Hadronic interaction and exotic nuclei

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Abstract. The shell evolution and drip line are discussed with links to the nuclear two- and three-body forces.

Exotic nuclei provide us with new phenomena which are not found in stable nuclei. One of them is the evolution of the shell structure over a wide change of the proton number (Z) or the neutron number (N) [1, 2]. While the conventional magic numbers are valid for stable nuclei [3], the evolution indeed ends up, at extreme cases, with the appearance of new magic numbers and/or the disappearance of conventional ones. The nuclear force plays crucial roles in this evolution. As Z increases, there are more exotic isotopes between the β-stability line and the drip line, creating a wider frontier to be challenged. Most of such exotic nuclei are far inside the drip line, being well bound [4]. The driving force of changing structure should be the combination of the unbalanced Z/N ratio and the nuclear force mainly of two- and three-body nature. Thus, it is crucial to see the basic robust features of the nuclear force in exotic nuclei. We shall present, in this talk, such features governing the shell evolution. The tensor force has been shown to change the spin-orbit splitting of exotic nuclei, resulting in shifts of magic numbers [5, 6]. The two-body nuclear force is comprised also of its central part. We shall extract basic but novel features of effective nucleon-nucleon (NN) interactions, and suggest that the central and tensor forces of the effective NN interaction seem to have simple structures with their own characteristic effects. We further discuss the basic nature of the effects of the three-body force.

We start with the shell-model NN interactions which are successful in reproducing experimental data. These interactions have been obtained from so-called microscopic interactions based on NN scattering data and by including a treatment of the short-range repulsion and core polarization effects. A good example of such microscopic interactions is G-matrix [7, 8]. For successful shell-model calculations, the microscopic interaction has to be modified. This modification has been carried out for the families of the KB interaction [9] and GXPF1 interaction [10, 11].

The change of the shell structure, or the shell evolution, may have different origins. We first focus upon the shell evolution by the tensor force. It is well-known that the one-pion exchange process is the major origin of the tensor force, which is written as

$$V_T = (\vec{\tau}_1 \cdot \vec{\tau}_2)\,(\,[\vec{s}_1 \vec{s}_2]^{(2)} \cdot Y^{(2)})\, f(r), \tag{1}$$

where $\vec{\tau}_{1,2}$ ($\vec{s}_{1,2}$) denotes the isospin (spin) of nucleons 1 and 2, $[\,]^{(K)}$ means the coupling of two operators in the brackets to an angular momentum (or rank) K, Y denotes the spherical harmonics for the Euler angles of the relative coordinate, and the symbol $(\cdot)$ means a scalar product. Here, $f(r)$ is a function of the relative distance, r. Eq. (1) is equivalent to the usual expression containing the S_{12} function. Because the spins $\vec{s}_1$ and $\vec{s}_2$ are dipole operators and are coupled to rank 2, the total spin S of two interacting nucleons must be S=1. If both of the bra and ket states of V_T have L=0, with L being the relative orbital angular momentum, their matrix element vanishes because of the $Y^{(2)}$ coupling. These properties are used later.

The (spherical) bare single-particle energy of an orbit j is given by its kinetic energy and the effects from the inert core (closed shell) on the orbit j. As some nucleons are added to another orbit j', the single-particle energy of the orbit j is changed. The nucleons on j' can form various many-body states, but we are interested in monopole effects independent of details of such many-body states. The monopole component of an interaction, V, is [12]:

$$V^T_{j,j'} = \frac{\sum_J (2J+1) < j\,j'|V|\,j\,j' >_{JT}}{\sum_J (2J+1)}, \tag{2}$$

where $< j\,j'|V|\,j\,j' >_{JT}$ stands for the (diagonal) matrix element of a state where two nucleons are coupled to an angular momentum J and an isospin T. In the summation in eq. (2), J takes values satisfying antisymmetrization. We then construct a two-body interaction, called V_M, consisting of two-body matrix elements $V^T_{j,j'}$ in eq. (2). Because the J-dependence is averaged out in eq. (2), the monopole interaction, V_M, represents the angular-free, i.e., monopole property of the original interaction, V, while it still depends on the isospin. If neutrons occupy j' and one looks into the orbit j ($\neq j'$) as a proton orbit, the shift of the single-particle energy of j is given by

$$\Delta\varepsilon_p(j) = \frac{1}{2}\{\,V^{T=0}_{j,j'} + V^{T=1}_{j,j'}\,\}\; n_n(j'), \tag{3}$$

where $n_n(j')$ is (the expectation value of) the number of neutrons in the orbit j'. The same is true for $\Delta\varepsilon_n(j)$ as a function of $n_p(j')$. The monopole effects from orbits j', j'', ... are added as these orbits are filled. The single-particle energy, including this monopole effect, is called the effective single-particle energy (ESPE), and it depends on the configurations. We shall discuss, in this talk, how the ESPE of an orbit j varies due to the tensor force as an orbit j' is filled.

