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PHASE TRANSITIONS IN NEUTRON STARS AND MAXIMUM MASSES

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ABSTRACT

Using the most recent realistic effective interactions for nuclear matter with a smooth extrapolation to high densities including causality, we constrain the equation of state and calculate maximum masses of rotating neutron stars. First- and second-order phase transitions to, e.g., quark matter at high densities are included. If neutron star masses of ∼2.3 $M_\odot$ from quasi-periodic oscillations in low-mass X-ray binaries are confirmed, a soft equation of state as well as strong phase transitions can be excluded in neutron star cores.

Subject headings: dense matter — stars: neutron

1. INTRODUCTION

The best determined neutron star masses are found in binary pulsars and all lie in the range 1.35 ± 0.04 $M_\odot$ (see Thorsett & Chakrabarty 1999) except for the nonrelativistic pulsar PSR J1012+5307 of mass[1] $M = 2.1 \pm 0.8\ M_\odot$ (van Paradijs 1998). Several X-ray binary masses have been measured, of which the heaviest are Vela X-1 with $M = 1.9 \pm 0.2\ M_\odot$ (Barziv et al. 1999) and Cygnus X-2 with $M = 1.8 \pm 0.4\ M_\odot$ (Orosz & Kuulkers 1999). The recent discovery of high-frequency brightness oscillations in low-mass X-ray binaries provides a promising new method for determining masses and radii of neutron stars (see Miller, Lamb, & Psaltis 1998). The kilohertz quasi-periodic oscillations (QPOs) occur in pairs and are most likely the orbital frequencies $\nu_{\rm QPO} = (1/2\pi) \times (GM/R^3_{\rm orb})^{1/2}$ of accreting matter in Keplerian orbits around neutron stars of mass M and its beat frequency with the neutron star spin $\nu_{\rm QPO} - \nu_s$. According to Zhang, Strohmayer, & Swank (1997) and Kaaret, Ford, & Chen (1997), the accretion can for a few QPOs be tracked to its innermost stable orbit, $R_{\rm ms} = 6GM/c^2$. For slowly rotating stars the resulting mass is $M \simeq 2.2\ M_\odot({\rm kHz}/\nu_{\rm QPO})$. For example, the maximum frequency of 1060 Hz upper QPO observed in 4U 1820−30 gives $M \simeq 2.25\ M_\odot$ after correcting for the $\nu_s \simeq 275$ Hz neutron star rotation frequency. If the maximum QPO frequencies of 4U 1608−52 ($\nu_{\rm QPO} = 1125$ Hz) and 4U 1636−536 ($\nu_{\rm QPO} = 1228$ Hz) also correspond to innermost stable orbits, the corresponding masses are 2.1 and 1.9 $M_\odot$. Such large masses severely restrict the equation of state (EOS) for dense matter as addressed in the following.

Recent models for the nucleon-nucleon interaction have reduced the uncertainty in the nuclear EOS, allowing for more reliable calculations of neutron star properties (see Akmal, Pandharipande, & Ravenhall 1998, hereafter APR98, and Engvik et al. 1997). Likewise, recent realistic effective interactions for nuclear matter obeying causality at high densities constrain the EOS severely and thus also the maximum masses of neutron stars (see APR98 and Kalogera & Baym 1996). We will here elaborate on these analyses by incorporating causality smoothly in the EOS for nuclear matter and allow for first- and second-order phase transitions to, e.g., quark matter. Finally, results are compared with observed neutron star masses and concluding remarks are made.

[1] 95% confidence limits or ∼2 σ.

2. THE NUCLEAR EQUATION OF STATE

For the discussion of the gross properties of neutron stars, we will use the optimal EOS of APR98 (specifically the Argonne $V18 + \delta v +$ UIX* model), which is based on the most recent models for the nucleon-nucleon interaction with the inclusion of a parametrized three-body force and relativistic boost corrections. The EOS for nuclear matter is thus known to some accuracy for densities up to a few times nuclear saturation density $n_0 = 0.16\ {\rm fm}^{-3}$. We parametrize the APR98 EOS by a simple form for the compressional and symmetry energies that gives a good fit around nuclear saturation densities and smoothly incorporates causality at high densities such that the sound speed approaches the speed of light. This requires that the compressional part of the energy per nucleon is quadratic in nuclear density with a minimum at saturation but linear at high densities

$$\begin{aligned}\mathcal{E} &= E_{\rm comp}(n) + S(n)(1-2x)^2\\ &= \mathcal{E}_0 u \frac{u-2-s}{1+su} + S_0 u^\gamma (1-2x)^2. \qquad (1)\end{aligned}$$

