Single Particle Entropy in Heated Nuclei

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Abstract. The thermal motion of single particles represents the largest contribution to level density (or entropy) in atomic nuclei. The concept of single particle entropy is presented and shown to be an approximate extensive (additive) quantity for mid-shell nuclei. A few applications of single particle entropy are demonstrated.

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INTRODUCTION

Atomic nuclei at low excitation energy are characterized by the motion of two and two nucleons moving in time reversed orbitals. Each of these Cooper pairs are forming spin $J = 0$, which strongly simplifies the description of the nuclear structure. However, the picture becomes much more complicated as Cooper pairs are broken by collective (Coriolis force) or intrinsic (temperature) excitations. In this work we will investigate the statistical properties of the system as function of the number of excited nucleons.

The experiments was conducted at the Oslo Cyclotron Laboratory (OCL) using a 45-MeV ^{3}He beam on self-supporting targets. Particle-γ coincidences were detected with an array of 28 collimated NaI γ-ray detectors and eight Si particle telescopes placed at 45° with respect to the beam. Details on the experimental set-up and data analysis are given elsewhere [1].

Figure 1 shows typical level densities ρ for well deformed rare earth nuclei as function of excitation energy E. The left panel also shows data based on counting known levels and neutron resonance spacing data. In the right panel the level density is transformed into entropy by $S(E) = \ln \rho(E)/\rho_0$, where $\rho_0 = 2.2$ MeV^{-1}. It is fascinating to see that the S curves are almost identical for ^{170,172}Yb. The odd ^{171}Yb system reveals an increase of $\Delta S = 1.5 - 2.0$, which is due to the entropy carried by the valence particle (or hole) outside the heated even-even core [2].

A systematic review [3] of nuclei having known resonance spacing data shows that the odd particle for mid-shell nuclei carries entropy of $1.5 - 2.0$. The fact that we find this value to be approximately constant for all E indicates that one could use the concept of single particle entropy, independent of the number of excited particles in the underlying

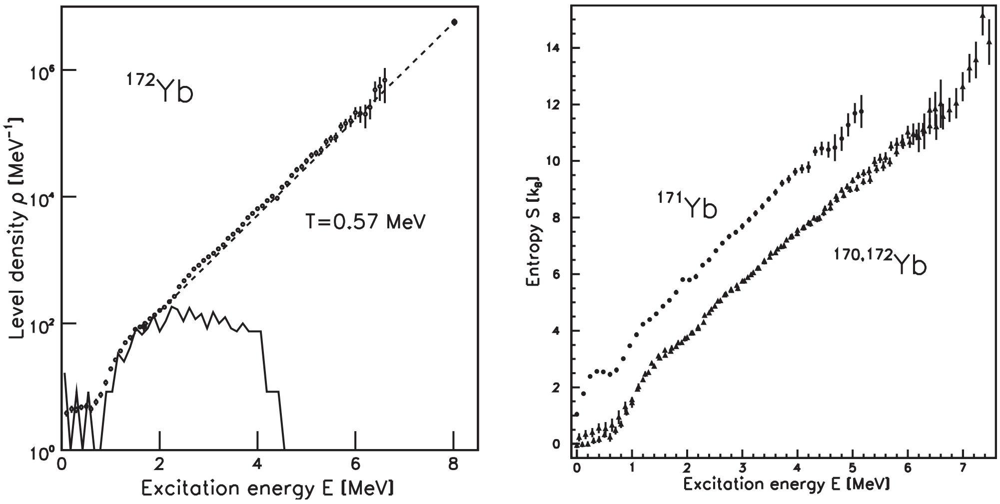

FIGURE 1. Experimental data based on the (^{3}He,α) reaction on ^{171,172,173}Yb targets. Left panel: experimental total level density (filled circles) and constant temperature extrapolation (solid line) to the level density based on neutron resonance data at the neutron binding energy B_n. The jagged line is based on counting known discrete levels. Right panel: entropies deduced for ^{170,171,172}Yb.

core. To further test this assumption, we have constructed a simple thermodynamic model within the canonical ensemble theory [3].

The total partition function includes thermal particle excitations, rotations, and vibrations according to $Z \sim Z_\pi Z_\nu Z_{\rm rot} Z_{\rm vib}$. The first two factor, which are the most important one, includes breaking of pairs and the scattering of protons and neutrons into equidistant single particle levels, where the Pauli principle for fermions is taken into account. The Helmholtz free energy is given by

$$F(T) = -T \ln Z(T), \qquad (1)$$

where thermodynamic quantities can be extracted like entropy $S(T) = -(\partial F/\partial T)_V$, average excitation energy $\langle E(T)\rangle = F + TS$, heat capacity $C_V(T) = (\partial\langle E\rangle/\partial T)_V$ and chemical potential $\mu(T) = \partial F/\partial n$, where n is the number of thermal quasiparticles. From these quantities, the saddle-point approximation gives back the level density by $\rho(\langle E\rangle) = \exp(S)/T\sqrt{2\pi C}$, which depends on the thermal average energy $\langle E\rangle$.

