Width of the Δ resonance in nuclei

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In this work we evaluate the imaginary part of the isobar Δ self-energy Σ_Δ from the two-body absorption process $\Delta + N \to 2N$. This contribution is calculated using a recently developed nonrelativistic scheme, which allows for an evaluation of the self-energy with a basis of single-particle states appropriate for both bound hole states and for particle states in the continuum. In order to test the medium dependence of the self-energy, we calculate the two-body absorption term Σ_Δ^{A2} for several finite nuclei with $N = Z$, i.e., ^{16}O, ^{40}Ca, and ^{100}Sn. The resulting self-energy, which is energy dependent and nonlocal, is compared with a simple parametrization derived from nuclear matter.

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The self-energy of the Δ resonance is central in the understanding of nucleus-nucleus collisions, pion-nuclear processes, and photonuclear reactions at intermediate energies, see e.g., Refs. [1–3] for recent reviews. The self-energy $\Sigma_\Delta(\omega, k, k')$ is in general a nonlocal and energy dependent operator where ω and k, k' are the energy and momenta of the isobar, respectively.

Within the framework of perturbative many-body theories, various contributions to the self-energy can pictorially be represented by so-called Feynman diagrams, of which examples are shown in Fig. 1. Experimental data for pion scattering [4] and absorption at various energies covering the isobar resonance region, suggest that the dominant process for absorption of pions at low energies is represented by diagram (a), which couples the isobar to two-nucleon one-hole states ($\Delta + N \to 2N$). Within the terminology of the isobar-hole model [3,5,6], diagram (b) is then supposed to represent the rescattering of e.g., a real pion, so-called reflection contributions to quasifree scattering. Diagram (c) is an example of a self-energy contribution arising from three-body absorption mechanisms.

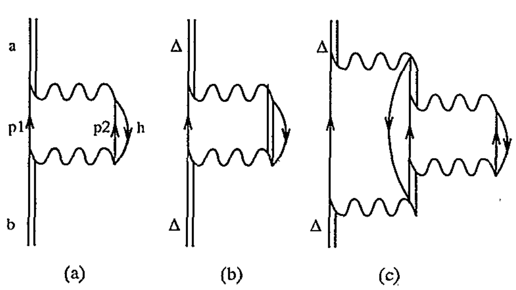

FIG. 1. Example of diagrams which arise in the evaluation of the self-energy of the isobar. (a) is the two-body absorption term evaluated in this work, (b) is an example of a so-called reflection contribution to quasifree scattering, and (c) stands for a three-body contribution. The wavy line represents the $G_{N\Delta}$ matrix, the single line is a nucleon, and the double line stands for the isobar Δ.

There is no such thing as an experimental measurement of the Δ self-energy, although indirect information about the self-energy can be derived from pion-nucleus scattering, see, e.g., Refs. [2,7,8] and references therein. In the extensive analyses of Ref. [9], contributions arising from diagrams like (a) and (b) in Fig. 1, are represented by way of a Δ-spreading potential, fitted to provide best results for pion-nucleus elastic scattering.

The authors of Ref. [10] evaluate the imaginary part of the Δ self-energy by considering the contributions from the diagrams in Fig. 1, accounting thus for quasielastic corrections, two-body and three-body absorption. The calculations were carried out in nuclear matter. The nuclear matter results are then compared to the Δ-spreading potential from the empirical determination in Ref. [9] by allowing for a density dependent self-energy, which is decomposed into various terms by

$$\mathrm{Im}\Sigma_\Delta = \mathrm{Im}\Sigma_\Delta^{A2} + \mathrm{Im}\Sigma_\Delta^{A3} + \mathrm{Im}\Sigma_\Delta^{Q}. \qquad (1)$$