Here, $n = n_p + n_n$ is the total baryon density, $x = n_p/n$ is the proton fraction, and $u = n/n_0$ is the ratio of the baryon density to nuclear saturation density. The compressional term is in equation (1) parametrized by a simple form which reproduces the saturation density and the binding energy per nucleon $\mathcal{E}_0 = 15.8$ MeV at n_0 of APR98. The "softness" parameter $s \simeq 0.2$, which gave the best fit to the data of APR98 (see Heiselberg & Hjorth-Jensen 1999), is determined by fitting the energy per nucleon of APR98 up to densities of $n \sim 4n_0$. For the symmetry energy term, we obtain $S_0 = 32$ MeV and $\gamma = 0.6$ for the best fit. The proton fraction is given by β-equilibrium at a given density.

The one unknown parameter s expresses the uncertainty in the EOS at high density, and we shall vary this parameter within the allowed limits in the following with and without phase transitions to calculate mass, radius, and density relations for neutron stars. The "softness" parameter s is related to the incompressibility of nuclear matter as $K_0 = 18\mathcal{E}_0/(1+s) \simeq 200$

MeV. It agrees with the poorly known experimental value (Blaizot et al. 1995), $K_0 \simeq 180$–250 MeV, which does not restrict it very well. From $(v_s/c)^2 = \partial P/\partial(n\mathcal{E})$, where P is the pressure, and the EOS of equation (1), the causality condition $v_s \leq c$ requires

$$s \gtrsim \sqrt{\frac{\mathcal{E}_0}{m_n}} \simeq 0.13, \tag{2}$$

where m_n is the mass of the nucleon. With this condition we have a causal EOS that reproduces the data of APR98 at densities up to 0.6–0.7 fm^{-3}. In contrast, the EOS of APR98 becomes superluminal at $n \approx 1.1$ fm^{-3}. For larger s values the EOS is softer, which eventually leads to smaller maximum masses of neutron stars. The observed $M \simeq 1.4\ M_\odot$ in binary pulsars restricts s to be less than 0.4–0.5 depending on rotation as shown in calculations of neutron stars below.

In Figure 1 we plot the sound speed $(v_s/c)^2$ for various values of s and that resulting from the microscopic calculation of APR98 for β-stable pn-matter. The form of equation (1), with the inclusion of the parameter s, provides a smooth extrapolation from small to large densities such that the sound speed v_s approaches the speed of light. For $s = 0.0$ ($s = 0.1$), the EOS becomes superluminal at densities of the order of 1 (6) fm^{-3}.

The sound speed of Kalogera & Baym (1996) is also plotted in Figure 1. It jumps discontinuously to the speed of light at a chosen density. With this prescription they were able to obtain an optimum upper bound for neutron star masses and obey causality. This prescription was also employed by APR98; see Rhoades & Ruffini (1974) for further details. The EOS is thus discontinuously stiffened by taking $v_s = c$ at densities above a certain value n_c which, however, is lower than $n_s = 5n_0$ where their nuclear EOS becomes superluminal. This approach stiffens the nuclear EOS for densities $n_c < n < n_s$ but softens it at higher densities. Their resulting maximum masses lie in the range $2.2\ M_\odot \lesssim M \lesssim 2.9\ M_\odot$. Our approach, however, incorporates causality by reducing the sound speed smoothly toward the speed of light at high densities. Therefore, our approach will not yield an absolute upper bound on the maximum mass of a neutron star but gives reasonable estimates based on modern EOSs around nuclear matter densities, causality constraints at high densities, and a smooth extrapolation between these two limits (see Fig. 1).

