FİZ112E: General Physics-2

Spring 2024

Gauss's Law

Dr. Serpil Yalcin Kuzu
skuzu@firat.edu.tr

Content: Gauss's Law

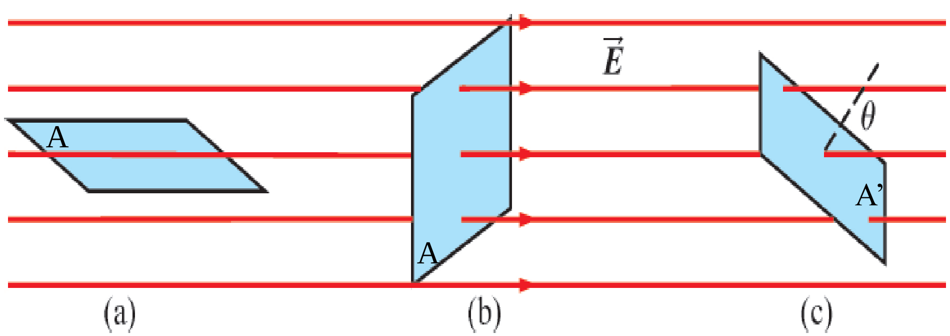
- Electric Flux
- Gauss's Law
- Application of Gauss's Law to Various Charge Distributions
- Conductors in Electrostatic Equilibrium

INTRODUCTION

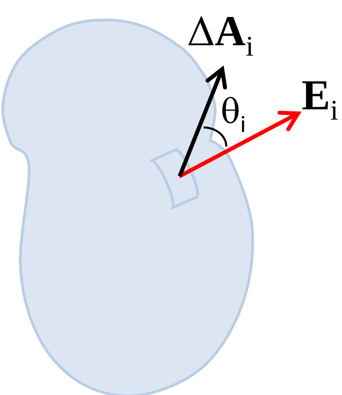
- We showed how to calculate the electric field generated by a given charge distribution.
- Today we will describe Gauss's law and an alternative procedure for calculating electric fields.
- The law is based on the fact that the fundamental electrostatic force between point charges exhibits an inverse-square behavior.
- Gauss's law is more convenient for calculating the electric fields of highly symmetric charge distributions and makes possible useful qualitative reasoning when dealing with complicated problems.

ELECTRIC FLUX

- Consider an electric field that is uniform in both magnitude and direction.
- The field lines penetrate a rectangular surface of area A, whose plane is oriented perpendicular to the field.
- the number of lines per unit area is proportional to the magnitude of the electric field.
- Therefore, the total number of lines penetrating the surface is proportional to the product EA.
- This product of the magnitude of the electric field **E** and surface area **A** perpendicular to the field is called the electric flux Φ_E.
- Φ_E has units of Nm^2/C.
- **Electric flux is proportional to the number of electric field lines penetrating some surface.**



$\Phi_E = 0$ $\Phi_E = EA$ $\Phi_E = EA' = EA\cos\theta$



$$\Delta\Phi_E = E_i \Delta A_i \cos\theta_i$$
$$= \mathbf{E_i} \cdot \Delta\mathbf{A_i}$$
$$\Phi_E = \lim_{\Delta A_i \to 0} \sum \mathbf{E_i} \cdot \Delta\mathbf{A_i}$$
$$\Phi_E = \int_{surface} \mathbf{E} . d\mathbf{A}$$
$$\Phi_E = \oint \mathbf{E} . d\mathbf{A} = \oint E_n . dA$$

Karaoglu, B. (2015). *fizik2-15 Gauss* [2]. Seckin Yayincilik.https://www.seckin.com.tr/kitap/941919955

ELECTRIC FLUX

EXAMPLE 1: What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of +2.00 μC at its center?

$$E = k_e \cdot \frac{q}{r^2} = (8.99 \times 10^{9}\ Nm^2/C^2) \cdot \frac{2 \times 10^{-6}\ C}{(1.0\ m)^2}$$

