PHY 321, MARCH 22, 2023

$$\vec{r} = \vec{r}_1 - \vec{r}_2$$

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{M}$$

$$M\ddot{\vec{R}} = 0 \qquad \mu = \frac{m_1 m_2}{M}$$

$$\mu\ddot{\vec{r}} = \vec{F}(\vec{r})$$

$$= -\gamma\frac{\vec{r}}{r^3}$$

$$r = |\vec{r}| = \sqrt{x^2+y^2+z^2}$$

$$\vec{r} = \vec{r}_1 - \vec{r}_2$$

$$r = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

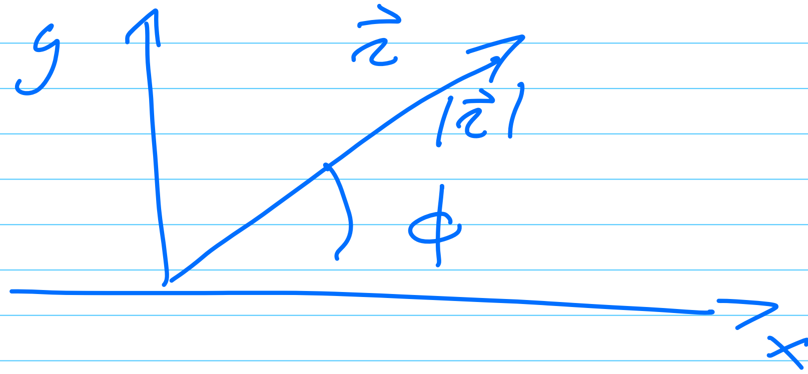
$$\frac{d\vec{L}}{dt} = 0 \quad \text{conserved } \vec{L}$$

$\vec{L} = \vec{r} \times \vec{p}$, we can redefine $\vec{r}$ and $\vec{p}$ to span the xy-plane

the z-axis is defined by L (L_z), constant

$\Rightarrow$ 2 dim, x, y

$r \in [0, \infty)$ $\phi \in [0, 2\pi]$



$x = r \cdot \cos\phi \;\wedge\; y = r \cdot \sin\phi$

$\mu \cdot \ddot{\vec{r}} = -\gamma\, \vec{r}/r^3 \longrightarrow$

$$\frac{d^2 r}{dt^2} = ? \;\wedge\; \frac{d\phi}{dt} = ?$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = ?$$

skip vector signs

$r = \sqrt{x^2 + y^2}$

$$\frac{dr}{dt} = \frac{dx}{dt} \frac{1}{r} \cdot x + \frac{dy}{dt} \frac{1}{r} \cdot y$$

$$\frac{d^2 r}{dt^2} = \ddot{r} = \boxed{\ddot{x}\,\frac{x}{r} + \ddot{y}\,\frac{y}{r} + \frac{\dot{x}^2 + \dot{y}^2}{r} - \frac{\dot{r}^2}{r}}$$

$$v^2 = \dot{x}^2 + \dot{y}^2$$

$$x = r \cdot \cos\phi \quad \wedge \quad y = r \cdot \sin\phi$$

$$\frac{dx}{dt} = \dot{x} = \frac{dr}{dt} \cdot \cos\phi - r\sin\phi\,\frac{d\phi}{dt}$$

$$\frac{dy}{dt} = \dot{y} = \frac{dr}{dt}\sin\phi + r\cos\phi\,\frac{d\phi}{dt}$$

$$\begin{aligned}
\dot{x}^2 + \dot{y}^2 = {} & \dot{r}^2\cos^2\phi + r^2\sin^2\phi\,\dot{\phi}^2 \\
& \cancel{- 2\dot{r}\cos\phi \cdot \sin\phi\,\dot{\phi}} \\
& + \dot{r}^2\sin^2\phi + r^2\cos^2\phi\,\dot{\phi}^2 \\
& \cancel{+ 2\dot{r}\cos\phi \cdot \sin\phi\,\dot{\phi}}
\end{aligned}$$

$$= \dot{r}^2 + r^2 \cdot \dot{\phi}^2$$

$$\frac{d^2 r}{dt^2} = \ddot{x}\,\frac{x}{r} + \ddot{y}\,\frac{y}{r} + r\dot{\phi}^2$$