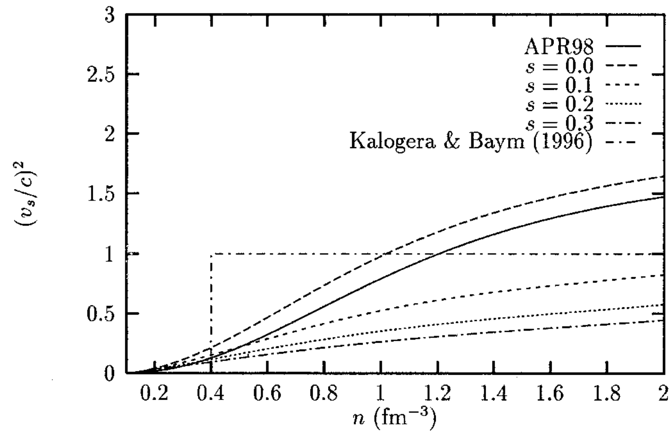

FIG. 1.—$(v_s/c)^2$ for β-stable pn-matter for s = 0, 0.1, 0.2, 0.3, the results of APR98, and for the patched EOS of Kalogera & Baym (1996), which shows a discontinuous $(v_s/c)^2$.

3. PHASE TRANSITIONS

The physical state of matter in the interiors of neutron stars at densities above a few times normal nuclear matter densities is essentially unknown, and many first- and second-order phase transitions have been speculated upon. We will specifically study the hadron–to–quark matter transition at high densities, but note that other transitions such as, e.g., kaon and/or pion condensation or the presence of other baryons like hyperons also soften the EOS and thus further aggravate the resulting reduction in maximum masses. Hyperons appear at densities typically of the order $2n_0$ and result in a considerable softening of the EOS (see, e.g., Balberg, Lichenstadt, & Cook 1998). Typically, most EOSs with hyperons yield masses around 1.4–1.6 $M_\odot$. Here however, in order to focus on the role played by phase transitions in neutron star matter, we will assume that a phase transition from nucleonic to quark matter takes place at a certain density. We will for simplicity employ the bag model in our actual studies of quark phases and neutron star properties. In the bag model the quarks are assumed to be confined to a finite region of space, the so-called "bag," by a vacuum pressure B. Adding the Fermi pressure and interactions computed to order $\alpha_s = g^2/4\pi$, where g is the QCD coupling constant, the total pressure for three massless quarks of flavor $f = u, d, s$, is (see Kapusta 1989)

$$P = \frac{3\mu_f^4}{4\pi^2}\left(1 - \frac{2}{\pi}\alpha_s\right) - B + P_e + P_\mu, \tag{3}$$

where $P_{e,\mu}$ are the electron and muon pressure, e.g., $P_e = \mu_e^4/12\pi^2$. A Fermi gas of quarks of flavor i has density $n_i = k_{\mathrm{F}_i}^3/\pi^2$ due to the three color states. The value of the bag constant B is poorly known, and we present results using two representative values, $B = 150$ and $B = 200$ MeV fm^{-3}. We take $\alpha_s = 0.4$. However, similar results can be obtained with smaller α_s and larger B (Madsen 1998, and references therein).

The quark and nuclear matter mixed phase described in Glendenning (1992) has continuous pressures and densities due to the general Gibbs criteria for two-component systems. There are no first-order phase transitions but at most two second-order phase transitions. Namely, at a lower density, where quark matter first appears in nuclear matter, and at a very high density (if gravitationally stable), where all nucleons are finally dissolved into quark matter. This mixed phase does, however, not include local surface and Coulomb energies of the quark and nuclear matter structures. If the interface tension between quark and nuclear matter is too large, the mixed phase is not favored energetically due to surface and Coulomb energies associated with forming these structures (Heiselberg, Pethick, & Staubo 1993). The neutron star will then have a core of pure quark matter with a mantle of nuclear matter surrounding it, and the two phases are coexisting by a first-order phase transition or Maxwell construction (see Fig. 2). For a small or moderate interface tension the quarks are confined in droplet, rodlike, and platelike structures as found in the inner crust of neutron stars (Lorenz, Ravenhall, & Pethick 1993).

