Parametric Matrix Models for General Machine Learning

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Facility for Rare Isotope Beams
Michigan State University

STREAMLINE Symposium
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Theory Alliance

Introduction

Picking up where Danny left off...

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- Parametric Matrix Models (PMMs) are powerful for sparse and noisy data

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Does this advantage generalize to general machine learning problems?

Why should it matter?

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- Training time and difficulty
- Dataset size

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These factors directly determine the applications of a model

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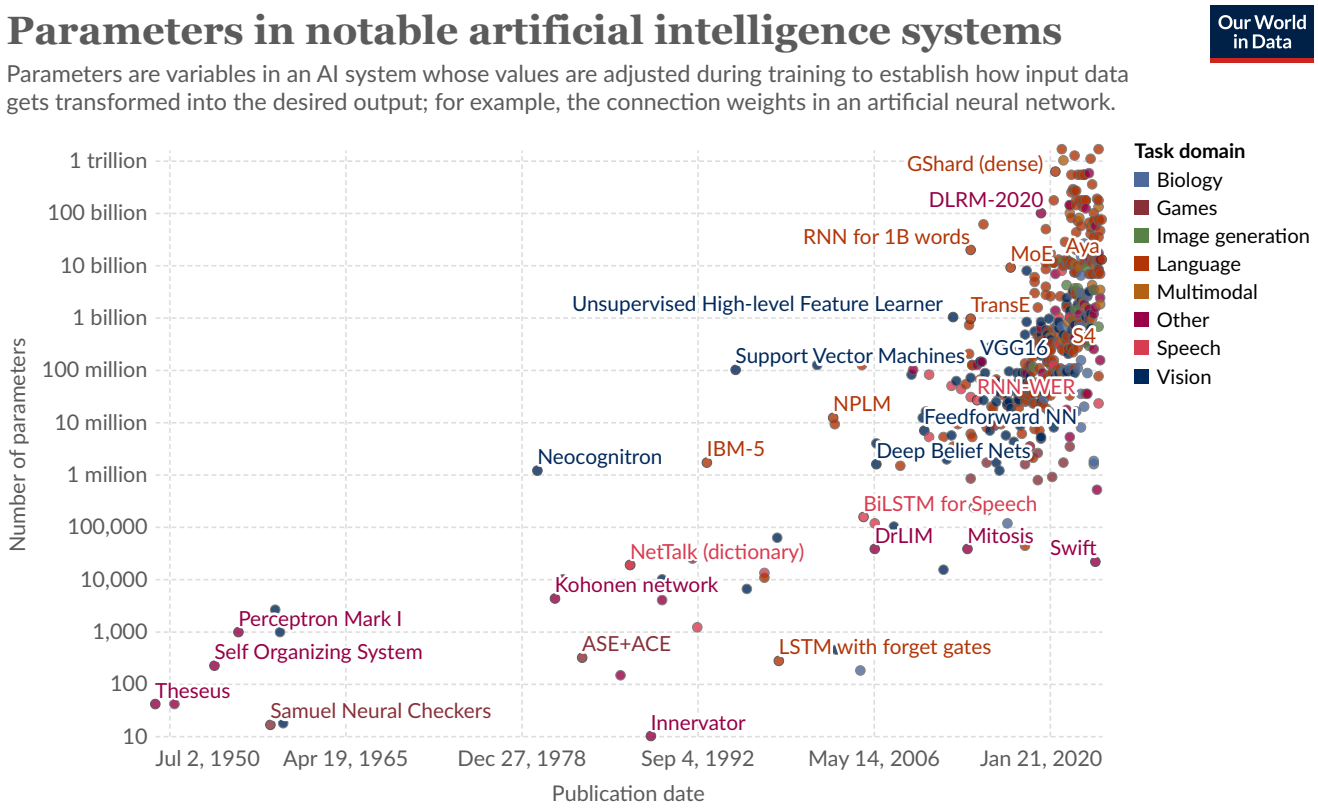
These factors directly determine the applications of a model

- Real-time applications
- Mobile platforms
- Resource-constrained environments

Current “Solution”

Current "Solution"

More parameters, more data, more compute, more time



Generalizing PMMs

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Make a "Hamiltonian"

Use its eigensystem

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$$H(\mathbf{x})$$

Generalizing PMMs

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Use its eigensystem

$$H(\mathbf{x}) \rightarrow V\Lambda V^{\dagger} \rightarrow \mathbf{z}$$

Generalizing PMMs

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$$H(\mathbf{x}) = \underline{H_0} + \sum_i x_i \underline{H_i}$$

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$$z_i = v_i^\dagger \underline{\Delta} v_i$$

Generalizing PMMs

Make a "Hamiltonian"	**Use its eigensystem**
$H(\mathbf{x})$	$H(\mathbf{x}) \rightarrow V\Lambda V^\dagger \rightarrow \mathbf{z}$
$H(\mathbf{x}) = \underline{H_0} + \sum_i x_i \underline{H_i}$	$\mathbf{z} = \mathrm{diag}\,(\Lambda)$
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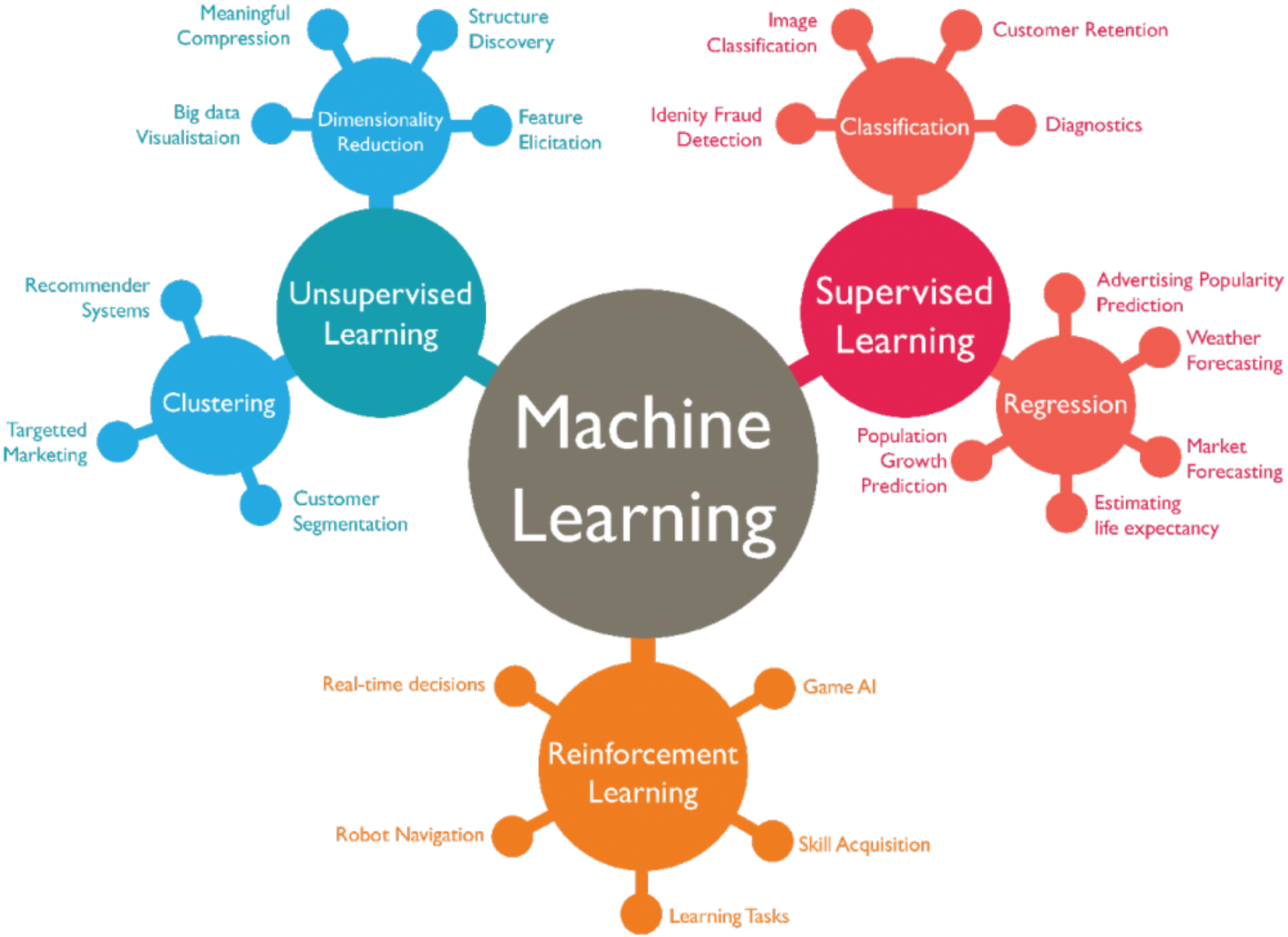
$H(\mathbf{X}) = \underline{H_0} + \underline{H_1}\mathbf{X}\underline{H_1}^\dagger$	$z_i = v_0^\dagger \underline{\Delta_i} v_0$