The term Q accounts for the quasielastic part while $A2$ is the two-body absorption part. $A3$ represents three-body absorption. A numerical parametrization for these terms was given by Oset and Salcedo [10], while analytical expressions based on the results of Ref. [10] were recently presented by Nieves *et al.* [11]. In the latter work the Σ_Δ^{A2} term of Eq. (1) has been calculated for a "typical" Δ, which is excited in the Δ-hole model, when a pion of a certain kinetic energy is absorbed. This parametrization reads

$$\mathrm{Im}\Sigma_\Delta^{A2}(x) = \mathrm{Im}\alpha_\Delta(x)\frac{1}{\beta(x)}\arctan\left[\beta(x)(\rho/\rho_0)\right], \qquad (2)$$

with $x = \frac{T_\pi}{m_\pi}$, T_π and m_π being the kinetic energy and mass of the pion, respectively. ρ is the density of particles and ρ_0 the corresponding density at nuclear matter saturation. Further, the function $\mathrm{Im}\alpha_\Delta$ is

$$\mathrm{Im}\alpha_\Delta(x) = -38.3\left(1 - 0.85x + 0.54x^2\right)\mathrm{MeV}, \quad (3)$$

and

$$\beta(x) = 2.72 - 4.07x + 3.07x^2. \quad (4)$$

The parametrization of Eq. (2) can be interpreted as the width of a Δ, which is excited when an energy $\omega = T_\pi + m_\pi$ is deposited at a nucleon in nuclear matter of density ρ.

The intention behind this Brief Report is to study the Σ_Δ^{A2} term directly for finite nuclei, since this contribution is expected to be the dominant one at energies below the isobar resonance [10]. For that purpose we will here employ a recently developed method to calculate the self-energy of the Δ in a finite nucleus. Microscopic calculations of the nucleon or Δ self-energy have commonly been carried out in nuclear matter, the results of Ref. [10] being one example. One of the advantages of studies in nuclear matter is the possibility to describe the single-particle wave functions by plane waves. For a microscopic calculation in finite nuclei one has to take into account the fact that one needs different representations for bound hole states and particle states in the continuum. A method which allows for this has recently been developed [12,13]. The bound hole states are described in terms of harmonic oscillator (HO) wave functions while particle states are given by plane waves. The basic ingredients in our microscopic calculation of Σ_Δ^{A2} are briefly outlined below. The finite nuclei we consider are ^{16}O, ^{40}Ca, and ^{100}Sn.

In order to calculate Σ_Δ^{A2}, we need first to define the transition potential $V_{NNN\Delta}$ for the $\Delta + N \to 2N$ process. For a nucleon and an isobar Δ interacting through the exchange of π plus ρ mesons, the transition potential $V_{NNN\Delta}$ is usually written, in the static nonrelativistic limit, as [3]

$$V_{NNN\Delta}(\mathbf{k}) = -\left\{D_\pi^{N\Delta}(\mathbf{k})\frac{f_{\pi NN}f_{\pi N\Delta}}{m_\pi^2}\boldsymbol{\sigma}\cdot\mathbf{k}\mathbf{S}\cdot\mathbf{k} + D_\rho^{N\Delta}(\mathbf{k})\frac{f_{\rho NN}f_{\rho N\Delta}}{m_\rho^2}\boldsymbol{\sigma}\times\mathbf{k}\cdot\mathbf{S}\times\mathbf{k}\right\}\boldsymbol{\tau}\mathbf{T}, \quad (5)$$

with $S(T)$ the transition matrix which creates a spin (isospin) 3/2 object from a spin (isospin) 1/2 one. The meson propagator $D_{\pi,\rho}(\mathbf{k})$ is defined as in Ref. [13] as

$$D_{\pi,\rho}^{N\Delta}(\mathbf{k}) = \frac{1}{2}\left(\frac{1}{m_{\pi,\rho}^2 + \mathbf{k}^2} + \frac{1}{m_{\pi,\rho}^2 + \mathbf{k}^2 + m_{\pi,\rho}(m_\Delta - m_N)}\right). \quad (6)$$

The coupling constant for the π meson is given by a relation obtained from the nonrelativistic quark model [14]

$$f_{\pi N\Delta} = \frac{6}{5}\sqrt{2}f_{\pi NN} = \frac{6}{5}\sqrt{2}g_{\pi NN}\frac{m_\pi}{2m_N}, \quad (7)$$

and similarly for the ρ meson we have

$$f_{\rho N\Delta} = \frac{f_{\pi N\Delta}}{f_{\pi NN}}g_{\rho NN}\frac{m_\rho}{4m_N}\left(1 + \frac{f_{\rho NN}}{g_{\rho NN}}\right), \quad (8)$$

with

$$f_{\rho NN} = \sqrt{4\pi}g_{\rho NN}\frac{m_\rho}{m_N}\left(1 + \frac{f_{\rho NN}}{g_{\rho NN}}\right). \quad (9)$$

