Mathematical and Physical Analysis of the Vibration Table

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1. Introduction

This document presents the mathematical and physical modeling of a small vibration table designed using **four springs** and a **DC motor with an offset mass**. The table is powered by a **9V, 1A motor operating at 5V**, controlled via Arduino Nano and an L298N motor driver.

The purpose of this analysis is to understand the vibration frequency, displacement, and forces involved. This enhances the engineering rigor of the project, making it suitable for academic or research applications.

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2. System Model

The vibration table can be modeled as a **mass-spring-damper system** excited by an unbalanced rotating mass.

- **Table mass (M):** effective mass of the platform (assume $M = 0.5\,kg$).
- **Spring constant (k):** stiffness of each spring. For four identical springs in parallel:

$$k_{eq} = 4k$$

- **Motor frequency (f):** related to motor speed (RPM).
- **Offset mass (m_e):** 20 g = 0.02 kg attached to the motor shaft.
- **Offset radius (r):** 3 cm = 0.03 m from the shaft center.

The excitation force is:

$$F(t) = m_e r \omega^2 \sin(\omega t)$$

where $\omega = 2\pi f$ is angular frequency.

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3. Natural Frequency of the System

The natural frequency of a spring-mass system is:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{M}}$$

For example, if each spring has stiffness $k = 200\,N/m$:

$$k_{eq} = 4 \times 200 = 800\, N/m$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{800}{0.5}} \approx 6.4\, Hz$$

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4. Motor Frequency

A 9V DC motor typically runs around **10,000 RPM at 9V**. At 5V, assume proportional speed:

$$\text{Speed} \approx 5000\, RPM = \frac{5000}{60} \approx 83\, Hz$$

Thus, the forcing frequency is much higher than the natural frequency ($f \gg f_n$). This creates rapid small-amplitude vibrations.

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5. Excitation Force Calculation

With the offset mass $m_e = 0.02\, kg$ and radius $r = 0.03\, m$, and angular frequency $\omega = 2\pi 83 \approx 521\, rad/s$:

$$F_0 = m_e r \omega^2 = 0.02 \times 0.03 \times 521^2 \approx 162.7\, N$$

This is the peak centrifugal force applied to the platform.

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6. Vibration Amplitude

For a lightly damped system, steady-state amplitude:

$$X = \frac{F_0}{|k_{eq} - M\omega^2|}$$

With $k_{eq} = 800\, N/m$ and $M = 0.5\, kg$:

$$X = \frac{162.7}{|800 - 0.5 \times 521^2|} \approx 0.0013\, m = 1.3\, mm$$

The table vibrates with ~1.3 mm amplitude.

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7. Power Consumption

Mechanical power delivered by the motor:

$$v = X \cdot \omega = 0.0013 \times 521 \approx 0.68\, m/s$$

$$P = F_0 \cdot v = 162.7 \times 0.68 \approx 110.6\, W$$

Since the motor is rated 9V × 1A = 9W, operating at 5V, actual delivered power is lower. This calculation shows the potential force without considering motor limits.

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8. Arduino Control

The Arduino adjusts motor speed using **PWM** via the L298N driver. The potentiometer allows smooth control of duty cycle, thus tuning vibration intensity.

Pseudo-equation:

$$V_{motor} = V_{supply} \times \frac{\text{PWM}}{255}$$

$$f_{motor} \propto V_{motor}$$