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Tensor force in effective interaction of nuclear force

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Abstract. The tensor force in effective interactions is discussed. We show that the tensor force in low-momentum effective interactions $V_{\text{low}k}$ and effective interactions for shell model studies are almost identical to that of the bare nucleon-nucleon interaction. We use realistic nuclear forces and microscopic theories of effective interactions. We calculate $V_{\text{low}k}$ by the technique of similarity transformation. The effective interaction for the nuclear shell model is calculated by folded diagram theory. The monopole contribution from the tensor force is analyzed by a spin-tensor decomposition.

1. Introduction

Radioactive beam accelerator facilities make it possible to perform experiments that test the limits of nuclear stability by studying exotic nuclei. The nuclear shell model has been one of the most successful models for describing nuclei. However, recent studies of neutron rich nuclei indicate that the so-called nuclear magic numbers may only be valid close to stable nuclei. Otsuka *et al.* [1] showed that the tensor force varies the spin-orbit splitting in exotic nuclei, where the proton number and the neutron number are very different. For instance, if neutrons occupy the orbit j' , the effective single particle energy of protons in the orbit j is described as follows

$$\Delta\epsilon_p(j) = \frac{1}{2} (V_{jj'}^{T=0} + V_{jj'}^{T=1}) n_n(j'), \quad (1)$$

where $\Delta\epsilon_p(j)$ represents the change in the effective single particle energy of protons in orbit j and $n_n(j')$ is the occupation number of neutrons in orbit j' . Here $V_{jj'}$ represents the monopole part of the Hamiltonian between two orbits j and j' , which is defined as

$$V_{jj'}^T = \frac{\sum_J (2J+1) \langle jj' | V | jj' \rangle_{JT}}{\sum_J (2J+1)}. \quad (2)$$

This is nothing but the angular averaged interaction between two orbits j and j' . The tensor-force contribution always has different sign between a pair of spin-orbit partners. For example, $V_{j>j'}$ and $V_{j<j'}$ have opposite sign, where $j>$ and $j<$ are the spin-orbit partners, that is,

$j_> = l + 1/2$ and $j_< = l - 1/2$. This effect breaks the conventional magic numbers believed to be universal during the last 50 years.

In Ref. [1], the tensor force in effective interactions for the nuclear shell model is taken as the exchange of $\pi + \rho$ mesons for simplicity. This is qualitatively the same as the tensor force in the bare realistic nuclear force. The tensor force of realistic interactions is fitted to reproduce phase shifts of NN scattering data and ground state properties of the deuteron. However, the effective interaction for the shell model is normally different from the bare realistic nuclear force. First, the free nuclear force exhibits a strong coupling between low-momentum and high-momentum degrees of freedom. This coupling is due to the short range properties of the interaction. For example, the Argonne interactions are defined by using a local operator form in coordinate space and has a strong short range repulsion [2]. Furthermore, the effective interaction for the shell model is defined in a restricted configuration space, normally called the model space. Therefore, it is far from trivial that the tensor force in an effective shell-model interaction can be simply considered to arise from the explicit exchange of $\pi + \rho$ mesons only, as seen in models for the nuclear force. To understand the validity of this assumption, theoretical studies based on realistic nuclear forces and the microscopic theory of effective interactions are needed.

In this work, we construct an effective interaction for the shell model starting from realistic nuclear forces and use perturbative many-body methods for deriving the effective interaction. Then, we analyze the tensor component to see if the tensor force survives under the renormalization.

2. How to construct an effective interaction from realistic nuclear forces

The original eigenvalue problem is

$$H|\Psi_i\rangle = E_i|\Psi_i\rangle. \quad (3)$$

We divide the Hilbert space into a model space, the so-called P -space and an excluded space, the Q -space for short. Here, P and Q are projection operators onto the P -space and the Q -space, respectively, i.e., $P^2 = P$, $Q^2 = Q$, $P + Q = \mathbf{1}$. The basic problem is to find an effective interaction \tilde{H} that reproduces some of the eigenvalues of the original eigenvalue problem of Eq. (3)

$$P\tilde{H}P|\phi_i\rangle = E_i|\phi_i\rangle, \quad (4)$$

where $|\phi_i\rangle = P|\Psi_i\rangle$. This can be accomplished by the following similarity transformation,

$$\tilde{H} = e^{-\omega} H e^{\omega} \quad Q\omega P = \omega. \quad (5)$$

Then it follows immediately that $\omega^2 = \omega^3 = 0 \dots$ and $e^{\omega} = 1 + \omega$. If $P\tilde{H}P$ is an effective interaction, the decoupling condition $Q\tilde{H}P = 0$ must be fulfilled.

