, 2022

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx$$

$$= F(t) = F_0\cos(\omega t)$$

ω_0 = Natural frequency

$$= \sqrt{k/m}$$

$$\gamma = \frac{b}{2m\omega_0}$$

$$\tau = \omega_0 \cdot t$$

$$\tilde{F}_0 = \frac{F_0}{m\omega_0^2} \qquad \tilde{\omega} = \frac{\omega}{\omega_0}$$

$$\frac{d^2x}{d\tau^2} + 2\gamma\frac{dx}{d\tau} + x = \tilde{F}_0\cos(\tilde{\omega}\tau)$$

$$x(\tau) = x_p(\tau) + x_h(\tau)$$

$$= \boxed{D\cos(\tilde{\omega}\tau - \delta)}$$

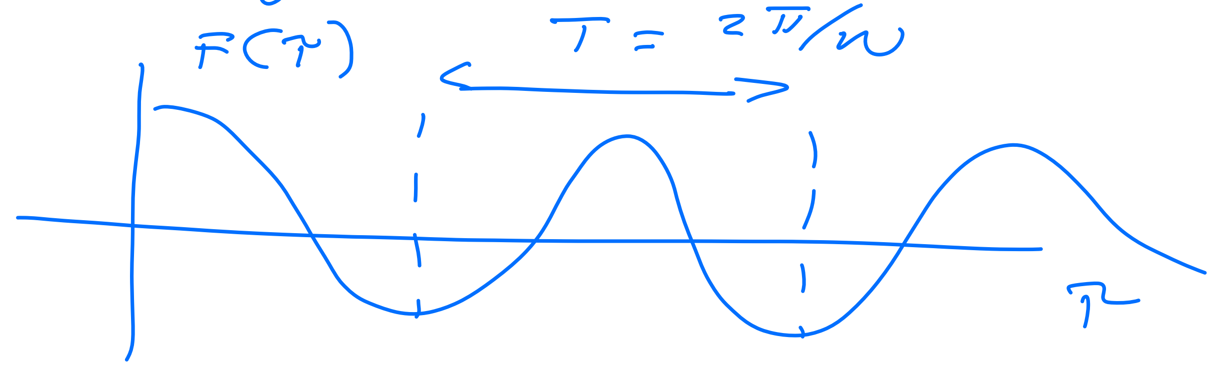
$$-r_1\tau \qquad -r_2\tau$$

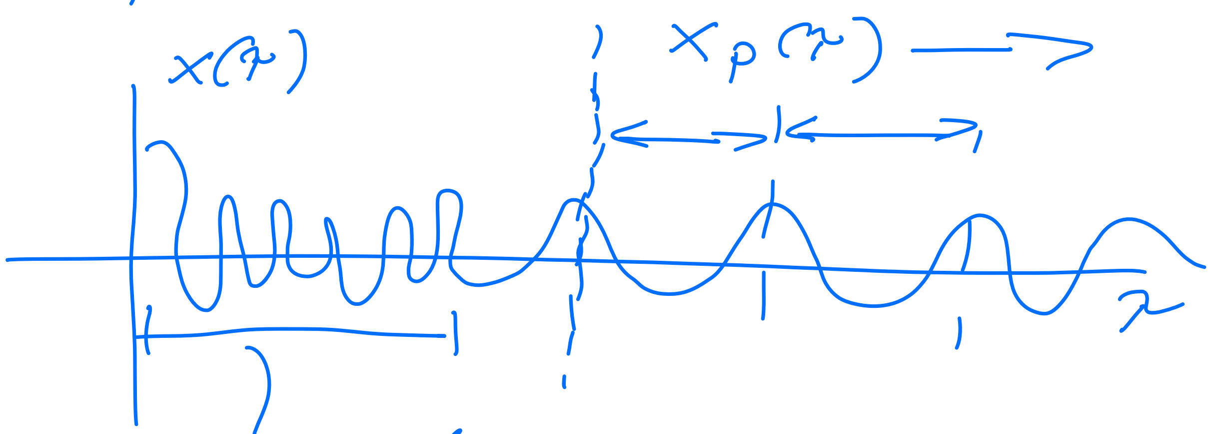
$$+ \left[C_1 e \quad + C_2 e \right]$$

$$r_1 = -\gamma + \sqrt{\gamma^2 - 1}$$

$$r_2 = -\gamma - \sqrt{\gamma^2 - 1}$$

After a time τ, then $x(\tau)$ is governed by $x_p(\tau)$





info from $x_h(\tau)$ (Transient solution)

