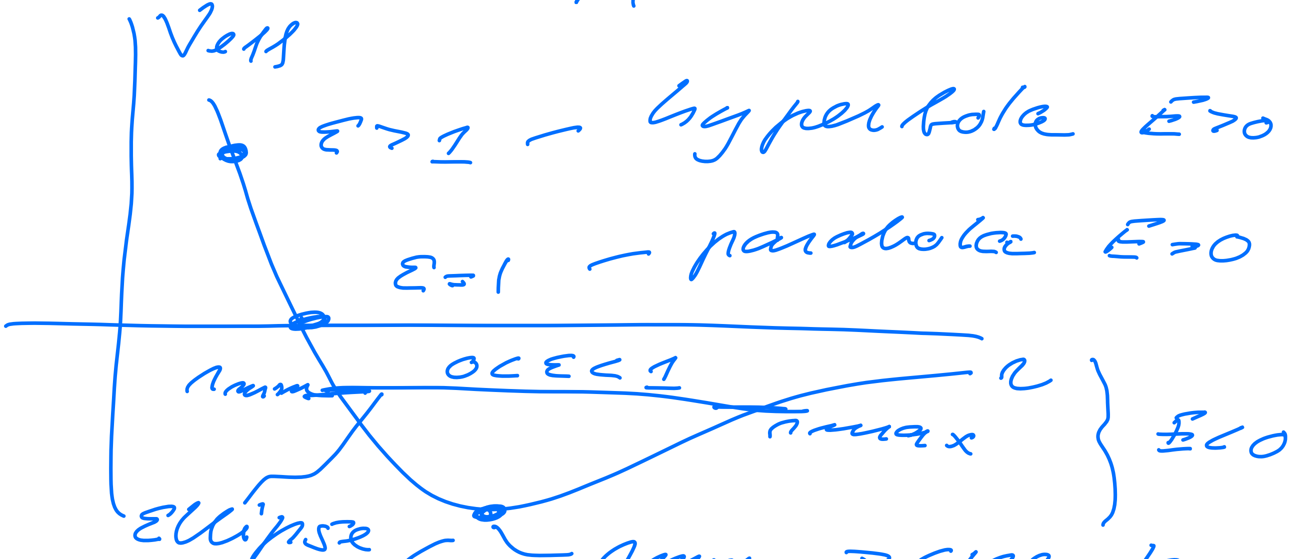
PHY 321, APRIL 6, 2022

$$V_{eff}(r) = V(r) + \frac{L^2}{2\mu r^2}$$

$$\mu = \frac{m_1 m_2}{M}$$



$r_{min} \Rightarrow$ circular

$$\varepsilon = 0$$

$$\frac{dr}{dt} = \dot{r} = 0$$

$$\varepsilon = \frac{AL^2}{\alpha\mu}$$

$V(r) = -\alpha/r$

$$r(\phi) = \frac{1}{\frac{\mu\alpha}{L^2} + A\cos\phi} = \frac{c}{1+\varepsilon\cos\phi}$$