Machine Learning
Unsupervised Learning
Dimensionality Reduction
Meaningful Compression
Structure Discovery
Big data Visualistaion
Feature Elicitation
Clustering
Recommender Systems
Targetted Marketing
Customer Segmentation
Supervised Learning
Classification
Image Classification
Customer Retention
Idenity Fraud Detection
Diagnostics
Regression
Advertising Popularity Prediction
Weather Forecasting
Population Growth Prediction
Market Forecasting
Estimating life expectancy
Reinforcement Learning
Real-time decisions
Game AI
Robot Navigation
Skill Acquisition
Learning Tasks

Regression PMM

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Using the eigensystem more effectively

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How can we use more of the information in the eigensystem?

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$$z_k = \sum_{i \leq j} v_i^\dagger \underline{\Delta_{ijk}} v_j$$

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$$z_k = \sum_{i \leq j} \left| v_i^\dagger \underline{\Delta_{ijk}} v_j \right|^2$$

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How can we use more of the information in the eigensystem?

$$z_k = \sum_{i \leq j} \left| v_i^\dagger \underline{\Delta_{ijk}} v_j \right|^2 - \frac{1}{2} \left\| \underline{\Delta_{ijk}} \right\|_2^2$$

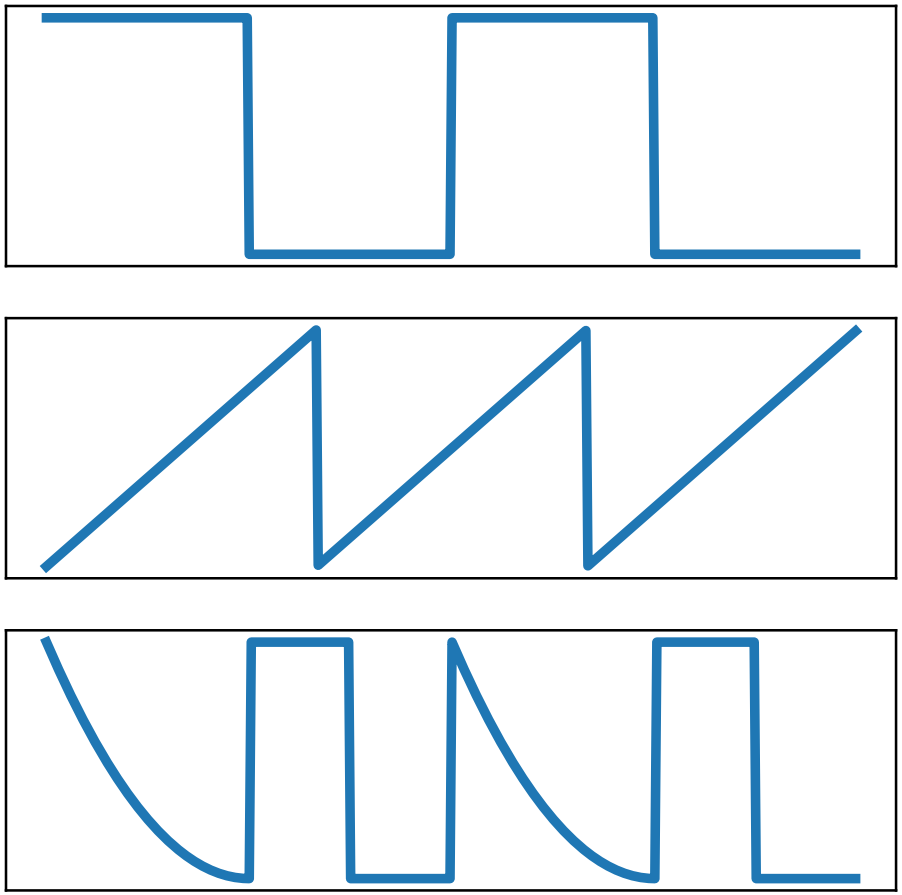
Using the eigensystem more effectively

How can we use more of the information in the eigensystem?

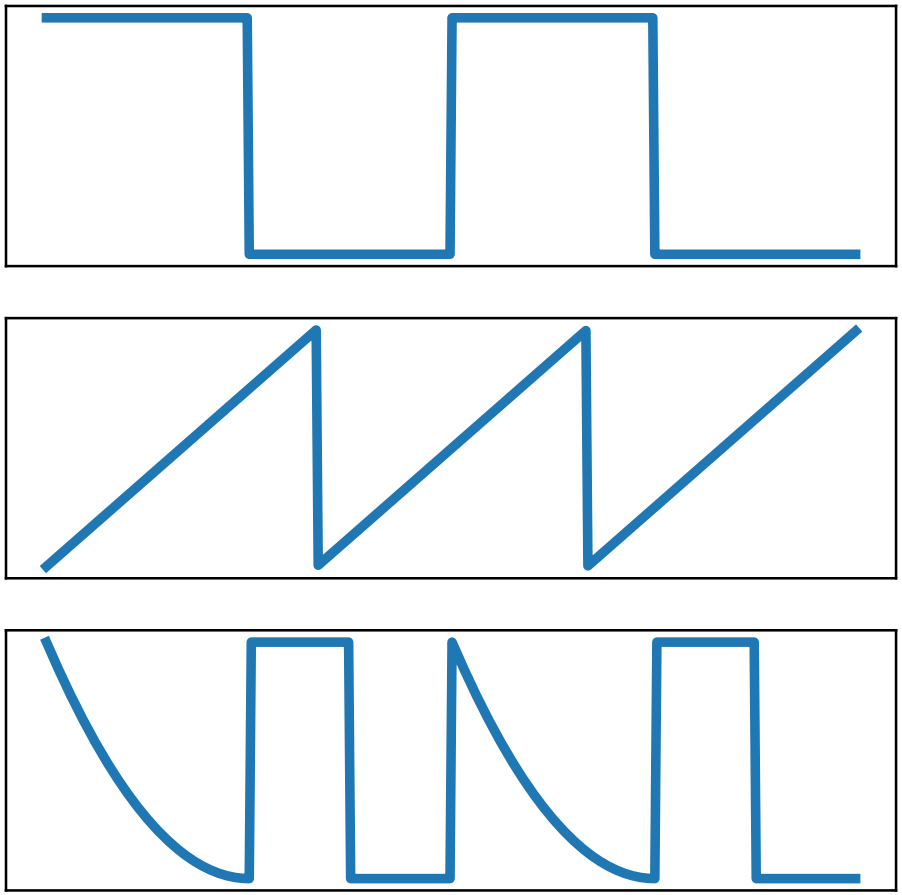
$$z_k = \sum_{i \leq j} \left| v_i^\dagger \underline{\Delta_{ijk}} v_j \right|^2 - \frac{1}{2} \left\| \underline{\Delta_{ijk}} \right\|_2^2$$