Left part of Fig. 2 shows the predicted entropy and heat capacity using parameters for the ^{162}Dy mass region [4]. The scattering of nucleons is by far the most important contribution to the entropy for $T > 0.4$ MeV. The right part of the figure shows that the model nicely describes the level density for various systems and mass numbers. The level densities for odd-odd, odd-even and even-even systems are parallel for all excitation energies giving support to the idea that each single particle carries a constant entropy of $1.5-2.0$. The property of extensivity is gratifying, and will be exploited in the next section.

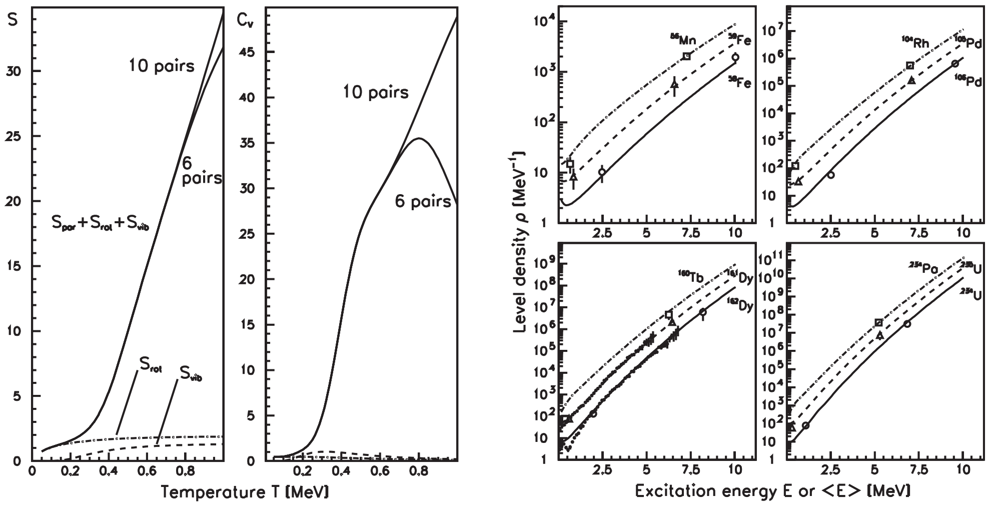

FIGURE 2. Two left panels: entropy and heat capacity for ^{162}Dy evaluated from simple canonical ensemble theory. The results depend strongly on the number of particles in the reservoir. The local bump in the C_V curve at $T \sim 0.5$ signalizes the critical temperature for breaking Cooper pairs. The four right panels: calculated level densities of even-even (solid lines), odd (dashed lines), and odd-odd (dashed-dotted lines) nuclei as function of thermal average excitation energies $\langle E \rangle$. The open data points are anchor points [3] at certain excitation energies. The solid circles are experimental data points.

APPLICATIONS

The data on level densities for different systems indicate that the entropy can be accounted for by adding single particle entropies, i.e. $S = nS_1$. The contribution from collective excitations is constant and has minor importance. In the following we present two applications for the concepts presented here.

In the first example we will estimate the number of valence particles, which are not coupled in pairs. We associate S_1 with the measured odd-even entropy difference ΔS. In Fig. 3 the experimental single particle entropy $\Delta S = S(\mathrm{odd} - \mathrm{even}) - S(\mathrm{even} - \mathrm{even})$ is evaluated for ^{161,162}Dy and ^{171,172}Yb. Above $E = 2\Delta$ the entropy differences are almost constant. In the right panels, the number of particles thermally excited is evaluated by $n(E) = S(\mathrm{even} - \mathrm{even})/\Delta S$ showing that the data follow reasonably well the behavior expected from breaking Cooper pairs at $E = 2$, 4 and 6Δ.

In the second application, we estimate the critical temperature at which the first nucleon pairs are thermally broken. The method is very similar to how the critical rotational frequency ω_c for pair breaking can be evaluated from routhians and spin alignment I_x along the rotational x-axis.

Due to the $-TS$ term of F, the unpaired system with the highest S will become favored at a critical temperature T_c. The T_c value depends on the pairing gap Δ and the quasiparticle entropy ΔS. If both these quantities would be independent of the number of excited quasiparticles, a break-up of all Cooper pairs should happen at one and the same critical temperature T_c, giving rise to a strong quenching of the pair correlations.

