FİZ112E: General Physics-2

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Electric Field – Gauss's Law

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Content: Solving Problems about

- Electric Field
- Gauss's Law

Summary - 1

1. Electric charges have the following important properties:

- Charges of opposite sign attract one another and charges of the same sign repel one another.
- Total charge in an isolated system is conserved.
- Charge is quantized.

2. Conductors are materials in which electrons move freely. Insulators are materials in which electrons do not move freely.

3. Coulomb's law states that the electric force exerted by a charge q_1 on a second charge q_2 is

$$\boxed{\boldsymbol{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\boldsymbol{r}}}$$

4. The smallest unit of free charge e known to exist in nature is the charge on an electron (-e) or proton (+e), where $e=1.60219\text{x}10^{-19}$ C.

5. The electric field **E** at some point in space is defined as the electric force $\mathbf{F_e}$ that acts on a small positive test charge placed at that point divided by the magnitude q_0 of the test charge:

$$\boxed{\mathbf{E} = \frac{\boldsymbol{F}_e}{q_0}}$$

Summary - 1

6. the electric force on a charge q placed in an electric field **E** is given by $\boldsymbol{F}_e = \text{q}\,\mathbf{E}$

7. At a distance *r* from a point charge q, the electric field due to the charge is given by $\mathbf{E} = k_e \frac{q}{r^2}\hat{\boldsymbol{r}}$

   where **r** is a unit vector directed from the charge toward the point in question. The electric field is directed radially outward from a positive charge and radially inward toward a negative charge. $\mathbf{E} = k_e \sum_i \frac{q_i}{{r_i}^2}\hat{\boldsymbol{r}}_i$

8. The electric field at some point due to a continuous charge distribution is

$$\mathbf{E} = k_e \int \frac{dq}{r^2}\hat{\boldsymbol{r}}$$

   where dq is the charge on one element of the charge distribution and r is the distance from the element to the point in question.

9. **Electric field lines** describe an electric field in any region of space. The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of **E** in that region.

10. A charged particle of mass m and charge q moving in an electric field **E** has an acceleration **a**=*q***E**/m

Summary - 2

1. Electric flux is proportional to the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle θ with the normal to a surface of area A, the electric flux through the surface is

$$\Phi_E = EA\cos\theta$$

2. In general, the electric flux through a surface is

$$\Phi_E = \int_{surface} \mathbf{E}.\,d\mathbf{A}$$

3. Gauss's law says that the net electric flux Φ_E through any closed gaussian surface is equal to the net charge q_{in} inside the surface divided by ε_0

$$\Phi_E = \oint \mathbf{E}.\,d\mathbf{A} = \frac{q_{in}}{\epsilon_0}$$

Summary - 2

4. A conductor in electrostatic equilibrium has the following properties:

a) The electric field is zero everywhere inside the conductor.

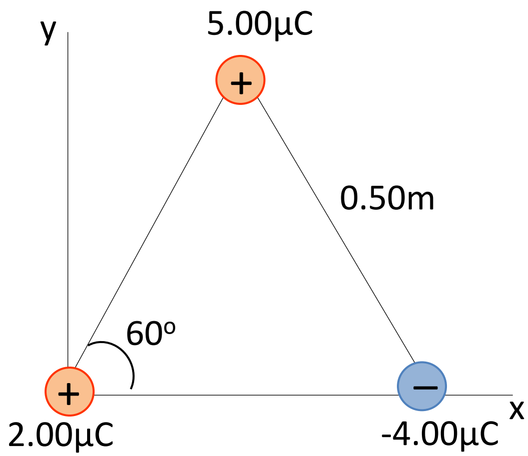
b) Any net charge on the conductor resides entirely on its surface.

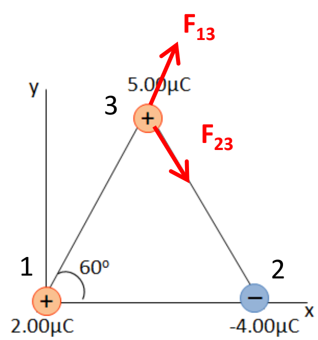
c) The electric field just outside the conductor is perpendicular to its surface and has a magnitude σ/ε_0, where σ is the surface charge density at that point.

d) On an irregularly shaped conductor, the surface charge density is greatest where the radius of curvature of the surface is the smallest.

Example 1

Three point charges are located at the corners of an equilateral triangle as shown in Figure. Calculate the resultant electric force on the 5.00μC charge.





Example 1

$\vec{F}_{13}$: force on (3) by (1)

$\vec{F}_{23}$: force on (3) by (2)

$q_1 = 2\mu C = 2\times10^{-6} C$

$q_2 = -4\mu C = -4\times10^{-6} C$

$q_3 = 5\mu C = 5\times10^{-6} C$

$r = 0.50 m$

$$F_{13} = k_e \frac{|q_1||q_2|}{r^2}$$

$$F_{13} = \frac{(8.99\times10^{9})(2\times10^{-6})(5\times10^{-6})}{(0.5)^2} = 0.360 N$$

$$F_{23} = k_e \frac{|q_2||q_3|}{r^2} = \frac{(8.99\times10^{9})(4\times10^{-6})(5\times10^{-6})}{(0.5)^2}$$

$$F_{23} = 0.719 N$$

now finding components of each force

$$\vec{F}_{13} = F_{13}\cos 60\,\hat{i} + F_{13}\sin 60\,\hat{j} = F_{13x}\hat{i} + F_{13y}\hat{j}$$

$$\vec{F}_{13} = (0.360)\cos 60\,\hat{i} + (0.360)\sin 60\,\hat{j}$$

$$\vec{F}_{13} = 0.180\,\hat{i} + 0.312\,\hat{j}$$

$$\vec{F}_{23} = F_{23}\cos 60\,\hat{i} - F_{23}\sin 60\,\hat{j} = F_{23x}\hat{i} + F_{23y}\hat{j}$$

$$\vec{F}_{23} = (0.719)\cos 60\,\hat{i} - (0.719)\sin 60\,\hat{j}$$

$$\vec{F}_{23} = 0.360\,\hat{i} - 0.623\,\hat{j}$$

Resultant force: $\vec{F} = \vec{F}_{13} + \vec{F}_{23}$

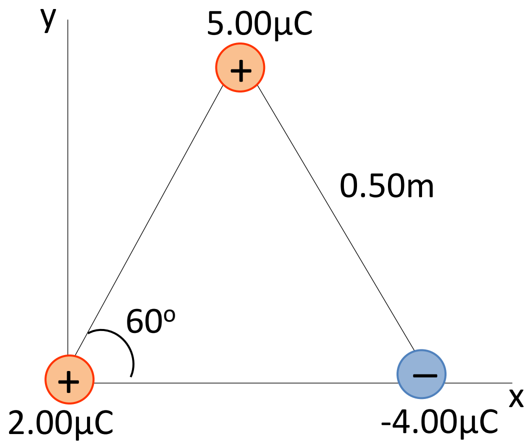
$$\boxed{\vec{F} = 0.540\,\hat{i} - 0.311\,\hat{j}}$$

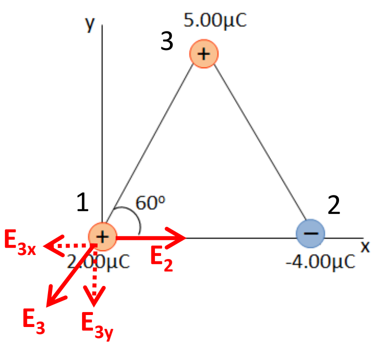
Example 2

Three charges are at the corners of an equilateral triangle as shown in Figure.

(a)Calculate the electric field at the position of the 2.00μC charge due to the 5.00μC and -4.00μC charges.

(b) Use your answer to part (a) to determine the force on the 2.00 μC charge.





$$\vec{E}_1 = \vec{E}_2 + \vec{E}_3$$

$$E_3 = k_e \frac{|q_3|}{r_3^2} = \frac{(8.99\times10^9)(5\times10^{-6})}{(0.5)^2} = (179.8\times10^3)\,N/C$$

$$\vec{E}_3 = E_3(-\cos 60)\hat{i} + E_3(-\sin 60)\hat{j}$$

$$\vec{E}_3 = [(179.8\times10^3)(-\cos 60)]\hat{i} + [(179.8\times10^3)(-\sin 60)]\hat{j}$$

$$\vec{E}_3 = -(89.9\times10^3)\hat{i} - (155.711\times10^3)\hat{j}\ N/C$$

$$E_2 = k_e \frac{|q_2|}{r_2^2} = \frac{(8.99\times10^9)(4\times10^{-6})}{(0.5)^2} = (143.84\times10^3)\,N/C$$

$$\vec{E}_2 = E_{2x}\hat{i} + \cancel{E_{2y}}\hat{j} = (143.84\times10^3)\hat{i}\ N/C$$

$$\vec{E}_1 = \vec{E}_2 + \vec{E}_3$$

$$\vec{E}_1 = [(143.84\times10^3) - (89.9\times10^3)]\hat{i} - (155.711\times10^3)\hat{j}$$

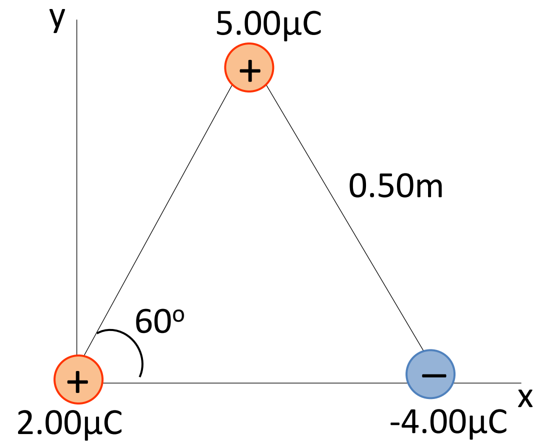
$$\vec{E}_1 = (53.940\times10^3)\hat{i} - (155.711\times10^3)\hat{j}\ N/C$$

Example 2

Three charges are at the corners of an equilateral triangle as shown in Figure.