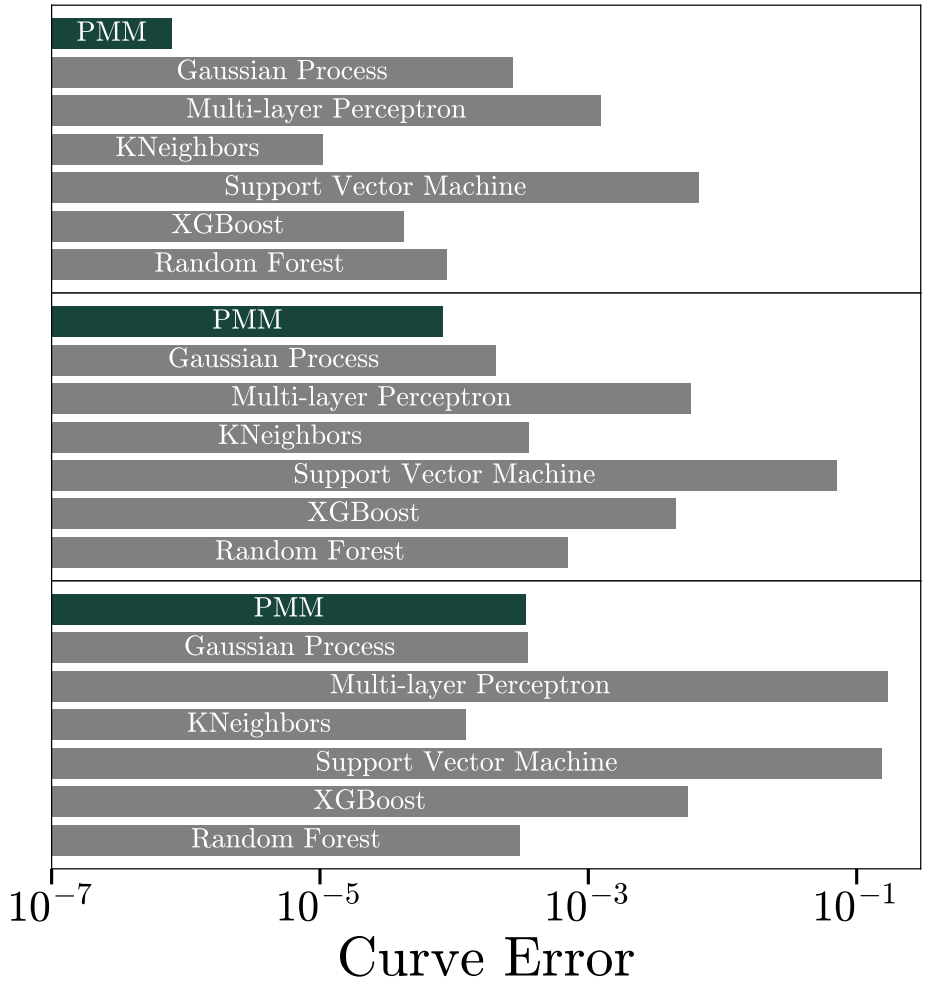
Each $\underline{\Delta_{ijk}}$ is known as a "decoder" matrix

1D Waveforms



1D Waveforms





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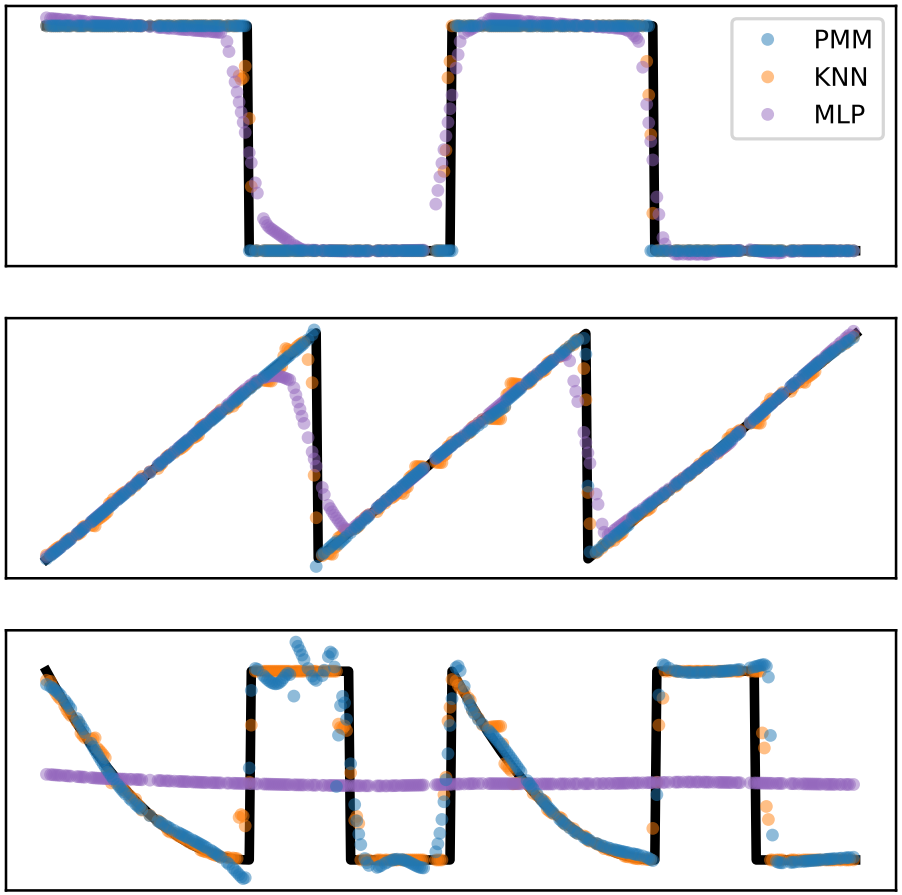