Let us assume that the partition function for an even-even nucleus is described by

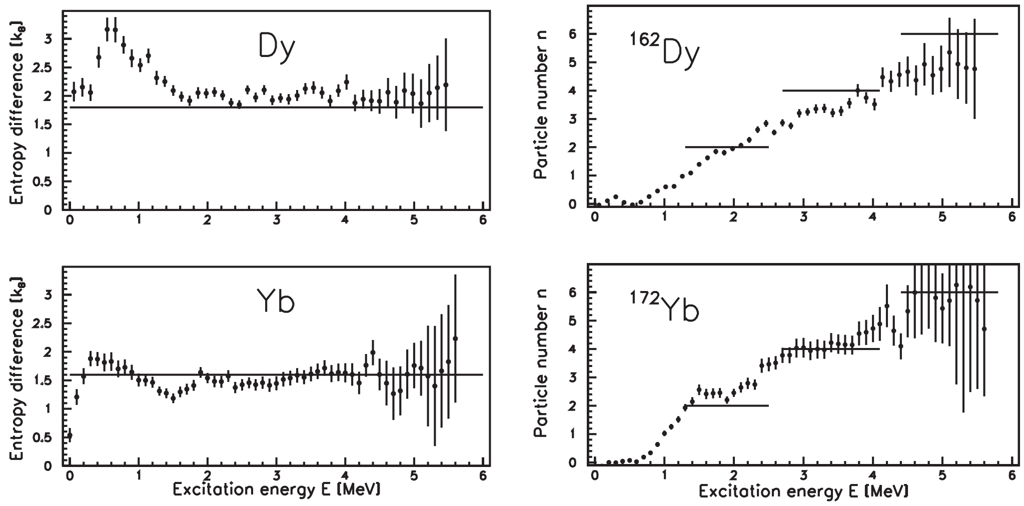

FIGURE 3. Left panel: entropy difference in ^{161}Dy compared to ^{162}Dy (upper panel) and in ^{171}Yb compared to ^{172}Yb (lower panel). The lines through the data points indicate the average values found, interpreted as the single particle entropy. Right panel: number of quasiparticles as function of excitation energy. The lines indicate the levels of two, four and six quasiparticles excited.

Z_{core}. We now introduce the breaking of one nucleon pair relative to this core[1]. Assuming the two quasiparticles to be independent of the core, we may describe the excited system by a factorization of the partition function into

$$Z^* \sim Z_{\mathrm{core}} Z_2 e^{-2\Delta/T}, \tag{2}$$

where Z_2 is the two quasiparticle partition function. Assuming extensivity using $Z_2 = Z_1^2$, the chemical potential is given by the difference in Helmholtz free energy by

$$\mu = \frac{\Delta F}{\Delta n} = \frac{-T(\ln Z^* - \ln Z_{\mathrm{core}})}{2} = \frac{-T\ln Z_2 + 2\Delta}{2} = F_1 + \Delta, \tag{3}$$

where $F_1 = -T\ln Z_1$ is the free energy of one quasiparticle.

The free energy of ^{161}Dy can be interpreted as the energy of an even-even system with one extra quasiparticle. We therefore identify F_1 in Eq. (3) with the difference $F(^{161}\mathrm{Dy}) - F(^{162}\mathrm{Dy})$. In the lower panel of Fig. 4 the chemical potential μ is shown as function of temperature. The higher entropy in ^{161}Dy compared to ^{162}Dy decreases the free energy relative to the even-even core with increasing temperature. When $\mu(T) \sim 0$, thermal quasiparticles can be formed without consuming free energy. Thus, from Fig. 4 we estimate the critical temperature for the quenching of pair correlations to be $T_c = 0.42$ MeV. The uncertainty of this number is mainly due to the extensivity assumed in the evaluation of F_1. Furthermore, the adoption of a chemical potential $\mu = \Delta$ at $T = 0$, rests on the assumption that the Fermi level coincides with single particle energies in both nuclei. Also the extrapolation of the experimental level density to higher excitation

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[1] The core described by Z_{core} is not necessary a cold core, but may already contain broken pairs.

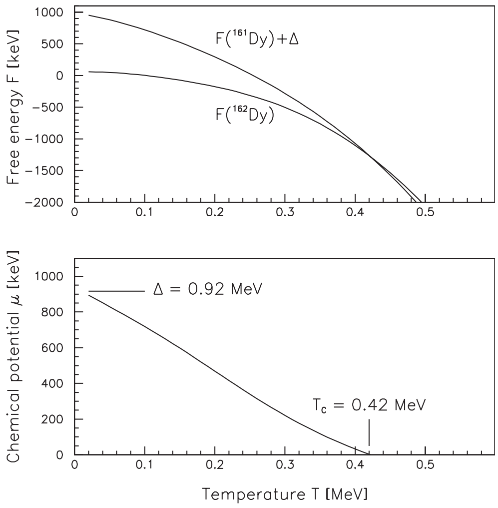

FIGURE 4. Experimental Helmholtz free energy for ^{160,161}Dy extracted in the canonical ensemble. The critical temperature for the quenching of pair correlations is found at $T_c = 0.42$ MeV.

energies gives systematical errors. From these considerations we estimate a 15 % error in the T_c value.

Finally, we point out a curious connection between the experimental quasiparticle entropy ΔS and the critical temperature T_c. If we assume a constant ΔS, we find

$$T_c = \frac{1}{\Delta S}\Delta, \tag{4}$$

giving $T_c = 0.56\Delta$ for $\Delta S = 1.8$. This estimate coincides almost exactly with the theoretical expression $T_c = 0.57\Delta$ deduced from a model based on Fermi gas with pairing [5]. The theoretical factor 0.57 is independent on the mass number, consistent with the concept of single quasiparticle entropy.

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