4. NEUTRON STAR PROPERTIES

In order to obtain the mass and radius of a neutron star, we have solved the Tolman-Oppenheimer-Volkov equation with and without rotational corrections, following the approach of Hartle (1967). The EOSs employed are given by the pn-matter EOS with s = 0.13, 0.2, 0.3, 0.4 with nucleonic degrees of

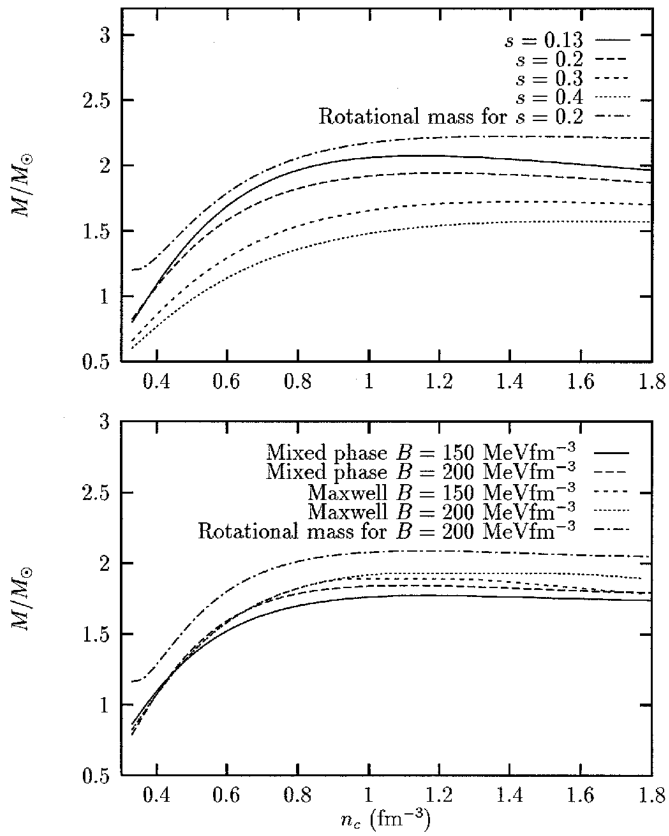

FIG. 2.—Total mass M as function of central density n_c for various values of s (*upper panel*) and the bag parameter B (*lower panel*) for both a mixed-phase and a Maxwell constructed EOS with $s = 0.2$ in eq. (1). In addition we include also the rotational corrections for the pure pn-case with $s = 0.2$ and the mixed phase construction for $B = 200$ MeV fm^{-3}. For the Maxwell construction which exhibits a first-order phase transition, in the density regions where the two phases coexist, the pressure is constant, a fact reflected in the constant value of the neutron star mass. All results are for β-stable matter. Note also that for the upper panel, the EOSs for $s = 0.3$ and $s = 0.4$ start to differ from those with $s = 0.13$, 0.2 at densities below 0.2 fm^{-3}.

freedon only. In addition we have selected two representative values for the bag-model parameter B, namely 150 and 200 MeV fm^{-3}, for our discussion on eventual phase transitions. The quark phase is linked with our pn-matter EOS from equation (1) with $s = 0.2$ through either a mixed-phase construction or a Maxwell construction; see Heiselberg & Hjorth-Jensen (1999) for further details. For $B = 150$ MeV fm^{-3}, the mixed phase begins at 0.51 fm^{-3} and the pure quark matter phase begins at 1.89 fm^{-3}. Finally, for $B = 200$ MeV fm^{-3}, the mixed phase starts at 0.72 fm^{-3} while the pure quark phase starts at 2.11 fm^{-3}. In case of a Maxwell construction, in order to link the pn and the quark matter EOS, we obtain for $B = 150$ MeV fm^{-3} that the pure pn phase ends at 0.92 fm^{-3} and that the pure quark phase starts at 1.215 fm^{-3}, while the corresponding numbers for $B = 200$ MeV fm^{-3} are 1.04 and 1.57 fm^{-3}.

