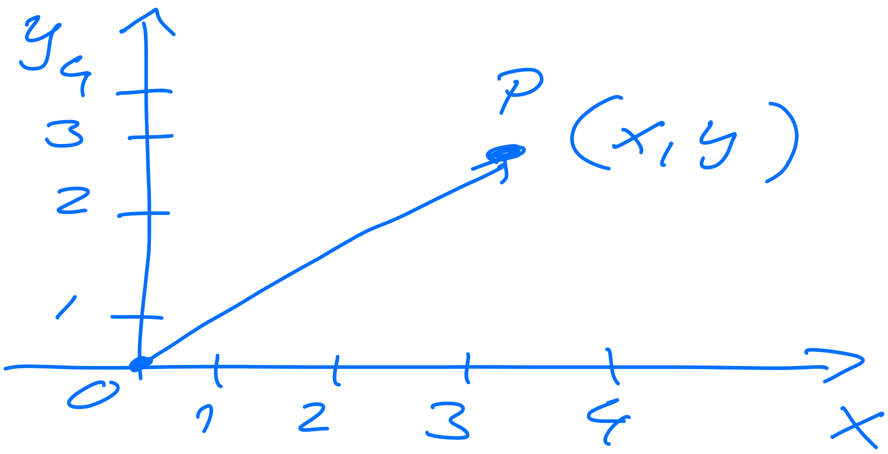
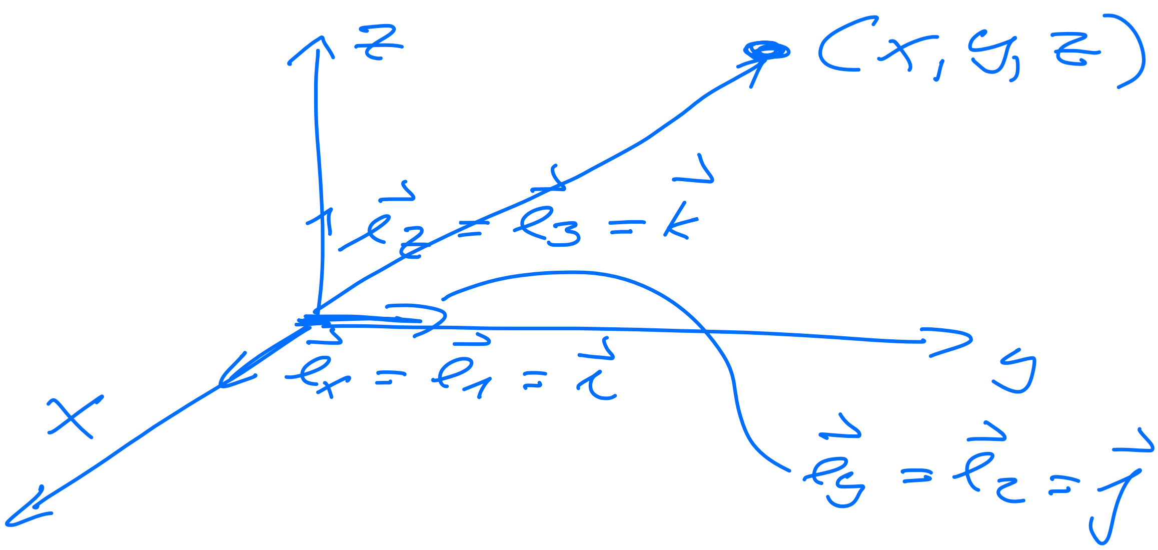
PHY 321, JANUARY 11, 2023

choose origin

2 Dim



3-Dim



$$\vec{a} = (x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{i}| = \underline{1} = |\vec{j}| = |\vec{k}|$$

$$\vec{i} \cdot \vec{j} = 0 = \vec{i} \cdot \vec{k} = 0 = \vec{j} \cdot \vec{k}$$

<u>Ex 3</u>

$$\vec{a} \cdot (\underline{\vec{b}} + \underline{\vec{c}}) = \vec{a}\vec{b} + \vec{a} \cdot \vec{c}$$

$$(a_x\vec{i} + a_y\vec{j})(b_x\vec{i} + b_y\vec{j} + c_x\vec{i} + c_y\vec{j})$$

$$= a_x b_x + a_y b_y + a_x c_x + a_y c_y$$

$$= \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

<u>Dimensions, scales & units</u>

$[\vec{r}] = \text{length}$

distance

$[t] = \text{time}$

$[\vec{v}] = \text{length}/\text{time}$

$[\vec{a}] = \text{length}/\text{time}^2$

$[\vec{F}] = \text{mass} \times \text{length}/\text{time}^2$

$\vec{F} \rightarrow F_x = F = ma_x = ma$

$\vec{F}/m = \vec{a}\ ; \quad \frac{F}{m} = a$

$$= \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$= \frac{d}{dt}\left[\overbrace{\frac{dx}{dt}}^{\text{length}}\right]$$