$$c = \frac{L^2}{\mu\alpha}$$

Ex 1

circular orbit

$$F = \alpha / r^2 \qquad \left(-\alpha / r^2 \right)$$

$$m v^2 / r = F \qquad \left(a_r = v^2 / r \right)$$

$$m v^2 / r = \alpha / r^2 \Rightarrow$$

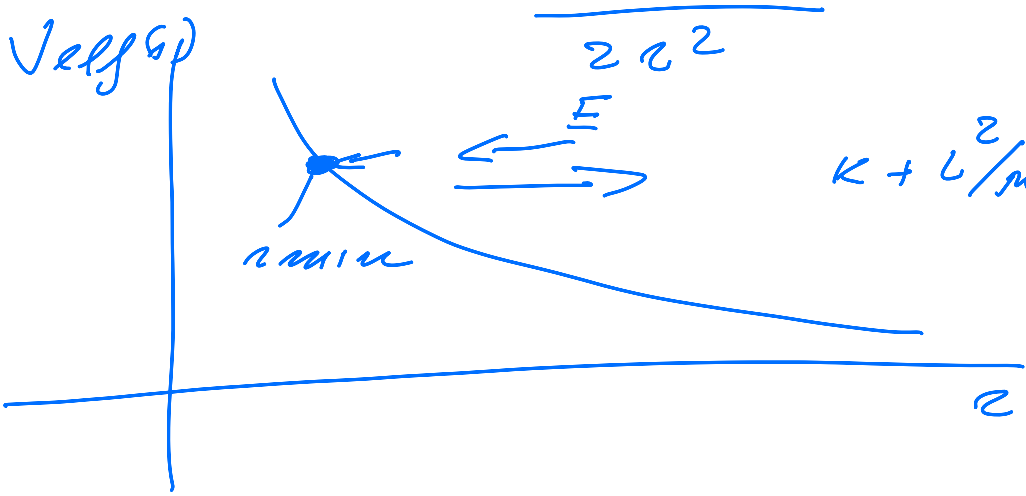
$$v = \sqrt{\alpha / m \cdot r}$$

Ex 2

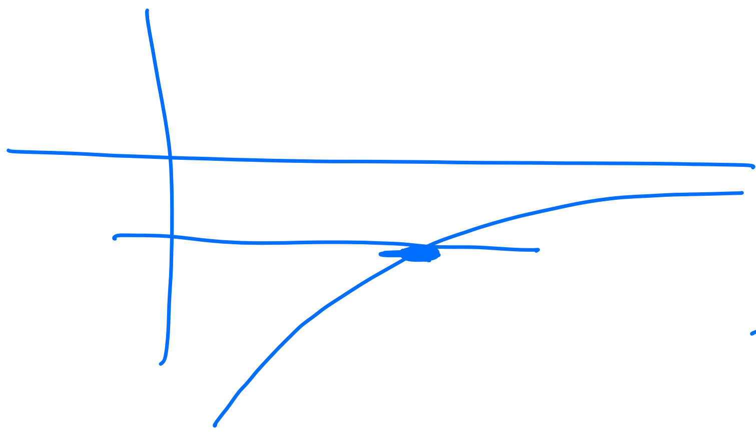
$$F = k / r^3 \qquad \left(V(r) = \frac{k}{2r^2} \right)$$

$$V_{eff}(r) = \frac{k}{2r^2} + \frac{L^2}{2\mu r^2}$$

$$= \frac{k + L^2/\mu}{2r^2}$$



$$k + L^2/\mu > 0$$



E < 0
bounded orbit.

$$u = \frac{1}{r}$$

$$\frac{d^2u}{d\phi^2} = -u - \frac{F\mu}{l^2u^2}$$

$$F = k/r^3 = ku^3$$

$$\frac{d^2u}{d\phi^2} = -\underbrace{\left(1 + \frac{k\mu}{l^2}\right)}_{\omega_0^2} u$$