$$(x = r \cdot \cos\phi \;\wedge\; y = r \cdot \sin\phi)$$

$$= \ddot{x} \cdot \cos\phi + \ddot{y} \sin\phi$$

$$+ r\dot{\phi}^2$$

$$\mu \frac{d^2 r}{dt^2} = \mu \cdot \ddot{r} = \mu a_x \cos\phi$$

$$+ \mu a_y \cdot \sin\phi + r\dot{\phi}^2 \mu$$

$$= \underbrace{F_x + F_y}_{F_r} + r\dot{\phi}^2 \mu$$

$$= F_r + r\dot{\phi}^2 \mu$$

$$\boxed{\frac{d\phi}{dt} = \dot{\phi} = \frac{L}{\mu r^2}}$$

<u>hw8</u>

$$\mu \cdot \ddot{r} = F_r + \frac{L^2}{\mu r^3}$$

kinetic energy

$$F_r = -\frac{dV}{dr}$$

$$= -\frac{d}{dr}\left(-\gamma/r\right)$$

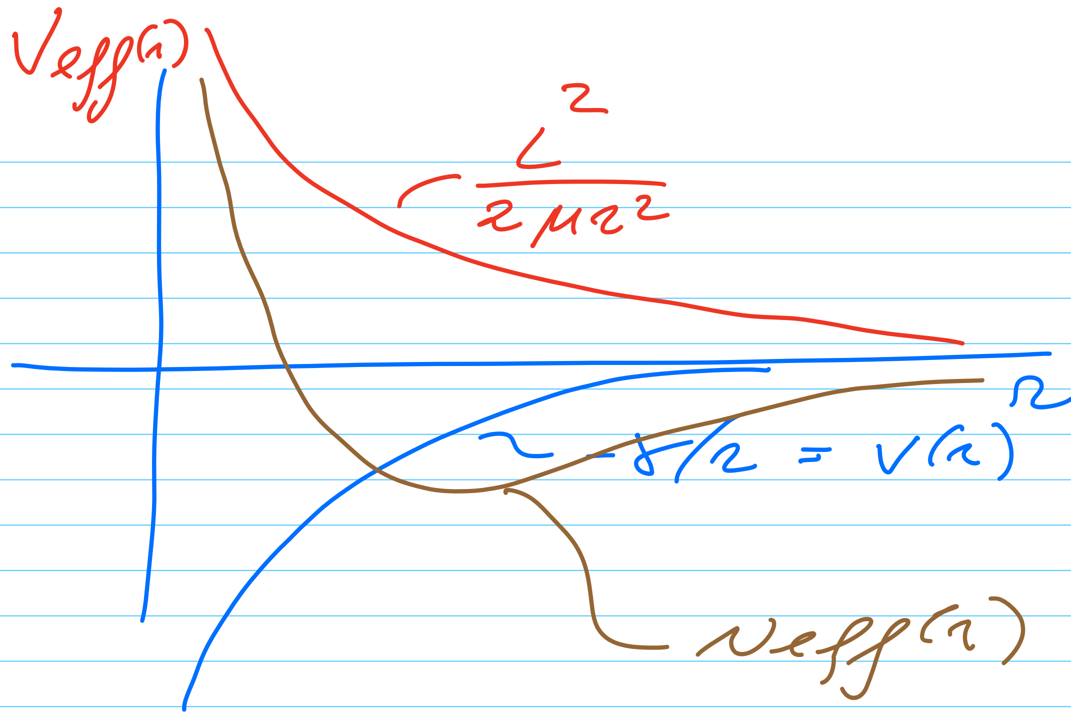
$$= -\gamma/r^2$$

$$\mu \ddot{r} = F_{eff}(r) = -\frac{dV}{dr} + \frac{L^2}{\mu r^2}$$

kinetic energy

$$= -\frac{dV_{eff}(r)}{dr}$$

$$V_{eff}(r) = V(r) + \frac{L^2}{2\mu r^2}$$



Monday (27th)

$$\ddot{r} = -\frac{\gamma}{\mu}\frac{1}{r^2} + \frac{L^2}{\mu^2 r^3}$$

$$u = \frac{1}{r}$$

$$\frac{d^2u}{d\phi^2} = -u + \frac{\mu\gamma}{L^2}$$

$$u = \frac{1}{r} = A\cdot\cos(\phi-\delta)$$

Example HO in 2 dim

$$\dot{\phi} = \frac{d\phi}{dt} = \frac{L}{\mu r^2}$$

$$\mu \ddot{r} = F(r) + \frac{L^2}{\mu r^3}$$

$$V(x,y) = \frac{1}{2} k (x^2 + y^2)$$

$$= \frac{1}{2} k r^2$$

$$F(r) = -\frac{dV}{dr} = -kr$$

$$\vec{F}(x,y) = -\vec{\nabla} V(x,y) =$$

$$-kx\vec{i} - ky\vec{j}$$

$$\mu \frac{dv_x}{dt} = F_x = -kx$$

$$\mu \frac{dv_y}{dt} = F_y = -ky$$

$$\frac{dx}{dt} = v_x \quad 1 \quad \frac{dy}{dt} = v_y$$

$$x = A\cos(\omega_0 \cdot t) + B\sin(\omega_0 \cdot t)$$

$$\omega_0 = \sqrt{k/\mu}$$

$$y = C\cos(\omega_0 t) + D\sin(\omega_0 t)$$

HO in 2 dim with r and ϕ

$$\mu \ddot{r} = -kr + L^2/\mu r^3$$

$$\dot{\phi} = \frac{L^2}{\mu r^2}$$

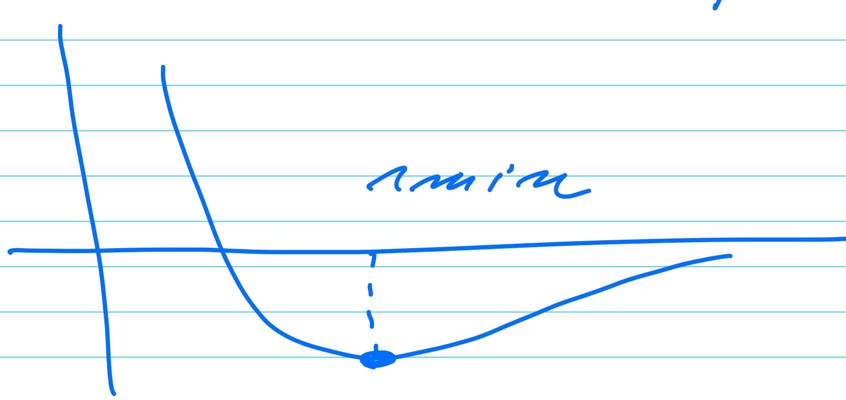
No advantage in transforming to r and ϕ from x and y, use x and y.

Analysis

$$V_{eff}(r) = \frac{1}{2} k r^2 + \frac{L^2}{2\mu r^2}$$

$$\frac{dV_{eff}}{dr} = 0 = kr - \frac{L^2}{\mu r^3}$$

$$\Rightarrow r_{min} = \left(\frac{L^2}{k\mu}\right)^{1/4}$$



$$\ddot{r}\Big|_{r=r_{min}} = 0 \Rightarrow$$

circular motion?