(a)Calculate the electric field at the position of the 2.00μC charge due to the 5.00μC and -4.00μC charges.

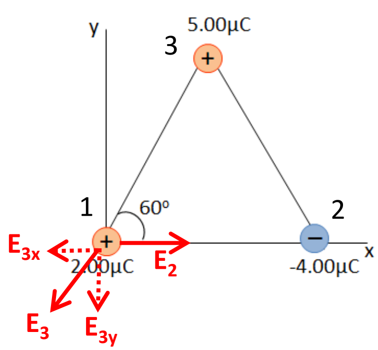
(b) Use your answer to part (a) to determine the force on the 2.00 μC charge.



Example 2 (b)

$$\vec{F}_1 = q_1 \cdot \vec{E}_1 = (2\times10^{-6}\,C)\,(53.940\times10^{3}\,\hat{i} - 155.711\times10^{3}\,\hat{j})\ N/C$$

$$\vec{F}_1 = (107.88\times10^{-3})\,\hat{i} - (311.422\times10^{-3})\,\hat{j}\ N$$

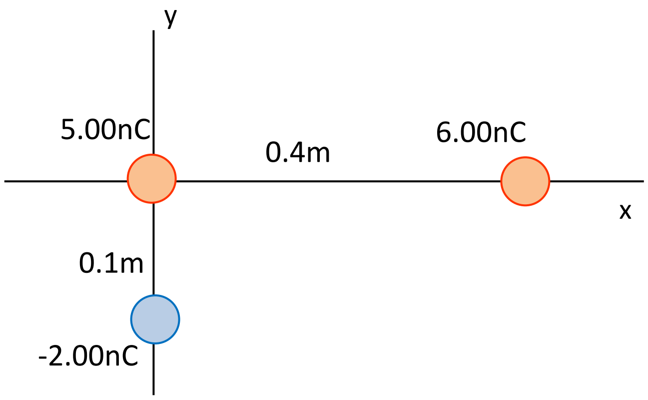


Example 3

Three point charges are arranged as shown in Figure.

(a) Find the vector electric field that the 6.00 nC and -2.00 nC charges together create at the origin.

(b) Find the vector force on the 5.00 nC charge.



Example 3(a)

Electric field at the origin: $\vec{E} = \vec{E}_1 + \vec{E}_2$

$\vec{E} = \vec{E}_1 + \vec{E}_2$ $\qquad q_1 = -2 \times 10^{-9}\,C$

$\vec{E} = k_e \dfrac{|q_1|}{r_1^2}(-\hat{j}) + k_e \dfrac{|q_2|}{r_2^2}(-\hat{i})$ $\qquad q_2 = 6 \times 10^{-9}\,C$

$$\vec{E} = \frac{(8.99 \times \cancel{10^9})(2 \times \cancel{10^{-9}})}{(0.1)^2}(-\hat{j}) + \frac{(8.99 \times \cancel{10^9})(6 \times \cancel{10^{-9}})}{(0.4)^2}(-\hat{i})$$

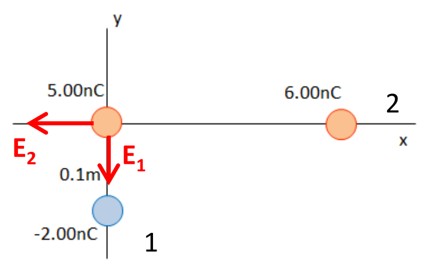
$$\vec{E} = -\left(337.125\,\hat{i} + 1.798 \times 10^3\,\hat{j}\right) N/C$$

Example 3(b)

Force acting on 5nC: $\vec{F} = (5 \times 10^{-9}\,C) \cdot \vec{E}$

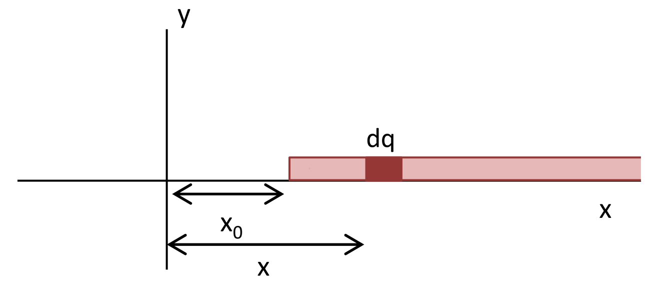
$$\vec{F} = (5 \times 10^{-9}\,C)\left(-337.125\,\hat{i} - 1.798 \times 10^3\,\hat{j}\right) N/C$$

$$\vec{F} = \left(-1.686 \times 10^{-6}\,\hat{i} - 8.99 \times 10^{-6}\,\hat{j}\right) N$$



Example 4

A continuous line of charge lies along the x axis, extending from $x=x_0$ to positive infinity. The line carries charge with a uniform linear charge density λ_0. What are the magnitude and direction of the electric field at the origin?



Example 4

Electric field at the origin $\vec{E}_0$

$$dq = \lambda_0 dx$$

$$dE_0 = \frac{k_e dq}{x^2}$$

$$E_0 = \int dE_0 = \int_{x_0}^{\infty} k_e \frac{dq}{x^2} = \int_{x_0}^{\infty} k_e \frac{\lambda_0}{x^2} dx$$

$$E_0 = k_e \lambda_0 \left(-\frac{1}{x}\right)\Bigg|_{x_0}^{\infty} = \frac{k_e \lambda_0}{x_0}$$ (magnitude of E field)

The charge is (+) therefore the electric field points away from its source, so $\leftarrow$ to the left! (direction of E field)

Example 5

A proton moves at 5.0×10^5 m/s in the horizontal direction. It enters a uniform vertical electric field with a magnitude of 9.60×10^3 N/C. Ignoring any gravitational effects, find

(a) the time interval required for the proton to travel 5.00 cm horizontally,
(b) its vertical displacement during the time interval in which it travels 5.00 cm horizontally, and
(c) the horizontal and vertical components of its velocity after it has traveled 5.00 cm horizontally.

a) $t = \dfrac{x}{v_{xi}} = \dfrac{0.05\,m}{5 \times 10^5\,s}$

$t = 1 \times 10^{-7}\,s = 100\,ns$

b) $\sum \vec{F} = m\vec{a}$ in here net force is force due to $\vec{E}$, in addition $\vec{E}$ is in y direction so $a_x = 0$

$qE = m a_y$

$\dfrac{qE}{m_p} = a_y$

$\dfrac{(1.602 \times 10^{-19}\,C)(9.6 \times 10^3\,N/C)}{1.67 \times 10^{-27}\,kg} = 9.21 \times 10^{11}\,m/s^2$

from 2D motion its known:

$y_f = \cancel{y_i}^{0} + \cancel{v_{yi} t}^{0} + \frac{1}{2} a_y t^2$ using values in the equation

$y_f = \frac{1}{2} (9.21 \times 10^{11}\,m/s^2)(100 \times 10^{-9}\,s)^2$

$y_f = 4.605 \times 10^{-3}\,m = 4.605\,mm$

Example 5

A proton moves at 5.0 x10^5 m/s in the horizontal direction. It enters a uniform vertical electric field with a magnitude of 9.60x10^3 N/C. Ignoring any gravitational effects, find

(a) the time interval required for the proton to travel 5.00 cm horizontally,

(b) its vertical displacement during the time interval in which it travels 5.00 cm horizontally, and

(c) the horizontal and vertical components of its velocity after it has traveled 5.00 cm horizontally.

c) $v_x = 5\times10^5\ m/s$

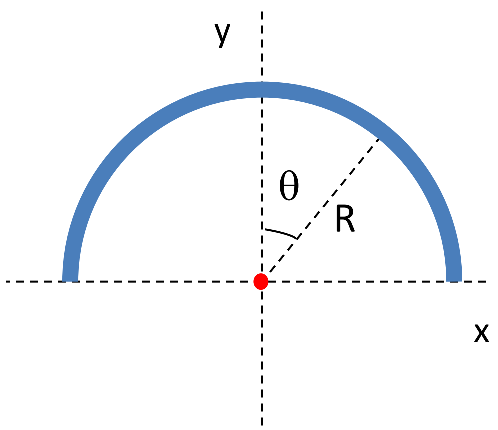
$v_{yf} = \underbrace{\cancel{v_{yi}}}_{0} + a_y t = (9.21\times10^{11}\ m/s^2)(100\times10^{-9}\ s)$

$v_{yf} = 92100\ m/s = 92.1\ km/s$

$\vec{v} = (500\,\hat{i} + 92.1\,\hat{j})\ km/s$

Example 6

A line of positive charge is formed into a semicircle of radius R=50.0 cm as shown in Figure. The charge per unit length along the semicircle is described by the expression $\lambda=\lambda_0\cos\theta$. The total charge on the semicircle is 12.0μC. Calculate the total force on a charge of 2.00μC placed at the center of curvature.