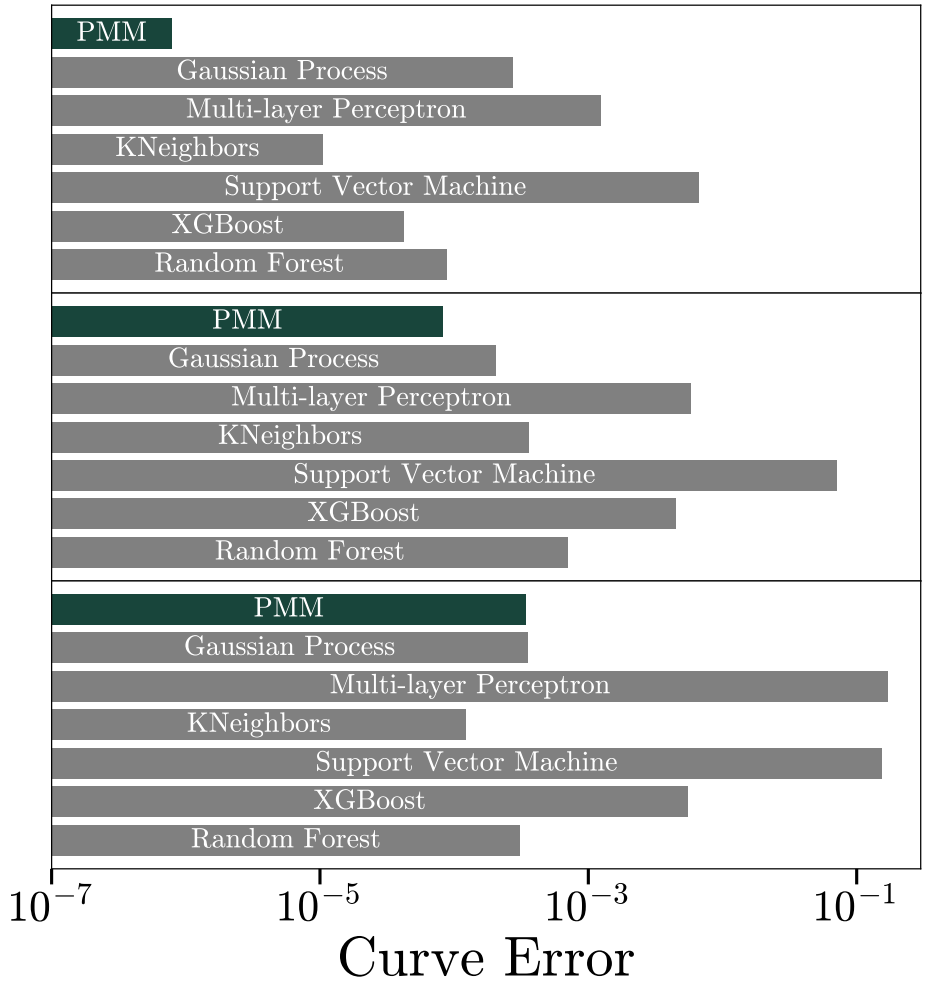

2D Surfaces

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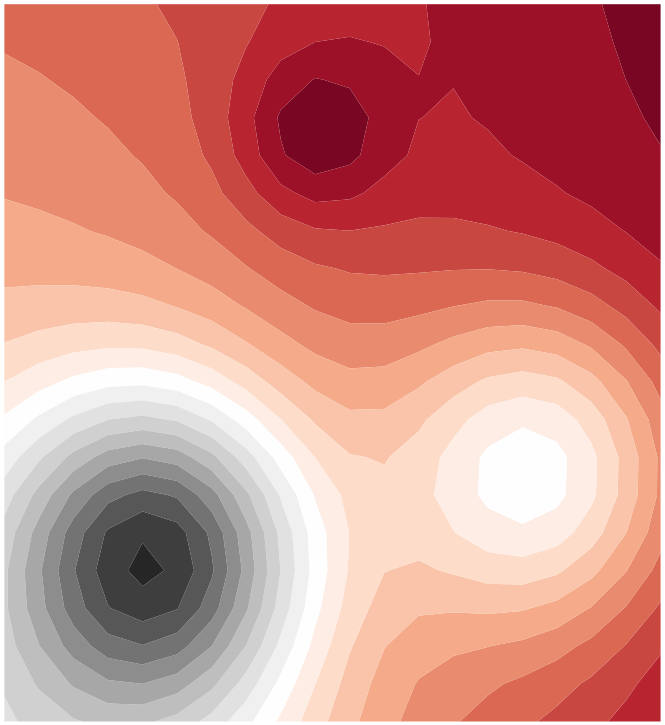
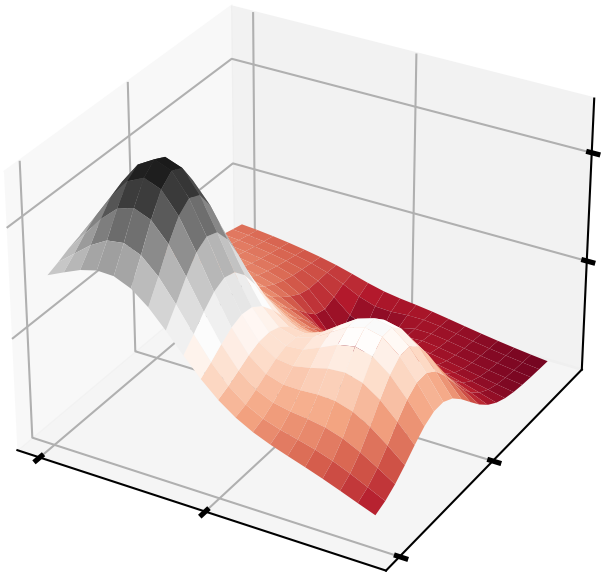
Franke's Bivariate Test Function





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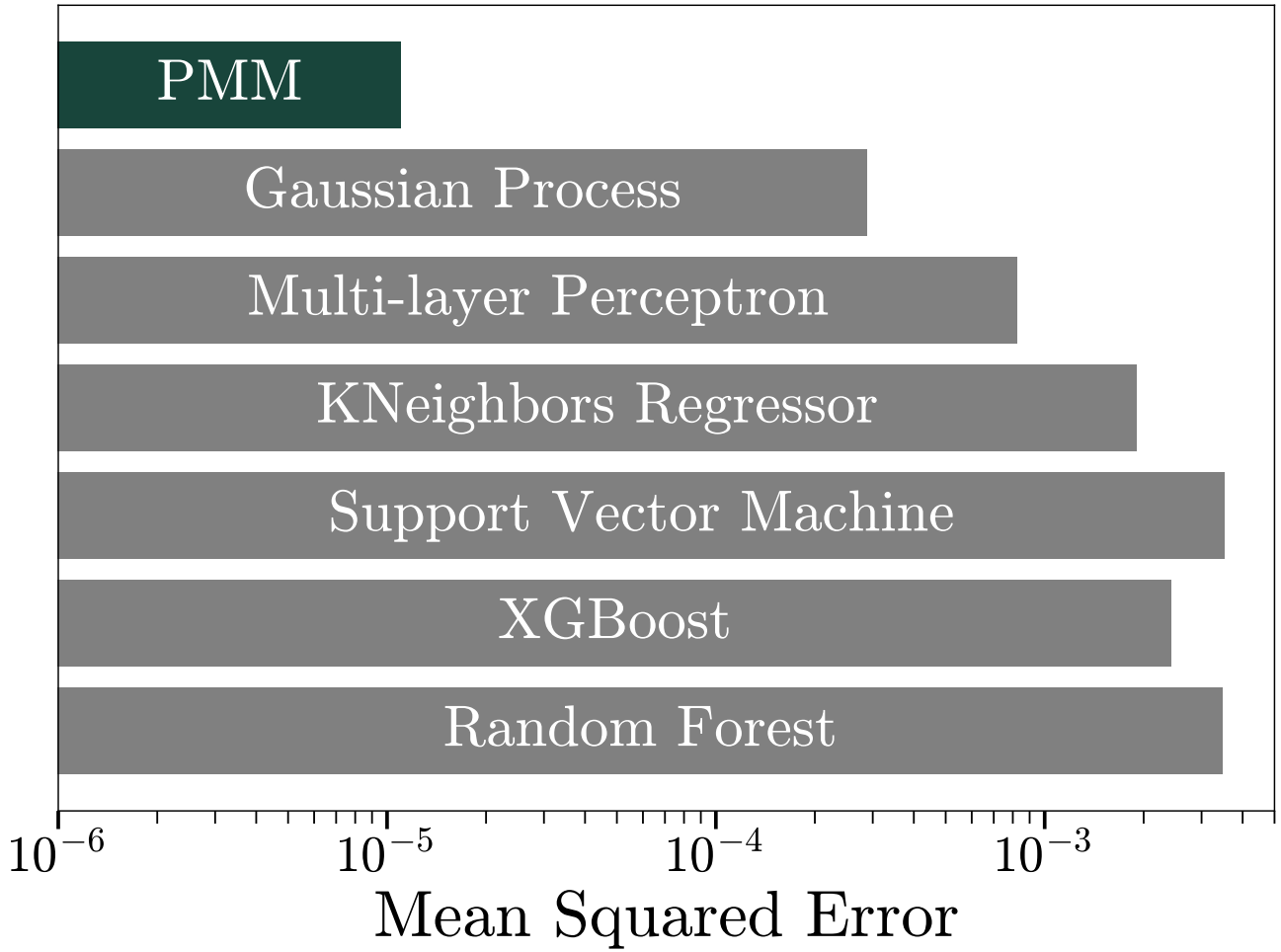
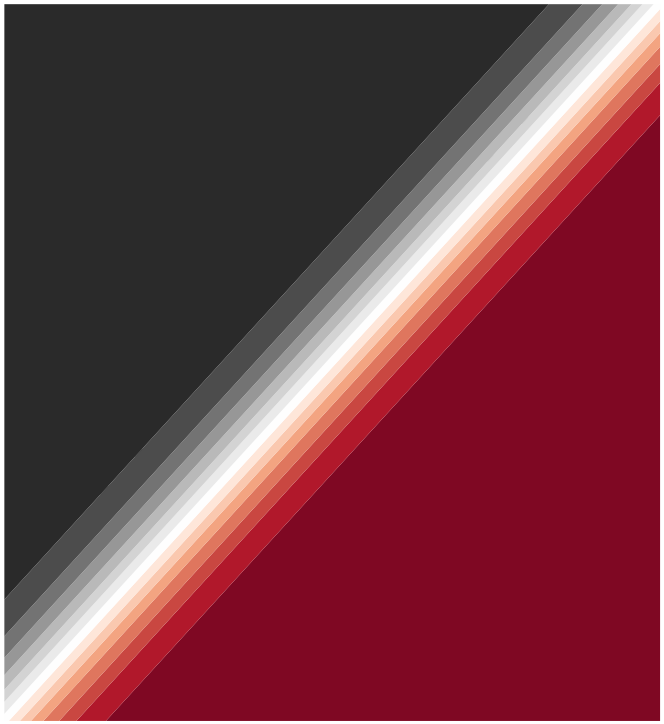
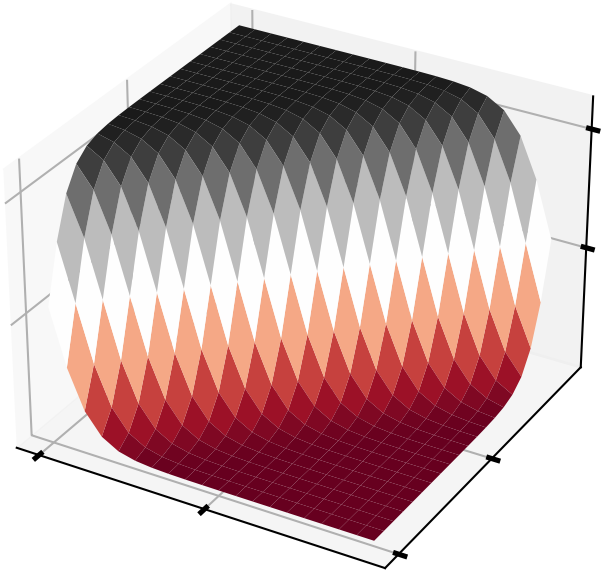


