PHY 321 JANUARY 19, 2022

$$y(t) = y_0 - \tfrac{1}{2} g t^2$$

$$y_0 = y(t_0)$$

initial time t_0

final time t_n

$$t \in [t_0, t_n]$$

$$t = \{t_0, t_1, t_2, \ldots, t_{n-1}, t_n\}$$

$$t_i = t_0 + i \cdot \Delta t$$

$$i = 0, 1, 2, \ldots n$$

predefine Δt

array of t - values

t = np.arange(t0, tn, Δt)

Define - n -

$$\Delta t = \frac{t_n - t_0}{n}$$

for $i = 0, n$

$$t[i] = t[0] + i \Delta t$$

[illegible]

Newton's law

$$F(t) = -gm$$

$$a = -g$$

$$m \cdot a = -m \cdot g$$

$$= m \cdot \frac{dv}{dt} = m \frac{d^2y}{dt^2}$$

$$a = \frac{d^2y}{dt^2}$$

$$a = \frac{dv}{dt} \quad \wedge \quad v = \frac{dy}{dt}$$

unknown

known

v_0, y_0

$$a \cdot dt = dv$$

$$\int_{t_0}^{t_n} a \cdot dt = \int_{v_0}^{v_n} dv$$

$-g$

$-g(t_n - t_0) = v_n - v_0$

$t_0 = 0 \qquad v_0 = 0$

$t_n \to t$

$v_n \to v(t) = -g \cdot t$

$v = \frac{dy}{dt} \Rightarrow$

$y(t) = y_0 - \frac{1}{2} g t^2$

Approach 2 (numerical)

$v = \frac{dy}{dt} \wedge a = \frac{dv}{dt}$

Taylor expansion:

$y(t+\Delta t) = y(t) + y'(t)\Delta t + \frac{\Delta t^2}{2!} y'' + O(\Delta t^3)$

$(v(t+\Delta t))$

Euler's methods

skip $\sim (\Delta t)^2$ and higher

$$y(t+\Delta t) = y(t_i + \Delta t)$$

$$= y_{i+1} = y_i + \Delta t \, y_i'$$

$$y_i' = \left.\frac{dy}{dt}\right|_{t_i} = v_i$$

$$y_{i+1} = y_i + \Delta t \, v_i$$

$$v_{i+1} = v_i + \Delta t \, a_i$$

v_0, y_0 are known.

How do we find the forces at play?

Analyze a problem

- Divide into system and environment
  - Forces are either long range or contact forces
  - only external force

- Draw a figure

$\vec{G}$ = gravitational force long range

$\vec{F}_B$

$\vec{N}$ = from the table

Buoyancy different air pressure

$\vec{F}_D$ = friction

Net external force

$$\vec{F}_{net} = m\vec{a} = \vec{G} + \vec{F}_B + \vec{N} + \vec{F}_D$$

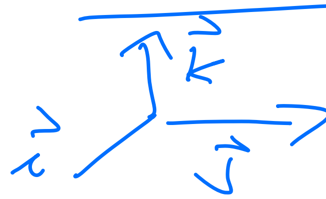
in general

$$\vec{F}_{net} = \sum_i \vec{F}_i$$

- name all forces
- identify long-range and contact/short range forces

– Decide upon coordinate system

$$\frac{d}{dt}\left(\vec{a}\cdot\vec{b}\right)$$



$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$\vec{i}\cdot\vec{k} = \vec{i}\cdot\vec{j} = 0 = \vec{j}\cdot\vec{k}$$

$$\vec{i}\cdot\vec{i} = 1 = \vec{j}\,\vec{j} = \vec{k}\cdot\vec{k}$$

$$\left(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}\right)\left(b_x\vec{i} + b_y\vec{j} + b_z\vec{k}\right)$$

$$\frac{d}{dt}\left(a_x b_x + a_y b_y + a_z b_z\right)$$

$$= a_x\frac{d b_x}{dt} + \frac{d a_x}{dt} b_x$$

$$+ a_y\frac{d b_y}{dt} + \frac{d a_y}{dt}\cdot b_y$$

$$+ a_z\frac{d}{dt}b_z + \frac{d a_z}{dt}b_z$$

$$\underbrace{a_x \frac{db_x}{dt} + a_y \frac{db_y}{dt} + a_z \frac{db_z}{dt}}_{\vec{a}\,\frac{d\vec{b}}{dt}}$$

$$\underbrace{\frac{da_x}{dt} b_x + \frac{da_y}{dt} b_y + \frac{da_z}{dt} b_z}_{\frac{d\vec{a}}{dt}\,\vec{b}}$$