As can be seen from Figure 2, none of the EOSs, from either the pure pn phase or with a mixed phase or Maxwell construction with quark degrees of freedom, result in stable configurations for densities above $\sim 10n_0$, implying thereby that none of the stars have cores with a pure quark phase. The EOS with pn degrees of freedom have masses $M \lesssim 2.2\ M_\odot$ when rotational corrections are accounted for. With the inclusion of the mixed phase, the total mass is reduced since the EOS is softer. For pure quark stars there is only one energy scale, namely B, which provides a homology transformation (Madsen 1998, and references therein), and the maximum mass is $M_{\rm max} = 2.0\ M_\odot(58\ {\rm MeV\ fm^{-3}}/B)^{1/2}$ (for $\alpha_s = 0$). However, for $B \gtrsim 58$ MeV fm^{-3}, a nuclear matter mantle has to be added and for $B \lesssim 58$ MeV fm^{-3} quark matter has lower energy per baryon than ^{56}Fe and is thus the ground state of strongly interacting matter. Unless the latter is the case, we can thus exclude the existence of 2.2–2.3 $M_\odot$ quark stars.

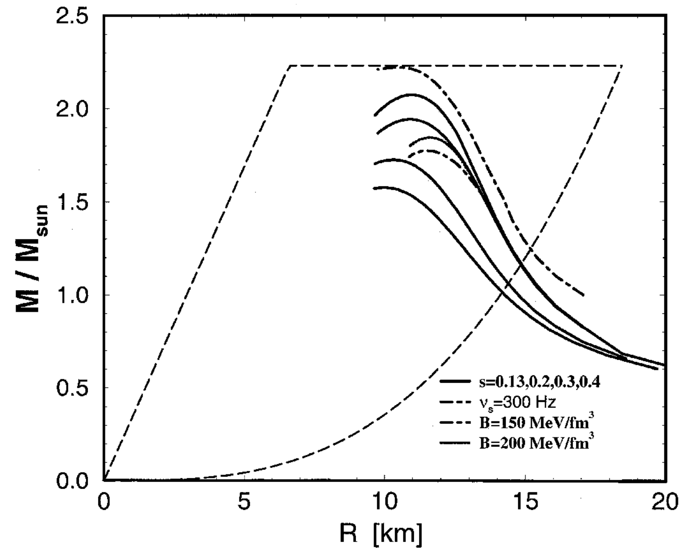

FIG. 3.—Neutron star masses vs. radius for the EOS of eq. (1) with softness $s = 0.13$, 0.2, 0.3, 0.4, with increasing values of s from top to bottom for the full curves. Phase transitions decrease the maximum mass, whereas rotation increases it. The shaded area represents the neutron star radii and masses allowed (see text) for orbital QPO frequencies 1060 Hz of 4U 1820−30.

In Figure 3 we show the mass-radius relations for the various EOSs. The shaded area represents the allowed masses and radii for $\nu_{\rm QPO} = 1060$ Hz of 4U 1820−30. Generally,

$$2GM < R < \left(\frac{GM}{4\pi^2\nu_{\rm QPO}^2}\right)^{1/3}, \quad (4)$$

where the lower limit ensures that the star is not a black hole, and the upper limit ensures that the accreting matter orbits outside the star, $R < R_{\rm orb}$. Furthermore, for the matter to be outside the innermost stable orbit, $R > R_{\rm ms} = 6GM$, requires that

$$M \lesssim \frac{1 + 0.75j}{12\sqrt{6}\pi G\nu_{\rm QPO}} \simeq 2.2\ M_\odot(1 + 0.75j)\,\frac{\rm kHz}{\nu_{\rm QPO}}, \quad (5)$$

where $j = 2\pi c\nu_s I/M^2$ is a dimensionless measure of the angular momentum of the star with moment of inertia I. The upper limit in equation (5) is the mass when $\nu_{\rm QPO}$ corresponds to the innermost stable orbit. According to Zhang et al. (1998), this is the case for 4U 1820−30 since $\nu_{\rm QPO}$ saturates at $\sim$1060 Hz with increasing count rate. The corresponding neutron star mass is $M \sim 2.2$–2.3 $M_\odot$, which leads to several interesting conclusions as seen in Figure 3. First, the stiffest EOS allowed by causality ($s \simeq 0.13$–0.2) is needed. Second, rotation must be included, which increases the maximum mass and corresponding radii by 10%–15% for $\nu_s \sim 300$ Hz. Third, a phase transition to quark matter below densities of order $\sim 5n_0$ can be excluded, corresponding to a restriction on the bag constant $B \gtrsim 200$ MeV fm^{-3}.