2D Surfaces

Cliff Function





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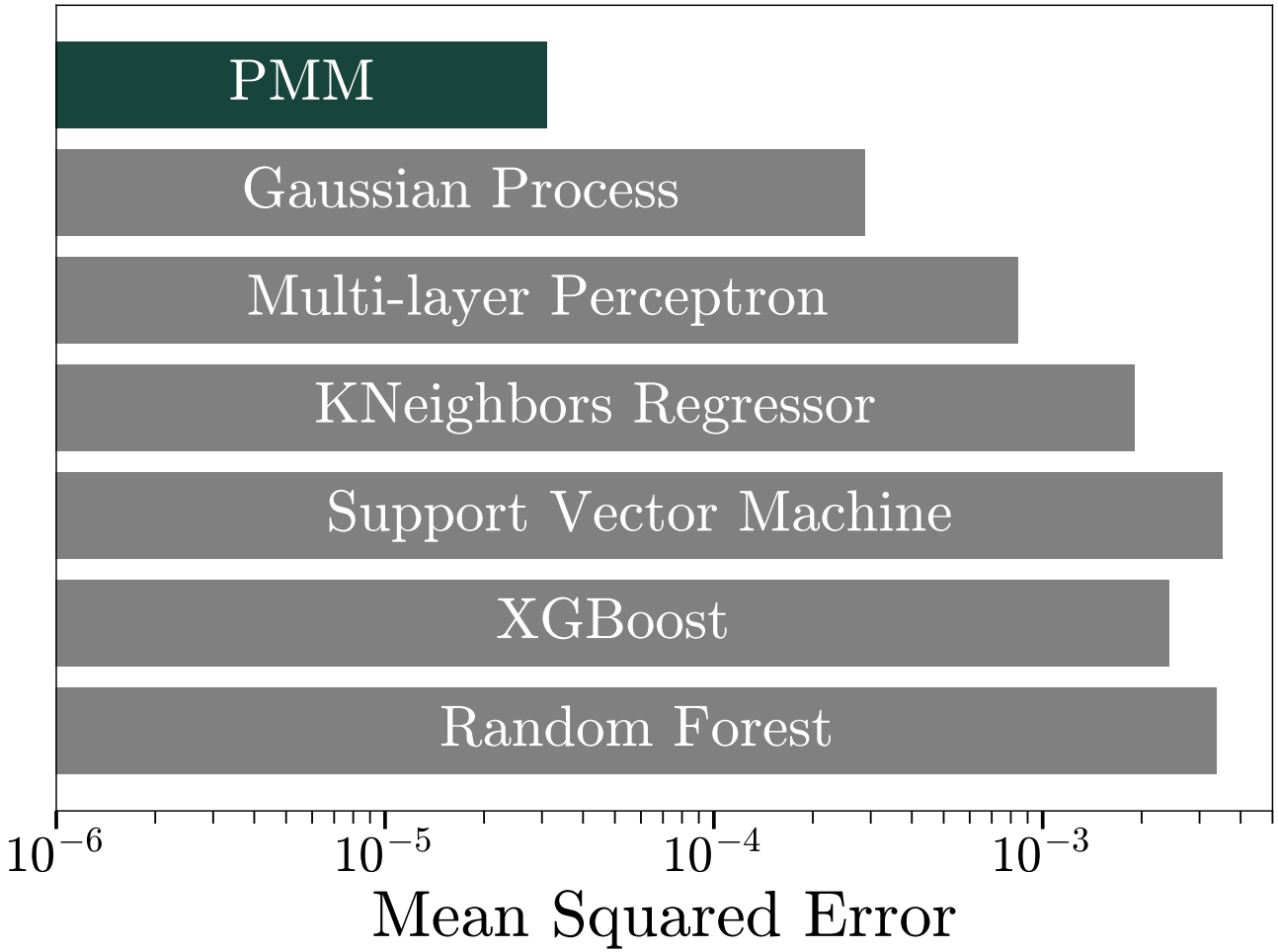

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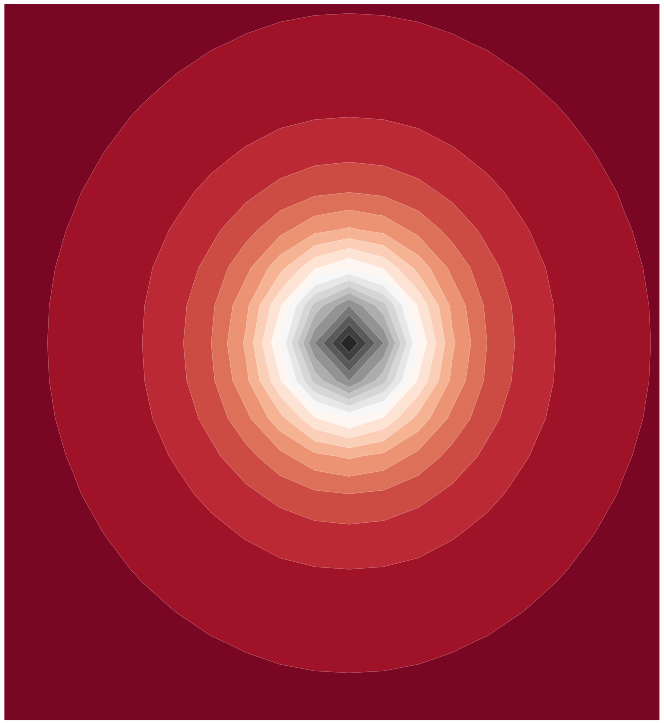

