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Phaseshift equivalent NN potentials and the deuteron

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Abstract

Different modern phase shift equivalent NN potentials are tested by evaluating the partial wave decomposition of the kinetic and potential energy of the deuteron. Significant differences are found, which are traced back to the matrix elements of the potentials at medium and large momenta. The influence of the localisation of the one-pion-exchange contribution to these potentials is analyzed in detail.

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During the past few years, considerable progress has been made in constructing realistic models for the nucleon-nucleon (NN) interaction. Several of these models describe an identical data base of NN scattering data with a χ^2 per datum $\approx$ 1 [1–3], meaning in turn that the on-shell matrix elements of the NN transition matrix T are essentially equal. This does, however, not imply that the models for the NN interaction underlying these descriptions are identical. Moreover, the off-shell properties of each potential may be rather different. All models for the NN interaction V include a one-pion exchange (OPE) term, using essentially the same πNN coupling constant, and account for the difference between the masses of the charged ($\pi_\pm$) and neutral (π_0) pion. However, even this long range part of the NN interaction, which is believed to be well understood, is treated quite differently in these models. As we will discuss below, the local approximation for the OPE contribution, employed in the non-relativistic Nijmegen [1] (hereafter abbreviated by Nijm) and Argonne [2] (abbreviated by Argon) potentials, yields significantly different contributions to the NN potential V as compared to the one derived in the relativistic framework underlying the Bonn potential [3]. The description of the short-range part is also different in these models. The NN potential Nijm2 [1] is a purely local potential in the sense that it uses the local form of the OPE potential for the long-range part and parametrizes the contributions of medium and short-range in terms of local functions (depending only on the relative displacement between the two interacting nucleons) multiplied by a set of spin-isospin operators. The same is true for the Argonne V_{18} potential [2]. The NN potential denoted by Nijm1 [1] uses also the local form of OPE but includes a $\boldsymbol{p}^2$ term in the medium- and short-range central-force (see Eq. (13) of Ref. [1]). This may be interpreted as a non-local

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contribution to the central force. The CD-Bonn potential is based consistently upon relativistic meson field theory [4]. Meson-exchange Feynman diagrams are typically non-local expressions that are represented in momentum-space in analytic form. It has been shown [3] that ignoring the non-localities in the OPE part leads to a larger tensor component in the bare potential. This is the origin of the fact that the CD-Bonn potential yields a smaller D-state probability for the deuteron ($P_D = 4.83\%$) as compared to other potentials ($P_D \approx 5.6\%$).

The aim of this note is to explore characteristic features of the various phase-shift equivalent NN potentials depending on the model constraints pertinent to the various potentials, such as the non-locality of the OPE part of the NN interaction as well as in the exchange terms of the heavier mesons. We have here chosen the simplest possible system to study these differences, namely the deuteron. Investigations along these lines, using the abovementioned NN interactions, have been made by studying the properties of the triton [5], the optical potential for nucleon-nucleus scattering [6], and the symmetry energy in nuclear matter [7]. Significant differences between non-local and local NN potentials have been observed in the study of the symmetry energy. As a side-remark, we ought to state that our aim, and that of the above studies, is different from earlier studies of phase-shift equivalent NN potentials, like e.g., Sauer et al. [8–10]. In those studies, NN scattering matrices were constructed to be phase-shift equivalent from a mathematical point of view, that is, the structure of the operators which determined the model for the potentials were kept fixed, while potential parameters were allowed to vary.

By construction, all realistic NN potentials reproduce the experimental value for the energy of the deuteron of −2.224 MeV. However, the various contributions to the total deuteron energy originating from kinetic energy and potential energy in the $^3S_1$ and $^3D_1$ partial waves of relative motion,

$$\begin{aligned} E &= \langle \Psi_S|T|\Psi_S\rangle + \langle \Psi_D|T|\Psi_D\rangle + \langle \Psi_S|V|\Psi_S\rangle \\ &\quad + \langle \Psi_D|V|\Psi_D\rangle + 2\langle \Psi_S|V|\Psi_D\rangle \\ &= T_S + T_D + V_{SS} + V_{DD} + V_{SD}, \end{aligned} \tag{1}$$

exhibit quite different results. This can be seen from the numbers listed in the upper part of Table 1. In that table, we display the various contributions to the deuteron binding energy employing the four potentials introduced above. For a comparison we also list in this table results obtained from older NN potentials.