In addition we include monopole form factors in order to regularize the potentials at short distances. The cutoff masses are $\Lambda_\pi = 1.2$ GeV and $\Lambda_\rho = 1.3$ GeV, while the coupling constants are $g_{\pi NN}^2/4\pi = 14.6$ and $g_{\rho NN}^2/4\pi = 0.95$, which are equal to the parameters which define the Bonn B nucleon-nucleon potential V_{NN} of Table A.2 in Ref. [15]. Further, $\frac{f_{\rho NN}}{g_{\rho NN}} = 6.1$. Thus, the parameters which define the $V_{NNN\Delta}$ potential, agree with those used in the Bonn B potential, since this potential is used to calculate the $G_{N\Delta}$ matrix, the next ingredient in our calculations. The reader should, however, observe that there is a great deal of uncertainty in the definitions of both the $N\Delta\pi$ coupling constant and the cutoff masses. Choices for the cutoff mass range from a few hundred MeV [16] up to 2 GeV [10]. Moreover, coupling constants deduced from Δ width are up to 20% larger than the one deduced from the quark model. Note that a reduction of the coupling constant by 20% reduces the result for the self-energy by almost 40%. Finally, the effects of two-particle correlations are taken into account in different ways. We try to treat all these ingredients consistently by employing the parameters of the Bonn potential and evaluating also the correlation effects from the same source.

To calculate the $G_{N\Delta}$ matrix, we need first to evaluate the nucleon-nucleon G-matrix G_{NN}. This is done by solving the Bethe-Goldstone equation

$$G_{NN}(\Omega) = V_{NN} + V_{NN}Q\frac{1}{\Omega - QH_0Q}QG_{NN}(\Omega). \quad (10)$$

Here V_{NN} is the free nucleon-nucleon interaction. In this work V_{NN} is defined by the parameters of the Bonn B potential in Table A.2 of Ref. [15]. The term H_0 is the unperturbed Hamiltonian. This equation is solved with an angle-average nuclear matter Pauli operator Q with a fixed starting energy $\Omega = -10$ MeV and a Fermi momentum $k_F = 1.4\ \mathrm{fm}^{-1}$. From the nucleon-nucleon G_{NN} matrix, we can evaluate the $G_{N\Delta}$ matrix [13] through the relation

$$G_{N\Delta}(\Omega) = V_{NNN\Delta} + V_{NNN\Delta}Q\frac{1}{\Omega - QH_0Q}QG_{NN}(\Omega). \quad (11)$$

Having accounted for the short-range correlations through the introduction of the $G_{N\Delta}$ matrix, we are then able to set up the expression for the imaginary part of Σ_Δ^{A2}

$$\mathrm{Im}\Sigma_{\Delta}^{A2}(j_b l_b k_b k_a \omega) = -\frac{1}{2(2j_b+1)} \sum_{n_h l_h j_h} \sum_{JT} \sum_{lLS\mathcal{J}} \int k^2 dk \int K^2 dK \hat{J}\hat{T} \langle k_a l_b j_b n_h l_h j_h JT | G_{N\Delta} | klKL(\mathcal{J})SJT\rangle \times \langle klKL(\mathcal{J})SJT | G_{N\Delta} | k_b l_b j_b n_h l_h j_h JT\rangle \times \pi\delta\Big(\omega + \varepsilon_h - \frac{K^2}{4M_N} - \frac{k^2}{M_N}\Big), \tag{12}$$

The single-hole energy ε_h is given by the eigenvalues of the harmonic oscillator minus a constant shift to place the Fermi energy at zero, while the energies of the particle states are represented by the pure kinetic energy. The variables k, K are the relative and center-of-mass momenta of the intermediate particle states p_1 and p_2 in Fig. 1. Further, l and L are the corresponding orbital momenta of the relative and center-of-mass motion. S, J, and T are the total spin, total angular momentum, and isospin, respectively. Finally, M_N is the average proton and neutron masses. A HO single-particle state is defined by the quantum numbers $n_h l_h j_h$, while plane waves are defined by $k_a l_a j_a$. For further details, see Refs. [13,17]. In Ref. [17] a prescription for orthogonalizing the intermediate particle states to the hole state is discussed. The authors of Ref. [17] find these corrections for the $2p1h$ diagram of the nucleon self-energy in ^{16}O to be rather small. Because of the uncertainty in coupling constants and cutoff masses, these corrections are neglected. The orthogonalization could lead to a sizable reduction in particular for the heavier nuclei.