Therefore, the central problem is now reduced to how to obtain ω which fulfills the decoupling condition. The physical meaning of ω can be understood as a mapping from the P -space wavefunction to the corresponding Q -space component. In general, there is no unique solution of the decoupling equation and ω depends on the set of original eigenfunctions which are selected to reproduce the corresponding eigenvalues.

If one chooses a set of d eigenfunctions, the decoupling equation has the following formal solution [3]

$$\omega = \sum_{i=1}^d Q|\Psi_i\rangle\langle\tilde{\phi}_i|P, \quad (6)$$

where $\langle \tilde{\phi}_i |$ refers to the bi-orthogonal state of $|\phi_i\rangle$, which is defined to be $\langle \tilde{\phi}_i | \phi_j \rangle = \delta_{ij}$. The original eigenfunction $|\Psi\rangle$ is orthogonal because of the hermiticity of the Hamiltonian. However, its projection onto the P -space $|\phi\rangle = P|\Psi\rangle$ is not orthogonal in general.

If we take the P -space as the low-momentum space and the Q -space as the high-momentum space and choose d low-energy eigenstates, we will obtain a low-momentum interaction by the above procedure. We call this potential $V_{\text{low}k}$ and the cutoff parameter Λ is defined as the boundary between the P -space and the Q -space [4, 5]. Using $V_{\text{low}k}$ can be interpreted as rewriting our problem from $H = T - T_{\text{CM}} + \sum V_{\text{bare}}$ to $H = T - T_{\text{CM}} + \sum V_{\text{low}k}$. This means that we change our resolution of the problem and solve the problem not seeing high-momentum details. The problem becomes much easier to solve. In this work, we will neglect induced many-body force.

Nevertheless, if we need to solve a many-body problem, it is still difficult to solve the Schrödinger equation directly even if we use $V_{\text{low}k}$. For example, if one wants to obtain the energy levels of ^{18}O with 18 interacting particles, solving the whole 18-body problem is too large a calculation to perform and we need to calculate a two-body effective interaction defined for a smaller space. A typical example is the sd -shell. However, to determine ω by Eq. (6) one needs the complete knowledge of the d true eigenfunctions $|\Psi_i\rangle$, which we do not know. Hence, what we want to do is to obtain the eigenvalues of the original problem without solving the original problem. In this case, one cannot know ω from Eq. (6) and have to obtain it by an alternative procedure. An efficient way to do this is the so-called \hat{Q} -box expansion [6].

If the unperturbed Hamiltonian is degenerate in the model space, that is, $H_0|\phi_i\rangle = E_0|\phi_i\rangle$, where $H = H_0 + H_1$, the effective interaction defined in P -space can be calculated by the following iterative formula

$$V_{\text{eff}}^{(n)} = \hat{Q}(E_0) + \sum_m \hat{Q}_m(E_0) \{V_{\text{eff}}^{(n-1)}\}^m \quad (7)$$

where

$$\hat{Q}(E_0) \equiv PH_1P + PH_1Q \frac{1}{E_0 - QHQ} QH_1P, \quad (8)$$

$$\hat{Q}_m(E_0) \equiv \frac{1}{m!} \frac{d^m \hat{Q}(E_0)}{dE_0^m}. \quad (9)$$

where the derivatives correspond to folded diagrams. By this prescription, we can obtain an effective interaction for the shell model, which gives equivalent results to the original eigenvalue problem within the range of approximation. Again, induced many-body forces are neglected.

