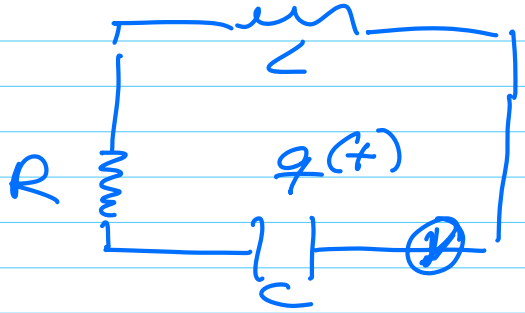
# PHY 321, MARCH 1, 2023

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos(\omega t)$$

similarity with RLC




charge $q(t)$

$$\frac{dq}{dt} = I$$

$$L\frac{d^2q}{dt^2} + R\cdot\frac{dq}{dt} + \frac{q}{C} = 0$$

$$\left(F_0\cos(\omega t)\right)$$

Scaling

$$\tau = \omega_0 t \qquad t = \frac{\tau}{\omega_0}$$

$$\omega_0 = \sqrt{k/m}$$

$$F_0\cos(\omega t) =$$

$$F_0\cos(\tilde{\omega}\tau)$$

$$\tilde{\omega} = \frac{\omega}{\omega_0}$$

$$m\omega_0^2 \frac{d^2x}{d\tau^2} + b\omega_0 \frac{dx}{d\tau}$$

$$+ kx = \tilde{F}_0 \cos(\tilde{\omega}\tau)$$

$$k/m = \omega_0^2$$

$$\frac{d^2x}{d\tau^2} + 2\gamma \frac{dx}{d\tau} + x = \tilde{F}_0 \cos(\tilde{\omega}\tau)$$

$$\tilde{F}_0 = \frac{F_0}{m\omega_0^2}$$

$$\gamma = \frac{b}{2m\omega_0}$$

$\tilde{F}_0 = 0$

$$x(\tau) = A_1 e^{r_1\tau} + A_2 e^{r_2\tau}$$

$$r_1 = -\gamma + \sqrt{\gamma^2 - 1}$$

$$r_2 = -\gamma - \sqrt{\gamma^2 - 1}$$

$\tilde{F}_0 \neq 0$

$$x(\tau) = A_1 e^{r_1\tau} + A_2 e^{r_2\tau} + x_p(\tau)$$

$$x_p(\tau) = D\cos(\omega^2\tau - \delta)$$

$$D = \frac{\bar{F}_0}{\sqrt{(1-\bar{\omega}^2)^2 + 4\bar{\omega}^2\gamma^2}}$$

$$\delta = \tan^{-1}\left(\frac{2\gamma\bar{\omega}^2}{1-\bar{\omega}^2}\right)$$