The energy variable ω refers to the energy of the Δ relative to the mass of a nucleon. Only positive energies ω contribute, as can be deduced from the δ function in Eq. (12).

To study the medium dependence of $\mathrm{Im}\Sigma_{\Delta}^{A2}$, we evaluate Eq. (12) for the nuclei ^{16}O, ^{40}Ca, and ^{100}Sn. The medium dependence of Eq. (12) is accounted for by the summation over single-hole states, represented by the $0s_{1/2}$, $0p_{1/2}$, and $0p_{3/2}$ single-hole states in ^{16}O, $0s_{1/2}$, $0p_{1/2}$, $0p_{3/2}$ $1s_{1/2}$, $0d_{3/2}$, and $0d_{5/2}$ single-hole states in ^{40}Ca, and $0s_{1/2}$, $0p_{1/2}$, $0p_{3/2}$ $1s_{1/2}$, $0d_{3/2}$, $0d_{5/2}$ $1p_{1/2}$, $1p_{3/2}$, $0f_{5/2}$, $0f_{7/2}$, and $0g_{9/2}$ single-hole states in ^{100}Sn. Moreover, the oscillator parameters used in the calculation of the single-hole wave functions are 1.72 fm for ^{16}O, 2.04 fm for ^{40}Ca, and 2.20 fm for ^{100}Sn.

In the discussion presented here, we only consider Δ isobar states with orbital angular momentum $l_b = 0$. Guided from our experience in evaluating the imaginary part of the nucleon self-energy we assume that the global features are similar for Δ isobar states with larger angular momenta. A Fourier transformation of $\mathrm{Im}\Sigma_{\Delta}^{A2}$ in Eq. (12) leads to an imaginary part, which depends on energy ω and is nonlocal in the coordinate r, the distance from the center of the nucleus. From the inspection of this function we observe that the nonlocality is weak in the sense that it is different from zero only for distances r and r', which are close to each other. Therefore it makes sense to look at the local component of $\mathrm{Im}\Sigma_{\Delta}^{A2}$ for the various energies as a function of the distance r [17]. As an example we present in the left part of Fig. 2 this local approximation obtained for ^{40}Ca. The shape of these functions is not really identical to a Woods-Saxon shape or a conventional density distribution. In particular at lower energies (ω below 200 MeV) one observes a clear surface contribution to $\mathrm{Im}\Sigma_{\Delta}^{A2}$. Similar results are also obtained for the other nuclei (see right side of Fig. 2).

Finally, in Fig. 3, we compare the imaginary part of the Δ self-energy calculated in the local approximation for a typical radius of r=1.5 fm at various energies ω with the parametrization of Nieves *et al.* [11]. This comparison must be considered with some care. As discussed above, the parametrization of Eq. (2) represents the average imaginary part of a Δ, which is typically excited, when a pion is absorbed in nuclear matter of density ρ, depositing an energy $\omega = T_\pi + m_\pi$. On the other hand, the results for finite nuclei show a nontrivial radial dependence and Fig. 3 just displays results for one "typical" radius. The nuclear matter parametrization is presented for a density ρ= 0.75 ρ_0 which is the average density of