Table 1
Contributions to the kinetic and potential energy of the deuteron originating from the $^3S_1$ and $^3D_1$ parts of the wave function as defined in Eq. 1. Results are listed for the charge-dependent Bonn potential (CD-Bonn [3]), the Argonne V_{18} [2], the Nijmegen potentials Nijm1 and Nijm2 [1], the Bonn potentials A,B,C [4], the Argonne V_{14} [11], the Reid soft-core potential [12] and the Urbana potential [13]. Also listed are the expectation values of the charge-dependent OPE part, using the non-local approach from the CD-Bonn potential ('π Bonn'), the local approximation ('π local'), and the local approximation with inclusion of the Argonne form factor ('π Argon'). In the latter three cases, these expectation values are calculated by employing the CD-Bonn deuteron wave function. The last two columns of this Table show the calculated D-state probabilities and the asymptotic ratios of the D/S parts of the wavefunction.

Pot.	T_S [MeV]	T_D [MeV]	V_{SS} [MeV]	V_{DD} [MeV]	V_{SD} [MeV]	P_D [%]	D/S
CD-Bonn	9.79	5.69	−4.77	1.34	−14.27	4.83	0.0255
Argon V_{18}	11.31	8.57	−3.96	0.77	−18.94	5.78	0.0250
Nijm1	9.66	7.91	−1.35	2.37	−20.82	5.66	0.0253
Nijm2	12.11	8.10	−5.40	0.59	−17.63	5.64	0.0252
Bonn A	10.05	4.41	−7.29	0.39	−9.78	4.38	0.0263
Bonn B	10.02	5.62	−5.39	1.01	−13.49	4.99	0.0264
Bonn C	10.13	7.44	−1.20	3.46	−22.05	5.62	0.0266
Argon V_{14}	10.53	8.68	−1.83	1.99	−21.59	6.08	0.0265
Reid	12.61	9.53	−0.47	4.55	−28.45	6.47	0.0262
Urbana	11.15	6.44	−6.91	−0.19	−12.73	5.20	0.0254
π Bonn			2.32	1.86	−21.48		
π local			−1.99	2.18	−26.76		
π Argon			−1.65	1.60	−18.28		

The kinetic energies are significantly larger for the local potentials V_{18} and Nijm2 than for the two interaction models CD-Bonn and Nijm1 which contain non-local terms. The corresponding differences in the S-wave functions can be seen in left part of Fig. 1, where the momentum distribution multiplied by the square of the momentum k is plotted for the S wave component of the wave function