$R = 50\,cm$

$\lambda = \lambda_0 \cos\theta$

$\vec{F} = F_x \hat{i} + F_y \hat{j} = (F \cdot \cos\theta)\hat{j}$

due to the symmetry x components of the force cancel out each other.

$q_1 = 2\times10^{-6}\,C$

$q_2 = \lambda\, dl$

$dl = R\, d\theta$

$F_y = F_1 \cos\theta = \int dF_y = \int k_e \frac{q_1 \lambda\, dl}{R^2} \cos\theta$

using definition of q_2 in the integral

$F_y = \int k_e \frac{q_1 (\lambda_0 \cos\theta)\cos\theta\, R\, d\theta}{R^2} = \int k_e \frac{q_1 \lambda_0 \cos^2\theta}{R} d\theta$

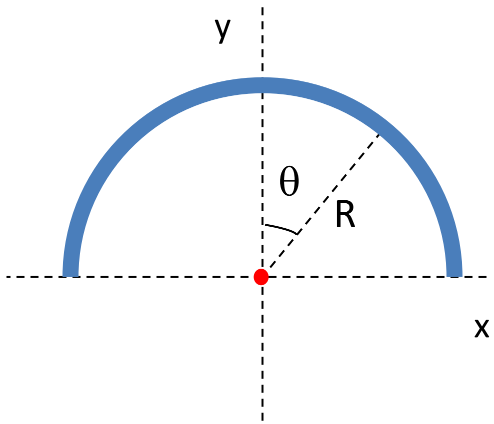
$\cos^2\theta = \frac{1+\cos 2\theta}{2}$ using this information in the integral

$F_y = \frac{k_e q_1 \lambda_0}{R} \int_{-\pi/2}^{+\pi/2} \frac{1+\cos 2\theta}{2} d\theta = \frac{k_e q_1 \lambda_0}{R} \left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta \right) \Big|_{-\pi/2}^{\pi/2}$

$F_y = \frac{k_e q_1 \lambda_0}{R} \left(\frac{\pi}{2}\right)$ we need to find λ_0

Example 6

A line of positive charge is formed into a semicircle of radius R=50.0 cm as shown in Figure. The charge per unit length along the semicircle is described by the expression $\lambda=\lambda_0\cos\theta$. The total charge on the semicircle is 12.0μC. Calculate the total force on a charge of 2.00μC placed at the center of curvature.



$$Q = \int \lambda \, dl = \int_{-\pi/2}^{\pi/2} \lambda_0 \cos\theta \, R \, d\theta = \lambda_0 R \sin\theta \Big|_{-\pi/2}^{\pi/2} = 2\lambda_0 R$$

$$Q = 2\lambda_0 R = 12 \times 10^{-6} C = 2\lambda_0 \cdot 0.5$$

$$\boxed{12 \times 10^{-6} = \lambda_0}$$ using this in Fy definition

$$F_y = \frac{(8.99 \times 10^{9})(2 \times 10^{-6})(12 \times 10^{-6})}{0.5} \cdot \left(\frac{\pi}{2}\right)$$

$F_y = 0.678N$ towards downward! $\vec{F} = F_y(-\hat{j})$

$$\vec{F} = -0.678 N \hat{j}$$

Example 7

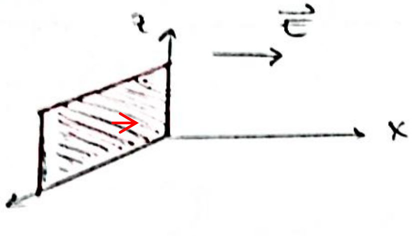
An electric field with a magnitude of 5.0 kN/C is applied along the x axis. Calculate the electric flux through a rectangular plane 0.500 m wide and 0.400 m long assuming that

(a) the plane is parallel to the yz plane; (b)the plane is parallel to the xy plane; (c) the plane contains the y axis, and its normal makes an angle of 53.0° with the x axis.

For a uniform $\vec{E}$ passing through a plane surface,

$\Phi_E = \vec{E} \cdot \vec{A} = E . A . \cos\theta$ — θ is the angle between $\vec{E}$ and the normal to the surface.

a)



plane is parallel to yz plane.

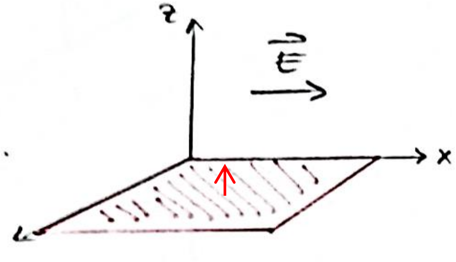
$\vec{E} \perp$ plane so $\theta = 0$.

$\Phi_E = \vec{E} . \vec{A} = E . A \underbrace{\cos\theta}_{1}$

$\Phi_E = (5 \times 10^3) N/C \cdot (0.5 \times 0.4) m^2$

$\Phi_E = 1000 \ Nm^2/C$

b)



plane is parallel to xy plane

$\vec{E} \parallel$ plane so $\theta = 90°$

$\cos\theta = 0$

so $\Phi_E = 0$

c) $\theta = 53°$ so $\Phi_E = \vec{E} . \vec{A} = E A \cos 53$

$\Phi_E = (5 \times 10^3) N/C . (0.5 \times 0.4) m^2 . 0.6$

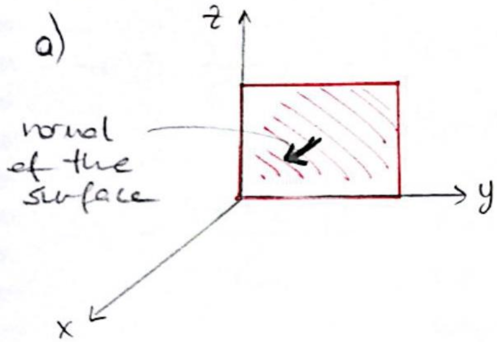
$\Phi_E = 1000 . (0.6) = 600 \ Nm^2/C$

Example 8

A uniform electric field $a\hat{\imath}+b\hat{\jmath}$ intersects a surface of area A. What is the flux through this area if the surface lies

(a) in the yz plane? (b) in the xz plane? (c) in the xy plane?

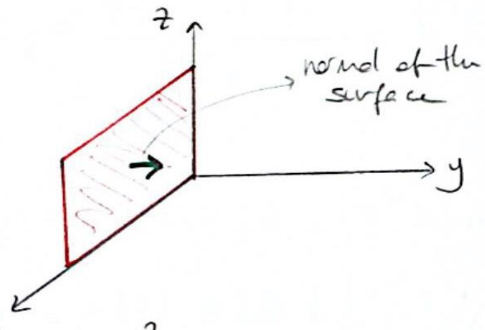
a)



$$\Phi_E = \vec{E}\cdot\vec{A}_{yz}$$

$$\Phi_E = (a\hat{i}+b\hat{j})\cdot A\hat{i} = aA$$

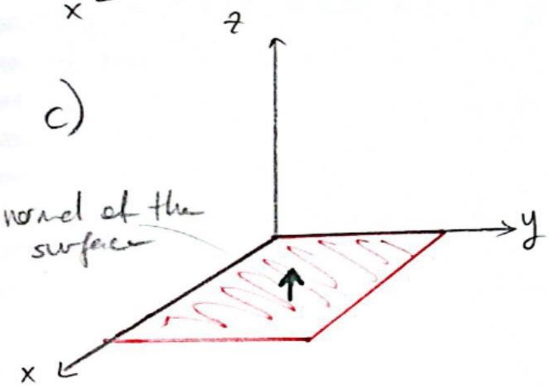
b)



$$\Phi_E = \vec{E}\cdot\vec{A}_{xz}$$

$$\Phi_E = (a\hat{i}+b\hat{j})\cdot A\hat{j} = bA$$

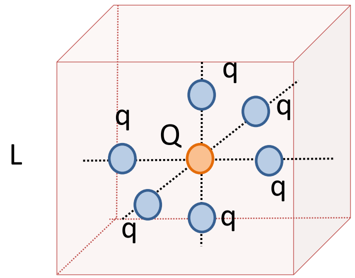
c)



$$\Phi_E = \vec{E}\cdot\vec{A}_{xy}$$

$$\Phi_E = (a\hat{i}+b\hat{j})A\hat{k} = 0$$

Example 9



A particle with charge Q=5μC is located at the center of a cube of edge L = 0.2m. In addition, six other identical negative point charges q=-2 μC are positioned symmetrically around Q as shown in Figure. Determine the electric flux through one face of the cube.

$$\Phi_E = \frac{q_{in}}{\epsilon_0}$$

q_{in} : total charge inside the closed surface

$q_{in} = Q - 6|q|$ using definition in equation

$$\Phi_E = \frac{Q - 6|q|}{\epsilon_0}$$

this is total flux outward from the cube.
we need to find flux for one surface so;
equation becomes

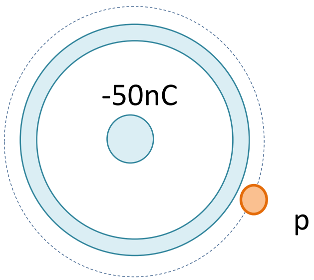
$$\Phi_E = \frac{Q - 6|q|}{6\epsilon_0} = \frac{(5\times10^{-6}C) - (6\times 2\times10^{-6}C)}{6\times \underbrace{8.85\times10^{-12}C^2}_{\text{value of } \epsilon_0}}$$

$$\Phi_E = -131.827\times10^{3}\ Nm^2/C$$

$$\Phi_E = -131.827\ kNm^2/C$$

Example 10

A particle with a charge of -50.0 nC is placed at the center of a nonconducting spherical shell of inner radius 20.0 cm and outer radius 25.0 cm. The spherical shell carries charge with a uniform density of -1.2 $\mu C/m^3$. A proton moves in a circular orbit just outside the spherical shell. Calculate the speed of the proton.