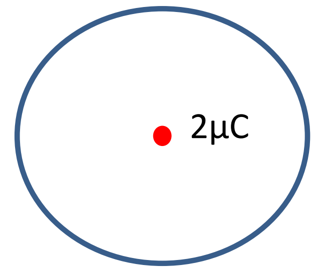
$$E = 17980\ N/C$$

$$\Phi_E = E . A \qquad A = 4\pi r^2 \text{ (area of sphere)}$$

$$A = 4 . (3.14) . (1.0m)^2 = 12.56 m^2$$

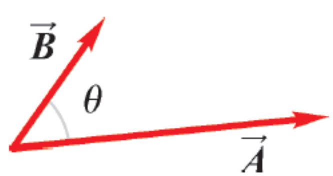
$$\Phi_E = (17980\ N/C) . (12.56\ m^2)$$

$$\Phi_E = 225828.8\ Nm^2/C$$



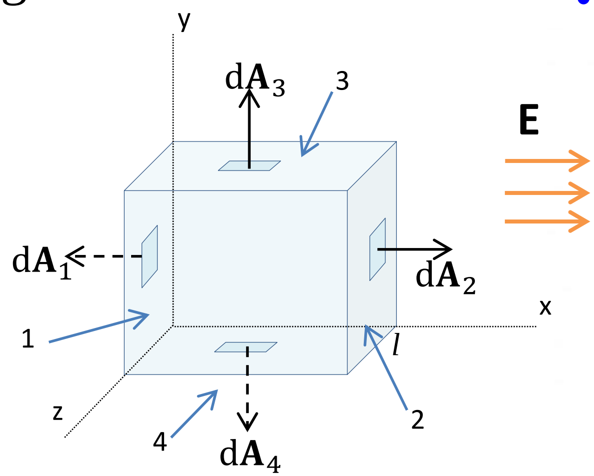
Reminder:

$$\vec{A} \cdot \vec{B} = A\, B \cos\theta$$



ELECTRIC FLUX

EXAMPLE 2: Consider a uniform electric field **E** oriented in the x direction. Find the net electric flux through the surface of a cube of edge length *l*, oriented as shown in Figure.



- For 3, 4 and unnumbered ones flux is zero because $\theta=90^0$

The net flux through faces 1 and 2:

$$\Phi_E = \int_1 \vec{E}\cdot d\vec{A} + \int_2 \vec{E}\cdot d\vec{A}$$

$$\int_1 \vec{E}\cdot d\vec{A} = \int_1 E(\cos 180^0)\,dA = -E\int_1 dA = -EA = -El^2$$

$$\int_2 \vec{E}\cdot d\vec{A} = \int_2 E(\cos 0^0)\,dA = E\int_2 dA = +El^2$$

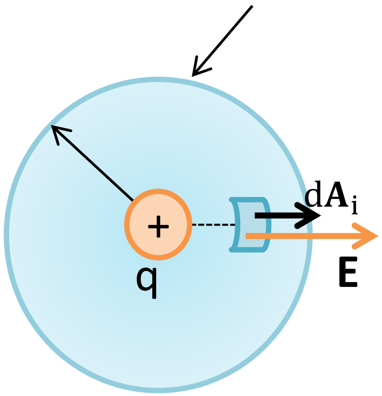
The net flux over all six faces;

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6$$
$$= -El^2 + El^2 + 0 + 0 + 0 + 0 = 0$$

Note: dA always perpendicular to the surface

Gauss's Law

- We will describe a **general relationship** between the **net electric flux** through a closed surface (often called a *gaussian surface*) and the **charge** enclosed by the surface.
- This relationship, known as Gauss's law, is of fundamental importance in the study of electric fields.



$$\mathbf{E_i} \cdot \Delta\mathbf{A_i} = E_i \Delta A_i$$

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E \cdot dA$$

$$\Phi_E = E \oint dA \quad \text{where } E = \frac{k_e q}{r^2} \text{ then}$$

$$\Phi_E = \frac{k_e q}{r^2}(4\pi r^2) = 4\pi k_e q$$

$$\boxed{\Phi_E = \frac{q}{\epsilon_0} \quad \text{where } \epsilon_0 = 1/\, 4\pi k_e}$$

*(**E** is constant due to symmetry)*

1. ***The net flux through any closed surface surrounding a point charge q is given by q/ε_0 and is independent of the shape of that surface.***
2. ***The net electric flux through a closed surface that surrounds no charge is zero.***

