Phys 321, January 31

work-energy

$K = \frac{1}{2} m \cdot v^2$

$\vec{F} = F(\vec{r}, \vec{v}, t)$

$v^2 = \vec{v} \cdot \vec{v}$ $\qquad \vec{a} = \vec{F}/m$

$\frac{dK}{dt} = \frac{1}{2} m \frac{d}{dt} (\vec{v} \cdot \vec{v})$

Ex 3, hw 1

$$\frac{dK}{dt} = \underbrace{m \cdot \frac{d\vec{v}}{dt}}_{\vec{F}} \cdot \vec{v}$$

$$= \vec{F} \cdot \frac{d\vec{r}}{dt}$$

Discretize

$$\frac{dK}{dt} = \lim_{\Delta t \to 0} \frac{K_2 - K_1}{t_2 - t_1}$$

$\Delta t = t_2 - t_1$

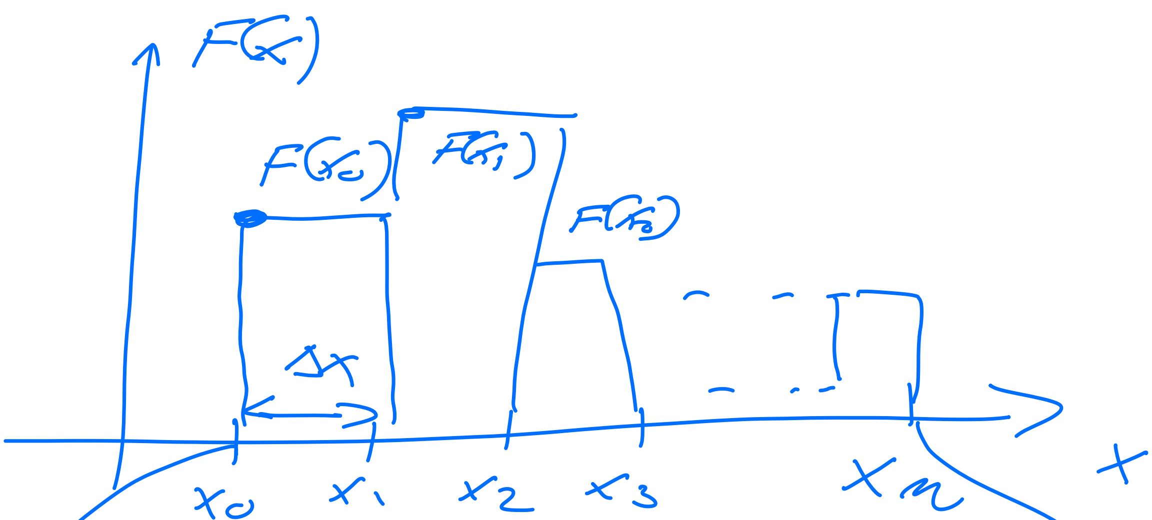
$$\frac{\Delta K}{\Delta t} = m\frac{\Delta \vec{v}}{\Delta t}\cdot \vec{v}$$

$$= \vec{F}\cdot\frac{\Delta\vec{r}}{\Delta t}$$

$$\Delta K = \vec{F}\cdot\Delta\vec{r}$$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

<u>1-Dim</u>



$$X = \{x_0, x_1, x_2, x_3, \ldots, x_n\}$$

$\frac{1}{2} m v_0^2$ $\qquad\qquad$ $\frac{1}{2} m v_n^2$

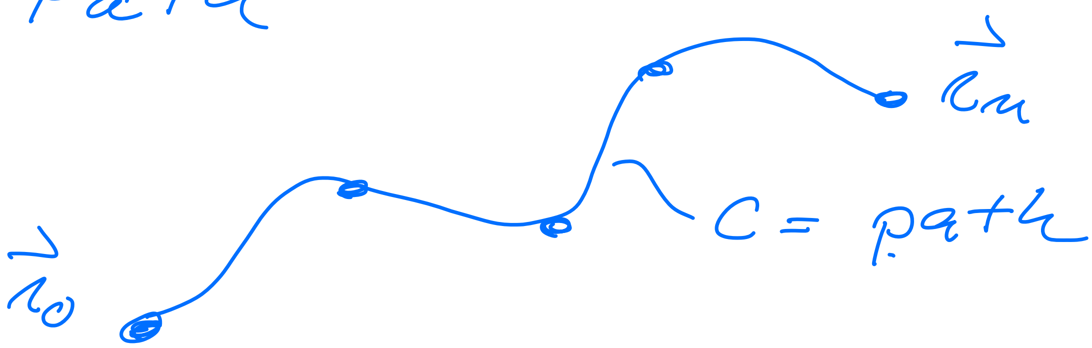
$$\frac{1}{2} m v_n^2 - \frac{1}{2} m v_0^2 = \sum_{i=0}^{n-1} F_i \Delta x$$