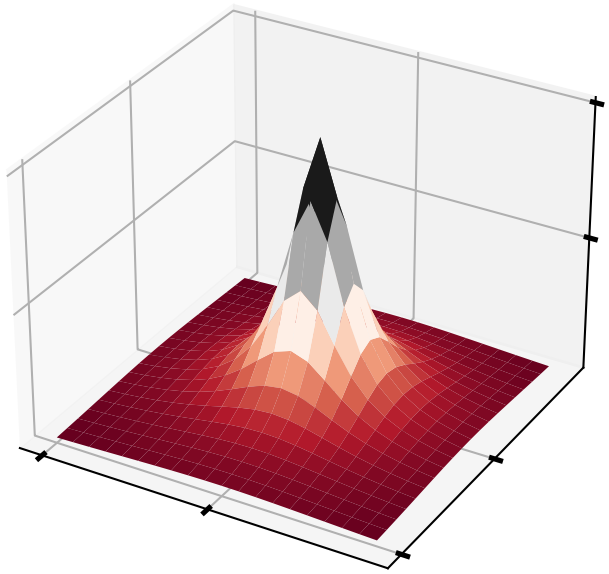

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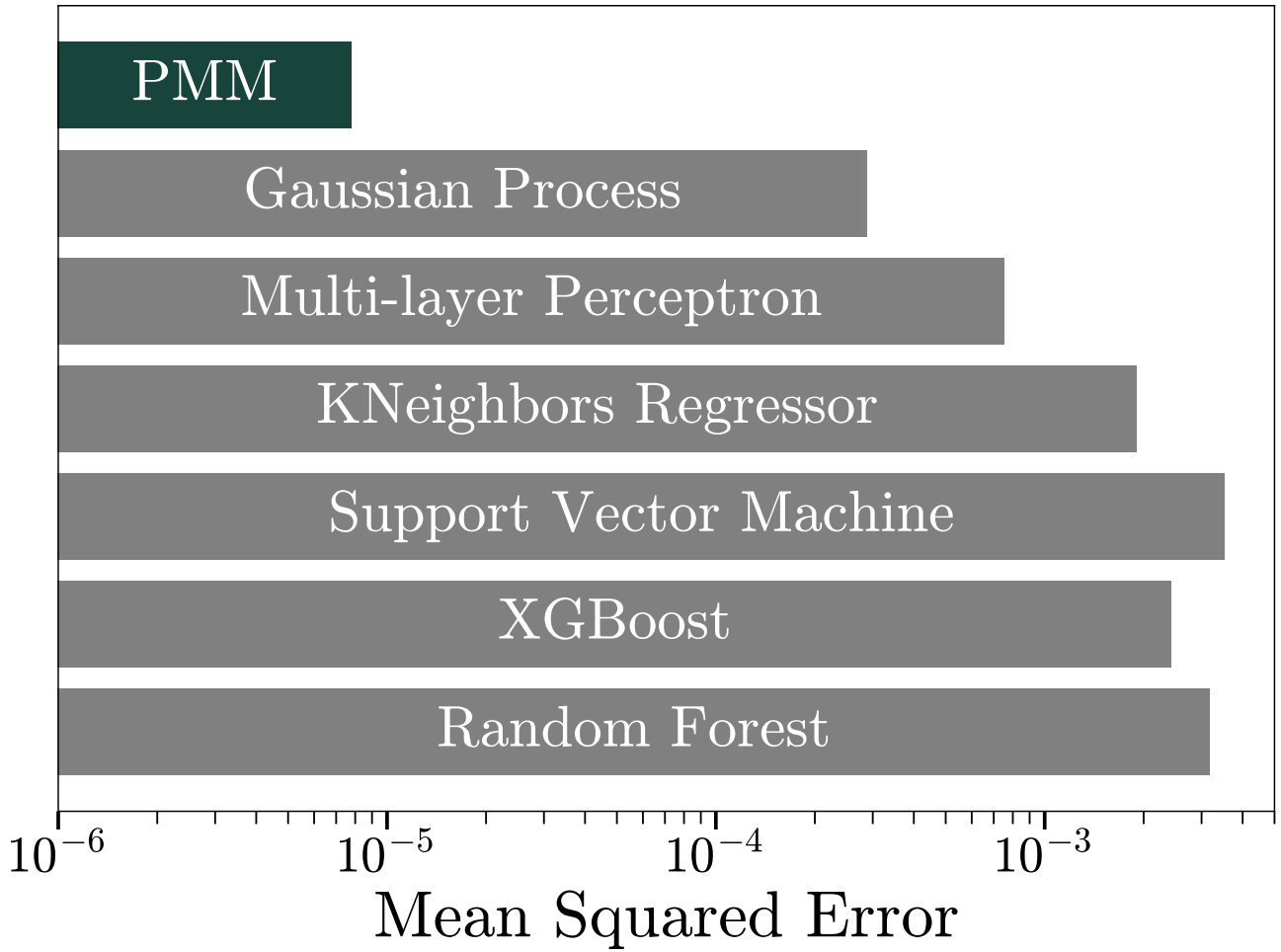

Image Classification PMM

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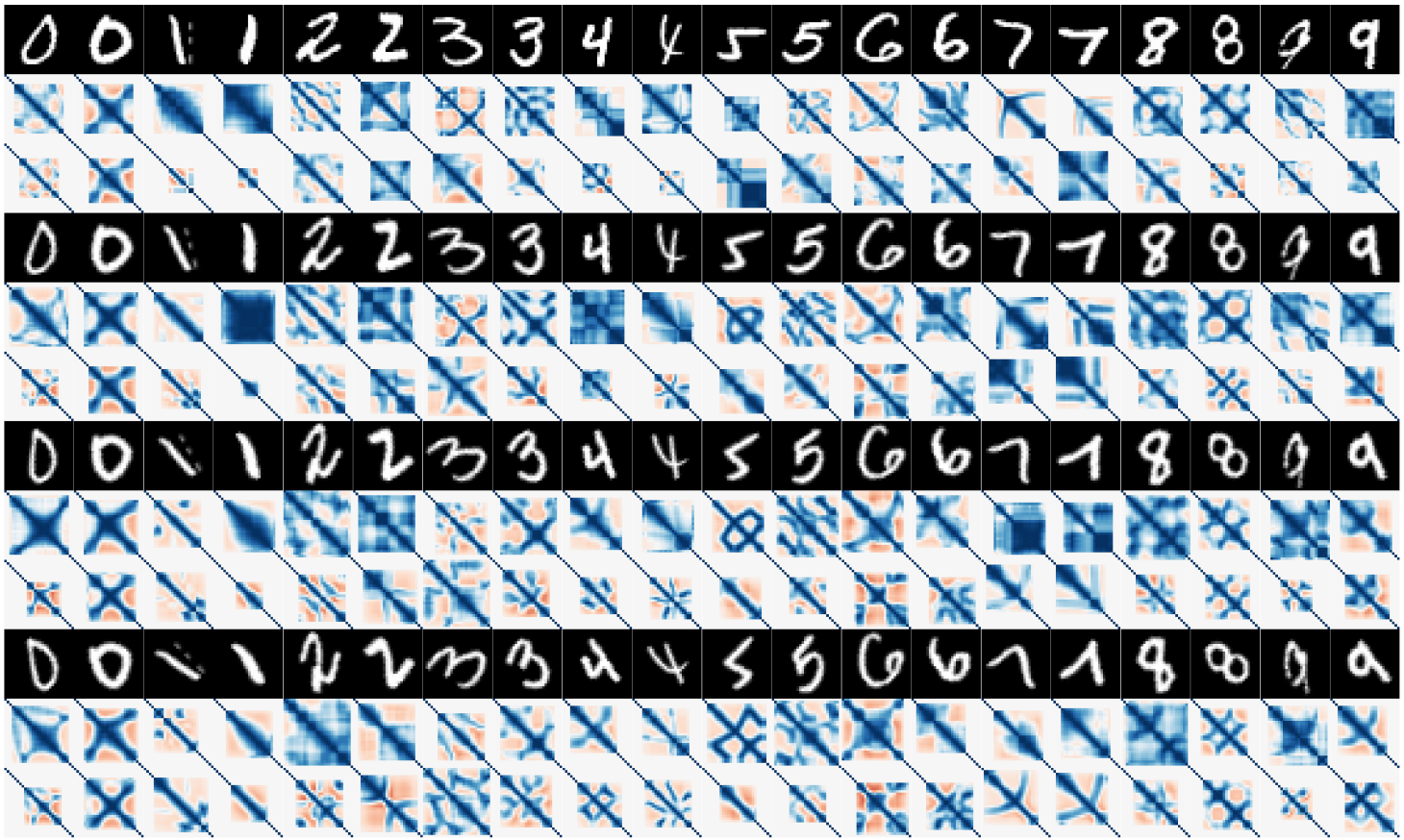
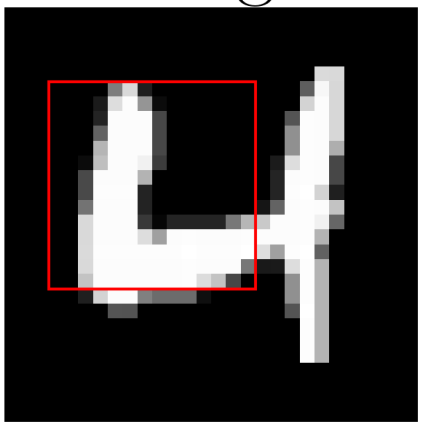
Decoder formalism

Images as Hermitian Matrices

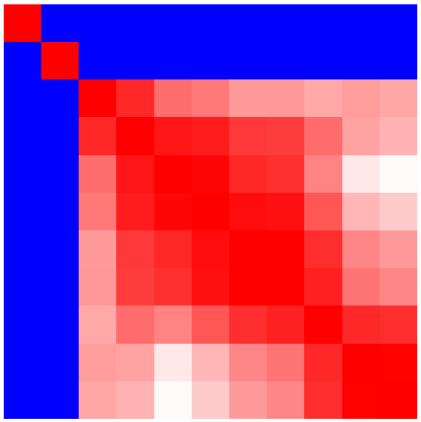


Images as Hermitian Matrices

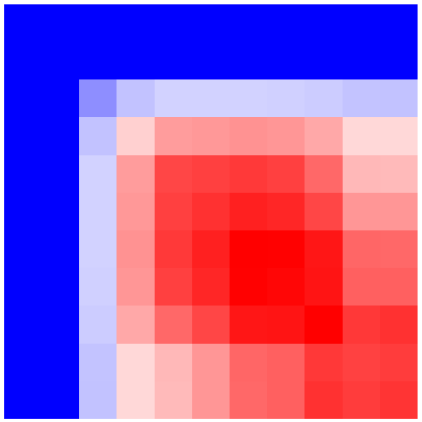
Image



C_R



$XX^\dagger$



Make a "Hamiltonian"