Gauss's Law

- **Gauss's law states that the net flux through any closed surface**

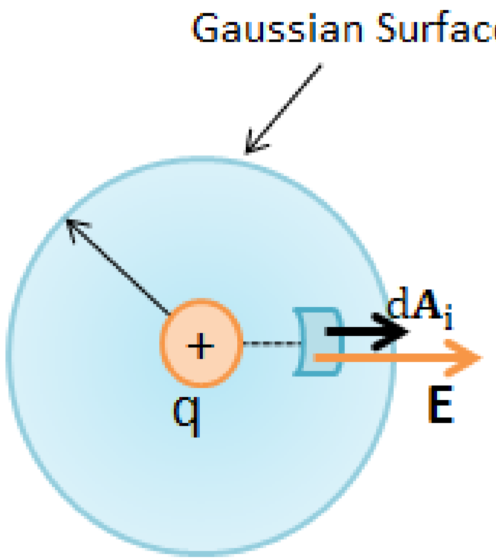
$$\Phi_E = \oint \mathbf{E}.\,d\mathbf{A} = \frac{q_{in}}{\epsilon_0}$$

- Where q_{in} represents the net charge inside the surface and **E** represents the electric field at any point on the surface.

- Gauss's law can be solved for **E** to determine the electric field due to a system of charges or a continuous distribution of charge.

Gauss's Law

EXAMPLE 3: Calculate the electric field due to an isolated point charge q.



$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \oint E\,dA = \frac{q}{\epsilon_0}$$

$$\oint E\,dA = E\oint dA = E\cdot(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2} = k_e\frac{q}{r^2}$$

Strategy:

1- Draw a Gaussian surface includes the charge

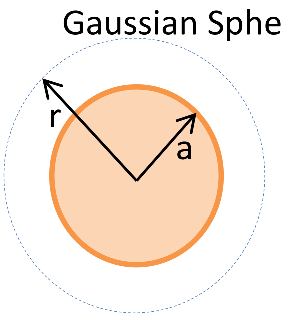
2- Show the E and dA vectors (remember that dA is always perpendicular to area) and find the angle between them.

3-Apply the Gauss's Law

Gauss's Law

EXAMPLE 4: An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge q

(A) Calculate the magnitude of the electric field at a point outside the sphere.



$$\Phi_E = \oint \mathbf{E}.\,d\mathbf{A} = \oint E.\,dA = \frac{q}{\epsilon_0}$$

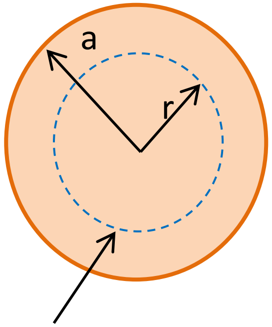
$$E = \frac{q}{4\pi\epsilon_0 r^2} = \frac{k_e q}{r^2}$$

for a uniformly charged sphere, the field in the region external to the sphere is equivalent to that of a point charge located at the center of the sphere.

Gauss's Law

EXAMPLE 4: An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q

(B) Find the magnitude of the electric field at a point inside the sphere.



$$\oint \mathbf{E}.\,d\mathbf{A} = \oint E.\,dA = E\oint dA = E4\pi r^2 = \frac{q_{in}}{\epsilon_0}$$

$$\oint \vec{E}.\,d\vec{A} = \oint E\,dA = E\oint dA = E\,(4\pi r^2) = \frac{q_{in}}{\epsilon_0}$$

$$E = \frac{q_{in}}{4\pi r^2 \epsilon_0} = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0} r$$

using value of ρ and ϵ_0 in E:

$$E = \frac{Q/\left(\frac{4}{3}\pi a^3\right)}{3\left(1/4\pi k_e\right)} r$$

$$\boxed{E = k_e \frac{Q}{a^3} r}$$

ρ: volume charge density

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi a^3}$$

$$q_{in} = \rho . V'$$

V' = volume of Gaussian surface

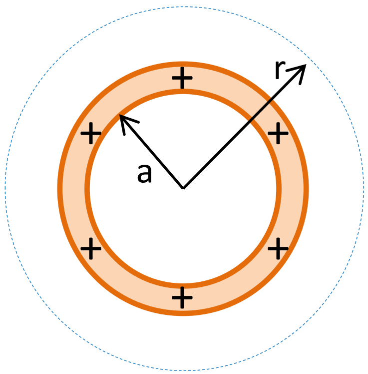
$$q_{in} = \rho . \frac{4}{3}\pi r^3$$

Gauss's Law

EXAMPLE 5: A thin spherical shell of radius a has a total charge Q distributed uniformly over its surface.

