Coherence length of neutron superfluids

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The coherence length of superfluid neutron matter is calculated from the microscopic BCS wave function of a Cooper pair in momentum space making use of recent nucleon-nucleon potential models and including polarization (RPA) effects. We find as our main result that the coherence length is proportional to the Fermi momentum to pairing gap ratio, in good agreement with simple estimates used in the literature, with a nearly interaction independent constant of proportionality. Our calculations can be applied to the problem of inhomogeneous superfluidity of hadronic matter in the crust of a neutron star. [S0556-2813(97)05210-2]

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Calculations based on BCS theory with phenomenological nucleon-nucleon (NN) forces indicate that neutron matter is superfluid in a wide region of densities and temperatures. In particular, at the densities corresponding to the interior of a neutron star crust (4×10^{11} g cm$^{-3}<\rho<10^{14}$ g cm^{-3}) neutrons couple in the singlet isotropic channel $^1S_0$ while at larger densities they pair in the $^3P_2$ state [1]. Although many investigations have been devoted to the superfluidity and superconductivity of neutron, β-stable, and nuclear matter, little attention has been paid to possible effects of inhomogeneities in hadronic superfluids or superconductors. In fact the interior of neutron stars, the subject of most of such calculations, is often referred to as the only existing example in nature of infinite superfluid neutron or β-stable matter, since in an atomic nucleus the wave functions of the Cooper pairs are limited in extension by the potential well. On the other hand, superfluidity in a neutron star crust represents a case intermediate between the nucleus and the idealized infinite system, since superfluid neutrons in the inner crust occupy a region where a lattice of nuclei creates strong inhomogeneities in the medium. From an astrophysical point of view, most of the observable effects of superfluids in a neutron star are probably due to phenomena in the crust [2].

An important length scale of the neutron superfluid is the coherence length. From a microscopic point of view the coherence length represents the squared mean distance of two paired particles (a Cooper pair of neutrons) on top of the Fermi surface. The magnitude of this quantity affects several of the physical properties of a neutron star crust. First of all, neutrons paired in a singlet state form quantized vortices induced by the rotational state of the star. These can pin to the nuclei present in the crust, possibly leading to the observed sudden release of angular momentum known as pulsar glitches. The magnitude of the pinning force depends on the size of the vortex cores, which is equal to the coherence length of the neutron superfluid. A second question is how properties of the neutron superfluid change due to the inhomogeneous environment of a neutron star crust, a problem related to the average thermodynamical properties of neutron matter [2,3]. In the inner crust, depending on density, nuclei of different shapes and sizes are present. At a density of $\sim10^{14}$ g cm^{-3}, spherical nuclei cease to be energetically favored and are replaced first by cylindrical nuclei, then slabs to end up with holes, where the roles of protons and neutrons are exchanged; see Refs. [2,4] for further details. Only at higher densities, corresponding to what is called the core of the star, do nuclei merge into the uniform medium. The fact that neutron superfluidity in neutron star matter is actually a problem of inhomogeneous superfluidity in hadronic matter has been noticed quite recently [2,3]. According to Anderson's theorem [5] the electron density of states in a superconductor is changed very little from a pure metal to an alloy of similar chemical properties. The physical situation we are examining here is quite different, since both the average density of states and the effective neutron-neutron matrix elements are changed compared to the uniform case when one considers the presence of nuclei.

The typical dimension of nuclei in the inner crust of a neutron star is $R_N\approx4-6$ fm. This number is, in an appropriate range of densities, comparable to the coherence length ξ as estimated from existing BCS calculations. If Anderson's theorem holds also for neutron star matter (limit where $R_N\ll\xi$), there will be no appreciable variations in the superfluid properties induced by the nuclei. On the other hand, if $R_N\gg\xi$ the superfluid will change its properties locally. This limit has been investigated in a recent series of papers [3] where it was found that some thermodynamical properties like, e.g., the neutron specific heat may change by a very large amount. Unfortunately, the situation is complicated by the fact that R_N and ξ are of the same order of magnitude.

Clearly, the coherence length represents a critical parameter by which one can establish the behavior of an inhomogeneous superfluid. It sets the scale for the possible spatial variation of the pairing properties of the system, and thus plays a role if some inhomogeneities are present in the system at a length scale comparable to it.