time time

It is common to introduce dimensionless variables

$$t = \tau/\alpha \qquad \tau \text{ - Dim less time}$$

$$[\alpha] = 1/\text{time}$$

$$\frac{F}{m} = \frac{d}{dt}\left[\frac{dx}{dt}\right]$$

$$= \frac{d}{d(\tau/\alpha)} \frac{dx}{d\,\tau/\alpha}$$

$$\frac{F}{m\alpha^2} = \frac{d}{d\tau}\left[\frac{dx}{d\tau}\right]$$

Gravitational Force

$$\vec{F}(\vec{r}_1,\vec{r}_2) = -\frac{G m_1 m_2}{|\vec{r}_1-\vec{r}_2|^2}$$

$$|\vec{r}_1-\vec{r}_2| = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$[\vec{F}] = \frac{\text{mass} \cdot \text{length}}{\text{time}^2}$$

$$= -G \cdot \frac{\text{mass} \times \text{mass}}{\text{length}^2}$$

$$\Rightarrow [G] = \frac{\text{length}^3}{\text{mass} \cdot \text{time}^2}$$

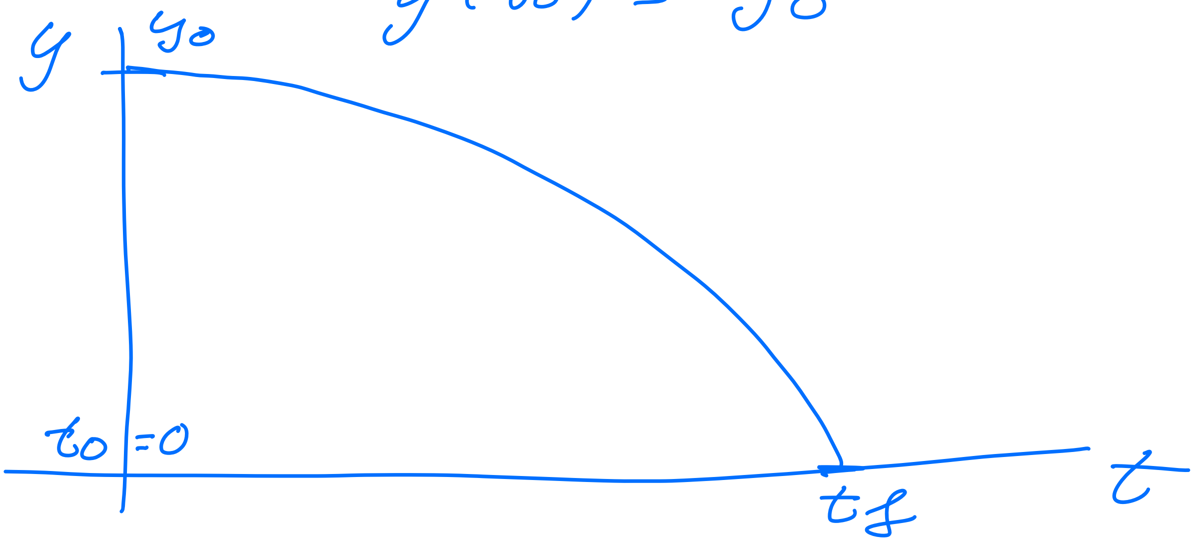
1-Dim, somebody gives

$$y(t) = y(t_0) - \frac{1}{2} g t^2$$

initial position

9.80665 m/s^2

$$y(t_0) = y_0$$



$$y(t_f) = 0 = y_0 - \frac{1}{2} g t_f^2$$

$$\Rightarrow \quad t_f = \sqrt{2y_0/g}$$

$$v(t) = \frac{dy}{dt} = -g \cdot t$$

$$a(t) = \frac{dv}{dt} = -g$$

Derive $y(t)$ from $a(t)$

$$\frac{\vec{F}}{m} = \vec{a}(t)$$

$$\frac{F}{m} = -g = a(t)$$

$$-g = \frac{dv}{dt}$$

$$\int_{t_0=0}^{t} (-g)\, dt' = \int_{t_0}^{t} dt' \frac{dv}{dt'}$$

$$-g \cdot t = v(t) - v(t_0)$$

$$= v(t) - v_0$$

$$v(t) = v_0 - g t$$

$$\frac{dy}{dt} = v(t)$$

$$\int_{t_0}^{t} \frac{dy}{dt'} dt' = \int_{t_0=0}^{t} (v_0 - g t') dt'$$

$$v_0 = 0$$

$$y(t) - y_0 = -\frac{1}{2} g t^2 \Rightarrow$$

$$y(t) = y_0 - \frac{1}{2} g t^2$$