Quantum Computing and Machine Learning

Quantum Lecture Series

Department of Quantum Information Science

April 3, 2025

Outline

What is Quantum Computing?

Quantum computing leverages principles of quantum mechanics to perform computations beyond classical capabilities.

Key Concepts:

- **Superposition:** Qubits can exist in a combination of states.
- **Entanglement:** Correlation between qubits regardless of distance.
- **Quantum Interference:** Probability amplitudes interfere to solve problems.

What is Quantum Computing?

Quantum computing leverages principles of quantum mechanics to perform computations beyond classical capabilities.

Key Concepts:

- **Superposition:** Qubits can exist in a combination of states.
- **Entanglement:** Correlation between qubits regardless of distance.
- **Quantum Interference:** Probability amplitudes interfere to solve problems.

Qubit Representation:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, \quad |\alpha|^2 + |\beta|^2 = 1$$

What is Machine Learning?

Machine Learning (ML) is the study of algorithms that improve through data experience.

Types of Machine Learning:

- **Supervised Learning:** Labeled data for classification or regression.
- **Unsupervised Learning:** No labels; discover hidden patterns.
- **Reinforcement Learning:** Learning through interaction with the environment.

What is Machine Learning?

Machine Learning (ML) is the study of algorithms that improve through data experience.

Types of Machine Learning:

- **Supervised Learning:** Labeled data for classification or regression.
- **Unsupervised Learning:** No labels; discover hidden patterns.
- **Reinforcement Learning:** Learning through interaction with the environment.

ML Workflow:

$$\text{Data} \rightarrow \text{Model Training} \rightarrow \text{Prediction}$$

What is Quantum Machine Learning?

Quantum Machine Learning (QML) integrates quantum computing with machine learning algorithms to exploit quantum advantages.

Motivation:

- High-dimensional Hilbert spaces for better feature representation.
- Quantum parallelism for faster computation.
- Quantum entanglement for richer data encoding.

What is Quantum Machine Learning?

Quantum Machine Learning (QML) integrates quantum computing with machine learning algorithms to exploit quantum advantages.

Motivation:

- High-dimensional Hilbert spaces for better feature representation.
- Quantum parallelism for faster computation.
- Quantum entanglement for richer data encoding.

Quantum Model Example:

$$U(\theta)\,|x\rangle \implies \text{Quantum Kernel for Classification}$$

1. Quantum Support Vector Machines (QSVM)

Quantum Kernel Estimation:

- Maps classical data to a quantum Hilbert space.
- Quantum kernel measures similarity in high-dimensional space.

Quantum Kernel Estimation:

- Maps classical data to a quantum Hilbert space.
- Quantum kernel measures similarity in high-dimensional space.

Quantum Kernel:

$$K(x, x') = |\left\langle \psi(x)|\psi(x')\middle|\psi(x)|\psi(x')\right\rangle|^2$$

Advantage: - Potentially exponential speedup over classical SVMs.

2. Quantum Neural Networks (QNNs)

Quantum Neural Networks replace classical neurons with parameterized quantum circuits.

Key Concepts:

- Quantum Gates as Activation Functions.
- Variational Quantum Circuits (VQCs) for optimization.

2. Quantum Neural Networks (QNNs)

Quantum Neural Networks replace classical neurons with parameterized quantum circuits.

Key Concepts:

- Quantum Gates as Activation Functions.
- Variational Quantum Circuits (VQCs) for optimization.

Parameterized Quantum Circuit:

$$U(\theta) = \prod_i R_y(\theta_i) \cdot CNOT \cdot R_x(\theta_i)$$

Advantage: - Quantum gradients enable exploration of non-convex landscapes.

3. Quantum Boltzmann Machines (QBMs)

Quantum Boltzmann Machines leverage quantum mechanics to sample from a probability distribution.

- Quantum tunneling aids in escaping local minima.
- Quantum annealing for optimization problems.

3. Quantum Boltzmann Machines (QBMs)

Quantum Boltzmann Machines leverage quantum mechanics to sample from a probability distribution.

- Quantum tunneling aids in escaping local minima.
- Quantum annealing for optimization problems.

Quantum Hamiltonian:

$$H = -\sum_i b_i \sigma_i^z - \sum_{ij} w_{ij} \sigma_i^z \sigma_j^z$$

Advantage: - Efficient sampling in complex probability distributions.

Quantum Speedups in ML

Why Quantum?

- **Quantum Parallelism:** Process multiple states simultaneously.
- **Quantum Entanglement:** Correlated states for richer information.
- **Quantum Interference:** Constructive and destructive interference to enhance solutions.

Quantum Speedups in ML

Why Quantum?

- **Quantum Parallelism:** Process multiple states simultaneously.
- **Quantum Entanglement:** Correlated states for richer information.
- **Quantum Interference:** Constructive and destructive interference to enhance solutions.

Example - Grover's Algorithm:

$$\text{Quantum Search Complexity: } O(\sqrt{N}) \text{ vs. } O(N)$$

Advantage: - Speedups in high-dimensional optimization and linear algebra problems.

Challenges and Limitations

1. **Quantum Hardware Limitations:**
   - Noisy Intermediate-Scale Quantum (NISQ) devices.
   - Decoherence and limited qubit coherence times.
2. **Data Encoding:**
   - Efficient embedding of classical data into quantum states.
3. **Scalability:**
   - Difficult to scale circuits to large datasets.

Applications of Quantum Machine Learning

1. **Quantum Chemistry:**
   - Simulate molecular structures with QML.
2. **Finance:**
   - Quantum optimization for portfolio management.
3. **Image Recognition:**
   - Quantum-enhanced convolutional neural networks.

Future Perspectives in QML

1. **Fault-Tolerant Quantum Computing:**
   - Overcoming noise for stable quantum circuits.
2. **Hybrid Quantum-Classical Models:**
   - Combining quantum circuits with classical neural networks.
3. **Quantum Internet:**
   - Distributed quantum machine learning over quantum networks.

References

- M. Nielsen and I. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, 2000.
- J. Biamonte et al., "Quantum Machine Learning", *Nature*, 2017.
- J. Preskill, "Quantum Computing in the NISQ Era", *Quantum*, 2018.