Entanglement-Enhanced Quantum Sensing

Singlet–Triplet Interferometry, QFI, and Decoherence

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Entangled probe state

We start from the spin singlet state

$$|S\rangle = \frac{1}{\sqrt{2}} \left(|{\uparrow\downarrow}\rangle - |{\downarrow\uparrow}\rangle \right),$$

which has total spin zero and is invariant under common-mode magnetic fields.

This makes the singlet a **noise-protected reference state** for differential sensing.

Phase accumulation by a local magnetic field

A magnetic field acting only on electron 1 induces a relative phase

$$|\uparrow\rangle_1 \to e^{+i\phi/2}\,|\uparrow\rangle_1\,, \qquad |\downarrow\rangle_1 \to e^{-i\phi/2}\,|\downarrow\rangle_1\,,$$

with

$$\phi(t) = \gamma \int_0^t B(t')\,dt'.$$

Applying this to the singlet gives

$$|\psi(\phi)\rangle = \frac{1}{\sqrt{2}}\left(e^{i\phi/2}\,|\uparrow\downarrow\rangle - e^{-i\phi/2}\,|\downarrow\uparrow\rangle\right).$$

Singlet–triplet decomposition

Rewrite $|\psi(\phi)\rangle$ in the singlet–triplet basis:

$$|\psi(\phi)\rangle = \cos\left(\frac{\phi}{2}\right)|S\rangle + i\sin\left(\frac{\phi}{2}\right)|T_0\rangle\,,$$

where

$$|T_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right).$$

Key point: the phase ϕ is converted into a measurable population transfer.

Relation to Ramsey interferometry

This protocol is mathematically equivalent to a Ramsey experiment:

- Singlet preparation $\leftrightarrow$ first $\pi/2$ pulse
- Phase accumulation under $B(t)$ $\leftrightarrow$ free evolution
- Singlet–triplet readout $\leftrightarrow$ final $\pi/2$ pulse

Unlike single-spin Ramsey interferometry:

- common-mode noise cancels automatically,
- the interferometer operates in an **entangled basis**.

Quantum Fisher information: definition

For a pure state $|\psi(\phi)\rangle$, the quantum Fisher information is

$$F_Q = 4\left(\langle \partial_\phi \psi | \partial_\phi \psi | \partial_\phi \psi | \partial_\phi \psi \rangle - |\langle \psi | \partial_\phi \psi | \psi | \partial_\phi \psi \rangle|^2\right).$$

It sets the ultimate precision bound via the quantum Cramér–Rao inequality:

$$\Delta\phi \geq \frac{1}{\sqrt{F_Q\, N}}.$$

QFI for singlet–triplet interferometry

Using

$$|\psi(\phi)\rangle = \cos\left(\frac{\phi}{2}\right)|S\rangle + i\sin\left(\frac{\phi}{2}\right)|T_0\rangle\,,$$

we compute

$$\partial_\phi|\psi\rangle = -\frac{1}{2}\sin\left(\frac{\phi}{2}\right)|S\rangle + \frac{i}{2}\cos\left(\frac{\phi}{2}\right)|T_0\rangle\,.$$

This yields

$$F_Q = 1 \quad \text{(per interrogation time unit).}$$

With optimized control and extended interaction time, F_Q scales with the effective generator variance, leading to large enhancements.

Physical meaning of QFI here

- The generator of ϕ acts *differentially* on the two spins.
- The singlet has maximal susceptibility to such differential phases.
- Entanglement amplifies phase sensitivity beyond separable probes.

This is the origin of the observed quantum metrological advantage.

Including decoherence

Decoherence (e.g. dephasing) suppresses singlet–triplet coherence. A simple phenomenological model gives

$$\rho_{ST}(\phi, t) = \begin{pmatrix} \cos^2(\phi/2) & ie^{-\Gamma t}\cos(\phi/2)\sin(\phi/2) \\ -ie^{-\Gamma t}\cos(\phi/2)\sin(\phi/2) & \sin^2(\phi/2) \end{pmatrix}.$$

Here Γ is the decoherence rate.

Triplet leakage with decoherence

The measured triplet population becomes

$$P_{T_0}(\phi, t) = \frac{1}{2}\left(1 - e^{-\Gamma t}\cos\phi\right).$$

For small ϕ:

$$P_{T_0} \approx \frac{\phi^2}{4}\, e^{-\Gamma t}.$$

Decoherence reduces contrast but preserves phase sensitivity.

Impact on quantum Fisher information

Decoherence reduces the QFI to

$$F_Q(t) = F_Q(0)\, e^{-2\Gamma t}.$$

- Short times: quantum advantage retained
- Long times: decoherence restores shot-noise scaling

This defines the optimal sensing time.

Summary

- Local magnetic fields induce singlet–triplet mixing.
- Phase information is converted into measurable populations.
- The protocol is an entanglement-based Ramsey interferometer.
- Quantum Fisher information quantifies the enhanced sensitivity.
- Decoherence limits, but does not invalidate, the quantum advantage.