3. Tensor force in low-momentum interaction $V_{\text{low}k}$

As mentioned above, the tensor force in the bare realistic nuclear force mainly comes from $\pi + \rho$ meson exchange. In this section, we discuss the effect of the renormalization procedure on the tensor force. For this purpose, we have performed a spin-tensor decomposition of the $V_{\text{low}k}$ interaction with various values of the cutoff Λ [7]. Figure 1 shows the monopole part of the tensor force in $V_{\text{low}k}$ derived using the Argonne V8' (AV8') potential [2]. Except for very low cutoff values like $\Lambda = 1.0 \text{ fm}^{-1}$, the monopole part of tensor force in $V_{\text{low}k}$ has almost no cutoff dependence. A low value $\Lambda = 1.0 \text{ fm}^{-1}$ in momentum space corresponds to a distance in coordinate space of approximately 1.0 fm. Since the Compton wavelength of a pion is approximately 0.7 fm, a cutoff value $\Lambda = 1.0 \text{ fm}^{-1}$ is too low. With such a cutoff, the renormalization induces strong and non-negligible higher-body forces, which make the problem much more complicated. Therefore, this value is not appropriate for our purpose.

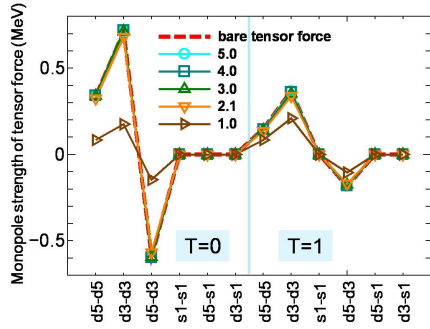


Figure 1. Monopole part of tensor force in low-momentum interaction V_{lowk} . Cutoff parameter Λ of V_{lowk} is taken as from 1.0 fm^{-1} to 5.0 fm^{-1} .

We may say that the tensor force survives in a low-momentum interaction V_{lowk} with usual cutoff values, at least for its monopole part. Although we have only showed results for the sd -shell, similar conclusions are also reached for the pf -shell.

4. Effect of renormalization of short-range tensor force

In this section, we will focus on how the tensor force renormalization of the short range part of the interaction is incorporated into an effective interaction. In our calculation of low-momentum interaction V_{lowk} , we consider a two-nucleon system. The tensor force plays a crucial role in the case of the deuteron. The deuteron has isospin $T = 0$ and a non-negligible $^3S_1 - ^3D_1$ mixing. There is no experimental data which indicates an existence of excited states or any other bound state of deuteron. The mixing of $^3S_1 - ^3D_1$ is due to the tensor force.

The Schrödinger equation for the deuteron can be written as the following coupled equations,

$$\begin{aligned} -\frac{\hbar^2}{M} \frac{d^2 u(r)}{dr^2} + V_C u(r) + \sqrt{8} V_T w(r) &= E_d u(r), \\ -\frac{\hbar^2}{M} \frac{d^2 w(r)}{dr^2} + \left(\frac{6\hbar^2}{Mr^2} + V_C - 2V_T - 3V_{LS} \right) w(r) + \sqrt{8} V_T u(r) &= E_d w(r), \end{aligned} \quad (10)$$

where $u(r)$ and $w(r)$ are the radial wave functions of the S -wave and the D -wave, respectively. Knowing the solution of Eq. (10), we integrate out the D -wave degrees of freedom to obtain the following effective central force

$$\begin{aligned} V_{\text{eff}}(r; ^3S_1) &= V_C(r; ^3S_1) + \Delta V_{\text{eff}}(r; ^3S_1) \\ \Delta V_{\text{eff}}(r; ^3S_1) &\equiv \sqrt{8} V_T(r) \frac{w(r)}{u(r)}. \end{aligned} \quad (11)$$

Here ΔV_{eff} is comparable to V_C in strength. This effective central force makes the deuteron bound. In this sense, the tensor force plays a crucial role in making the deuteron bound. This effect is at least a second order effect in terms of the tensor force, since both the initial and the final state have zero orbital angular momentum. To see how the tensor force is renormalized into a central force, we derive V_{lowk} starting from the full Argonne V8' potential (AV8' full) and tensor subtracted Argonne V8' (AV8' TS) potential. In Fig.2, the red line and the blue line represent the monopole part of V_{lowk} from AV8' full and AV8' TS, respectively, and the black line represents the bare AV8' potential. The difference between V_{lowk} from AV8' full and V_{lowk} from AV8' TS shows the effect of the renormalization of the short-range tensor force. The effect is strongly attractive in the $T = 0$ channel, which is consistent with the considerations from Eq. (11).