$$k^2 n_S(k) = k^4 \Psi_S^2(k). \tag{2}$$

Note that, besides a factor $1/M$ with M the nucleon

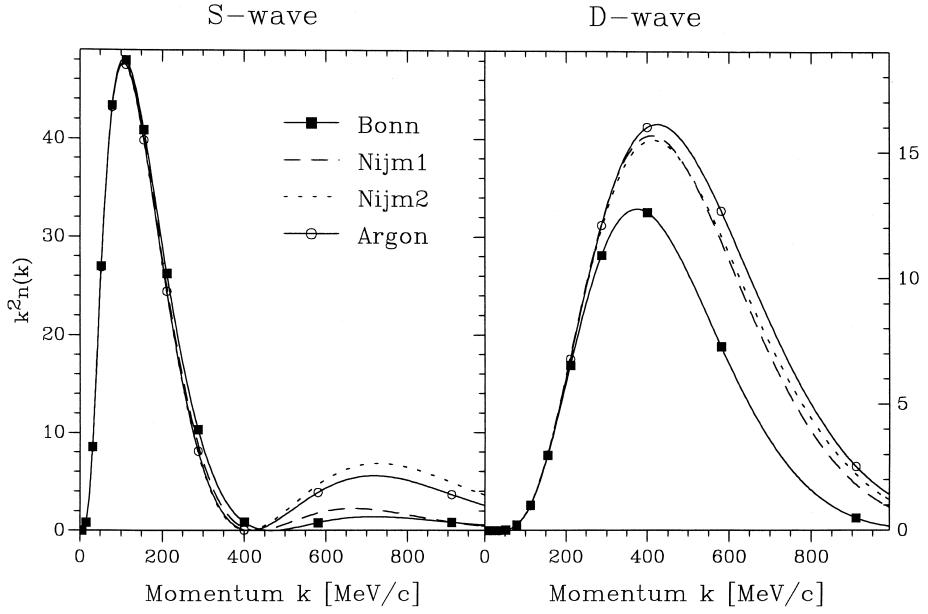

Fig. 1. Momentum distribution in the *S*-wave (left part) and *D*-wave (right part of the figure) of the deuteron derived from various NN interactions. To enhance the high momentum components the distribution has been multiplied by k^2 as outlined in Eq. (2).

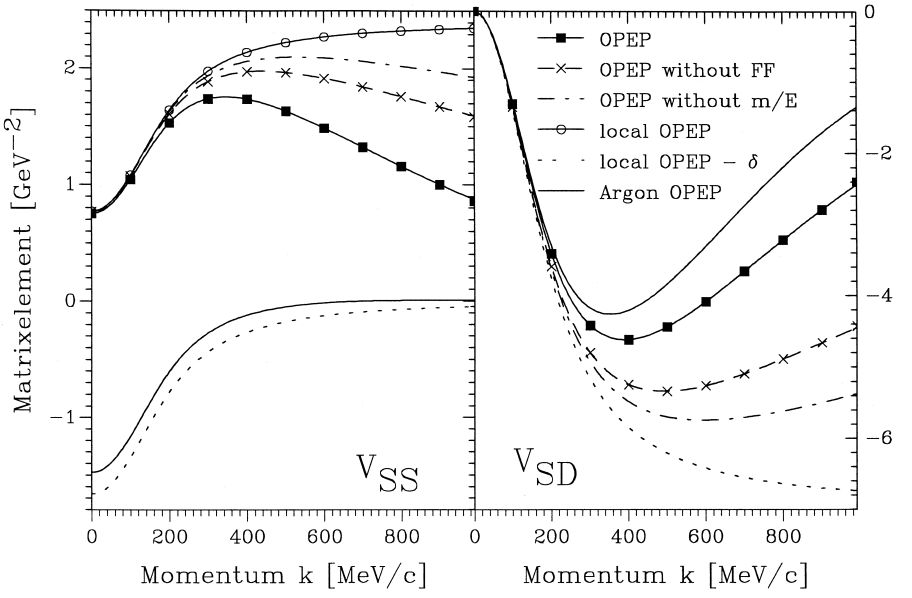

Fig. 2. Plane-wave matrix elements of the one-pion-exchange potential (OPEP) using various approximations. As an example, the matrix elements in momentum space $\langle k'|V|k\rangle$ are shown as functions of k for a fixed value of $k' = 95$ MeV/c. The left part of the figure exhibits matrix elements for the partial waves $^3S_1-^3S_1$, while the right part shows the tensor component in the $^3S_1-^3D_1$ channel with k refering to the momentum in the $^3D_1$ partial wave. Note that the subtraction of the δ component of the local OPE does not affect the matrix elements of V_{SD}.

mass, this function corresponds to the integrand for evaluating the kinetic energy T_S. The corresponding momentum distributions for the D-wave also reflect the D-state probability calculated for the various potentials (see Table 1).