Examples

$\omega = 2\pi$ $\qquad \omega_0 = 5\omega$

$m = 1$ $\qquad F_0 = 1000$

$$\tilde{F}_0 = \frac{1000}{m\omega_0^2} = \frac{1000}{100\pi^2} \approx 1.01$$

$$T = 2\pi/\omega$$

$$\tilde{\omega} = \frac{\omega}{\omega_0} = \frac{1}{5} = 0.2$$

$$\gamma = \frac{b}{2m\omega_0} \qquad b = \frac{\omega_0}{20}$$

$$\gamma = 0.025$$

$$x_0 = v_0 = 0$$

$$t = \tau/\omega_0 = \frac{\tau}{2\pi \cdot 5}$$

$$= \frac{\tau}{10\pi}$$

$$b \ll \omega_0$$

– Resonances

$$x_p(\tau) = D\cos(\tilde{\omega}\tau - \delta)$$

$$D = \frac{\tilde{F}_0}{\sqrt{(1-\tilde{\omega}^2)^2 + 4\tilde{\omega}^2\gamma^2}}$$

$$\tilde{F}_0^2 = \frac{F_0^2}{m^2\omega_0^4}$$

$$\frac{F_0^2}{m^2} = f_0^2$$

$$D^2 = \frac{f_0^2}{(\omega_0^2-\omega^2)^2 + 4\beta^2\omega^2}$$

$$\beta = \frac{b}{2m} \qquad \gamma = \frac{b}{2m\omega_0^2}$$

(i) can tune ω and keep ω_0 fixed and find max D

(ii) can tune ω_0 and keep ω fixed,

To find max D, we minimize the denominator

$$d\,[(\omega_0^2-\omega^2)^2 + 4\beta^2\omega^2] = 0$$

