Name:

PROJECT: BURGERS Equation
Due November 6th 11.59 pm

Inviscid Burgers equation was simplified from the Navier-Stokes equation by just dropping the pressure term and diffusion term. The equation is shown below:

$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$ Which also could be written as $\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial x} = 0$

1. Solve the inviscid Burger's equations using Lax method. Use 6th-order finite difference for the interior points and 5th-order finite difference for the boundary points. The exact solution of the inviscid Burger's equation is given as:

$$u(x,t) = -U \tanh\left[k(x - ut)\right]$$

The initial conditions are given as : $u(x,0) = -U \tanh\left[k\, x(i)\right]$

Where k is the wave number. Solve for k= 1 and 5 . The domain is x = (0,1)

a. Show the numerical stencil for the interior and boundary points.
b. Pick CFL = 0.8, Compare the exact and numerical solution at different times for k=1. Repeat for k=5.
c. perform the convergence analysis at a fixed time, and show the order of the scheme is 6 (Use Richardson's extrapolation method with 3 grids).

   Explain the results.

2. Add the viscous term and solve the Viscous Burger's equation. Discretize the viscous term with a 6th order accurate scheme.

   The viscous term is given as: $\vartheta \frac{\partial^2}{\partial x^2} u(x,t)$ Pick, υ =0.01

The initial condition are:

$$u_0(x) = e^{-(2(x-1))^{\wedge}2}$$

The boundary conditions are: $u(0,t) = u(L,t) = 0.$, L=1

The domain is x = (0, 1)

a. Show the numerical stencil for the interior and boundary points.
b. Plot the solution vs. time at x= 0.5.
c. Plot the solution vs. x at different times of the solution.

Explain the results.

3. Use the viscous Burger's equation solver, assume υ=0 and solve the linear advection equation with c= 0.5

The initial conditions are as follows

$$u(x,t=0) = 2 + u_o(x)[1 + 0.3\sin(\frac{2\pi x}{9\Delta x})][1 + 0.4\sin(\frac{2\pi x}{10\Delta x})]$$

$$u_o(x) =$$

$$-1\ldots 8 \le x \le 28$$

$$= 1..28 < x \le 39$$

$$= 0..otherwise$$

Solve the linear advection (wave equation) using 1. Upwind method. 2. Leap Frog method for the domain x= (0,50).
Plot the solution vs. x at different times.