Use its eigensystem

$$\mathbf{X} \xrightarrow{\text{blocks}} X_i$$

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$$R_i \mathrel{\widetilde{\propto}} X_i X_i^\dagger$$
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$$M = \sum_i \underline{K_i} R_i \underline{K_i}^\dagger + \underline{L_i} C_i \underline{L_i}^\dagger$$

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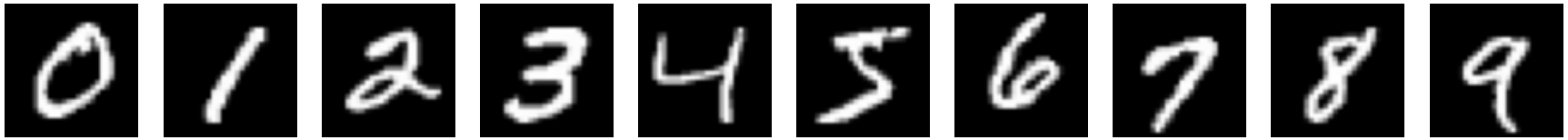
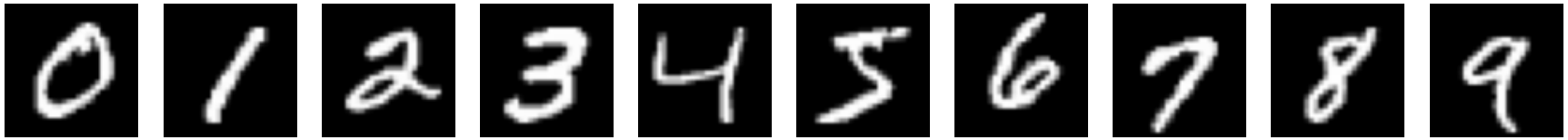
$$p_c = \frac{\exp\left\{\left(z_c + \underline{b_c}\right)/\tau\right\}}{\sum_k \exp\left\{\left(z_k + \underline{b_k}\right)/\tau\right\}}$$

MNIST Digits

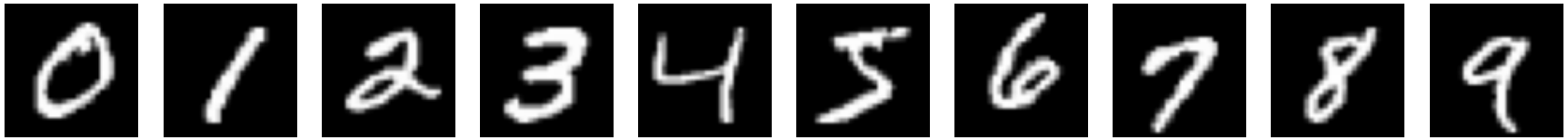


MNIST Digits



DNN-2 96.4% with 5500 parameters

MNIST Digits



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PMM 97.4% with 2560 parameters

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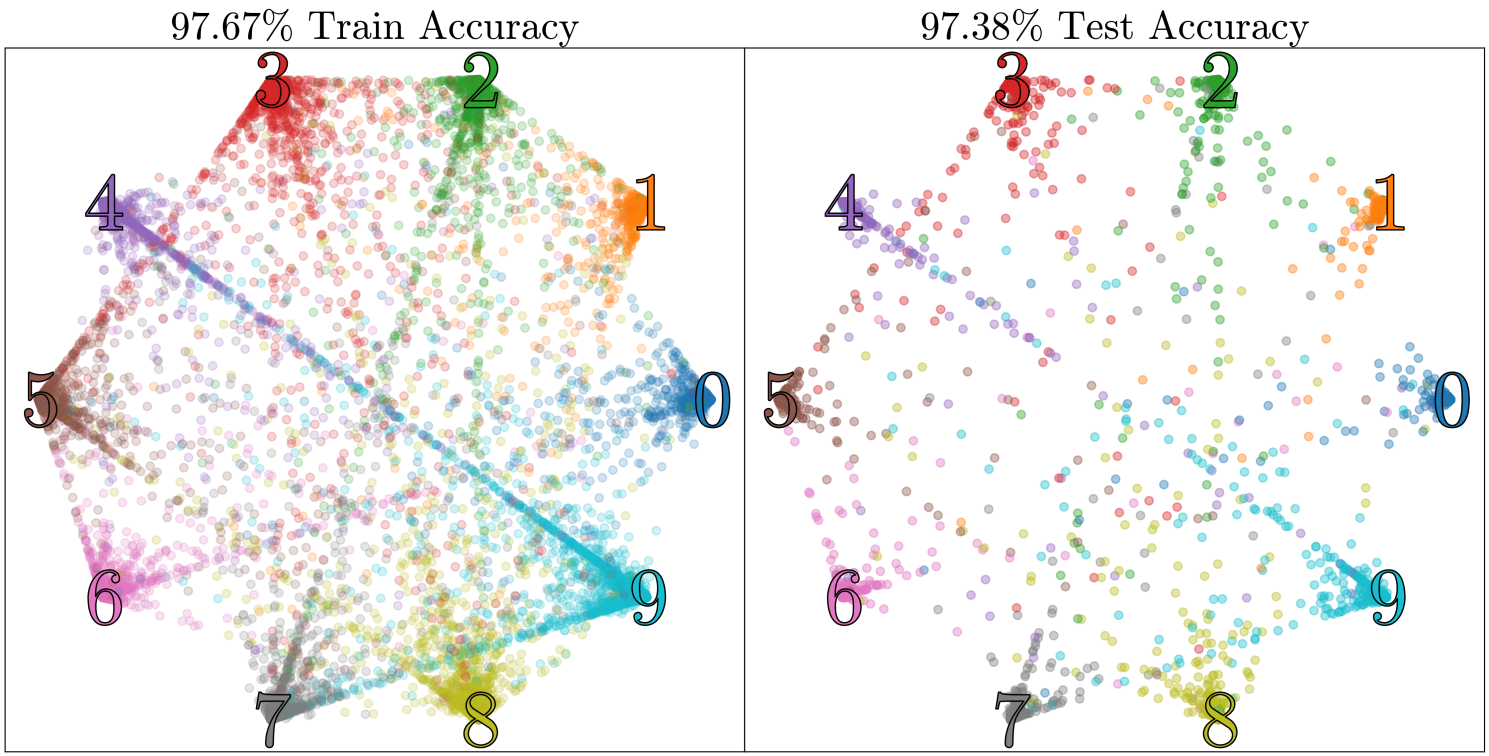

Fashion MNIST

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GECCO 88.09% with 19 000 parameters

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PMM 87.61% with 4210 parameters