Volume of the spherical shell: V

$$V = \frac{4}{3}\pi(R^3 - r^3) = \frac{4}{3}\pi(0.25^3 - 0.20^3) = 3.19\times10^{-2}\,m^3$$

calculating charge of the shell: Q

$$Q = \rho \cdot V = (-1.2\times10^{-6}\,C/m^3)\cdot(3.19\times10^{-2}\,m^3)$$

$$Q \cong -3.83\times10^{-8}\,C$$

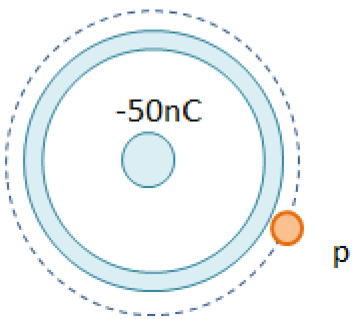
The net charge inside a sphere containing proton's path:

$$q_{net} = -50\times10^{-9}\,C - 3.83\times10^{-8}\,C$$

$$Q_{net} = -50.383\times10^{-9}\,C$$

Example 10

A particle with a charge of -50.0 nC is placed at the center of a nonconducting spherical shell of inner radius 20.0 cm and outer radius 25.0 cm. The spherical shell carries charge with a uniform density of -1.2 $\mu C/m^3$. A proton moves in a circular orbit just outside the spherical shell. Calculate the speed of the proton.



The electric field is radially inward since q_{net} is (−) and the magnitude of $\vec{E}$.

$$E = k_e \frac{|q_{net}|}{r^2} = \frac{(8.99\times 10^{9})(+50.383\times 10^{-9})}{(0.25)^2}$$

$$E = 7.247\times 10^{3}\ N/C$$

for speed of proton (circular motion)

mass of proton, charge of proton

$$\Sigma F = ma = m\frac{v^2}{r} = q\cdot E$$

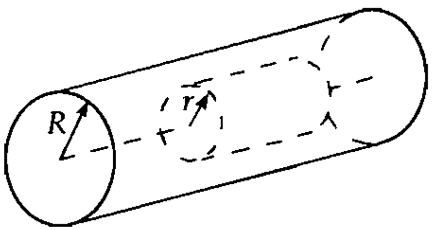
$$v = \left(\frac{qEr}{m}\right)^{1/2} = \left[\frac{(1.6\times 10^{-19}\,C)(7.247\times 10^{3}\,N/C)(0.25\,m)}{(1.67\times 10^{-27}\,kg)}\right]^{1/2}$$

$$v \cong 4.17\times 10^{5}\ m/s$$

Example 11

Consider a long cylindrical charge distribution of radius R with a uniform charge density ρ. Find the electric field at distance r from the axis where r<R.

if ρ is (+) then the field must be radially outward!
define a Gaussian surface inside charged rod which is cylinder of length L and radius r.



Volume of Gaussian Surface $= V = \pi r^2 L$

Gauss Law $\Rightarrow \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$

$$\oint \vec{E}\cdot d\vec{A} = \underbrace{\int_{\substack{\text{surface 1}\\ \text{(side 1)}}} \vec{E}\cdot d\vec{A}}_{\substack{\vec{E}=E\hat{r}\\ d\vec{A}=dA(-\hat{i})\\ \theta=90^\circ\\ \text{so this term}=0}} + \underbrace{\int_{\substack{\text{surface 2}\\ \text{(side 2)}}} \vec{E}\cdot d\vec{A}}_{\substack{\vec{E}=E\hat{r}\\ d\vec{A}=dA\,\hat{i}\\ \theta=90^\circ\\ \text{so this term}=0}} + \int_{\text{curved}} \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$\int_{\text{curved}} \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} = \int E\hat{r}\cdot dA\,\hat{r} = \int E\cdot dA \underbrace{\cos 0}_{1} = \frac{q_{in}}{\epsilon_0}$$

$$E \underbrace{\int dA}_{\substack{\text{area of}\\ \text{Gaussian Surface}}} = \frac{q_{in}}{\epsilon_0} \Rightarrow E\cdot 2\pi r L = \frac{\int \rho\, dV}{\epsilon_0}$$

$$E\cdot 2\cancel{\pi} \cancel{r} \cancel{L} = \frac{\rho\, \cancel{\pi} r^{\cancel{2}} \cancel{L}}{\epsilon_0}$$

$$E = \frac{\rho r}{2\epsilon_0}, \quad \vec{E} = \frac{\rho r}{2\epsilon_0}\hat{r}$$

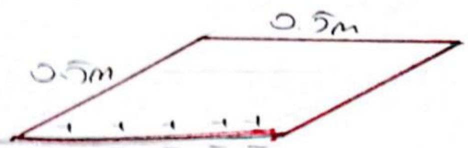
(PC) since the charge distribution is long, no electric flux passes through the circular endcaps.

Example 12

A square plate of copper with 50.0 cm sides has no net charge and is placed in a region of uniform electric field of 100.0 kN/C directed perpendicularly to the plate. Find

(a) the charge density of each face of the plate and

(b) the total charge on each face.



no net charge!

$\uparrow \; \uparrow \; \uparrow \quad \vec{E} = 100 \times 10^3 \, N/C$

(a)

$$\int E \cdot dA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma \cancel{A}}{\epsilon_0} = E \cdot \cancel{A} \qquad E = \frac{\sigma}{\epsilon_0}$$

$$\sigma = \epsilon_0 E$$

$$\sigma = (8.85 \times 10^{-12})(100 \times 10^{3})$$

$$\sigma = 8.85 \times 10^{-7} \, C/m^2,$$

σ is (+) on upper face

σ is (−) on lower face

(b) $Q = \sigma A = (8.85 \times 10^{-7})(0.5)^2$

$Q = \sigma A = 2.21 \times 10^{-7} \, C,$

$Q = +22.1 \, \mu C$ on upper surface

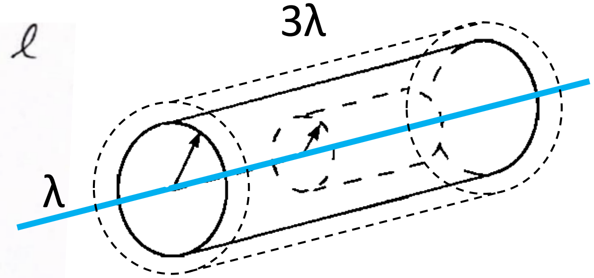
$Q = -22.1 \, \mu C$ on lower surface

Example 13

A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire. The wire has a charge per unit length of λ, and the cylinder has a net charge per unit length of 3λ. From this information, use Gauss's law to find

(a) the charge per unit length on the inner and outer surfaces of the cylinder and

(b) the electric field outside the cylinder, a distance r from the axis.



a) for the inner surface defining a gaussian surface of lenghth l

for line charge $dq = \lambda \, dl$

so $q = \lambda l$

applying Gauss' law : $\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$

0 inside the conducting shell

So equation becomes $0 = \frac{(\lambda + \lambda_{inner}) l}{\epsilon_0}$

$\lambda = -\lambda_{inner\ surface}$

for the outer surface defining gaussian surface of length l

for the total charge inside : $q_{wire} + q_{cylinder}$

$q_{wire} + q_{cylinder} = q_{wire} + (q_{inner\ surface} + q_{outer\ surface})$

$\cancel{\lambda l} + 3\lambda l = \cancel{\lambda l} + (-\lambda l + \lambda_{outer\ surface} l)$

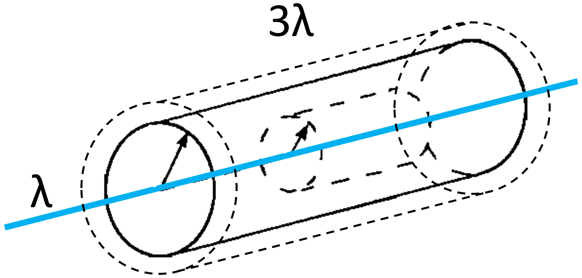
$\boxed{4\lambda = \lambda_{outer\ surface}}$

Example 13

A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire. The wire has a charge per unit length of λ, and the cylinder has a net charge per unit length of 3λ. From this information, use Gauss's law to find

(a) the charge per unit length on the inner and outer surfaces of the cylinder and

(b) the electric field outside the cylinder, a distance r from the axis.



