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Project 2
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1.a Define:

$$a_{ij} = \begin{cases} 1, & \text{i, j don't know each other} \\ 0, & \text{otherwise,} \end{cases} \qquad x_i = \begin{cases} 1, & \text{i is in the gang} \\ 0, & \text{otherwise.} \end{cases}$$

Model:

$$\begin{aligned} \text{Max:} \quad & \sum_{i=1}^{n} x_i \\ \text{s.t.} \quad & x_i + x_j \leq 1 \quad \text{for all } a_{ij} = 0, \quad i, j = 1, \cdots, n. \end{aligned}$$

1.b Define:

$$b_{ij} = \begin{cases} 1, & \text{i, j know each other} \\ 0, & \text{otherwise,} \end{cases} \qquad y_i = \begin{cases} 1, & \text{i is informant} \\ 0, & \text{otherwise.} \end{cases}$$

Model:

$$\begin{aligned} \text{Min:} \quad & \sum_{i=1}^{n} y_i \\ \text{s.t.} \quad & \sum_{i} b_{ij} y_j \geq 1, \quad \forall j = 1, \cdots, n. \end{aligned}$$

2. Let V denote the set of all vertices of G and E denote the set of all edges of G. Define:

$$a_{ij} = \begin{cases} 1, & \text{edge i is connected to vertex j} \\ 0, & \text{otherwise,} \end{cases} \qquad x_i = \begin{cases} 1, & \text{vertex i is in the vertex-cover S} \\ 0, & \text{otherwise.} \end{cases}$$

Model:

$$\begin{aligned} \text{Min.:} \quad & \sum_{i \in V} x_i \\ \text{s.t.} \quad & \sum_{j \in V} a_{ij} x_j \geq 1, \quad \forall i \in E. \end{aligned}$$

Note that for any two edges i, j in the matching M, i, j cannot be connected to the same vertex. Therefore, each vertex in the graph G can only be connected to at most one edge in M. We know that vertices in S are connected to all edges in G, and vertices in $G \setminus S$ share same edges with vertices in S. Since each of them can only be connected to at most one edge in M, we conclude that

$$max|M| \leq min|S|.$$

3.

$$\begin{aligned} \text{Max.:} \quad & x + y \\ \text{s.t.} \quad & y > \frac{1}{4} \\ & y < \frac{3}{4} \\ & x, y \in \mathbb{Z}. \end{aligned}$$

4. Suppose we have a powerful software that solves TPS problem. To solve problem 4, we add weights to edges. To be more specific, we define:

$$a_{ij} = \begin{cases} 1, & \text{if city i and j are connected by a highway} \\ \infty, & \text{otherwise} \end{cases}$$

Wlog, assume we have n cities and let K_n be the set contains all n cites as vertices. We then do the same thing as in the traveling sales man problem. That is: define

$$x_{ij} = \begin{cases} 1, & \text{edge i-j is in the cycle} \\ 0, & \text{otherwise.} \end{cases}$$

Model:

$$\begin{aligned} \text{Min.:} \quad & \sum_{i,j=1}^{n} a_{ij} x_{ij} \\ \text{s.t.} \quad & \sum_{i=1}^{n} x_{ij} = 2, \forall j = 1, \cdots, n \\ & \sum_{\{i,j\} \in S} x_{ij} \leq |S| - 1, \text{ for all } S \text{ in } K_n \text{ with } 3 \leq |S| \leq n - 3. \end{aligned}$$

7. The output will be of the form $x\#m\#n\#k$, which stands for the position $\{m+1, n+1\}$ with its possible value $k+1$
For example, x#0#0#3 means that the possible value for the position $\{1,1\}$ is 4.