These maximum masses are smaller than those of APR98 and Kalogera & Baym (1996) who, as discussed above, obtain upper bounds on the mass of neutron stars by discontinuously setting the sound speed to equal the speed of light above a certain density n_c. By varying the density $n_c = 2 \rightarrow 5n_0$, the maximum mass drops from 2.9 to 2.2 $M_\odot$. In our case, incorporating causality smoothly by introducing the parameter s in equation (1), the EOS is softened at higher densities in order to obey causality and yields a maximum mass which instead is slightly lower than the 2.2 $M_\odot$ derived in APR98 for nonrotating stars.

If the QPOs are not from the innermost stable orbits and one finds that even accreting neutron stars have small masses, say like the binary pulsars, $M \lesssim 1.4\ M_\odot$, this may indicate that heavier neutron stars are not stable. Therefore, the EOS is soft at high densities $s \gtrsim 0.4$ or a phase transition occurs at a few times nuclear matter densities. For the nuclear-to-quark matter transition, this would require $B < 80$ MeV fm^{-3} for $s = 0.2$. For such small bag parameters, there is an appreciable quark and nuclear matter mixed phase in the neutron star interior, but even in these extreme cases a pure quark matter core is not obtained for stable neutron star configurations.

A third QPO frequency referred to as horizontal branch oscillations (HBOs) around $\nu_{\rm HBO} \simeq 20$–50 Hz has been suggested to be caused by Lense-Thirring precession at the inner border of the accretion disk (Stella & Vietri 1998):

$$\nu_{\rm LT} = \frac{8\pi^2 I}{c^2 M}\nu_s \nu_{\rm QPO}^2$$
$$\simeq \frac{13.2}{50\ {\rm km}^2}\frac{I}{M}\frac{\nu_s}{300\ {\rm Hz}}\left(\frac{\nu_{\rm QPO}}{1\ {\rm kHz}}\right)^2 . \tag{6}$$

However, even for the stiffest EOS $s \simeq 0.13$–0.2, we calculate moment of inertia and Lense-Thirring frequencies from equation (6), which are a factor ~ 4 below the observed $\nu_{\rm HBO}$, thus confirming analyses of Schaab & Weigel (1999), Psaltis et al. (1999), and Kalogera & Psaltis (1999).

5. SUMMARY

Modern nucleon-nucleon potentials have reduced the uncertainties in the calculated EOS. Using the most recent realistic effective interactions for nuclear matter of APR98 with a smooth extrapolation to high densities including causality, the EOS could be constrained by a "softness" parameter s, which parametrizes the unknown stiffness of the EOS at high densities. Maximum masses were calculated for rotating neutron stars with and without first- and second-order phase transitions to, e.g., quark matter at high densities.

The calculated bounds for maximum masses leaves two natural options when compared to the observed neutron star masses:

Case I: *The large masses of the neutron stars in QPO 4U 1820−30* ($M = 2.3\ M_\odot$), *PSR J1012+5307* ($M = 2.1 \pm 0.4\ M_\odot$), *Vela X-1* ($M = 1.9 \pm 0.1\ M_\odot$), *and Cygnus X-2* ($M = 1.8 \pm 0.4\ M_\odot$) *are confirmed and complemented by other neutron stars with masses around* $\sim 2\ M_\odot$. As a consequence, the EOS of dense nuclear matter is severely restricted and only the stiffest EOSs consistent with causality are allowed, i.e., softness parameter $0.13 \leq s \lesssim 0.2$. Furthermore, any significant phase transition at densities below $5n_0$ can be excluded. That the radio binary pulsars all have masses around 1.4 $M_\odot$ is then probably due to the formation mechanism in supernovae in which the Chandrasekhar mass for iron cores is $\sim 1.5\ M_\odot$. Neutron stars in binaries can subsequently acquire larger masses by accretion as X-ray binaries.

Case II: *The heavy neutron stars prove erroneous by more detailed observations, and only masses like those of binary pulsars are found.* If accretion does not produce neutron stars heavier than $\gtrsim 1.4\ M_\odot$, this indicates that heavier neutron stars simply are not stable which in turn implies a soft EOS, either $s > 0.4$ or a significant phase transition must occur already at a few times nuclear saturation densities.

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