PHY 321, APRIL 1, 2022

$$\mu \ddot{r} = F(r) + \frac{L^2}{\mu r^3}$$

$$\frac{d\phi}{dt} = \frac{L}{\mu r^2}$$

HW 8, Ex 2 & 3

$$F(r) = -kr$$

March 30

$$F(r) = -\frac{\alpha}{r^2}$$

$$r(\phi) = \frac{c}{1+\varepsilon\cos\phi}$$

$$\varepsilon = \frac{AL^2}{\alpha\mu} \qquad c = \frac{L^2}{\mu\alpha}$$

ε = eccentricity

$\varepsilon = 0 \Rightarrow$ circular motion

$0 < \varepsilon < 1 \Rightarrow$ elliptical motion

$$r_{min} = \frac{c}{1+\varepsilon}$$

$$r_{max} = \frac{c}{1-\varepsilon}$$

$$x = r\cos\phi \qquad y = r\sin\phi$$

$$r(\phi) = r = \frac{c}{1+\varepsilon\cos\phi}$$

$$r(1+\varepsilon\cos\phi) = c$$

$$(r\cos\phi = x)$$

$$r + \varepsilon x = c$$

$$r = c - \varepsilon x$$

square both sides

$$r^2 = x^2 + y^2 = c^2 + \varepsilon^2 x^2 - 2x\varepsilon c$$

$$x^2(1-\varepsilon^2) + 2c\varepsilon x + y^2 = c$$

Divide by $1-\varepsilon^2$

and define $d = \frac{c\varepsilon}{1-\varepsilon^2}$

$$x^2 + 2dx + \frac{y^2}{1-\varepsilon^2} = \frac{c^2}{1-\varepsilon^2}$$

add d^2 to both sides

$$(x+d)^2 + \frac{y^2}{1-\varepsilon^2} = \frac{c^2}{1-\varepsilon^2} + d^2$$

$$= \frac{c^2}{(1-\varepsilon^2)^2} = a^2$$

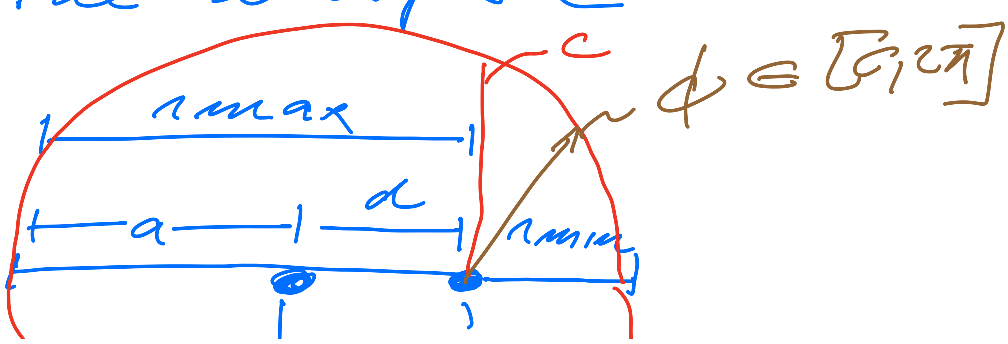
Divide by a^2 and define $b = a\sqrt{(1-\varepsilon^2)}$

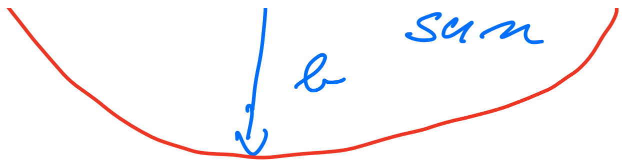
$d = a \cdot c$

$$\frac{(x+d)^2}{a^2} + \frac{y^2}{b^2} = \underline{1}$$

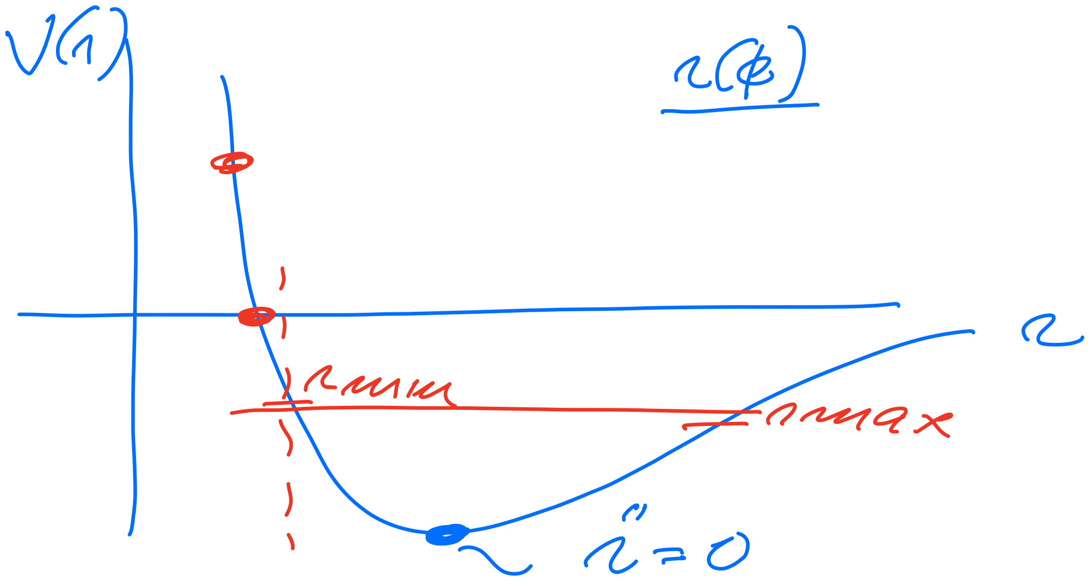
Ellipse!

$x+d$ reflects the fact that our origin, the sun, is not at the center of the ellipse





Monday



$r(\phi)$

$\varepsilon = 0$	circle	$E < 0$	bound motion
$0 < \varepsilon < 1$	ellipse	$E < 0$	
$\varepsilon = 1$	parabola	$E = 0$	
$\varepsilon > 1$	hyperbola	$E > 0$	