Comparing the contributions to the potential energy, one finds large differences particularly for the tensor contribution V_{SD}. The dominant part of this tensor contribution should originate from the tensor component of the OPE potential. This can also be seen from the last three lines of Table 1 where the expectation value of the OPE part of the NN interaction is calculated with the deuteron wave function derived from the Bonn potential. These OPE contributions to V_{SS}, V_{DD} and V_{SD} have been evaluated using three different approximations to the OPE potential. The first one denoted by ''π Bonn'' in Table 1 corresponds to the relativistic non-local OPE of the CD-Bonn approach, in which the matrix elements in the $^3S_1-^3S_1$ channel between plane-wave states of momenta k' and k are given by

$$\langle k'|V_{SS}^{\pi}|k\rangle = -\frac{g_\pi^2}{4\pi}\frac{1}{2\pi M^2}\sqrt{\frac{M^2}{E_k E_{k'}}} \times \int_{-1}^{1} d\cos\theta \left(\frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + q^2}\right)^2 \times \frac{k'k\cos\theta - \left(E_k E_{k'} - M^2\right)}{q^2 + m_\pi^2}, \qquad (3)$$

where q^2 denotes the momentum transfer,

$$q^2 = (\boldsymbol{k} - \boldsymbol{k}')^2 = k^2 + k'^2 - 2kk'\cos\theta,$$

$$E_k = \sqrt{k^2 + M^2}, \qquad (4)$$

and m_π is the mass of the pion (note that in order to shorten the notation we ignore in these equations the charge dependence of m_π and the nucleon mass M); Λ stands for the cut-off parameter, which was chosen to be $\Lambda = 1.7$ GeV [3]. Results for such matrix elements as a function of k, keeping $k' = 95$ MeV/c fixed, are displayed in Fig. 2. If we introduce now the nonrelativistic approximation for

$$E_k E_k' - M^2 \approx \tfrac{1}{2}k^2 + \tfrac{1}{2}k'^2, \qquad (5)$$

we obtain (ignoring the form factor $(\Lambda^2 - m_\pi^2)/(\Lambda^2 + q^2)$ and the relativistic square-root factors)

$$\langle k'|V_{SS}^{\pi}|k\rangle_{\rm local} = -\frac{g_\pi^2}{4\pi}\frac{1}{2\pi M^2} \times \int_{-1}^{1} d\cos\theta \left(\frac{m_\pi^2}{2\left(q^2 + m_\pi^2\right)} - \tfrac{1}{2}\right). \qquad (6)$$

This can be viewed as the local approximation to the OPE since the matrix element depends on the momentum transfer q^2, only. It can easily be transformed into the configuration-space representation, resulting in a Yukawa term plus a δ function, which originates from the Fourier transform of the constant $1/2$ in Eq. (6). The various steps leading to this result are shown in Fig. 2. It is obvious from this figure that all of the steps leading to the local expression (6) are not really justified for momenta k around and above 200 MeV, a region of relative momenta which is of importance in the deuteron wave function (see Fig. 1). This is true for the matrix elements V_{SS} as well as V_{SD}. It is remarkable that the regularization of the local OPE by the local Gaussian form factor introduced in the Argonne V_{18} potential leads to matrix elements in the SD-channel which are close to those derived from the relativistic expression of the Bonn potential. This is not the case in the SS channel, where the removal of the δ function term is very significant. This comparison of the various approximations demonstrates that even the OPE part of the NN interaction is by no means settled. The local approximation and the regularization by form factors have a significant effect, modifying the short-range part of this contribution.

