**Fall 2024** Due: October 4 th 2024

**Computational Fluid Dynamics ME 5653**
**Project- I Parabolic 1D Heat Conduction**

1. **(1) and (2) questions of your homework needs to be submitted electronically as a report. It should include the matlab code with comments. The figures needs to be labeled (variables on axis, legend, etc). Make sure you include brief explanation of the results.**
2. **You can submit Question (3) either electronically or hard-copy.**

1. For the following 1-D heat transfer equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

There exists a 0.3m thick wall of infinite lateral extent that is initially at a uniform temperature To= 100K. The surface temperature of the two sides is suddenly increased to and maintained at 300K. $\alpha = 3*10^{-6} m^2 s^{-1}$. The analytical solution is given as

$$T(x,t) = T_{side} + 2(T_{initial} - T_{side}) \sum_{m=1}^{\infty} [e^{-(m\pi/L)^2 \alpha t} \frac{1-(-1)^m}{m\pi} \sin(\frac{m\pi x}{L})]$$

Where $0<x<L$. The size of $m$ determines the accuracy of the solution.

(a) Using FTCS (forward-time central-space) solve heat equation to t=30minutes. Plot your solution for all x points every 5 minutes using $\Delta x$=0.015m and $\Delta t$ = 20 and 60 seconds. Explain your results.

(b) Using the exact solution given above, determine the convergence rate (|error|) for this scheme by running a series of experiments in which $\Delta x$ is successively refined over a wide range of values. Remember to vary $\Delta t$ such that **s =1/2** Here $s = \alpha \Delta t/(\Delta x)^2$. Plot absolute value error vs. spatial resolution on the log-plot.

(c) Compare the truncation error vs grid size s=1/2 and s=1/6. Compare the solution with s=1/2, s=1/6 and s=1 at t=30 minutes (plot on same figure). You can pick any grid size. Comment on the results.

(d) Plot the *contours of the solution* with $x$ on x-axis and $t$ on the y-axis. Show the contour legend. Interpret the figure.

2. Repeat the above problem (a,b,c,d) using Crank-Nicolson (C-N) scheme. Write down your conclusions on the all the differences between the FTCS and C-N scheme. What are your recommendations for solving 1-D heat diffusion problem?

3. For the 1D heat equation, show the stencil and the matrix form for Dufort-Frankel scheme. Perform Von-Neumann stability analysis and shown that it is unconditionally stable.