A simple estimate of the coherence length is obtained by assuming constant matrix elements between particle states within a shell centered at the Fermi momentum and zero outside (see for example Ref. [5]). This gives

$$\xi \approx K \frac{k_F}{\Delta_F}, \tag{1}$$

where k_F is the Fermi momentum and Δ_F is the pairing gap at the Fermi momentum. The value of the constant K depends on the approximations used, and different values have been reported in the literature. However, the estimate (1) does not consider thoroughly the matrix elements of the particle-particle interactions, and in principle can be too rough if one wants to calculate the coherence length at all values of the density. In view of the important astrophysical considerations discussed above, it is desirable to calculate the coherence length from a microscopic study of the wave function of a Cooper pair. The aim of this Brief Report is thus twofold: first we check the validity of the simple estimate (1) and search for possible deviations or fluctuations from its average behavior. Secondly, in those cases where Eq. (1) is well reproduced, we determine the best value of K and check the dependence of that formula on the neutron interaction in the particle-particle channel. To this end, we shall make use of the Bonn A [6] and the Argonne V_{14} potentials [7], and the Gogny force. The Bonn A meson-exchange potential is defined by the parameters of Table A.1 in Ref. [6]. The Gogny force is an effective interaction fitted to reproduce various nuclear data, and contains therefore effects of nucleon-nucleon correlations not included in a bare nucleon-nucleon interaction like the Bonn A potential.

Without polarization effects, the Bonn A and the Argonne potentials yield rather similar gaps for the $^1S_0$ state [1,8]. Thus, in the discussion below we will limit the presentation of results without the polarization diagrams to those obtained with the Bonn A potential. However, such contributions are known to be important. In Ref. [8] the authors find a substantial reduction of the gap. We will therefore also include results for the coherence length obtained with the Argonne potential including polarization effects, as discussed in Ref. [8].

Our microscopic calculations of coherence lengths have been carried out as follows. Let $\phi(\mathbf{r})$ be the wave function of the relative motion of the two neutrons in a Cooper pair, $\mathbf{r}$ being the relative coordinate of the two particles. The coherence length ξ is given by

$$\xi^2 = \frac{\int d^3r |\phi(\mathbf{r})|^2 r^2}{\int d^3r |\phi(\mathbf{r})|^2} = \frac{\int d^3k \left| \frac{\partial}{\partial \mathbf{k}} \chi(\mathbf{k}) \right|^2}{\int d^3k |\chi(\mathbf{k})|^2}$$

$$= \frac{\int_0^\infty dk k^2 |\partial \chi(k)/\partial k|^2}{\int_0^\infty dk k^2 |\chi(k)|^2}, \tag{2}$$

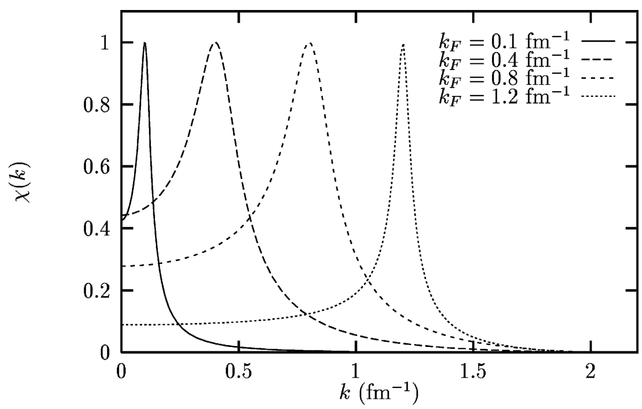

FIG. 1. Wave functions in momentum space, $\chi(k)$, for four different values of the Fermi momentum, $k_F = 0.1$ fm^{-1}, $k_F = 0.4$ fm^{-1}, $k_F = 0.8$ fm^{-1}, and $k_F = 1.2$ fm^{-1}. The wave functions peak at the corresponding value of the Fermi momentum.

where in the second equation the expectation value is calculated in momentum space, $\chi(\mathbf{k})$ being the Fourier transform of $\phi(\mathbf{r})$. Equation (2) is particularly suited for numerical computation, since the BCS equations for a uniform system are solved in momentum space. The wave function of the Cooper pair in momentum space is given by (apart from an unimportant normalization constant)

$$\chi(k) = \frac{\Delta(k)}{E(k)}, \tag{3}$$

where $\Delta(k)$ is the k-dependent pairing gap, while $E(k) = \sqrt{\Delta^2(k) + [\epsilon(k) - \mu]^2}$ is the quasiparticle energy, and $\epsilon(k)$ and μ are the neutron single-particle energy and chemical potential, respectively. Notice that due to the isotropic nature of $^1S_0$ pairing all quantities depend only on the magnitude of the momentum.