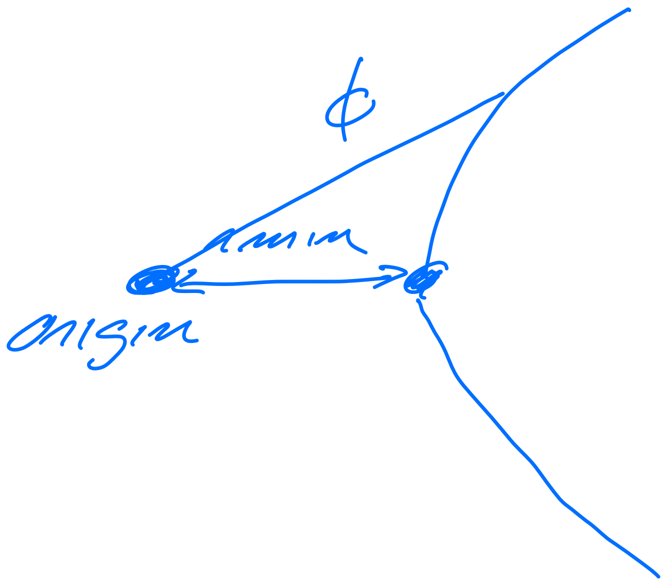
$$= -\omega_0^2 u$$

$$u(\phi) = C\cos(\omega_0\phi) + D\sin(\omega_0\phi)$$

$$= A\cos(\omega_0\phi - \delta)$$

$$\boxed{r = \frac{1}{u} = \frac{1}{A\cos(\omega_0\phi - \delta)}}$$

$K > L^2/\mu$ $\qquad$ $K\mu/L^2$ is larger than zero



Ex 3

$$F = -\alpha/r^2$$

L is given $\qquad$ μ is known

$$\ddot{r} = a_r = -\frac{1}{\mu}\frac{dV}{dr} + r\dot{\phi}^2$$

$$\dot{\phi} = \frac{L}{\mu r^2}$$

$$V_{eff}(r) = \underbrace{-\frac{\alpha}{r}}_{V(r)} + \frac{L^2}{2\mu r^2}$$

... the radius of a

find the radius of a
circular orbit

$$\frac{dV_{eff}}{dr} = 0 = \ddot{r} \Rightarrow$$

$$\frac{1}{\mu}\frac{dV}{dr} = r\dot{\phi}^2 = \frac{L^2}{\mu^2 r^4} r$$

$$\frac{\alpha}{\mu r^2} = \frac{L^2}{\mu^2 r^3} \Rightarrow$$

$$r_{min} = \frac{L^2}{\mu \alpha}$$

$$\ddot{r} = 0 = \frac{F}{\mu} + \dot{\phi}^2 r$$

$$\dot{\phi}^2 r = \frac{\alpha}{r^2 \mu} \Rightarrow$$

$$\dot{\phi}^2 = \frac{\alpha}{\mu r^3}$$

$$\dot{\phi} = \pm \sqrt{\frac{\alpha}{\mu r^3}}$$

r_{min}

$$\boxed{\dot{\phi} = \pm \frac{\alpha^2 \mu}{L^3}}$$

Can we find an effective spring constant at r_{min}?

Harmonic oscillations

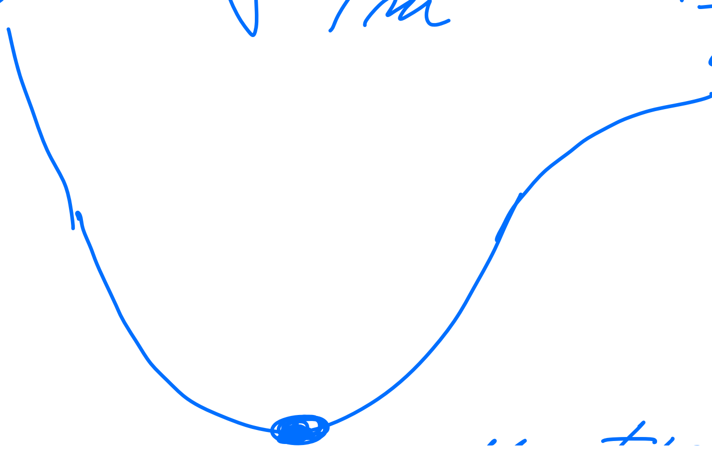
$$K = \left.\frac{d^2 V_{eff}}{dr^2}\right|_{r=r_{min}}$$

$$\left(\left.\frac{d V_{eff}}{dr}\right|_{r=r_{min}} = 0 \right)$$

$$K = -\frac{2\alpha}{r_{min}^3} + \frac{3L^2}{\mu r_{min}^4}$$

$$= \frac{\mu^3 \alpha^4}{L^6} \Rightarrow$$

$$\omega_0 = \sqrt{K/m} = \frac{\mu \alpha^2}{L^3} = \dot{\phi}$$



small oscillations around 1min

2-Dim (Earth)

$$F_x = -\frac{G M_G M_E}{\left(\sqrt{x^2+y^2}\right)^3} x$$

$$\vec{r} = x\vec{i} + y\vec{j}$$

$$F_y = -\frac{G M_G M_E}{\left(\sqrt{x^2+y^2}\right)^3} y$$

3-Dim problem

$$F_z = -\frac{G M_G M_E \, z}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$$

same for F_x and F_y

$$\frac{dv_x}{dt} = a_x = -\frac{G M_G x}{r^3}$$

$$\frac{dx}{dt} = v_x$$

$$\frac{dv_y}{dt} = a_y = -\frac{GM_\odot y}{r^3}$$

$$\frac{dy}{dt} = v_y$$

$$\frac{dv_z}{dt} = a_z = -\frac{GM_\odot z}{r^3}$$

$$\frac{dz}{dt} = v_z$$