To understand this, consider the Schrödinger equation written as

$$\begin{pmatrix} PHP & PHQ \\ QHP & QHQ \end{pmatrix} \begin{pmatrix} P|\Psi\rangle \\ Q|\Psi\rangle \end{pmatrix} = E \begin{pmatrix} P|\Psi\rangle \\ Q|\Psi\rangle \end{pmatrix}. \quad (12)$$

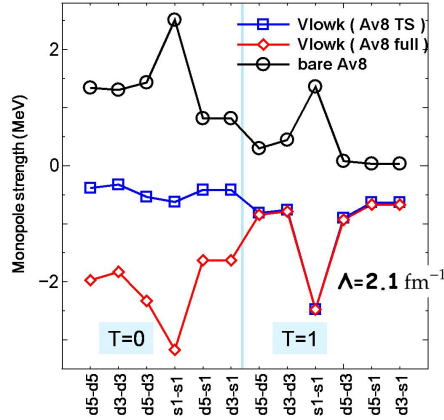


Figure 2. Effect of renormalization of short-range tensor force.

By integrating out the Q -space degrees of freedom, we obtain

$$PHP|\Psi\rangle + PHQ \frac{1}{E - QHQ} QHP|\Psi\rangle = EP|\Psi\rangle. \quad (13)$$

From this equation, we can consider an effective interaction defined in the P -space as

$$H_{\text{eff}} = PHP + PHQ \frac{1}{E - QHQ} QHP. \quad (14)$$

Equation (14) is so general that we can choose any definition of the P -space and the Q -space. If we choose the P -space and the Q -space as 3S_1 and 3D_1 respectively, the resulting H_{eff} is exactly $V_{\text{eff}}(r; ^3S_1)$ of Eq. (11), that is the effective central force in the 3S_1 channel. On the other hand, if we choose P -space and Q -space as low-momentum and high-momentum space, we obtain the low-momentum interaction $V_{\text{low}k}$.

5. Tensor force in effective interaction for the shell model

In this section we discuss the tensor force derived from the effective interaction for the shell model. We have calculated shell model effective interactions for the sd -shell and the pf -shell, by considering the \hat{Q} -box to second and third order, with folded diagram correction as explained in Sec. 2, starting from $V_{\text{low}k}$.

The \hat{Q} -box is calculated by considering valence-linked and connected diagrams with unperturbed single particle energy of the harmonic oscillator. Folded diagrams are obtained by numerical derivatives of the \hat{Q} -box with respect to the starting energy, the sum of single particle energies of two valence particles, as given in Eq. (7).

Since the Q -space is defined as the complement of the P -space, the intermediate states should be taken up to infinitely high oscillator shells, in principle. As an approximation, we define a sufficiently large Q -space in the evaluation of the \hat{Q} -box. Here the \hat{Q} -box is calculated by $V_{\text{low}k}$ with cutoff $\Lambda = 2.1 \text{ fm}^{-1}$.

Figure 3 shows the monopole part of tensor force of the effective interaction for the sd -shell. We label this effective interaction as V_{eff} . One can see again that the monopole part of tensor force of V_{eff} is fairly similar to that of the bare realistic nuclear force. These results are not trivial again, because they mean that the effects of the renormalization of the medium contributions and the truncation of the model space are not predominantly affecting the tensor force. The first-order \hat{Q} -box is just the $V_{\text{low}k}$ interaction, whose tensor force is almost equal to that of the nuclear force as discussed already in Sec. 3. Therefore, these results mean that the monopole

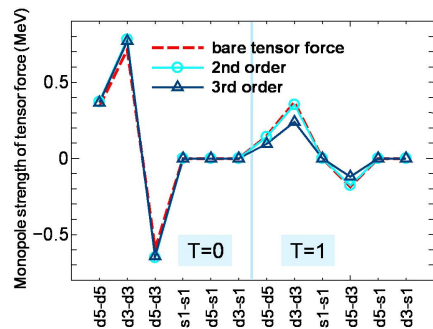


Figure 3. Monopole part of tensor force of the effective interaction for the shell model calculated with the \hat{Q} -box expansion.

part of the tensor force is dominated by the first order \hat{Q} -box term and the contributions from the other terms are comparatively small. Again, although we have only showed results from calculations in the *sd*-shell, we have obtained similar results for the *pf*-shell as well.

6. Conclusion

We conclude that the tensor force survives under both steps of the renormalization procedure, that is, the renormalization of the high-momentum component of realistic nuclear forces and the medium-renormalization needed to obtain an effective two-body interaction for a specific model space.

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