We calculate the pairing gaps and wave functions in momentum space from the BCS gap equation [1]

$$\Delta(p) = -\frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \tilde{v}(p,k) \frac{\Delta(k)}{E(k)} \tag{4}$$

for different values of the Fermi momentum. Here $\tilde{v}(p,k)$ are the matrix elements of the $^1S_0$ NN interaction, obtained from the Bonn A potential (or Argonne V_{14} potential), following the procedure outlined in Ref. [1]. The single-particle energies $\epsilon(k)$ were obtained from Brueckner-Hartree-Fock calculations with the same potential; see Refs. [1,9] for details. The chemical potential μ was set equal to the Fermi energy $\epsilon(k_F)$. The inclusion of particle-hole correlations is accounted for by following the scheme of Ref. [8].

Notice that for our purposes the pairing gap has to be calculated at all wave numbers and not only at the Fermi surface, as in the simple formula (1). The wave functions $\chi(k)$ in momentum space for four different values of the Fermi momentum ($k_F = 0.1$, 0.4, 0.8, and 1.2 fm^{-1}) are shown in Fig. 1. The square of the derivatives of the respective wave functions are displayed in Fig. 2. It is seen that the

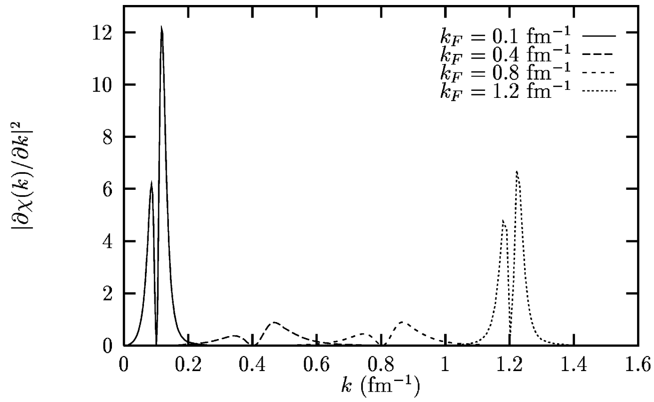

FIG. 2. Plot of $|\partial\chi(k)/\partial k|^2$ proportional to the spread in space of the Cooper pair. Legend as in Fig. 1.

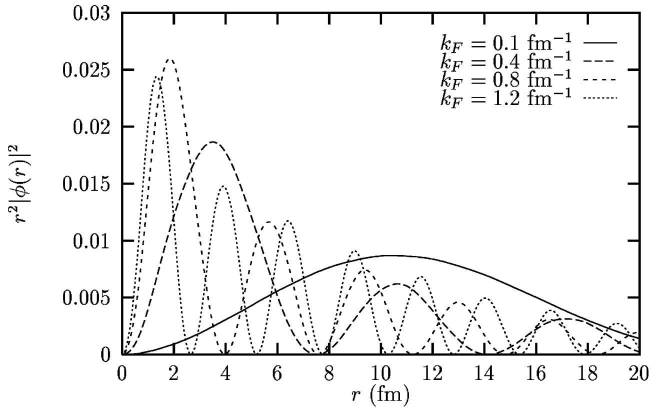

FIG. 3. $r^2|\phi(r)|^2$ as function of r.

spread of the wave function in momentum space $\langle \mathbf{k}^2\rangle$ varies considerably according to the value of the Fermi wave number k_F. The distribution in momentum space is particularly wide for intermediate values of k_F and, since we have $\langle \mathbf{k}^2\rangle = 1/\xi^2 \approx 1/\langle \mathbf{r}^2\rangle$ through the uncertainty relation, a smaller value of the coherence length for these values is expected. Due to the limited, but significant variation of the pairing gap $\Delta(k)$ within a single peak (each peak occurs at the Fermi momentum), the wave function is distorted with respect to a simple Gaussian and therefore deviations from the simple relation (1) for a Gaussian distribution should be expected. This becomes evident by looking at the squared derivative of the wave function $(\partial\chi(k)/\partial k)^2$ which shows two unequal peaks.

In Fig. 3 we present the squared wave functions $r^2|\phi(r)|^2$ for the same values of the Fermi momentum as in Figs. 1 and 2. The wave functions oscillate with a wavelength of the order k_F^{-1} and extend for quite long distances compared to the range of nuclear forces (a few fermis at most). Not surprisingly, this behavior is due to the relatively small value of the energy of the pair compared to the Fermi energy (a comparison can be made with another weakly bound state with a very spread wave function, the deuteron [10]).