(A) Calculate the magnitude of the electric field at a point outside the shell.

Gaussian Sphere



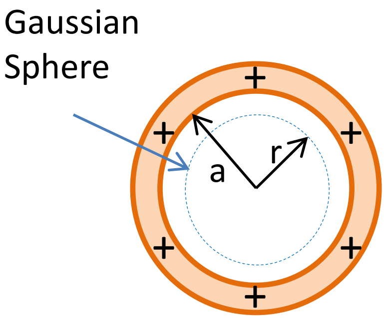
$$\Phi_E = \oint \mathbf{E}.\, d\mathbf{A} = \oint E.\, dA = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{k_e Q}{r^2}$$

Gauss's Law

EXAMPLE 5: A thin spherical shell of radius a has a total charge Q distributed uniformly over its surface.

(B) Find the magnitude of the electric field at a point inside the shell.

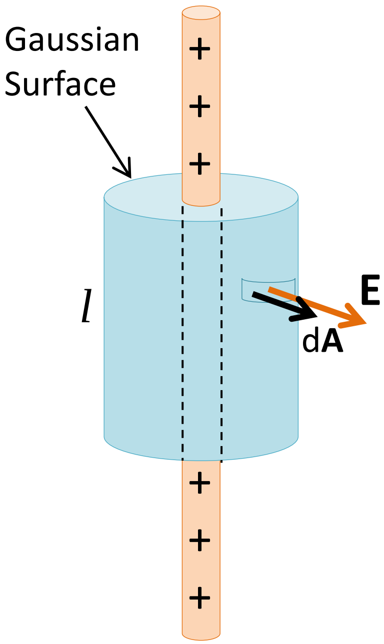


$q_{in}=0$

The electric field inside the spherical shell is zero.

Gauss's Law

EXAMPLE 6: Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ



$\mathbf{E} \perp \mathbf{A}$ for the upper and lower edges. Only curved surface has the value.

$$\Phi_E = \oint \mathbf{E}.\, d\mathbf{A} = \oint E.\, dA = E \oint dA = EA = \frac{q_{in}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

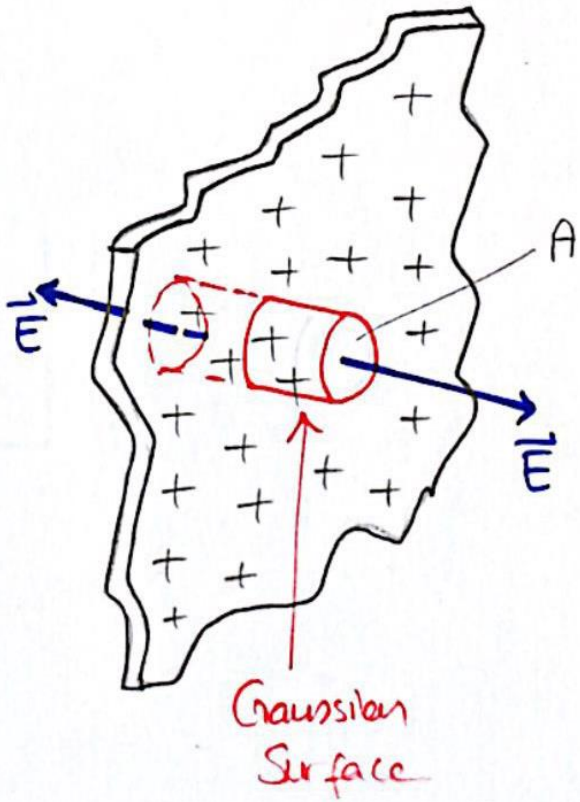
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E.dA = E \oint dA$$

$$\Phi_E = E.A = \frac{q_{in}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0} \qquad A = 2\pi r l$$

$$E.\, 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda \cancel{l}}{2\pi \epsilon_0 r \cancel{l}} = \frac{\lambda}{2\pi \frac{1}{4\pi k_e} . r} = 2k_e \frac{\lambda}{r}$$

EXAMPLE 7: Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ.