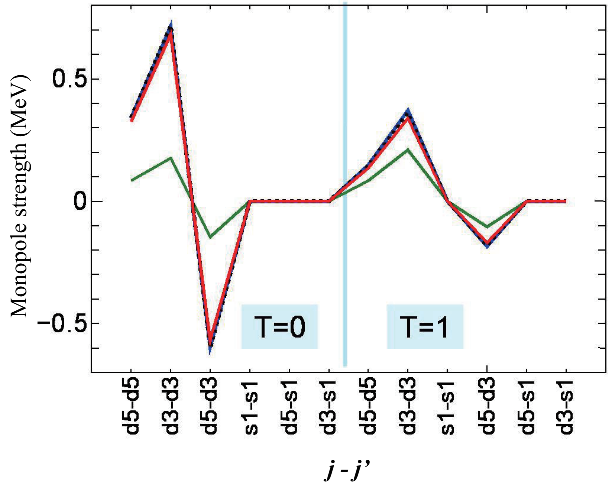

FIGURE 1. Monopole interaction of the tensor component obtained by the spin-tensor decomposition of V_{lowk} interaction from AV8' interaction [13]. The green and red solid line indicate the results of the cut-off 1.0 and 2.1 (fm^{-1}), respectively. The black dashed line is for the cut-off 4.0 (fm^{-1}), while the blue solid line denotes the bare AV8' potential. Left (Right) panel shows the results for isospin T=0 (1).

If the orbit j' is fully occupied by neutrons in eq. (3), only the monopole effect remains over the other multipoles and eq. (3) gives the shift of the bare single-particle energy for this shell closure. If protons and neutrons are occupying the same orbit, the change of ESPE becomes slightly more complicated due to isospin symmetry [9, 12].

We begin with cases with orbital angular momenta l or l', protons are in either $j_> = l + 1/2$ or $j_< = l - 1/2$, while neutrons are in either $j'_> = l' + 1/2$ or $j'_< = l' - 1/2$. In results to be presented, the radial wave functions are given by the harmonic oscillator potential for simplicity.

From now on, V is the tensor force. For the orbits j and j', the monopole component in eq. (2) is calculated. We begin with the effect of the treatment of the short-range correlation. For this purpose, we use the Argonne V8' (AV8') potential [13], while the following argument is not specific to a particular choice of the interaction. The AV8' potential contains the tensor part explicitly, and therefore is suitable for the present purpose. The AV8' potential in fact include both isoscalar and isovector tensor components, and we keep them. This is not a problem, because the following discussion is made in the isospin formalism.

We obtain a low-momentum potential V_{lowk} based on Ref. [14] from the AV8' potential [13]. By transforming a given NN interaction this way, the short-range correlation effect is renormalized, and only low-momentum properties remain. Thus, the interaction becomes free from difficulties of the short-range strong repulsion and suitable for studies of the ground and low-lying excited states, *i.e.*, states of the shell model.

We then carry out the spin-tensor decomposition [15] for the obtained potential. We thus obtain the tensor part of the V_{lowk} potential. Figure 1 indicates monopole component of the tensor part of such V_{lowk} potentials in comparison to the original AV8'. The cut-off parameter is taken to be 1.0, 2.1 and 4.0 (fm^{-1}).

The cut-off 1.0 (fm^{-1}) is too small in the momentum space, or, too large in the

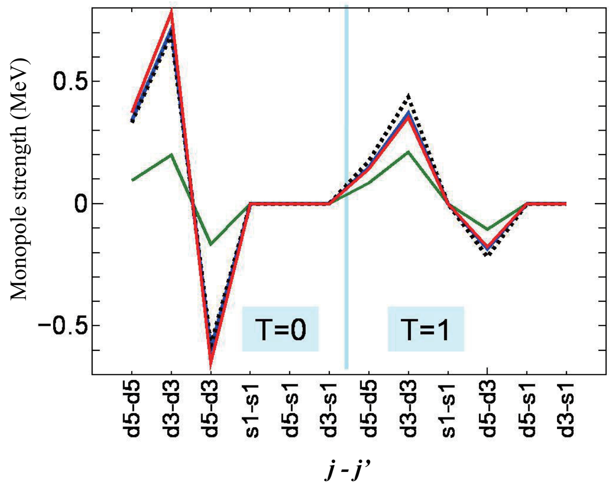

FIGURE 2. Monopole interaction of the tensor component obtained by the spin-tensor decomposition of the second order Q-box treatment of the interaction discussed in Fig. 1. See the caption of Fig. 1 for details.

coordinate space. We loose much of the tensor force effect, and the monopole component is quite small. This is, however, taken for the sake of comparison. The cut-off 2.1 (fm^{-1}) is taken from a typical low-momentum range. We can find a good agreement to the result of the AV8'. We also note that cut-off 4.0 (fm^{-1}) produces the basically identical result to the one by the cut-off 2.1 (fm^{-1}). We here conclude that the V_{lowk} treatment of the *NN* interaction conserves the tensor component to a great extent. This feature should be found in results of UCOM formalism, where the short-range contributions are treated by unitary transformations [16].