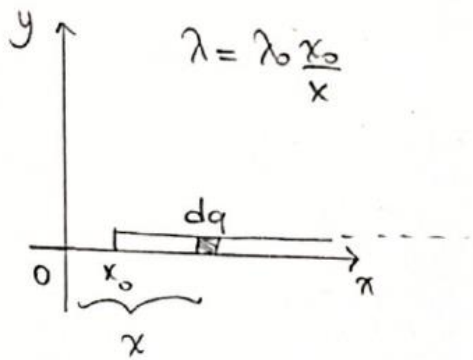
b) Applying Gauss's Law to find $\vec{E}$:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} = \frac{\int 4\lambda \, dl}{\epsilon_0}$$

$$E \cdot \cancel{2}\pi r \cancel{l} = \frac{\cancelto{2}{4}\lambda \cancel{l}}{\epsilon_0} \Rightarrow \boxed{E = \frac{2\lambda}{\pi r \epsilon_0}}$$

Example 14

A line of charge starts at $x=x_0$ and extends to positive infinity. The linear charge density is $\lambda=(\lambda_0 x_0)/x$. Determine the electric field at the origin.



Electric field at the origin: E_0

$$E_0 = \int dE_0 = \int k_e \frac{dq}{x^2}$$

$dq = \lambda . dx$ using in the equation

$$E_0 = \int k_e \frac{\lambda \, dx}{x^2}$$ using definition of λ

$$E_0 = \int_{x_0}^{\infty} k_e \frac{\lambda_0 x_0}{x^3} dx$$

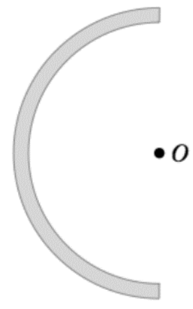
$$E_0 = k_e \frac{\lambda_0}{2x_0}$$

at the origin direction of electric field is $-\hat{i}$ so the definition

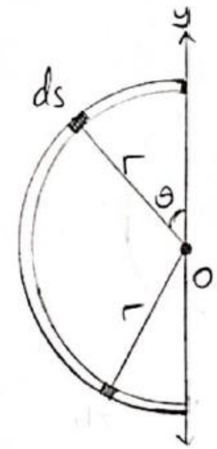
$$\vec{E}_0 = k_e \frac{\lambda_0}{2x_0}(-\hat{i})$$

Example 15

A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown in Figure. The rod has a total charge of -5.0μC. Find the magnitude and direction of the electric field at O, the center of the semicircle.



Example 15:



Electric field at the origin $\vec{E} = \vec{E}_x + \vec{E}_y$

$$\vec{E} = \vec{E}_x + \vec{E}_y = \int dE_x \hat{i} + \underbrace{\int dE_y \hat{j}}_{=0 \text{ due to symmetry}}$$

$dE_x = dE . \sin\theta$ using in the definition

$$\vec{E} = \int dE_x \hat{i} = \int dE . \sin\theta \, \hat{i} = \int k_e \frac{dq \sin\theta}{r^2} \hat{i}$$

$q = \lambda . s$

$dq = \lambda . ds \qquad ds = r \, d\theta$

$dq = \lambda . r \, d\theta$ using in the equation

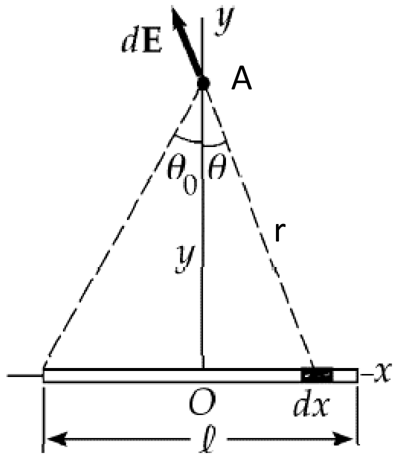
$$\vec{E} = \int_0^{\pi} k_e \frac{\lambda . \cancel{r}}{r^{\cancel{2}}} \sin\theta \, d\theta (\hat{i}) = \frac{k_e \lambda}{r} (-\cos\theta) \Big|_0^{\pi} (\hat{i}) = \frac{2 k_e \lambda}{r} (\hat{i})$$

$\lambda = \frac{q}{L}$ and $r = \frac{L}{\pi}$ then $\vec{E} = \frac{2 (8.99 \times 10^9)(-5 \times 10^{-6}) \pi}{(0.14 m)^2} (\hat{i})$

$\vec{E} = -1.44 \times 10^{7} \hat{i} \, N/C = -14.4 \, \hat{i} \, MN/C$

Example 16

A thin rod of length *l* and uniform charge per unit length λ lies along the axis, as shown in Figure. Calculate the electric field at point A.



due to each element of dx finding dE

$$dE = k_e \frac{dq}{r^2} = k_e \frac{dq}{x^2+y^2}$$

the total field at point A $\vec{E} = E_x \hat{i} + E_y \hat{j}$

due to symmetry $\vec{E}_x$ will be 0! then $\vec{E} = E_y \hat{j}$

$$E = \int dE_y = \int dE \cos\theta = \int k_e \frac{dq}{x^2+y^2} \cos\theta$$

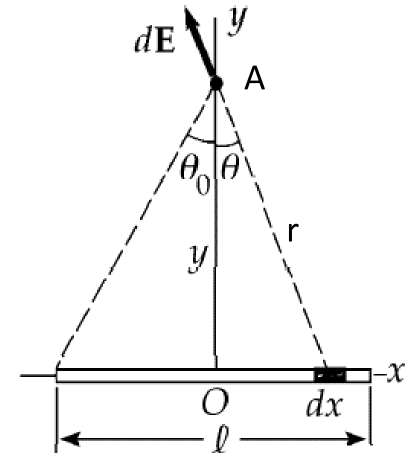
$dq = \lambda . dx$. $\cos\theta = \frac{y}{(x^2+y^2)^{1/2}}$ using these definitions

$$E = \int_{-l/2}^{+l/2} k_e \frac{\lambda . y}{(x^2+y^2)^{3/2}} dx = 2 k_e \lambda y \int_0^{l/2} \frac{dx}{(x^2+y^2)^{3/2}}$$

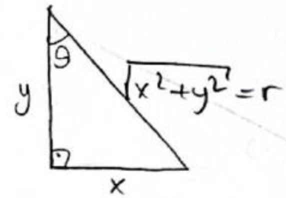
doing some calculation

Example 16

A thin rod of length *l* and uniform charge per unit length λ lies along the axis, as shown in Figure. Calculate the electric field at point A.



$x = y\tan\theta \qquad dx = y\sec^2\theta\, d\theta$



then the integral becomes

$$\int_0^{\theta_0} \frac{y\sec^2\theta}{(y^2\tan^2\theta + y^2)^{3/2}}\, d\theta = \int_0^{\theta_0} \frac{1}{y^2}\, \frac{\sec^2\theta}{(\tan^2\theta+1)^{3/2}}\, d\theta$$

$$= \frac{1}{y^2}\int_0^{\theta_0} \frac{\sec^2\theta}{\sec^3\theta}\, d\theta = \frac{1}{y^2}\int_0^{\theta_0} \frac{1}{\sec\theta}\, d\theta = \frac{1}{y^2}\int_0^{\theta_0} \cos\theta\, d\theta$$

$$= \frac{1}{y^2}\sin\theta \Big|_0^{\theta_0} = \frac{1}{y^2}\sin\theta_0$$ using in the equation

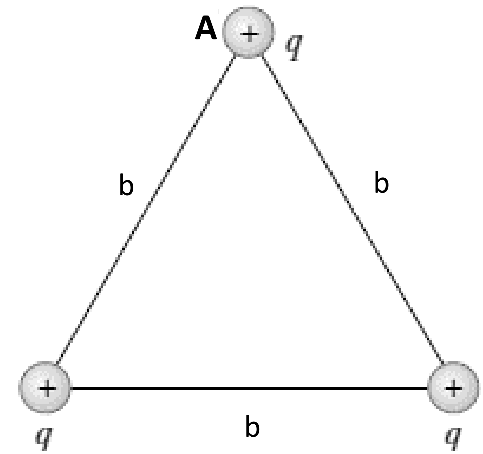
$$E = 2k_e\lambda\frac{\cancel{y}}{y^{\cancel{2}}}\sin\theta_0 = \frac{2k_e\lambda\sin\theta_0}{y}$$

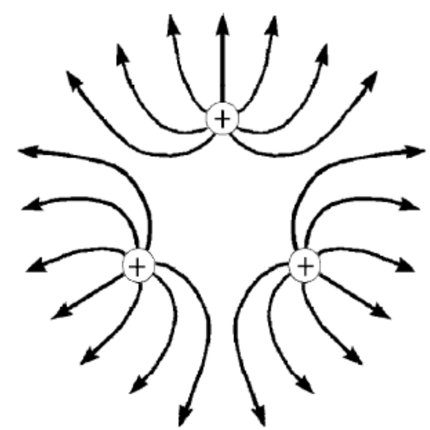
Example 17

Three equal positive charges q are at the corners of an equilateral triangle of side b as shown in Figure.

(a) Assume that the three charges together create an electric field. Sketch the field lines in the plane of the charges. Find the location of a point (other than ∞) where the electric field is zero.

(b) What are the magnitude and direction of the electric field at A due to the two charges at the base?





a) It will be zero at the center. Because due to symmetry each three charges produces $\vec{E}$ that cancel out.

b) each charge produces $\vec{E}$ field having same magnitude but different direction.

