Comment on "$0_{gs}^+ \rightarrow 2_1^+$ Transition Strengths in ^{106}Sn and ^{108}Sn"

Recently results of a difficult and challenging experiment were published [1] by a galaxy of authors and institutions. The aim of this comment is just to correct some statements about generalized seniority [2] made in the paper. The following considerations apply to valence neutrons in the semi-magic Sn isotopes .

A Hamiltonian with single nucleon terms and two-body interactions, is diagonal in the scheme of seniority if it constructed from generators of the SU(2) Lie algebra.Eigenstates and eigenvalues of such Hamiltonians are characterized by the seniority v and have some simple properties. Ground states of an even number n of nucleons in one or several j-orbits, with J=0, are the following, unnormalized states

$$H(S^+)^n|0\rangle=\{nV_0+n(n-1)W/2\}(S^+)^n|0\rangle$$
$$S^+=\Sigma\,\alpha_j S_j^+ \qquad (1)$$

In (1) the vacuum state |0> is the state with closed shells and the S_j^+ create pairs of j-particles coupled to J=0

$$S_j^+=\Sigma\,(-1)^{j-m}a_{j,m}^+a_{j,-m}^+$$

The Hamiltonian is diagonal in the simple seniority scheme if and only if all the α_j coefficients, more precisely all α_j^2, are equal. In this case, S^+ in (1) is a generator of SU(2). An important special case is when only one α_j does not vanish and states belong to the j^n configuration.

In the case of SU(2) symmetry, excited eigenstates of the n=2 configuration with various values of J have seniority v=2. If several j-orbits are involved, there are also states with J=0 which are orthogonal to the state (1). The v=2 states can be generated by acting on the vacuum by creation operators D_J^+. Then all states

$$(S^+)^{n-1}D_J^+|0\rangle \qquad (2)$$

are eigenstates and the spacings between them and the state (1) are constant, independent of n. The lowest state with J=2 is created by one of the $D_{J=2}^+$ operators which will be denoted by D^+.

In the j^n configuration, all quadrupole single particle operators Q are proportional. The dependence on n of E2 transition rates is thus given by the dependence of the matrix elements of Q on n. In the general case of SU(2) symmetry, with several j-orbits, there are several independent Q operators. They have different matrix elements between ground states and the lowest J=2 state but the dependence of their matrix elements on n is the same. These matrix elements are between |gs>, the ground state (1), properly normalized, and |2>, the state (2) after normalization.

For a single j-orbit or in the general case of SU(2) symmetry, that dependence is given by

$$\langle 2|Q|gs\rangle_n=2\sqrt{n(\Omega-n)/(\Omega-1)} \qquad (3)$$

In (3), the sum $2\Omega=\Sigma\,(2j+1)$ is over all states with non-vanishing coefficients in (1). Indeed, in this case, the seniority scheme formulae "predict that the B(E2) value follow a parabolic trend, that peaks at midshell"[1]. In the case of generalized seniority, however, there is no simple prediction of the dependence of B(E2) values on nucleon number. Matrix elements of any quadrupole operator, like the E2 one, depend on n in a rather complicated way.

In generalized seniority the coefficients in (1) are no longer required to be equal. Thus, (1) is no longer a generator of SU(2) and some simple features of the seniority scheme are lost. Still, under rather general conditions, some important features survive this generalization. Ground states and their eigenvalues of the one and two-body Hamiltonian for even values of n are given by (1). Also a special state (2) can be constructed so that its eigenvalue for any n minus the energy (1) of the ground state is constant, independent of n.

Unlike the case of simple seniority, constant spacings do not occur for other excited states. In the case of Sn isotopes, experimental binding energies and rather constant 0-2 spacings indicate that generalized seniority may give a good description of these nuclei. Still. energy spacings in odd Sn nuclei change dramatically with n. Had the coefficients in (1) been equal, these spacings should have also been independent of nucleon number.

Any quadrupole operator is determined by its reduced matrix elements between states of single nucleons in orbits which appear in the expansion (1). Between N=50 and N=82 there are 9 such elements (j||Q||j') which may have arbitrary values. In the case of the E2 operator, these coefficients depend also on the various effective charges. The latter are due to polarization of the core by the valence neutrons.

In spite of the possible complexity of such operators, if the coefficients in (1) are equal, the dependence on n of their matrix elements is given exactly by (3). If, however, the coefficients (or their values squared), are not all equal, then the dependence on n of the matrix elements is very complicated. The coefficients α_j and their powers appear explicitly. Thus, there is no "symmetric trend predicted by the seniority model" nor is peaking in midshell generally predicted.

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[1] A.Ekstrom et al, Phys. Rev. Lett. **101**, 012502 (2008).
[2] Seniority and generalized seniority are discussed in detail in I.Talmi, *Simple Models of Complex Nuclei*, Harwood (1993).