PHY 321, APRIL 5, 2023

Midterm discussion

($\hat{i}$) Part 1, (d+e)

$$\mu \ddot{r} = F(r) + \frac{L^2}{\mu r^3}$$

$$V(r) = -\gamma / r^{\beta+1}$$

gravitation $\beta = 0$

$\beta = 1$

$\beta = 2$

$$F(r) = -\frac{dV(r)}{dr}$$

gravitational force

$$F(r) = -\gamma / r^2$$

$\beta = 1$

$$F(r) = -\gamma / r^3 = -\gamma u^3$$

$$r = \frac{1}{u} \qquad u(\phi) \Rightarrow r(\phi)$$

$$\frac{d^2 u}{d\phi^2} = -\frac{F(u)\mu}{L^2 u^2} - u$$

$$= \frac{\gamma u \mu}{l^2} - u$$

$$= u\left(\frac{\gamma\mu}{l^2} - 1\right)$$

$$= -u \cdot A$$

$$A = 1 - \gamma\mu/l^2$$

$$\frac{d^2u}{d\phi^2} = -uA$$

$$u = B\cos(\phi - \delta)$$

$$r(\phi) = 1/u = \frac{1}{B\cos(\phi-\delta)}$$

(ii) Part (c)

$$\frac{dA}{dt} = \frac{L}{2\mu} = \frac{1}{2\mu}|\vec{r}\times\vec{p}|$$

Taylor chap 3.

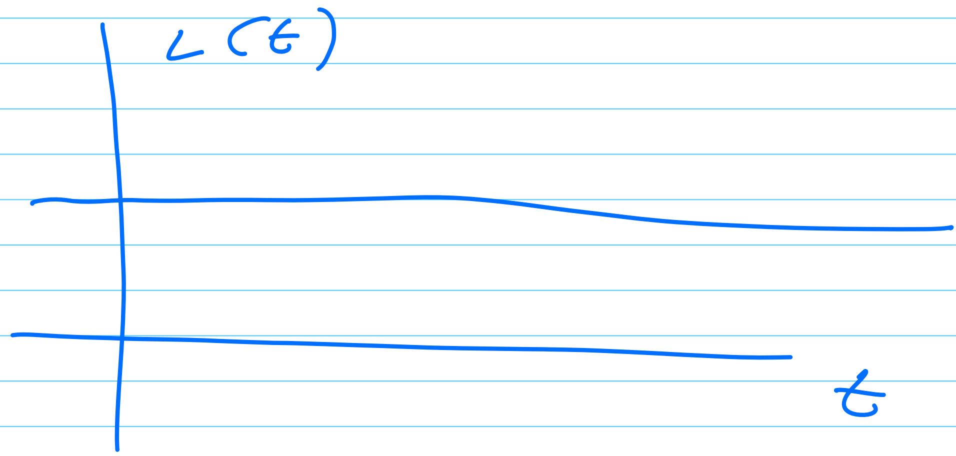
$$\vec{p} = \mu \cdot \vec{v}$$

Don't need to show

$$\frac{L}{2\mu} = |\vec{r}\times\vec{v}|\frac{1}{2}$$

$\vec{r}(t)$, $\vec{v}(t)$ from ODE solver.

$$L(t) \propto |\vec{r}(t) \times \vec{v}(t)|$$



part 1

Scaling the equations

$$\vec{F}_E = - \frac{G M_\odot M_E \, \vec{r}}{r^3}$$

2-Dim

$$r = \sqrt{x^2+y^2}$$

3-Dim

$$r = \sqrt{x^2+y^2+z^2}$$

$$a_x = \frac{F_x}{M_E} = -\frac{GM_\odot}{r^3}\,x = \frac{dv_x}{dt}$$

$$a_y = \frac{F_y}{M_E} = -\frac{GM_\odot}{r^3}\,y = \frac{dv_y}{dt}$$

$$\frac{dx}{dt} = v_x \quad \wedge \quad \frac{dy}{dt} = v_y$$

$$GM_\odot = ?$$

circular motion

$$\cancel{M_E}\, v^2/r = F = \frac{GM_\odot \cancel{M_E}}{r^2}$$

$$v^2 r = GM_\odot$$

units : $1\,AU = 1{,}5 \cdot 10^{11}\,m$

time : $1\,Yr$.

$$v = 2 \cdot \pi \cdot r / 1\,Yr$$

$$= 2\pi\, 1\,AU / 1\,Yr$$

$$G \cdot M_\odot = 4\pi^2 (AU)^3 / Yr^2$$