The magnitude of each $\vec{E}$ field is $\vec{E} = k_e \frac{q}{r^2}\hat{r}$

for the charge at left side: $|\vec{E_1}| = k_e \frac{q}{b^2}$ to the right upward at 60°

for the charge at right side $|\vec{E_2}| = k_e \frac{q}{b^2}$ to the left upward at 60°

x component of each field cancels out each other!

then the total field becomes

$$\vec{E} = \vec{E_1} + \vec{E_2} = E_1\hat{j} + E_2\hat{j} = 2k_e\frac{q}{b^2}(\sin 60\,\hat{j})$$

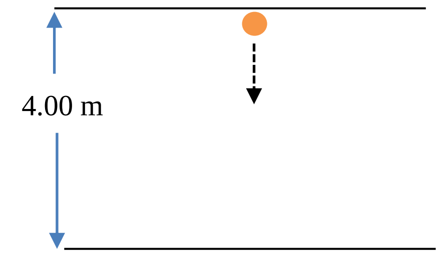
$$= 1.73\, k_e \frac{q}{b^2}\hat{j}$$

Example 18

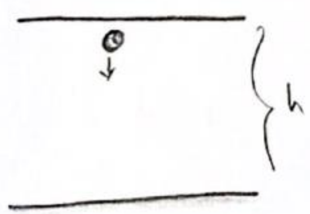
A positively charged bead having a mass of 1.00 g falls from rest in a vacuum from a height of 4.00 m in a uniform vertical electric field with a magnitude of 1.00×10^4 N/C. The bead hits the ground at a speed of 21.0 m/s. Determine

(a) The direction of the electric field (up or down), and

(b) the charge on the bead.



Example 18: (a)



$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

$$v_f^2 = 0 + 2a(-h) \qquad a = -\frac{v_f^2}{2h}$$

$$\vec{F}_{net} = m.\vec{a} = \underbrace{\vec{F}_g}_{-m.g\hat{j}} + \underbrace{\vec{F}_e}_{q.\vec{E}} = -\frac{m v_f^2}{2h}\hat{j}$$

$$q.\vec{E} = \left(-\frac{m v_f^2}{2h} + mg\right)\hat{j}$$

due to only gravity $\Rightarrow$ $v_f = \sqrt{2gh} = (2 \cdot 4 \cdot 9.8)^{1/2} = 8.85$ m/s,

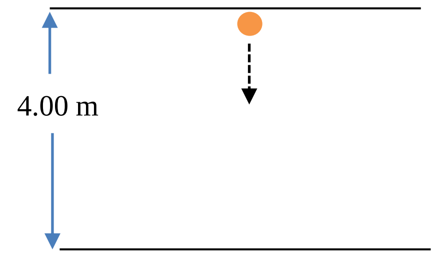
to change this to 21.0 m/s electric field $\vec{E}$ must be downward!

Example 18

A positively charged bead having a mass of 1.00 g falls from rest in a vacuum from a height of 4.00 m in a uniform vertical electric field with a magnitude of $1.00\text{x}10^4$ N/C. The bead hits the ground at a speed of 21.0 m/s. Determine

(a) The direction of the electric field (up or down), and

(b) the charge on the bead.



Example 18(b)

$$qE(-\hat{j}) = \left(-\frac{m v_f^2}{2h} + mg\right)\hat{j}$$ leaving charge term alone

$$q = \frac{m}{E}\left\{\frac{v_f^2}{2h} - g\right\} = \frac{(1\times10^{-3})}{(1\times10^{4})}\left\{\frac{(21.0)^2}{2.4} - 9.8\right\}$$

$$q = (10^{-7} \times 45.32)\,C = 4.532\,\mu C$$

Example 19

A proton moves at 5.0x10^5 m/s in the horizontal direction. It enters a uniform vertical electric field with a magnitude of 9.60x10^3 N/C. Ignoring any gravitational effects, find

(a) the time interval required for the proton to travel 5.00 cm horizontally,

(b) its vertical displacement during the time interval in which it travels 5.00 cm horizontally, and

(c) the horizontal and vertical components of its velocity after it has traveled 5.00 cm horizontally.

(a) $t = \frac{x}{v_x} = \frac{0.05}{5\times10^5} = 1\times10^{-7}\,s = 100\,ns$

(b) $ma_y = q\cdot E \Rightarrow a_y = \frac{qE}{m} = \frac{(1.602\times10^{-19})(9.6\times10^3)}{(1.67\times10^{-27})} = 9.21\times10^{11}\,m/s^2$

$\Delta y = y_f - y_i = v_{iy}\,t + \frac{1}{2}a_y t^2$

$y_f = \cancel{y_i} + \cancel{v_{iy}t} + \frac{1}{2}a_y t^2$

$y_f = \frac{1}{2}(9.21\times10^{11})(1\times10^{-7})^2 = 4.605\times10^{-3}\,m$

$y_f = 460.5\,mm$

(c) $v_x = v = 5\times10^{+5}\,m/s$ $v_{yf} = v_{yi} + a_y t$

$= (9.21\times10^{11})(1.0\times10^{-7})$

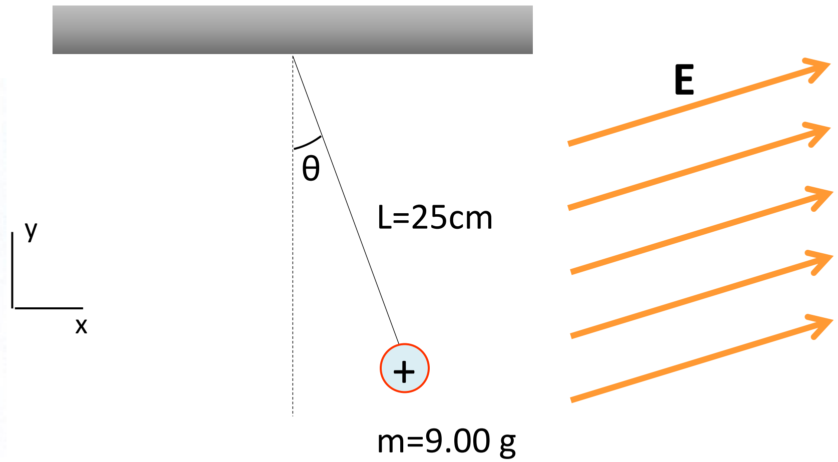
$= 9.21\times10^4\,m/s$

Example 20

A charged cork ball of mass m is suspended on a light string in the presence of a uniform electric field as shown in Figure. When **E**=(Aî+Bĵ) N/C, where A and B are positive numbers, the ball is in equilibrium at the angle θ. Find

(a) the charge on the ball and

(b) the tension in the string.



$$\Sigma F_x = qE_x - T\sin\theta = 0$$
$$\Sigma F_y = qE_y + T\cos\theta - mg = 0$$

from these two equations

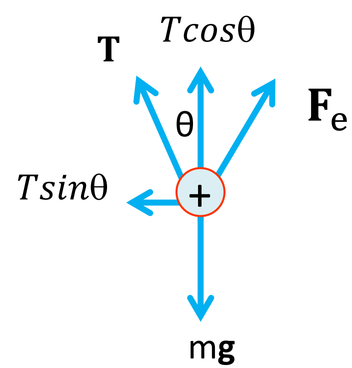
$$T = \frac{qE_x}{\sin\theta} \qquad qE_y + \frac{qE_x}{\sin\theta}\cos\theta - mg = 0$$

$\cot\theta$

$$q = \frac{mg}{E_x\cot\theta + E_y} = \frac{mg}{(A\cot\theta + B)}$$

Examp 20 (b)

$$T = \frac{qE_x}{\sin\theta} = \frac{qA}{\sin\theta} = \frac{mgA}{\sin\theta\,(A\cot\theta + B)}$$



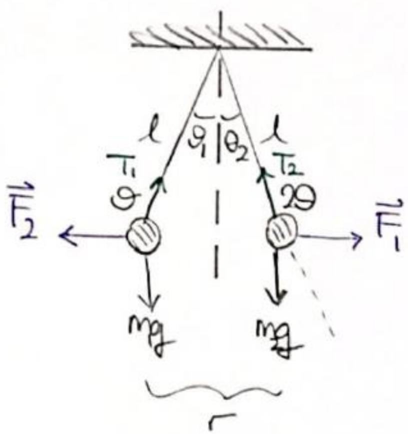
Example 21

Two small spheres of mass m are suspended from strings of length *l* that are connected at a common point. One sphere has charge Q; the other has charge 2Q. The strings make angles θ_1 and θ_2 with the vertical.

(a) How are θ_1 and θ_2 related?

(b) Assume θ_1 and θ_2 are small. Find that the distance r between the spheres.

Example 21 : (a)



$$|\vec{F}_1| = |\vec{F}_2| = F$$

for x direction

$$F - T_2 \sin\theta_2 = 0 \quad (I) \qquad F = T_2 \cdot \sin\theta_2$$

$$T_1 \cdot \sin\theta_1 - F = 0 \quad (II) \qquad F = T_1 \cdot \sin\theta_1$$

for y direction

$$T_2 \cos\theta_2 - m_2 g = 0 \Rightarrow T_2 = \frac{m_2 g}{\cos\theta_2} \quad (III)$$

$$T_1 \cos\theta_1 - m_1 g = 0 \Rightarrow T_1 = \frac{m_1 g}{\cos\theta_1} \quad (IV)$$

for T_1 and T_2 information $\quad F = m_2 g \tan\theta_2$

$$F = m_1 g \tan\theta_1$$

$$m_1 = m_2 = m \quad \text{so} \quad \boxed{\theta_1 = \theta_2}$$

Example 21

Two small spheres of mass m are suspended from strings of length *l* that are connected at a common point. One sphere has charge Q; the other has charge 2Q. The strings make angles θ_1 and θ_2 with the vertical.

