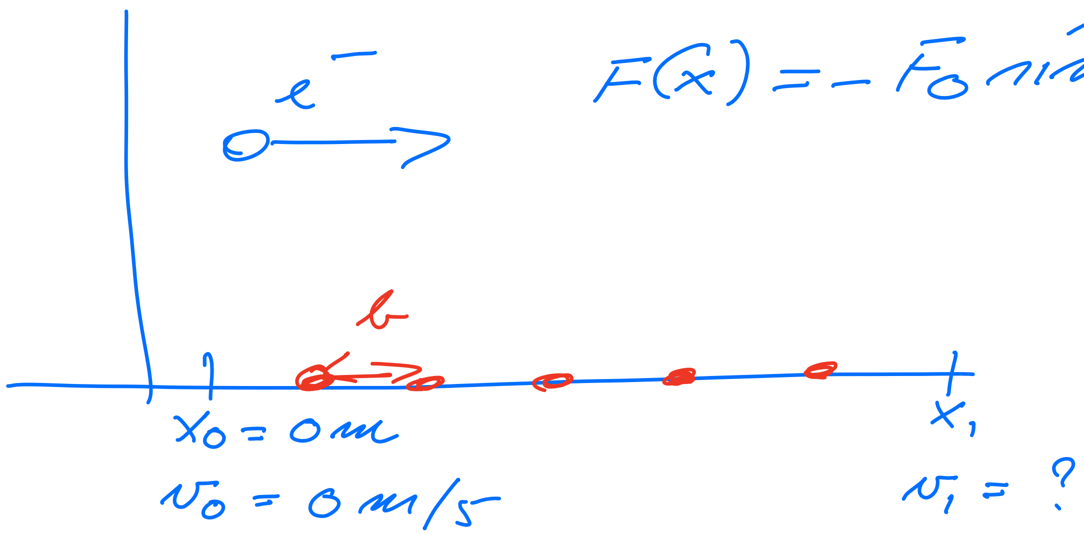
PHY 321 February 2

Example

$$F(x) = -F_0 \sin\left(\frac{2\pi x}{b}\right)$$



work-energy theorem

$$\frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 = -\int_{x_0}^{x_1} F_0 \sin\left(\frac{2\pi x}{b}\right) dx$$

(change of variables

$$u = \frac{2\pi x}{b} \qquad du = \frac{2\pi\, dx}{b}$$

)

$$= \frac{F_0 b}{2\pi}\left[\cos\left(\frac{2\pi x_1}{b}\right) \right.$$

$$- \cos\left(\frac{2\pi x_0}{b}\right)\Big]$$

$v_0 = 0 \qquad x_0 = 0 \qquad \Rightarrow$

$$\frac{1}{2} m v_1^2 = \frac{F_0 b}{2\pi}\left[\cos\left(\frac{2\pi x_1}{b}\right) - 1\right]$$

$$v_1 = \pm\sqrt{\frac{F_0 b}{2\pi}\left[\cos\left(\frac{2\pi x_1}{b}\right) - 1\right]}$$

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conservative forces $\Rightarrow$ conservation of energy

$$E = \underbrace{K}_{\text{kinetic energy}} + \underbrace{V}_{\text{potential energy}}$$

$$\frac{dE}{dt} = 0$$

(i) $\vec{F} = \vec{F}(\vec{r})$

(ii) $\vec{\nabla} \times \vec{F} = 0$

(iii) $\vec{F} = -\vec{\nabla} V(\vec{r})$

$$\vec{F}^{\,total} = \sum_{i=1}^{N} m_i \vec{a}_i$$

$$= \sum_{i=1}^{N} \vec{F}_i$$

$$\vec{F}_i = \vec{F}_i^{\,ext} + \vec{F}_i^{\,int}$$

$$\vec{F}_i^{\,int} = \sum_{j \neq i}^{N} \vec{F}_{ij}$$

$N = 2$, and internal forces only:

$$\sum_{i=1}^{N} \vec{F}_i^{\,int} = 0$$

$$N = 2 \qquad \sum_{i=1}^{N=2} \vec{F}_i^{\,int} = \sum_{i=1}^{N=2} \sum_{j \neq i}^{N=2} \vec{F}_{ij}$$

$$= \vec{F}_{12} + \vec{F}_{21} = 0$$

since $\vec{F}_{21} = -\vec{F}_{12}$

$$\sum_{i=1}^{N} \vec{F}_i^{\,int} = \sum_{i=1}^{N} \sum_{j \neq i}^{N} \vec{F}_{ij}$$

