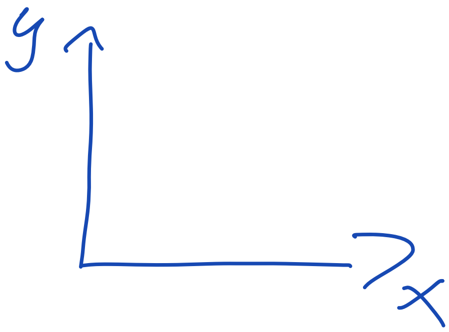
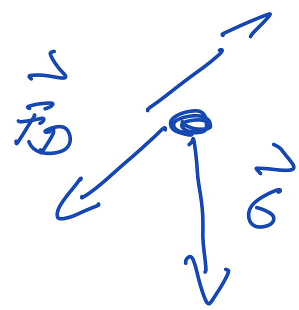
PHY 321 JANUARY 24, 2022

2 Dim





$\vec{v} = v_x \vec{i} + v_y \vec{j}$ $\vec{i} \perp \vec{j}$

initial velocity

$v_x(t_0) = v_{0x}$ | y_0 at t_0

$v_y(t_0) = v_{0y}$ | x_0

$\vec{F}_D = -m\gamma \vec{v}(t)$

$\vec{G} = -mg\vec{j}$

$\vec{F}_D = -m\gamma (v_x \vec{i} + v_y \vec{j})$

$\vec{F}_{net} = \vec{F}_D + \vec{G}$

$a_x = \dfrac{dv_x}{dt} \wedge a_y = \dfrac{dv_y}{dt}$

$$m a_x = -m\gamma v_x \Rightarrow$$

$$\boxed{\begin{aligned} a_x &= -\gamma v_x = \frac{dv_x}{dt} \\ a_y &= -\gamma v_y - g = \frac{dv_y}{dt} \\ v_x &= \frac{dx}{dt} \\ v_y &= \frac{dy}{dt} \end{aligned}}$$

x & y degrees of freedom are decoupled $\Rightarrow$ possible analytical solution.

HW3

$$\vec{F}_D = -D\vec{v}\,|\vec{v}|$$

in 2 Dim

$$\vec{F}_D = -D\left(v_x \vec{i} + v_y \vec{j}\right)\sqrt{v_x^2+v_y^2}$$

$$a_x = -D v_x \sqrt{v_x^2+v_y^2}$$

no longer decoupled.

$$\frac{dv_x}{dt} = -\gamma v_x$$

$$\frac{dv_x}{v_x} = -\gamma\, dt$$

$$\int_{v_{0x}}^{v_x} \frac{dv_x'}{v_x'} = -\gamma \int_0^t dt'$$

$$\ln \frac{v_x}{v_{0x}} = -\gamma \cdot t$$

$$\frac{v_x}{v_{0x}} = e^{-\gamma \cdot t} \Rightarrow$$

$$v_x(t) = v_{0x}\, e^{-\gamma \cdot t}$$

$$\frac{dx}{dt} = v_x = v_{0x} e^{-\gamma t}$$

$$\int_{x_0}^{x} dx' = \int_0^t v_{0x}\, e^{-\gamma \cdot t'} \cdot dt'$$

$$x(t) = x_0 + \frac{v_{0x}}{\gamma}\left(1 - e^{-\gamma t}\right)$$

y- direction

$$\frac{dv_y}{dt} = -\gamma v_y - g = a_y$$

$$\frac{dv_y}{+\gamma v_y + g} = -dt$$

$$\int_{v_{0y}}^{v_y} \frac{dv_y'}{+\gamma v_y' + g} = -\int_0^t dt' = -t$$

$$\int_{v_{0y}}^{v_y} \frac{dv_y'}{v_y' + g/\gamma} = -\gamma \cdot t$$

$$\ln\left(\frac{v_y + g/\gamma}{v_{0y} + g/\gamma}\right) = -\gamma \cdot t$$

$$\frac{v_y + g/\gamma}{v_{0y} + g/\gamma} = e^{-\gamma \cdot t}$$

$$v_y(t) = -g/\gamma + \left(v_{0y} + g/\gamma\right)e^{-\gamma t}$$

$$\frac{dy}{dt} = v_y(t)$$

$$\int_{y_0}^{y} dy' = \int_0^t v_y(t')\,dt'$$

$$y(t) = y_0 - g\cdot t/\gamma + \left(v_{0y} + g/\gamma\right)\left(1 - e^{-\gamma t}\right)$$

Numerical approach

arrays for $v_x(t)$, $v_y(t)$
$x(t)$ and $y(t)$

Brute force