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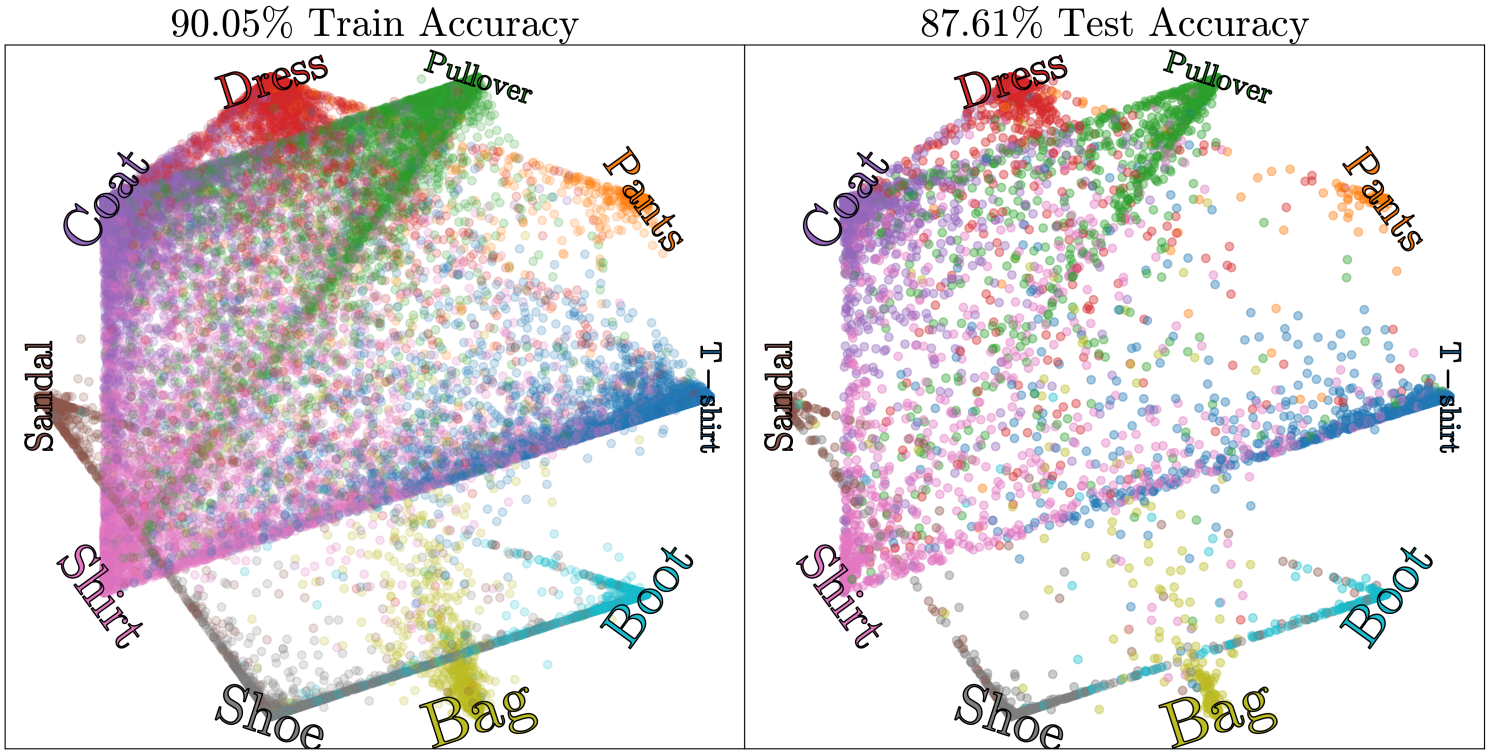
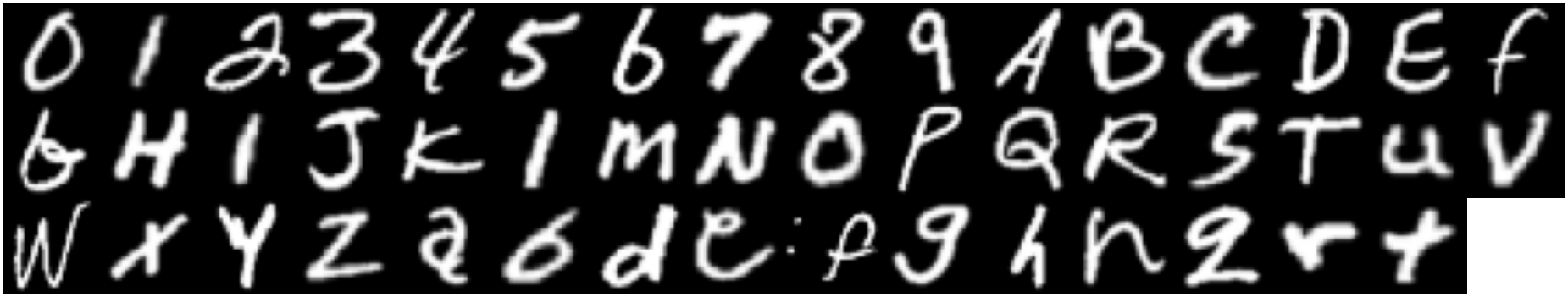
GECCO 88.09% with 19 000 parameters

PMM 87.61% with 4210 parameters



Extended MNIST



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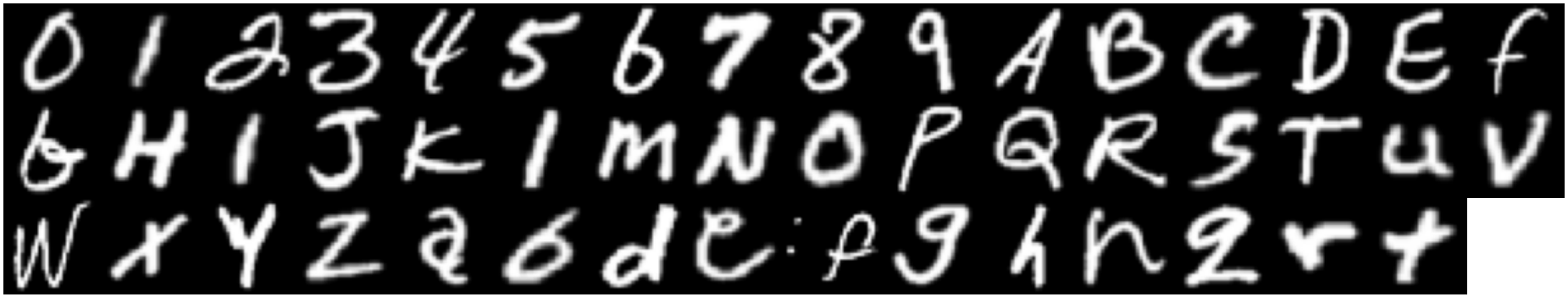

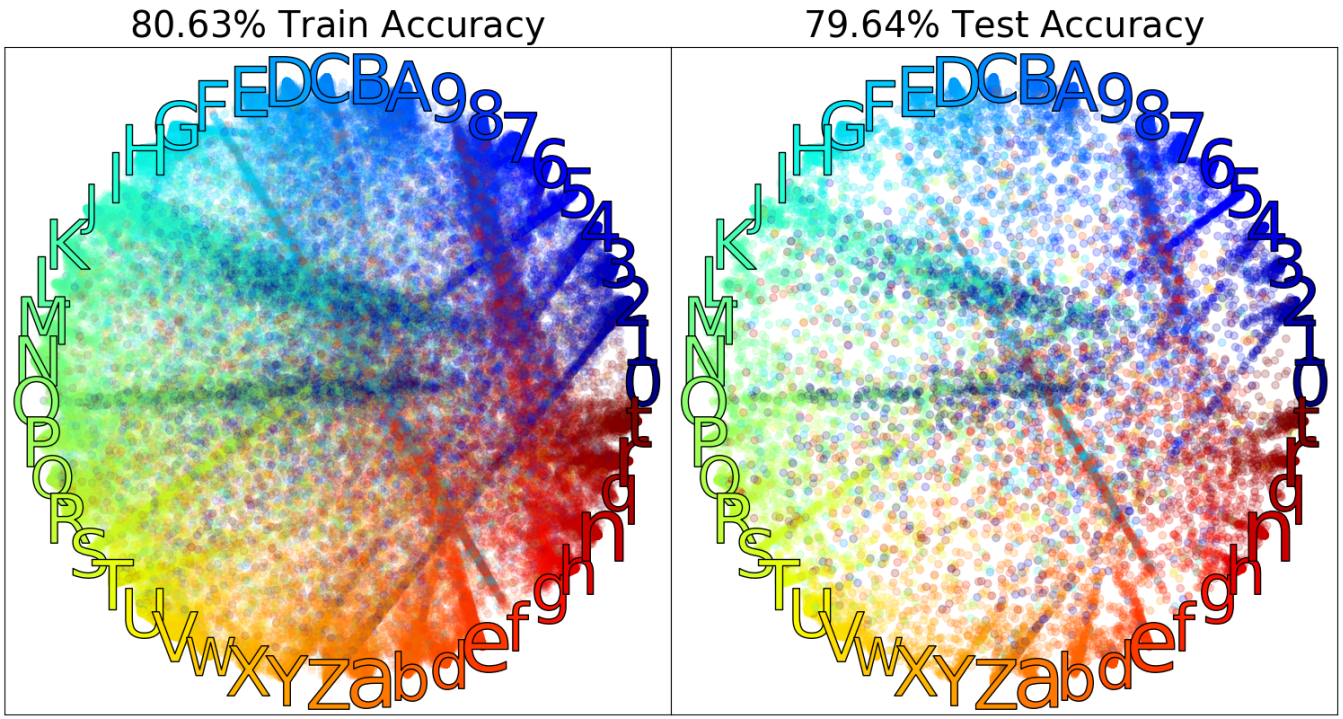

5567 Trainable Parameters

Conclusions and Future Work

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