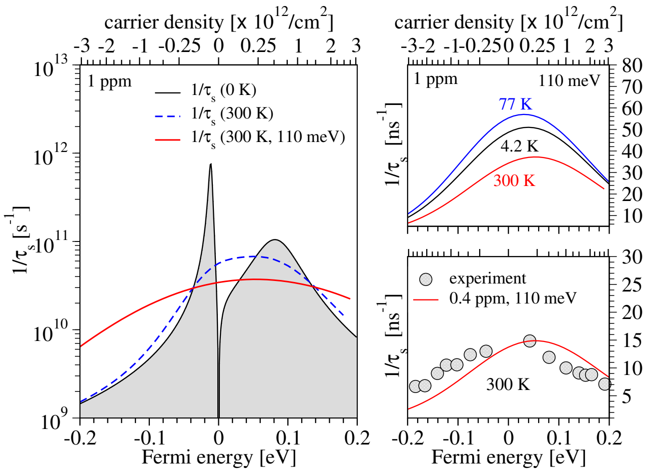

FIG. 2. Same as Fig. 1 of the paper, but with $J = 0.4$ eV. (a) Spin relaxation rate $1/\tau_s$ at zero temperature, at 300 K, and at 300 K and broadened by puddles with 110 meV Dirac point fluctuations. The rates are plotted as functions of energy and (top scale) electron density. The impurity concentration is 1 ppm. (b) Spin relaxation rates, broadened by puddles, at different temperatures. (c) Comparison with experiment, for 0.4 ppm of impurities.

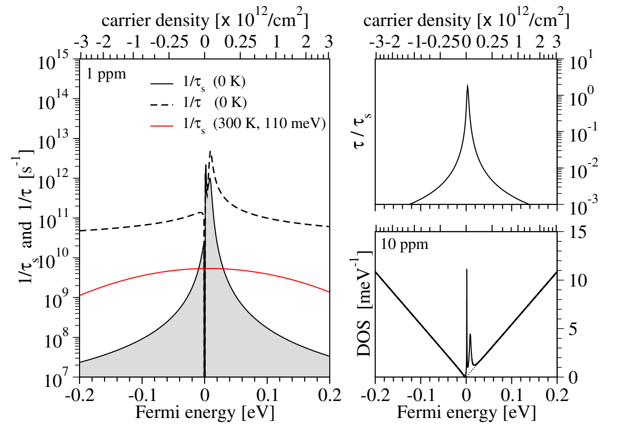

FIG. 3. Spin relaxation rate for $J = -0.04$ eV. (a) Spin relaxation $1/\tau_s$ and spin-preserving $1/\tau$ rates as functions of energy and electron density (top scale). The spin relaxation rate is also shown at 300 K and broadened by puddles with 110 eV energy broadening. The calculations are for 1 ppm impurities. (b) The ratio of the spin-flip and spin-preserving rates. (c) Density of states for 10 ppm of impurities.

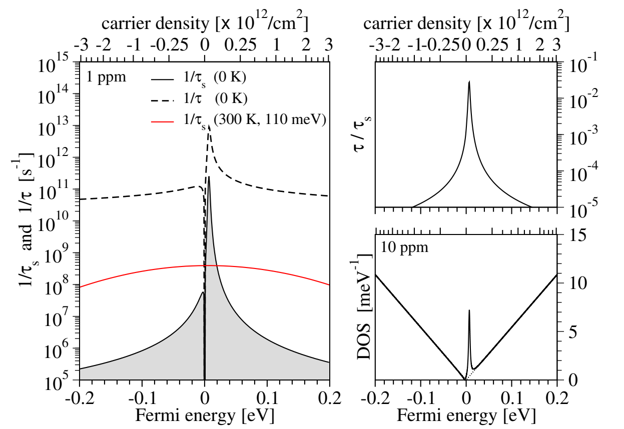

FIG. 4. Same as in Fig. 3 but for $J = -0.004$ eV.

Robustness of spin flip resonant scattering.

In Fig. 2 we show the results for the spin relaxation rates for a ferromagnetic coupling of $J = 0.4$ eV. The singlet and triplet peaks are flipped relative to Fig. 1 of the paper, which is for the antiferromagnetic coupling. Although in a clean system at low temperatures the peaks could be observed, the presence of puddles (or different types of magnetic impurities) wash out the peak structure. Apart a minor detail the results are quantitatively similar to the antiferromagnetic case, namely, the averaged spin relaxation rate is skewed towards positive energies, reflecting the flipped singlet and triplet peaks.

In the paper we claim that the spin relaxation rate is not sensitive to the precise value of J as long as $|J| \gtrsim \Gamma$, where Γ is the resonance width. In Figs. 3, 4, and 5, we plot $1/\tau_s$ and spin-preserving rate $1/\tau$ for exchange couplings $J = -0.04$ eV, $J = -0.004$ eV, and $J = -0.0004$ eV. For $J = -0.04$ eV, the spin relaxation rate for 1 ppm of impurities is still about 100 ps, since at resonance the value of $1/\tau_s$ is unchanged (compared to $J = -0.4$ eV). This lower value of J could also be used to explain the experiment! The singlet and triplet peaks are still resolved. In Fig. 3(b) we plot the ratio of the spin-flip to spin-conserving rates. At resonances, this ratio is about one, meaning that we are still in the regime of $J \gtrsim \Gamma$, and not in the perturbative regime (which is visible outside of the resonance peaks). In Fig. 3(c) we plot the density of states around the Dirac point. The split peak is visible.

If the exchange J further decreases (Γ is of order of 10 meV), although the resonance peaks are present, the spin-flip scattering rates become smaller than the spin-preserving rates, as expected from conventional perturbative (Fermi golden rule) scattering. In particular, it is expected that if $|J| < \Gamma$, the ratio τ/τ_s decreases as J^2 with decreasing J. This is evident in Figs. 4 and 5. *This is also the regime of spin-orbit coupling.* Say, if an adatom induces local spin-orbit coupling of 1 meV, which is reasonable, then resonance scattering would enhance the spin-orbit spin relaxation rate roughly as 1/16 ($1/4^2$) compared to what is shown in Fig. 4 (which is for $|J| = 4$ meV). For a 1 ppm of adatoms the spin relaxation time would be 10-100 nanoseconds. The experimental rates could then be achieved by $\eta \approx 10^{-4} - 10^{-3}$ spin-orbit coupling inducing resonant adatoms.