As a next step we would like to discuss the importance of the OPE contribution as compared to the total NN interaction in the partial waves relevant for the deuteron. As an example we consider the meson-exchange model of the Bonn potential (figures in the left column of Fig. 3) and the local Argonne V_{18} potential considering matrix elements in momentum space between $^3S_1-^3S_1$ (upper part) and $^3S_1-^3D_1$ partial waves (lower part). The predominant modification of the π-exchange in the OBE model is caused by the exchange of the ρ-me-

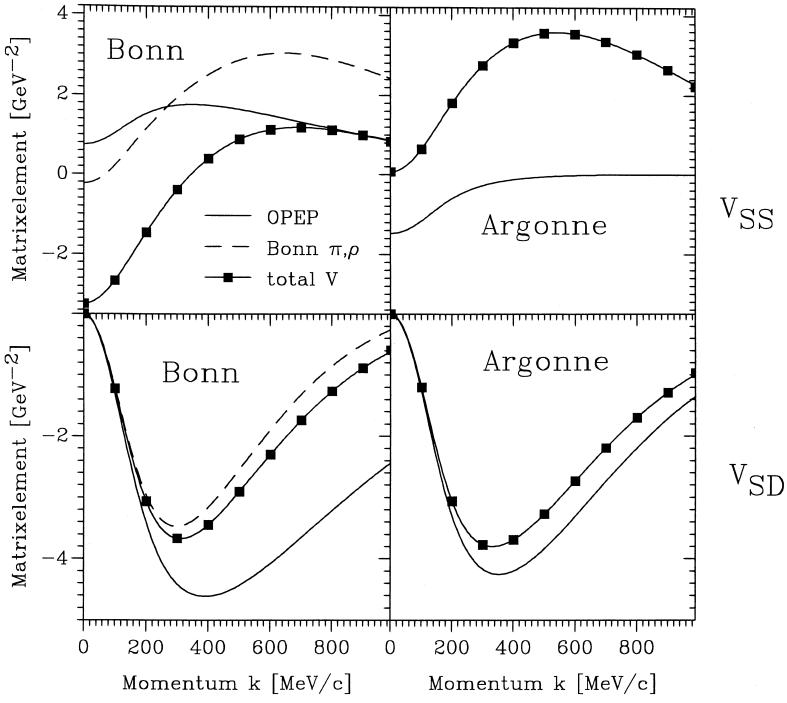

Fig. 3. Plane-wave matrix elements of the one-pion-exchange potential (OPEP, solid lines) are compared to those for the total interaction (solid lines with squares) for the CD-Bonn potential (left part) and the Argonne V_{18} potential (right part) in the $^3S_1-^3S_1$ (upper part) and $^3S_1-^3D_1$ partial waves (lower part). For the Bonn potential also the result of π plus ρ exchange (dashed line) is shown. For the definition of the matrix elements see Fig. 2.

son. This leads to a considerable reduction of the tensor component (see V_{SD} in the lower part of Fig. 3) and enhancement of the central contribution (see V_{SS}). The effect of other mesons is very weak in the tensor channel and leads to a global attraction for the central part of the NN interaction. The modification of the OPE part in the Argonne potential is quite different. The tensor part of the NN interaction is reduced much less than in the OBE model. In the central *SS* channel the short range components of the V_{18} potential introduce considerable repulsion but lead to a momentum dependence which, apart from a repulsive constant, is similar to that of the Bonn potential.

Finally, we compare in Fig. 4 the momentum space matrix elements of all four phase-shift equivalent charge-dependent NN potentials. Substantial differences are observed in the central *SS* channel as well as in the tensor channel *SD*. The local approximation of the OPE contribution to the tensor term is highly suppressed at large momenta (compare Fig. 2). The differences between the local approximation of the OPE tensor term and the non-local treatment shown in Fig. 2 is much larger than the final differences between the various potentials shown in Fig. 4. In the OBE model the final shape of the tensor potential is achieved by means of relativistic corrections, form factors, and ρ exchange. The local potentials describe the reduction of the local OPE mainly in terms of local form factors. From the remaining differences we see that the requirement of identical NN scattering phase shifts provides only a moderate constraint on this tensor component. As a consequence one obtains different D-state probabilities in the deuteron wave function (see Table 1). It seems that the range of possible D-state probabilities has been reduced to some extent by the improvements in NN scattering data analysis (as compared to the