(a) How are θ_1 and θ_2 related?

(b) Assume θ_1 and θ_2 are small. Find that the distance r between the spheres.

Example 21 : (b)

$\sin\theta_1 = \frac{r_1}{l}$ $\quad \sin\theta_2 = \frac{r_2}{l}$ $\quad \theta_1 = \theta_2 \Rightarrow r_1 = r_2$

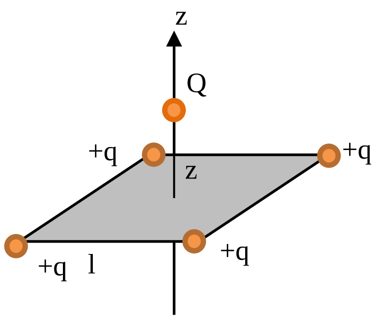
electric force $F = mg\tan\theta = k_e \frac{Q \cdot 2Q}{r^2}$

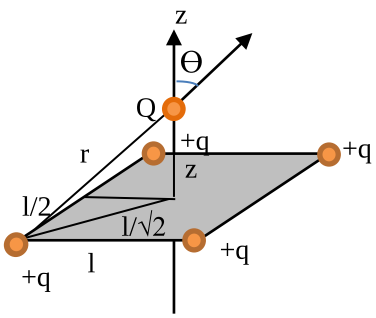
$$mg\frac{r}{2l} = 2k_e\frac{Q^2}{r^2}$$

$$r = \left(\frac{4k_e Q^2 l}{mg}\right)^{1/3}$$

Example 22

Four identical particles, each having charge +q, are fixed at the corners of a square of side l. A fifth point charge Q lies a distance z along the line perpendicular to the plane of the square and passing through the center of the square. Find that the force exerted by the other four charges on Q.





Example 22(a)

the distance from one corner to the center of the square is

$$\left(\left(\frac{l}{2}\right)^2+\left(\frac{l}{2}\right)^2\right)^{1/2} = \frac{l}{\sqrt{2}}$$

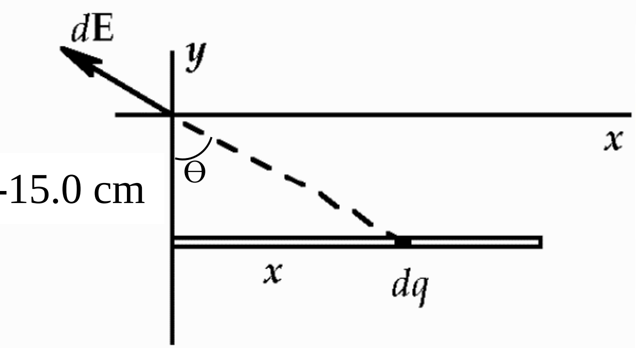
$r=\left(\left(\frac{l}{\sqrt{2}}\right)^2+z^2\right)^{1/2}$ the distance between each +q and +Q

due to the symmetry of the electric field at that point only z component of the field exists. And we have 4 charges;

$$\vec{F} = 4F\cos\theta\,\hat{k} = 4k_e\frac{qQ}{r^2}\frac{z}{r}\hat{k} = 4k_e\frac{qQz}{\left(\left(\frac{l}{\sqrt{2}}\right)^2+z^2\right)^{3/2}}\hat{k}$$

Example 23

A line of charge with uniform density 30.0 nC/m lies along the line y=-15.0 cm, between the points with coordinates x=0 and x=40.0 cm. Find the electric field it creates at the origin.



15cm = 0.15 m   40cm = 0.4 m

$$d\vec{E} = k_e \frac{dq}{r^2}\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)$$

$$\vec{E} = \int_0^{0.4} d\vec{E} = k_e \int \left(\frac{\lambda\,dx}{x^2+0.15^2}\right)\left\{\left(\frac{x}{(x^2+0.15^2)^{1/2}}\,(-\hat{i})\right) + \left(\frac{0.15}{(x^2+0.15^2)^{1/2}}\,\hat{j}\right)\right\}$$

$$\vec{E} = k_e\lambda\left\{\underbrace{\int_0^{0.4}\frac{x\,dx}{(x^2+0.15^2)^{3/2}}\,(-\hat{i})}_{\frac{1}{(x^2+0.15^2)^{1/2}}\Big|_0^{0.4}\hat{i}} + \underbrace{\int_0^{0.4}\frac{0.15\,dx}{(x^2+0.15^2)^{3/2}}\,\hat{j}}_{\frac{x}{(0.15)^2(x^2+0.15)^{1/2}}\Big|_0^{0.4}\hat{j}}\right\}$$

$$\vec{E} = k_e\lambda\left\{\frac{1}{(x^2+0.15^2)^{1/2}}\Bigg|_0^{0.4} + \frac{(0.15)\,x}{(0.15)^2(x^2+0.15^2)^{1/2}}\Bigg|_0^{0.4}\right\}$$

using values

$$\vec{E} = (8.99\times\cancel{10^9})(30\times\cancel{10^{-9}})\left(\hat{i}(2.34-6.67)+\hat{j}(6.24-0)\right)$$

$$\vec{E} = 269.7\left(-4.33\,\hat{i} + 6.24\,\hat{j}\right) = \left(-1167.8\,\hat{i} + 1682.93\,\hat{j}\right)\ N/C$$

$$\vec{E} = \left(-1.1678\,\hat{i} + 1.68293\,\hat{j}\right)\ kN/C$$

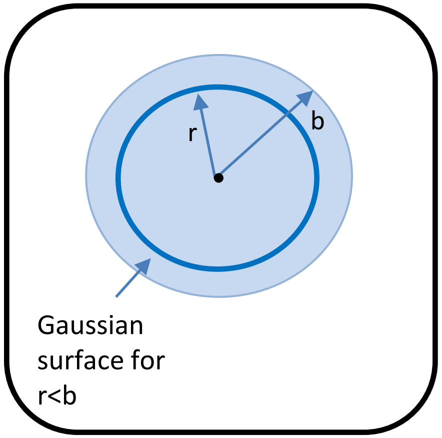
Example 24

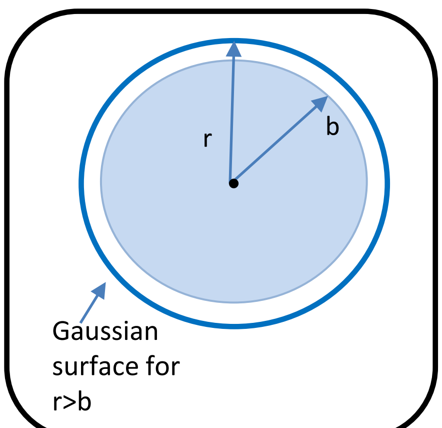
An insulating solid sphere of radius b has a uniform volume charge density and carries a total positive charge Q. A spherical gaussian surface of radius r, which shares a common center with the insulating sphere, is inflated starting from r=0.

(a) Find an expression for the electric flux passing through the surface of the gaussian sphere as a function of r for r<b.

(b) Find an expression for the electric flux for r>b.

(c) Plot the flux versus r.





Example 24(a)

the volume charge density $\rho = \frac{q}{V} \Rightarrow q = \rho V = \frac{4}{3}\pi b^3 \rho$

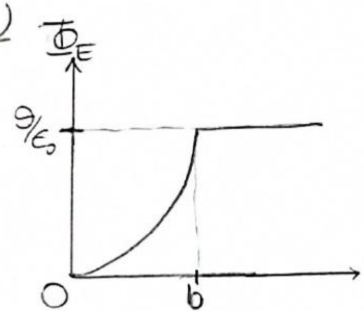
$\rho = \frac{3q}{4\pi b^3}$

the flux $\Phi_E = \frac{q_{in}}{\epsilon_0} = \frac{\rho}{\epsilon_0} \cdot V' = \frac{3q}{4\pi b^3 \epsilon_0} \cdot \frac{4}{3}\pi r^3 = \frac{q r^3}{\epsilon_0 b^3}$

Example 24(b)

the flux $\Phi_E = \frac{q_{in}}{\epsilon_0} = \frac{q}{\epsilon_0}$

Example 24(c)

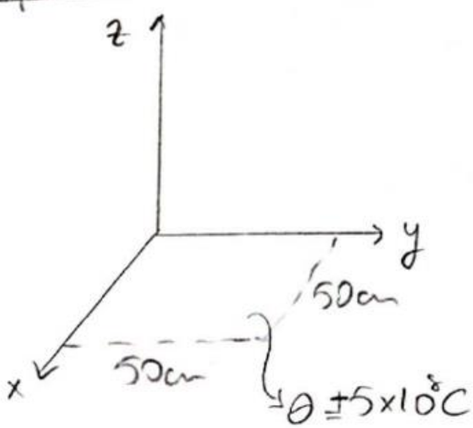


the flux will be maximum at the surface after that it will be constant!

Example 25

A thin square conducting plate 50.0 cm on a side lies in the xy plane. A total charge of 5.00×10^{8} C is placed on the plate. Find
(a) the charge density on the plate,
(b) the electric field just above the plate, and
(c) the electric field just below the plate.
You may assume that the charge density is uniform.