Upper and lower surfaces have the value.

$$\Phi_E = \oint \mathbf{E}.\,\mathrm{d}\mathbf{A}$$

$$\Phi_E = 2EA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

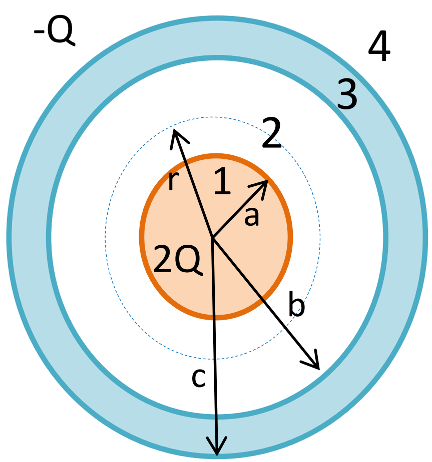
Conductors in Electrostatic Equilibrium

- A good electrical conductor contains charges (electrons) that are not bound to any atom and therefore are free to move about within the material.
- When there is no net motion of charge within a conductor, the conductor is in **electrostatic equilibrium**.
- A conductor in electrostatic equilibrium has the following properties:

1. The electric field is zero everywhere inside the conductor.
2. If an isolated conductor carries a charge, the charge resides on its surface.
3. The electric field just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude σ/ε_0, where σ is the surface charge density at that point.
4. On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest.

Conductors in Electrostatic Equilibrium

EXAMPLE 8: A solid conducting sphere of radius a carries a net positive charge 2Q. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and carries a net charge -Q. Using Gauss's law, find the electric field in the regions labeled (1), (2), (3), and (4) and the charge distribution on the shell when the entire system is in electrostatic equilibrium.



① region: $r < a, \quad q_{in} = 0, \quad E_{in} = 0$

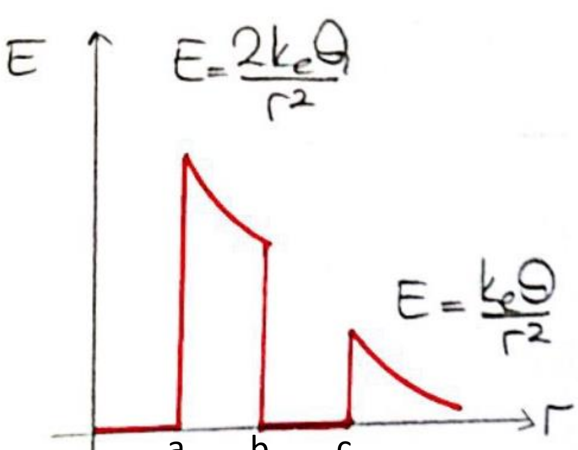
② region: $E_2 A = E_2 (4\pi r^2) = \dfrac{q_{in}}{\epsilon_0} = \dfrac{2Q}{\epsilon_0}$

$$E_2 = \frac{2Q}{4\pi\epsilon_0 r^2} = \frac{2k_e Q}{r^2} \quad (\text{for } a < r < b)$$

③ region: $b < r < c, \quad q_{in} = 0, \quad E_n = 0$

④ region: $r > c \quad q_{in} = 2Q - Q = Q$

$$E_4 = \frac{k_e Q}{r^2} \quad (\text{for } r > c)$$



Summary

1. Electric flux is proportional to the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle θ with the normal to a surface of area A, the electric flux through the surface is

$$\Phi_E = EA\cos\theta$$

2. In general, the electric flux through a surface is

$$\Phi_E = \int_{surface} \mathbf{E}.\,d\mathbf{A}$$

3. Gauss's law says that the net electric flux Φ_E through any closed gaussian surface is equal to the net charge q_{in} inside the surface divided by ε_0

$$\Phi_E = \oint \mathbf{E}.\,d\mathbf{A} = \frac{q_{in}}{\epsilon_0}$$

Summary

4. A conductor in electrostatic equilibrium has the following properties:

a) The electric field is zero everywhere inside the conductor.

b) Any net charge on the conductor resides entirely on its surface.

c) The electric field just outside the conductor is perpendicular to its surface and has a magnitude σ/ε_0, where σ is the surface charge density at that point.

d) On an irregularly shaped conductor, the surface charge density is greatest where the radius of curvature of the surface is the smallest.

Demo: https://www.youtube.com/watch?v=DrkyWwyp6wI

Demo: https://www.youtube.com/watch?v=vSEjxR56w28

Demo: https://www.youtube.com/watch?v=ULJTDCzKcZA