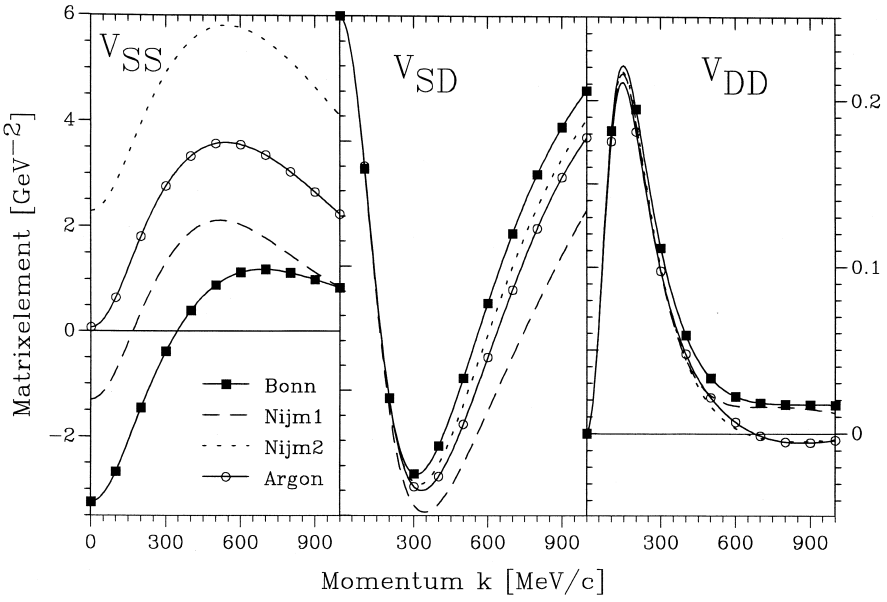

Fig. 4. Plane-wave matrix elements of various realistic NN interactions in momentum space for the partial wave channels $^3S_1-^3S_1$, $^3S_1-^3D_1$, and $^3D_1-^3D_1$. For the definition of the matrix elements see Fig. 2.

range produced by the older potentials). The new potentials also yield rather similar predictions for the asymptotic ratio of the D to S wave component of the wavefunction (see last column of Table 1). They are all in good agreement with the experimental value for the D/S ratio of 0.0256(4) [14]. This supports the conclusion that it is the short-range part of the wavefunction and the underlying NN interactions, which is different.

The momentum dependence of the various NN potentials in the central SS channel is rather similar. Here the main differences can be described in terms of a constant in momentum space or a corresponding δ function in configuration space. It is worth mentioning that the two non-local potentials Nijm1 and CD-Bonn yield matrix elements which are closer to each other as compared to the other two potentials. The same is true for the V_{DD} matrix elements. This may also be the origin of the differences in the high momentum components of the wave functions (cf. Fig. 1).

In summary, although the modern NN potentials yield the same binding energy for the deuteron, there are significant differences in the contributions to both the kinetic and potential energy in the various partial waves. Speaking in general terms, these differences can be traced back to off-shell differences between the potentials. It is well known that the off-shell behavior of NN potentials cannot be pinned down by on-shell data. On the other hand, within realistic physical models, not everything is possible that mathematics may allow. Moreover, within a given model, pinning down the on-shell point limits the range of variation for the off-shell behavior. The four modern high-precision models considered in this study pin down the on-shell T-matrix as much as by all means possible since they fit the NN scattering data with the perfect $\chi^2/\text{datum} \approx 1$. Furthermore, it may be reasonable to believe that the four models cover about the range of diversity that there is to realistic physical models for the NN interaction. Based upon these premises, one may then conclude that the off-shell uncertainties revealed in this study are what we have to deal with, at the current status of theoretical nuclear physics. At this time, we do not know of any other objectively verifiable aspects that could further reduce the off-shell uncertainties. Tighter constraints for the off-shell behavior of NN

potentials may emerge in the future from our growing confidence in a particular theory or from the many-body problem.

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