$$\frac{d}{d\omega}\left[(\omega_0^2-\omega^2)^2+4\beta^2\omega^2\right]=0$$

$$= -4\cancel{\omega}(\omega_0^2-\omega^2)+8\beta^2\cancel{\omega}=0$$

$(\omega \neq 0)$

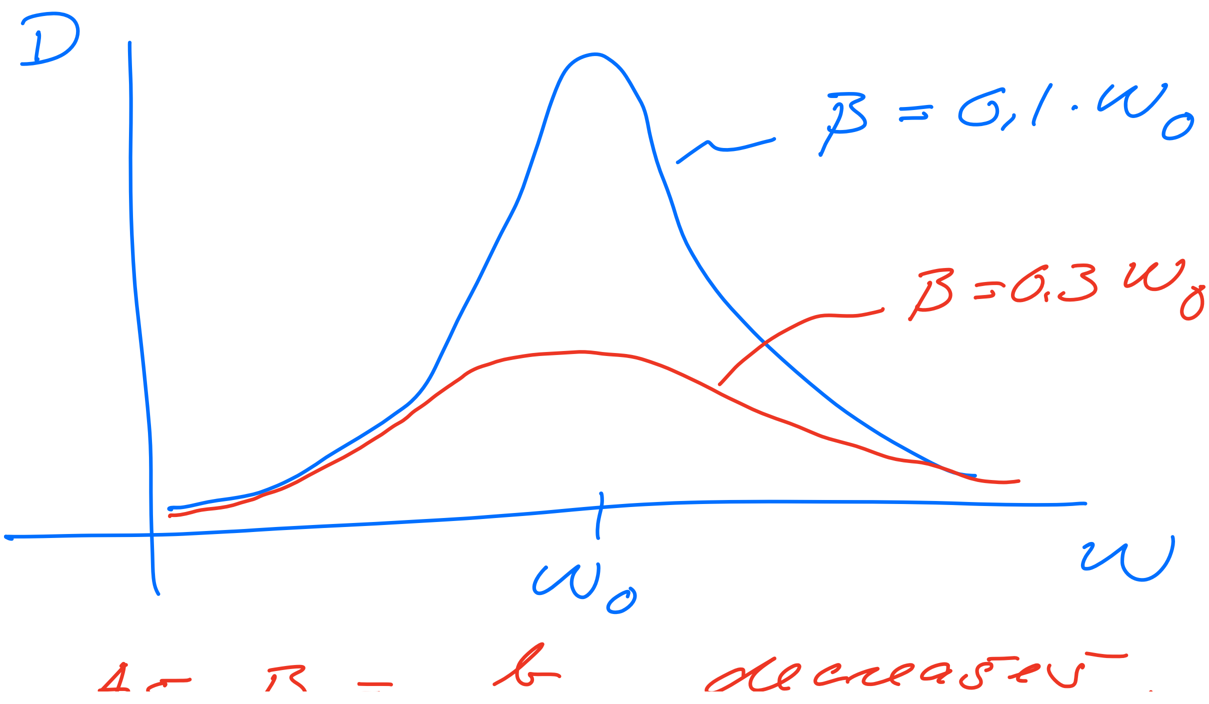
$$\omega = \sqrt{\omega_0^2-2\beta^2}$$

$$\beta \ll \omega_0 \Rightarrow \omega \simeq \omega_0$$

$$\omega_0 = \omega_{\sim} \Rightarrow$$

$$D = \frac{F_0}{2\beta\omega_0}$$

$$D = \frac{\tilde{F}_0}{\sqrt{(\omega_0^2-\omega^2)^2+4\beta^2\omega^2}}$$



As R – b decreases.

As $\beta = \frac{\quad}{2m}$

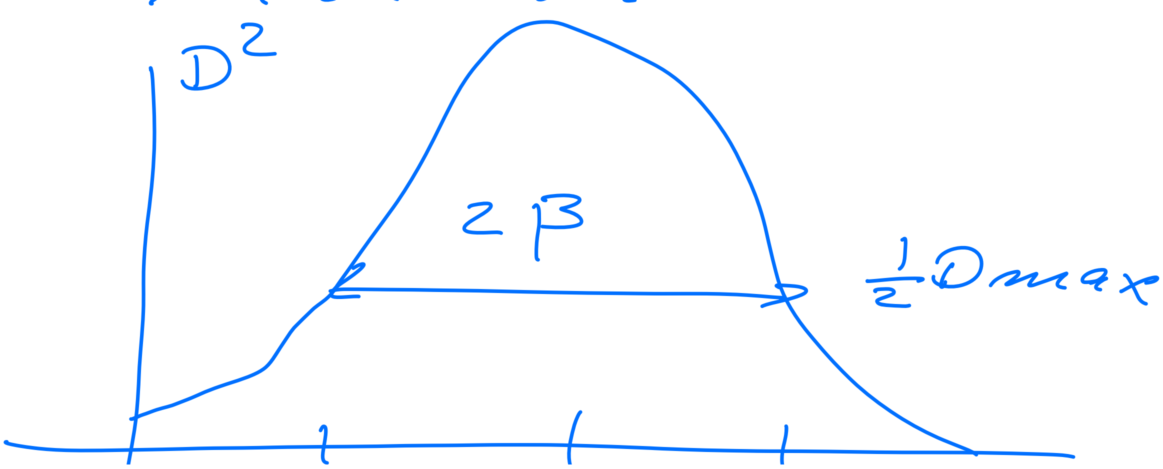
the resonance gets sharper and higher.

$\omega = \omega_0 \Rightarrow D_{max} = \frac{F_0}{2\beta\omega_0}$

The value of the D =

$\frac{1}{2} D_{max} = \frac{F_0}{4\beta\omega_0}$

defines a width (or full width at half maximum) which is defined by the interval between the two points where D is equal to half its max value.



$\omega_0 - \beta \quad \omega_0 \quad \omega_0 + \beta$

Defines the quality

$$Q = \frac{\omega_0}{2\beta}$$