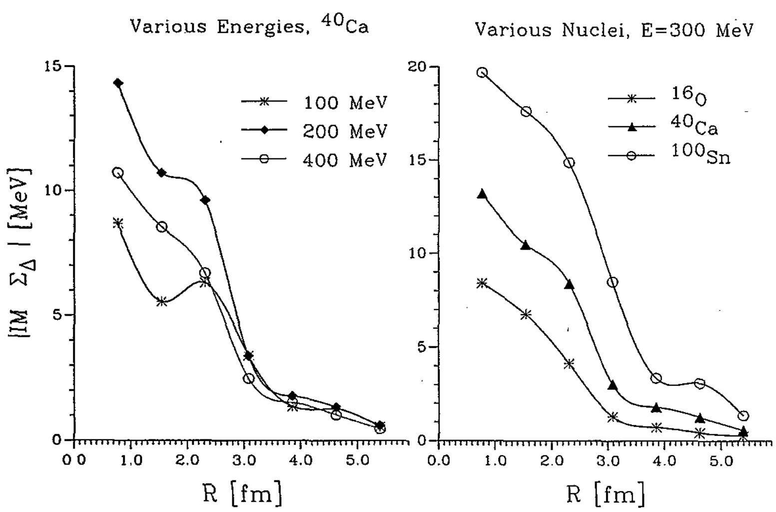

FIG. 2. Local representation of the imaginary part of $-\Sigma_{\Delta}^{A2}$ as function of the distance r from the center of the nucleus. In the left part of the figure results are shown for the nucleus ^{40}Ca, considering the energies ω = 100, 200, and 400 MeV. In the right part of the figure the results are displayed for the nuclei ^{16}O, ^{40}Ca, and ^{100}Sn assuming an energy ω=300 MeV.

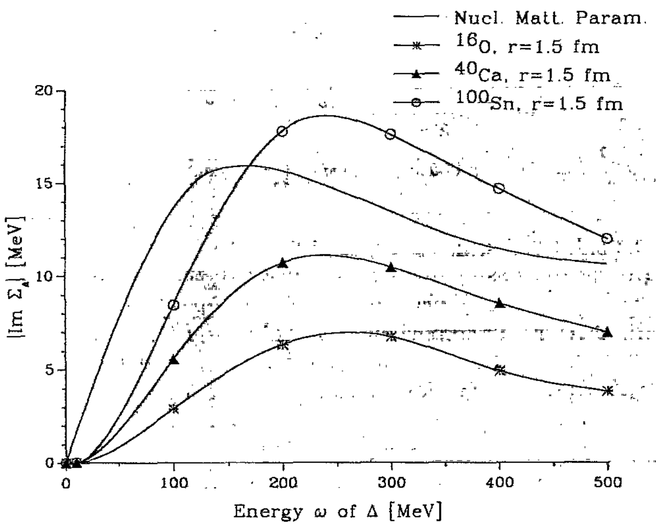

FIG. 3. Strength of the imaginary part of $-\Sigma_{\Delta}^{A2}$ at various energies. Results obtained for the local representation at r=1.5 fm, derived from the microscopic calculation of the Δ self-energy in the finite nuclei ^{16}O, ^{40}Ca, and ^{100}Sn are compared to the parametrization of Nieves *et al.* [11], displayed in Eq. (2) for a density $\rho = 0.75\rho_0$. This parametrization has been extrapolated from $\omega = m_\pi$ to ω=0.

nucleons in ^{100}Sn. The agreement between the microscopic calculation for this nucleus and the parametrization, which is based on studies of nuclear matter is remarkable. This is true for both the absolute value as well as the shape of the energy-dependence. Only for the lightest nucleus which we considered, ^{16}O, the calculated width lies considerably below the parametrization. These results should, however, be considered with some care. Although Oset and Salcedo [10] use the same coupling constants and cutoff mass for the pion, they differ in Λ_ρ, which in Ref. [10] is set to 2.0 GeV. With such a choice in our calculations, the results for the lightest nuclei would agree better with the parametrization of Ref. [10]. This parametrization has also been rather successful in the description of pion-nucleus scattering for light nuclei, see also [7,8]. An enhancement of our results due to the use of the stronger $N\Delta$ transition potential of Ref. [10] may partly be compensated for the heavy nuclei by the orthogonalization of the plane wave particle states to the HO hole states, discussed above. However, in order to be consistent with the parameters which define our NN potential, we adopt the value $\Lambda_\rho = 1.3$ GeV.

In conclusion we would like to point out that our microscopic evaluation of the Δ-spreading potential in finite nuclei supports the parametrization of [11], which is based on studies of nuclear matter. For low energies, however, it may be important to consider a surface enhancement of the imaginary part in the self-energy of the Δ isobar.

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