Example 25



a) it is 2D so surface charge density:

$$\sigma_{total} = \frac{Q}{A} = \sigma_{up} + \sigma_{down}$$

$$\sigma_{up} = -\sigma_{down} = \frac{\sigma_{total}}{2} = \frac{Q}{2A}$$

$$\sigma_{up} = -\sigma_{down} = \frac{+5 \times 10^{-8}}{2.(0.5)^2} = 1 \times 10^{-7}\ C/m^2$$

$$\sigma_{up} = -\sigma_{down} = 1\,nC/m^2$$

b) $$\Phi_E = \oint \vec{E}_{up} \cdot d\vec{A} = E_{up} A = \frac{q_{in}}{\epsilon_0} = \frac{\sigma_{up} \cancel{A}}{\epsilon_0} \qquad E_{up} = \frac{1 \times 10^{-7}}{(8.85 \times 10^{-12})}$$

$$\vec{E}_{up} = (11.3)\hat{k}\ kN/C \leftarrow E_{up} = (11,3)\ kN/C$$

Example 25

A thin square conducting plate 50.0 cm on a side lies in the xy plane. A total charge of 5.00x10^8 C is placed on the plate. Find

(a) the charge density on the plate,

(b) the electric field just above the plate, and

(c) the electric field just below the plate.

You may assume that the charge density is uniform.

Exaple 25(c)

c) $\Phi_E = \oint \vec{E}_{down} \cdot d\vec{A} = E_{down} \cdot \cancel{A} = \frac{q_{in}}{\epsilon_0} = \frac{\sigma_{down} \cdot \cancel{A}}{\epsilon_0}$

$$E_{down} = \frac{\sigma_{down}}{\epsilon_0} = -\frac{\sigma_{up}}{\epsilon_0}$$

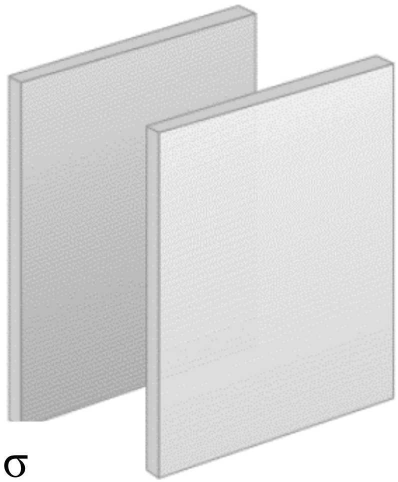
$$E_{down} = -(11.3)\, kN/C$$

$$\vec{E}_{down} = -(11.3)\hat{k}\ kN/C$$

Example 26

Two infinite, nonconducting sheets of charge are parallel to each other, as shown in Figure. The sheet on the left has a uniform surface charge density σ, and the one on the right has a uniform charge density -σ. Calculate the electric field at points

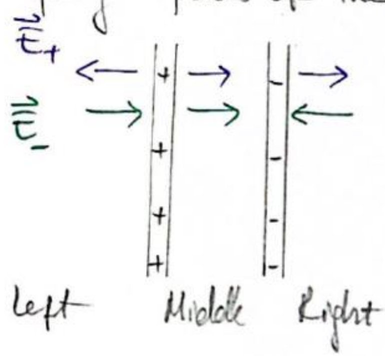
(a) to the left of, (b)in between, and (c)to the right of the two sheets.



σ

-σ

defining fields of the plates:



for a single sheet $|\vec{E}_+| = |\vec{E}_-| = \frac{\sigma}{2\epsilon_0}$

a) left: $|\vec{E}_+| = |\vec{E}_-|$

$\vec{E}_+$ and $\vec{E}_-$ are in opposite direction

so the total field $\vec{E} = \vec{E}_+ + \vec{E}_- = 0$

b) middle: $|\vec{E}_+| = |\vec{E}_-|$

they are in the same direction

the total field $\vec{E} = 2|E_+| = \frac{\sigma}{\epsilon_0}$

direction is towards right.

c) right: $|\vec{E}_+| = |\vec{E}_-|$

$\vec{E}_+$ and $\vec{E}_-$ are in opposite direction

total field $\Rightarrow \vec{E} = \vec{E}_+ + \vec{E}_- = 0$

Example 27

A spherically symmetric charge distribution has a charge density given by ρ=b/r, where b is constant. Find the electric field as a function of r.

Example 27

applying Gauss law: $\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$

$$E \underbrace{\int dA}_{A=4\pi r^2} = \frac{\int \rho \cdot dV}{\epsilon_0} \qquad \int dV = \int 4\pi r^2 dr$$

$$E \cdot 4\pi r^2 = \frac{1}{\epsilon_0} \int_0^r \frac{b}{\cancel{r}} 4\pi r^{\cancel{2}} dr = \frac{4\pi b}{\epsilon_0} \int r\, dr$$

$$E \cdot \cancel{4\pi} \cancel{r^2} = \frac{\cancel{4\pi} b}{\epsilon_0} \cdot \frac{\cancel{r^2}}{2} \Rightarrow E = \frac{b}{2\epsilon_0} \quad \text{"constant"}$$

the direction of the field is radially outward for $b>0$
radially inward for $b<0$

Example 28

A solid insulating sphere of radius R has a nonuniform charge density that varies with r according to the expression $\rho = Cr^2$, where C is a constant and $r<R$ is measured from the center of the sphere. Find the magnitude of the electric field

(a) outside ($r>R$)

(b) inside of the sphere.

Example 28

a) Gauss Law $\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$ $\qquad \int dV = 4\pi r^2 dr$ for sphere

for $r>R \Rightarrow q_{in} = \int_0^R \rho \, dV = \int_0^R Cr^2(4\pi r^2)\,dr = \frac{4\pi C R^5}{5}$

using this in gauss law:

$$\oint \vec{E}\cdot d\vec{A} = E\int_0^r dA = E\,\cancel{4\pi} r^2 = \frac{\cancel{4\pi} C R^5}{5\epsilon_0} \Rightarrow E = \frac{CR^5}{5\epsilon_0 r^2}$$

b) for $r<R$ $\quad q_{in} = \int_0^R Cr^2(4\pi r^2)\,dr = \frac{4\pi C r^5}{5}$

using this in gauss law:

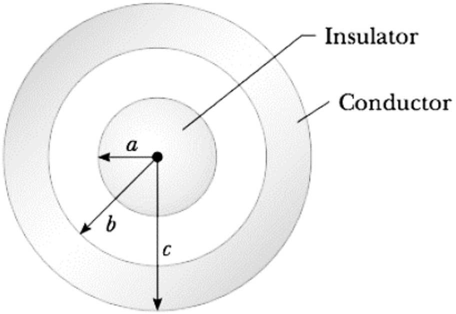
$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \Rightarrow E\cdot \cancel{4\pi}\cancel{r^2} = \frac{\cancel{4\pi} C r^{\cancel{5}\,3}}{5} \Rightarrow E = \frac{Cr^3}{5\epsilon_0}$$

Example 29

A solid, insulating sphere of radius a has a uniform charge density ρ and a total charge Q. Concentric with this sphere is an uncharged, conducting hollow sphere whose inner and outer radii are b and c, as shown in Figure.

(a) Find the magnitude of the electric field in the regions r<a, a<r<b, b<r<c, and r>c.

(b) Determine the induced charge per unit area on the inner and outer surfaces of the hollow sphere.



Example 29

a) for $r<a$: $\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} = \frac{\int \rho\, dV}{\epsilon_0} \Rightarrow \int \rho\, dV = \rho \frac{4}{3}\pi r^3$

$$E \cdot \cancel{4\pi r^2} = \frac{\cancel{4\pi} \rho r^{\cancel{3}}}{3\epsilon_0} \Rightarrow E = \frac{\rho r}{3\epsilon_0}$$

for $a<r<b$: $\oint E \cdot d\vec{A} = E \cdot 4\pi r^2 = \frac{q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{4\pi r^2 \epsilon_0}$

for $b<r<c$: $\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$ $\Rightarrow E = 0$

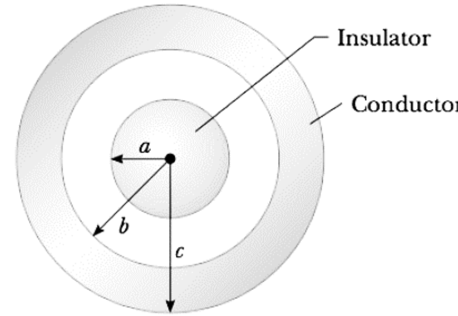
for $c<r$: $\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$ $q_{in} = Q \Rightarrow E = \frac{Q}{4\pi r^2 \epsilon_0}$

Example 29

A solid, insulating sphere of radius a has a uniform charge density ρ and a total charge Q. Concentric with this sphere is an uncharged, conducting hollow sphere whose inner and outer radii are b and c, as shown in Figure.

(a) Find the magnitude of the electric field in the regions r<a, a<r<b, b<r<c, and r>c.

(b) Determine the induced charge per unit area on the inner and outer surfaces of the hollow sphere.



b) q_{inner} is induced charge! of the inner surface of hollow sphere.

E=0 inside the hollow sphere so the total charge of the hollow sphere is 0!

$q_{inner} + Q = 0$ $\quad q_{inner} = -Q$ $\quad$ then $\quad \sigma_{inner} = \frac{q_{inner}}{A} = \frac{-Q}{4\pi b^2}$

q_{outer} is the induced charge of the outer surface of the hollow sphere.

$q_{inner} + q_{outer} = 0$ $\quad$ then $\quad \sigma_{outer} = \frac{q_{outer}}{A} = \frac{Q}{4\pi c^2}$