The Q-box formalism is applied to the V_{lowk} potential, in order to take into account effects of the inert core, including its polarization effect [17]. Figure 2 indicates monopole component of the tensor part of such effective interactions in comparison to the original AV8'. Here, the second order Q-box calculation has been carried out. The cut-off parameter is taken to be 1.0, 2.1 and 4.0 (fm^{-1}). It is remarkable that the results are very close to each other. except for the case with cut-off parameter 1.0 (fm^{-1}). We have already discussed for Fig. 1 that the cut-off parameter 1.0 (fm^{-1}) is extreme, and it is included in Fig. 2 for comparison. We thus find that the renormalization of various effects of the core does not change the monopole part of the tensor force in the valence shell. The V_{lowk} formalism is basically for the short-range correlation, while the monopole properties are of long-range within a shell. As for the Q-box, the particle-hole excitation due to the tensor force changes mainly the central part. The present consequences can be verified for the pf shell.

We investigate the monopole properties of phenomenologically improved shell-model effective interactions, the SDPF-M for the *sd* shell [18] and the GXPF1A for the pf shell [11]. The results cannot be shown here due to the space limitation, but one can see that the orbital variation of the monopole strength comes almost entirely from the tensor force. The rest can be explained by a simple central force with a Gaussian dependence on the relative distance. The tensor force here is the one obtained from $\pi + \rho$ meson exchange potential [19], which is the bare tensor force between free nucleons. Namely, the tensor force in the shell model effective interactions are nothing but the bare tensor force to a good extent. This remarkable conclusion is consistent with the observations made with V_{lowk} and Q box calculations, presented above.

The combination of simple Gaussian central forces and the bare tensor forces is somewhat similar to what Weinberg proposed for the interaction between free nucleons[20], which led to the Chiral Perturbation theory. In this model, the *NN* interaction is comprised of the zero-range central force and the one-pion exchange tensor force. In the shell-model effective interaction, the former becomes Gaussian central force with the range of about 1 fm, while the latter is the $\pi + \rho$ meson exchange potential. This similarity between two simple modelings is of certain interest.

The second part of this talk is about effects of the three-body force. The three-body force has been discussed, for instance, in Refs. [21, 22, 23]. We, however, present a robust repulsive effect of the three-body force. The dominant contribution to this effect originates in the Fujita-Miyazawa force [24], which incorporates the virtual excitation from a nucleon to a Delta particle. In fact, it has been argued that although the shell-model effective interaction should be repulsive in its T=1 monopole part except for the contribution from the pairing force, this property cannot be explained if one includes *NN* forces only [25].

We show [26] that the three-body force explains the drip line of oxygen isotopes at the right place, whereas the calculations including only microscopic *NN* forces, G-matrix type [8] or more modern Chiral EFT type [27, 28], predict the drip line too far. In other words, these forces are too attractive as a common general feature. This is because the Fujita-Miyazawa-type three-body force produces effective two-body interaction between valence neutrons which is robustly repulsive. This can be seen in terms of single-particle energies and also in terms of the energies of the ground states, calculated by conventional pion-nucleon-Delta coupling [29] and by Chiral EFT [30]. As the mechanism for repulsive valence-shell effective interaction is robust and general, we can predict similar effects on other regions of the nuclear chart.

On the other hand, by adding a proton to oxygen, one can create fluorine isotopes which have the drip line much further away. This is because the added proton is mainly in the $0d_{5/2}$ orbit, and produces strong attraction with neutrons in the $0d_{3/2}$ orbit, making it bound [31]. As the tensor force plays a crucial role here also, the drastic change between oxygen and fluorine isotopes is one of the prominent examples of the importance of the nuclear forces in exotic nuclei.

In summary, the shell evolutions due to the tensor and three-body forces are presented with the underlying mechanisms. The tensor and three-body forces produce general and robust effects on the shell and (sub-)magic structures from the *p*-shell to the superheavy regions. The significant role of the tensor force as a direct consequence of π and ρ meson exchange is discussed in the comparison to the well-established phenomenologically improved shell-model interactions, and is analyzed in the light of V_{lowk} and Q box theories. We end up with a simple Ansatz that the shell-model effective interaction is comprised of simple Gaussian central and bare tensor forces [32]. This picture can be related to the Chiral Perturbation idea of Weinberg [20].

The shell evolution is also due to the three-body force. The three-body force is dominated by Fujita-Miyazawa force in its long-range part, and indeed produces decisive repulsive effects on the structure of exotic nuclei. As an example, we presented the case of oxygen isotopes [26].

This work was supported in part by a Grant-in-Aid for Scientific Research (A) (20244022) from the MEXT, and also by the JSPS core-to-core program “International Research Network for Exotic Femto Systems (EFES)”. KT thanks JSPS for a fellowship. This work was supported in part also by the Natural Sciences and Engineering Research Council of Canada (NSERC). TRIUMF receives funding via a contribution through the National Research Council Canada.

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