$$= \frac{1}{2} \sum_{i,j}^{N} \left(\vec{F}_{ij} + \vec{F}_{ji} \right)$$

$(j \neq i)$

$$= \sum_{i}^{N} \sum_{j>i}^{N} \left(\vec{F}_{ij} + \vec{F}_{ji} \right)$$

$$\left(\vec{F}_{ij} = - \vec{F}_{ji} \right)$$

$$= 0 \ ! \quad \Rightarrow$$

$$\vec{p}^{\,total} = \sum_{i=1}^{N} m_i \vec{v}_i = \sum_{i=1}^{N} \vec{p}_i$$

$$\frac{d\vec{p}^{\,total}}{dt} = \sum_{i=1}^{N} \underbrace{m_i \frac{d\vec{v}_i}{dt}}_{\vec{F}_i^{net} = \vec{F}_i^{int}}$$

internal forces only:

$$\frac{d\vec{p}}{dt} = 0 \ !$$

linear momentum is a constant of motion.

if however

$\vec{F}^{net} \quad \vec{F}^{ext} \quad \vec{F}^{int}$

$$F_i^{net} = F_i^{ext} + F_i^{int}$$

then

$$\vec{F}^{total} = \sum_{i=1}^{N} \vec{F}_i^{ext} + 0$$

Angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

$$= m(\vec{r} \times \vec{v})$$

$$\frac{d\vec{L}}{dt} = 0 \ ?$$

$$\frac{d\vec{L}}{dt} = m\left\{\left(\frac{d\vec{r}}{dt} \times \vec{v}\right) + \left(\vec{r} \times \frac{d\vec{v}}{dt}\right)\right\}$$

$$= m \underbrace{\vec{v} \times \vec{v}}_{=0} + \underbrace{\vec{r} \times \vec{F}}_{\substack{\text{if } \vec{r} \parallel \vec{F} \\ =0}}$$

if not, then

$\vec{\tau}$ = torque

$\vec{\tau} = \vec{r} \times \vec{F} \neq 0$, ang. mom not conserved.

$$\vec{L} = \sum_{i=1}^{N} \vec{l_i}$$

N - isolated objects

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{N} \frac{d}{dt} \vec{l_i}$$

$$= \sum_{i=1}^{N} \vec{r_i} \times \vec{F_i}$$

(internal - forces)

$$= \sum_{i=1}^{N} \vec{r_i} \times \left(\sum_{j \neq i}^{N} \vec{F_{ij}} \right)$$

$$= \frac{1}{2} \sum_{\substack{ij \\ i \neq j}} \left\{ \vec{r_i} \times \vec{F_{ij}} + \vec{r_j} \times \underbrace{\vec{F_{ji}}}_{-\vec{F_{ij}}} \right\}$$

$$= \frac{1}{2} \sum_{\substack{ij \\ i \neq j}} (\vec{r_i} - \vec{r_j}) \times \vec{F_{ij}}$$

$j \neq i$

$i, j \qquad \vec{F}_{ij} \propto (\vec{r}_i - \vec{r}_j)$

Then

$$\frac{dL}{dt} = 0$$

Gravitational force

$$\vec{G}(\vec{r}_i, \vec{r}_j) \propto \frac{M_i M_j}{|\vec{r}_i - \vec{r}_j|^3} (\vec{r}_i - \vec{r}_j)$$

Exercises 5 + 6

$$\vec{r}(t_0) = \vec{r}_0 = h\vec{j} \qquad t_0 = 0$$

$$\vec{v}(t_0) = v_{0x}\vec{i} + v_{0y}\vec{j}$$

Gravitational force

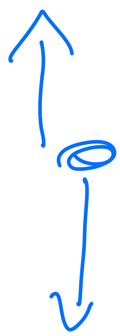
$$\vec{G} = -m \cdot g \vec{j}$$

Air resistance

$$\vec{F}_D = -D\vec{v}|\vec{v}(t)|$$

$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

specialize to object falling in the y-direction



$\vec{F}_D = +Dv^2\vec{j}$

$\vec{G} = -mg\vec{j}$

$$a_y = -g \pm \overbrace{\frac{D}{m}}^{\tilde{D}} v_y |\vec{v}|$$

$$= -g \pm \tilde{D} v_y |\vec{v}|$$

<u>in ex 5</u>

$$a_y = -g + \tilde{D} v_y^2 = \frac{dv_y}{dt}$$

$a_x = 0$?

else $a_x = -\tilde{D} v_x |\vec{v}| \vec{i}$

slides from week 3, in Taylor 2.

$$v_y(t) = v_T \tanh\left(\frac{-gt}{v_T}\right)$$

$$v_T = \sqrt{g/\tilde{D}}$$

$$y(t) = y(t_0) - \frac{v_T^2}{g} \log\left[\cosh\left(\frac{gt}{v_T}\right)\right]$$