$$\lim_{\Delta x \to 0} \sum F_i \Delta x = W$$

$$= \int_{x_0}^{x_n} dx\, F(x)$$

$$= \frac{1}{2} m v_n^2 - \frac{1}{2} m v_0^2$$

Path



$$W = \int_C F(\vec{r})\, d\vec{r}$$

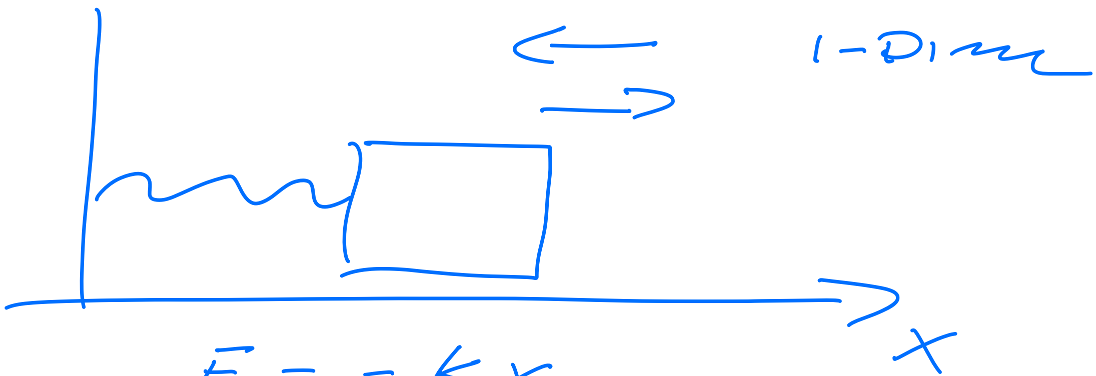
1) $\vec{F} \cdot \Delta\vec{r}$ is negative ?
$(\vec{F} \cdot d\vec{r})$

Reduced kinetic energy

2) $\vec{F} \perp \Delta \vec{r} = 0$

does not change kinetic energy

Example 1



$F = -kx$

we move from x_0 and end x_1

$$\frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 =$$

$$\int_{x_0}^{x_1} (-kx)\, dx$$

$$= -\frac{k x_1^2}{2} + \frac{k x_0^2}{2}$$

$$\frac{1}{2} m v_1^2 + \frac{k x_1^2}{2} = \frac{1}{2} m v_0^2 + \frac{k x_0^2}{2}$$

Two important conditions

(i) $F = F(\vec{r})$

(only dependence on $\vec{r}$)

The path chosen in the integral for W, leads to a result which is independent of path.

$\Rightarrow$ energy conservation

(ii) To have a path independent work

$$W = \int_{\vec{r}_0}^{\vec{r}_n} F(\vec{r})\, d\vec{r}$$

we need to have

$$\vec{\nabla} \times \vec{F} = 0$$

These are conservative forces.

Conservation of linear momentum

N-object with velocity

$\vec{v}_i \quad i = 1, 2, 3, \ldots N$

$\vec{P}_i = m_i \vec{v}_i$

Total momentum

$$\vec{P} = \sum_{i=1}^{N} \vec{P}_i = \sum_{i=1}^{N} m_i \vec{v}_i$$

<u>$N=2$</u>

Total force on <u>1</u>

$$\vec{F}_1^{\,net} = \vec{F}_1^{\,ext} + \vec{F}_{12}$$

internal force

Force on 2

$$\vec{F}_2^{\,net} = \vec{F}_2^{\,ext} + \vec{F}_{21}$$

Newton's 3rd Law

$$\vec{F}_{21} = -\vec{F}_{12}$$

Total force

$$\vec{F} = \sum_{i=1}^{2} \vec{F}_i^{net}$$

$$= \vec{F}_1^{ext} + \vec{F}_2^{ext} + \vec{F}_{12} - \vec{F}_{12}$$

$$= \vec{F}_1^{ext} + \vec{F}_2^{ext}$$

$$\vec{F}_i^{net} = m_i \vec{a}_i = m_i \frac{d\vec{v}_i}{dt}$$

$$= \frac{d\vec{p}_i}{dt}$$

$$\vec{p}_1 + \vec{p}_2 = \vec{p}$$

$$\frac{d\vec{p}}{dt} = \vec{F}_1^{ext} + \vec{F}_2^{ext}$$

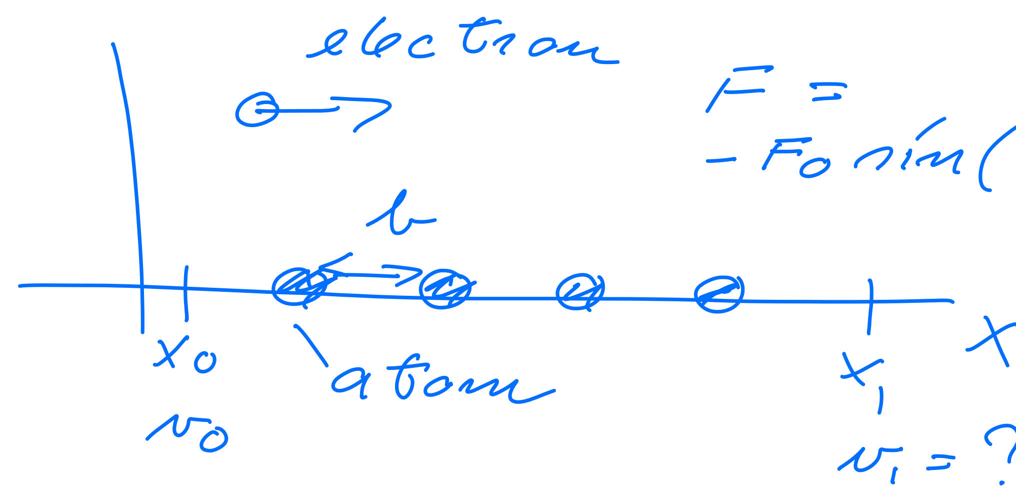
(i) If the total external $\neq 0$, then $\frac{d\vec{p}}{dt} \neq 0$

$\vec{p}$ not conserved.

(ii) if only internal forces, then

$$\frac{d\vec{p}}{dt}=0,$$ and $\vec{p}$ is conserved.

Example 2



$$F = -F_0 \sin\left(\frac{2\pi x}{b}\right)$$