Advantages of Quantum Entanglement

Quantum Information Lecture Series

Department of Quantum Computing

April 3, 2025

Outline

What is Quantum Entanglement?

Quantum Entanglement is a quantum phenomenon where two or more particles become correlated in such a way that the state of one particle directly affects the state of the other, regardless of distance.

Key Features:

- Non-local correlations
- No classical analog
- Violates Bell's inequalities

What is Quantum Entanglement?

Quantum Entanglement is a quantum phenomenon where two or more particles become correlated in such a way that the state of one particle directly affects the state of the other, regardless of distance.

Key Features:

- Non-local correlations
- No classical analog
- Violates Bell's inequalities

Entangled State Example:

$$\left|\Phi^+\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

- Measurement of the first qubit immediately defines the state of the second.

Mathematical Representation

In quantum mechanics, a system of two qubits is represented by a tensor product:

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle .$$

A state is **entangled** if it cannot be factorized:

$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle .$$

Mathematical Representation

In quantum mechanics, a system of two qubits is represented by a tensor product:

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle .$$

A state is **entangled** if it cannot be factorized:

$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle .$$

Bell State Example:

$$\left|\Phi^-\right\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

- Maximal entanglement state. - Violation of local realism (Bell's inequality).

Quantum Teleportation:

- Entanglement enables the transmission of quantum states using classical communication.
- No need to send the physical quantum particle.

Quantum Teleportation:

- Entanglement enables the transmission of quantum states using classical communication.
- No need to send the physical quantum particle.

Formula:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle \implies \text{(Teleportation)}$$

Advantage: - Instantaneous state transfer within quantum mechanics constraints. - Quantum networks rely on entanglement for secure communication.

2. Quantum Cryptography

Quantum Key Distribution (QKD):

- Entanglement ensures secure communication.
- Eavesdropping disturbs quantum states, revealing interception attempts.

BBM92 Protocol (Entangled Version of BB84):

$$\left|\Phi^+\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

- Any measurement by a third party collapses the wavefunction. - Ensures security based on quantum mechanics, not computational hardness.

2. Quantum Cryptography

Quantum Key Distribution (QKD):

- Entanglement ensures secure communication.
- Eavesdropping disturbs quantum states, revealing interception attempts.

BBM92 Protocol (Entangled Version of BB84):

$$\left|\Phi^+\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

- Any measurement by a third party collapses the wavefunction. - Ensures security based on quantum mechanics, not computational hardness.

Advantage: - Unconditional security guaranteed by the laws of physics.

3. Quantum Computing

Speedup in Quantum Algorithms:

- Entanglement provides exponential state space.
- Quantum parallelism arises from entangled qubits.

Grover's Algorithm:

$$\mathcal{O}(\sqrt{N}) \text{ vs. } \mathcal{O}(N)$$

Shor's Algorithm:

$$\text{Factoring in } \mathcal{O}((\log N)^3)$$

Speedup in Quantum Algorithms:

- Entanglement provides exponential state space.
- Quantum parallelism arises from entangled qubits.

Grover's Algorithm:

$$\mathcal{O}(\sqrt{N}) \text{ vs. } \mathcal{O}(N)$$

Shor's Algorithm:

$$\text{Factoring in } \mathcal{O}((\log N)^3)$$

Advantage: - Solves certain problems exponentially faster than classical computers. - Exploits entanglement for quantum parallelism.

Quantum Metrology:

- Uses entangled states for ultra-precise measurements.
- Overcomes the classical shot-noise limit.

Heisenberg Limit:

$$\Delta\theta \geq \frac{1}{N},$$

where N is the number of entangled particles.

4. Quantum Sensing and Metrology

Quantum Metrology:

- Uses entangled states for ultra-precise measurements.
- Overcomes the classical shot-noise limit.

Heisenberg Limit:

$$\Delta\theta \geq \frac{1}{N},$$

where N is the number of entangled particles.

Advantage: - Quantum entanglement improves sensitivity beyond classical limits. - Applications in gravitational wave detection and atomic clocks.

Challenges of Quantum Entanglement

Decoherence:

- Entangled states are fragile.
- Interaction with the environment collapses the wavefunction.

Scalability:

- Difficult to entangle large numbers of qubits.
- Error correction requires complex protocols.

Measurement Problem:

- Measurement destroys entanglement.
- Trade-off between information gain and entanglement preservation.

Future Perspectives

Quantum Internet:

- Entanglement as a resource for global quantum networks.

Fault-Tolerant Quantum Computing:

- Quantum error correction leveraging entanglement.

Advanced Quantum Sensors:

- Improved sensitivity for medical and scientific applications.

Future Perspectives

Quantum Internet:

- Entanglement as a resource for global quantum networks.

Fault-Tolerant Quantum Computing:

- Quantum error correction leveraging entanglement.

Advanced Quantum Sensors:

- Improved sensitivity for medical and scientific applications.

Conclusion: - Quantum entanglement is a fundamental resource.
- It enables quantum supremacy in communication, computation, and sensing.

References

M. Nielsen and I. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, 2000.

A. Einstein, B. Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?", Physical Review, 1935.

J. S. Bell, "On the Einstein-Podolsky-Rosen paradox", Physics, 1964.