In the second column of Table I we show our microscopic calculations of the coherence length for pure neutron matter performed using Eqs. (2)–(4). The microscopically calculated points can be fitted by a function linear in the parameter k_F/Δ_F, where Δ_F is the pairing gap at the Fermi surface. We find as a best fit, see Ref. [11] for further details on the form for ξ,

$$\xi = \theta \frac{\hbar^2}{2m}\frac{k_F}{\Delta_F}x, \tag{5}$$

where $x = m/m^*$ is the ratio between the bare and the effective mass, and the dimensionless parameter θ is found by a direct fit to our data to be $\theta = 0.814$ (for $x=1$) and $\theta = 0.836$ (with the correct density dependent value of x from the microscopic calculation). The choice $x=1$ is commonly used in the literature. The third and fourth column of Table I show the results of these two fits.

The simple linear behavior of the momentum to gap ratio is in very good agreement with Eq. (1) and shows that a Cooper pair represents a packet having a nearly well-defined relative momentum. As seen from Table I, the use of $x=1$ (effective mass equal to bare mass) does not change the picture very much, as is evident from the fact that at, e.g., the

TABLE I. Results for the coherence length in neutron matter. The column marked ‘‘Bonn‘‘ contains the results of a microscopic calculation with the bare Bonn potential. The columns marked $\theta = 0.814$ and $\theta = 0.836$ show the results using Eq. (5) with the corresponding values of θ and energy gaps calculated with the Bonn potential. The column marked ‘‘polar.’’ gives the results of a microscopic calculation with the Argonne V14 potential and including polarization effects. The last column shows the results using Eq. (5) with $\theta = 0.714$ and energy gaps calculated with the Argonne potential. Where no results are shown, this means that the energy gap $\Delta_F = 0$ at the corresponding value of k_F.

k_F			ξ (fm)		
(fm^{-1})	Bonn	$\theta = 0.814$	$\theta = 0.836$	Polar.	$\theta = 0.714$
0.1	17.431	17.350	17.812		
0.2	7.975	7.337	7.532		
0.4	4.877	4.375	4.492	194.177	208.947
0.6	4.301	3.921	4.025	102.758	105.895
0.8	5.176	4.392	4.509	30.881	31.357
1.0	5.174	6.193	6.357	21.332	20.681
1.2	13.244	12.285	13.355	18.076	17.418
1.4	66.281	77.497	79.560	14.264	12.651
1.6				232.054	234.240

relatively high Fermi momentum of $k_F = 1.0\ \mathrm{fm}^{-1}$ the effective mass differs from the bare one by less than 4%. However, deviations from Eq. (1) are found, especially at higher densities. We also calculated the coherence length from an approximation often used in neutron star studies, namely the choice of a coefficient $\theta = 2/\pi = 0.637$ in Eq. (5) [11]. Our values were on the average more than 20% larger than the results obtained with this choice. It is however interesting to see that our microscopic calculation of the gap and single-particle properties through a complicated many-body scheme, yields a qualitatively similar result as that of Eq. (1), derived originally in a solid state context [5].

A relevant point is whether the value of the coefficient θ depends on the interaction chosen. Besides, it might be that the good agreement with Eq. (5) gets worse with other interactions. To partially answer these questions, we repeated the same calculations making use of an effective force, the Gogny force, which pairing properties have been investigated in several works [3,12]. We find that with the D1 parametrization of the Gogny force a very good fit can be made for Eq. (3) with a coefficient $\theta = 0.815$, very close to the value found with Bonn A. This near equality of the coefficient θ for the two different interactions is not predictable on the basis of simple arguments. However, it ought to be stressed that we have not included so-called polarization effects in the calculations of the pairing gaps of Ref. [1]. Such effects are expected to reduce by at least a factor of 2 the value of the pairing gap, as can be seen in Ref. [8]. Thus, in the fifth column of Table I we show the corresponding results obtained with the Argonne potential including polarization effects.[1] The parametrization of Eq. (5) is shown in the sixth column of Table I. A good fit to the microscopically calculated coherence length is given by $\theta = 0.714$. The reader should however note that the value of the coherence length is rather different from that obtained when no polarization effects are included. Already in this approach, the coherence length is for several Fermi momenta larger than the typical size of known nuclei. When polarization effects are included, it is larger than (but still comparable to) the size of known nuclei for all Fermi momenta. This poses several constraints on the use of local density approximations in the study of neutron star crust properties [2,3].

To conclude, we calculated microscopically the coherence length of superfluid neutron matter. Equation (5) with the coefficient $\theta \approx 0.7-0.8$ reproduces rather well the microscopically calculated coherence length for $^1S_0$ pairing. Without polarization effects we find an almost interaction independent value of $\theta = 0.81 - 0.83$, while with polarization effects we find $\theta = 0.714$. The data reported in this Brief Report may be useful in quantitative calculations of the superfluid properties of neutron star crusts.

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[1] The qualitative behavior of the wave functions with polarization effects is similar to that reported in Figs. 1–3 with the Bonn A potential excluding these effects.

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