

Roth and Navrátil Reply: In the preceding Comment [1] Dean *et al.* criticize our calculations for the ground-state energy of ^{40}Ca within the importance-truncated no-core shell model (NCSM) [2]. In particular, they address the role of configurations beyond the $3p3h$ level, which have not been included in our ^{40}Ca calculations for large model spaces.

Before responding to this point, the following general statements are in order. For the atomic nucleus as a self-bound system, translational invariance is an important symmetry. The only possibility to preserve translational invariance when working with a Slater determinant basis is to (i) use the harmonic oscillator (HO) single-particle states and (ii) truncate the model space according to the total HO excitation energy $N_{\text{max}}\hbar\Omega$, as done in the *ab initio* NCSM. This is important not only for obtaining proper binding or excitation energies, but also for a correct extraction of physical wave functions.

The minimal violation of the translational invariance was one of the main motivations for developing the importance-truncation scheme introduced in [2]. In this scheme, we start with the complete $N_{\text{max}}\hbar\Omega$ HO model space and select important configurations via perturbation theory. All symmetries are under control and our importance-truncated NCSM calculations are completely variational, i.e., they provide an upper bound for the ground-state energy of the system.

The restriction to the $3p3h$ level, made for computational reasons in the $N_{\text{max}} > 8$ calculations for ^{40}Ca , is not inherent to the importance-truncation scheme. The explicit inclusion of $4p4h$ configurations—though computationally more demanding—is straightforward, even for the largest $N_{\text{max}}\hbar\Omega$ model spaces discussed. To demonstrate this we have performed full $4p4h$ calculations for ^{40}Ca in a $14\hbar\Omega$ no-core model space at $\hbar\Omega = 24$ MeV using the V_{lowk} interaction employed in [2]. The resulting ground-state energy of $E_{4p4h} = -471.0$ MeV can be compared with $E_{3p3h} = -461.2$ MeV for the $3p3h$ calculation reported in Fig. 5(b) with an uncertainty of typically 1 MeV due to the extrapolation $\kappa_{\text{min}} \rightarrow 0$. Thus the $4p4h$ configurations change the resulting ground-state energy of ^{40}Ca by approximately 2%.

In addition to the explicit inclusion of $4p4h$ or higher-order configurations one can use perturbative corrections based on the eigenstates obtained in the importance truncated space, to account for excluded higher-order configurations. Alternatively, Davidson-type corrections, which are employed successfully in quantum chemistry [3], can be employed. These techniques even restore size extensivity in an approximate way.

The coupled-cluster method (CCM) used by the authors of [1] lacks the above discussed features important for the nuclear many-body problem, in particular, it violates translational invariance from the very beginning and it does not fulfill the variational principle. The problem of a spurious center-of-mass contamination of the many-body states in

CCM is not resolved and often not even mentioned (see, e.g., Ref. [4]). In [1] it is stated that the CCM Hamiltonian is translationally invariant. By inspecting Eqs. (4) and (6) in [5], one can see that it is not the case as the parameter β_{CoM} is fine-tuned and thus nonzero in general. Moreover, noniterative triples corrections like CCSD(T), as referred to in [1], tend to overestimate the correlation energy or even collapse. It is claimed that the CCM is very accurate. A closer inspection of recently published nuclear many-body results does not support this statement. In Ref. [6], the ^4He binding energy with the chiral nucleon-nucleon (NN) potential was determined with an uncertainty of several MeV. The same is obtained in the *ab initio* NCSM with accuracy of 10 keV [7]. The CCM binding energy results for ^{16}O with the identical chiral N^3LO NN potential obtained in Refs. [5,8] differ by more than 5 MeV.

The CCSD(T) ground-state energy for ^{40}Ca with V_{lowk} reported in Ref. [4] is about 30 MeV lower than our $4p4h$ result (CCSD is about 20 MeV lower). Taking into account the small 2% difference of our $3p3h$ and $4p4h$ results and the violation of the variational principle and the translational invariance in the CCM method, we believe that the CCM result overestimates the exact ground-state energy in this case.

In conclusion, the importance truncation provides an efficient way to extend the domain of NCSM calculations to medium-heavy nuclei while preserving translational invariance and the variational principle and allowing for systematic and controlled improvements.

Supported by the Deutsche Forschungsgemeinschaft through contract SFB 634 and by the Department of Energy under Grant DE-FC02-07ER41457. Prepared by LLNL under Contract DE-AC52-07NA27344.

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Received 8 April 2008; published 10 September 2008

DOI: [10.1103/PhysRevLett.101.119202](https://doi.org/10.1103/PhysRevLett.101.119202)

PACS numbers: 21.60.Cs, 21.30.Fe, 27.40.+z

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