Students Performances Dataset

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August 29, 2020

1 Summary

In this study, models were built to predict the final grade in for students in mathematics class of Portuguese secondary school pupils based on some variables (i.e. pupils demographics, pupils activities at school and at home..etc.). Feature selection techniques were used to identify the best features to predict the target variable like sequential forward selection, sequential backward selection and select KBest. Regression methods like Linear Regression, Partial Least Square (PLS), Ridge Regression, Lasso Regression, Elastic Net and Random Forest Regression were used to predict the grade of the students. Random Forest gives the best (Rˆ2) score which is 0.87 on the test data. The data holds valuable information, such as trends and patterns, which can be used to improve decision-making. Hence, automated tools were deployed to analyze the raw data and extract interesting high-level information for decision making.

2 Introduction

Education is one of the most important investments, which a country must do for its future development. Education management is important to improve the education system and there is increasing demands for good support tools to manage the pupils' study. The application of Machine Learning (ML) may be a tool, which can help countries to manage their education systems. The dataset used in this report consisted of 33 variables and 395 instances (observations) and was retrieved from the website of University of California Irvine Machine Learning Repository. A description of variables is presented in appendix 1.

Portuguese education system was ranked in Top 20 education system in the third quarter, 2013. This data was also studied and published by Cotez & Silva, 2008. The variables of the dataset give a good insight of the student's personal and academic life. The dataset consists of scores for three periods of study that were represented by G1, G2 and G3. The target was to come up with a model that predicts the score of the G3 (The final period) based on the scores of the first two periods i.e. G1 and G2 and other variables that were provided in the dataset.

3 Methods

The raw data set was in csv format. We started with data visualisation using matplotlib, pandas and seaborn libraries. Null data was checked and histograms, correlation matrix, box-plot of numerical features were built (Fig. 1,2,3). Data was scaled using scikit-learn.

Different feature selection techniques used in this report were Sequential Forward Selection, Backward Selection and Select K Best. Mlextend library was used for sequential features selection. Sequential feature selection algorithms are greedy search algorithms. It's used to select a subset of features that is most relevant to the problem and removing irrelevant features or noise. The usage of features selection is important in this case study to remove the noises which leads to better prediction and to reduce the computation cost.

KBest feature selection method takes two arguments (f-regression and number of features to be selected). The idea behind f-regression in this context is that it uses f-score metric to tell if there is a regression relationship between each of features and the target. Select KBest technique proved to be the best features selection technique in this case study. Multiple regression methods were used to come up with a model that was best suited to predict the target variable (G3). The regression methods used were Linear regression, Partial Least Square (PLS) regression, Ridge regression, Lasso regression, Elastic Net and Random Forest Regression. All regressions implemented in this paper were using scikit-learn library.

Linear regression is best-fitting line through the training examples. Ordinary least squares (OLS) or Linear least squares method is to to estimate the coefficients of the regression line that minimizes the sum of the squared residuals or errors to the training data points. Partial Least Squares takes into account the decomposition of original variables into latent variables that describes the maximum variances.

Another family of regressions are the regularisation regressions (Ridge regression, Lasso, and elastic Net). Those methods are usually used to handle over-fitting by adding additional penalty term against complexity of the model which shrinks the parameter values of the model. The loss function will be sum of squared residuals of OLS equation plus the penalty term which is governed by the hyper-parameter lambda as shown the following equations.

$$L_{ridge} = \sum_{i=1}^{n} (y_i - a * x_i) + \lambda * \sum_{j=1}^{m} (a_j)^2$$

$$L_{lasso} = \sum_{i=1}^{n} (y_i - a * x_i) + \lambda * \sum_{j=1}^{m} |a_j|$$

$$L_{elasticnet} = \sum_{i=1}^{n} (y_i - a * x_i) + \lambda_1 * \sum_{j=1}^{m} *|a_j| + \lambda_2 * \sum_{j=1}^{m} *(a_j)^2$$

While a is the slope of the line, n is number of training examples and m is number of features. Ridge and Lasso use square and absolute value of the slope respectively. Elastic Net is just a combination of both penalties (Lasso and Ridge) in one loss function.

Another robust regression algorithms are decision tree. Decision tree splits its nodes until the leaves are pure which is defined by entropy as a measure of impurity. The impurity is what determine which feature split that maximizes the information gain. Combination or ensemble of several random decision trees gives better generalization performance and more robust model than an individual decision tree which is called "Random Forest".

Using these methods, It was not possible in this study to train all regression models on all kinds of selected features data. So, we train the regression models only on the data that gives the best validation score as shown in the results. For example, Lasso regression was only trained on forward selected features as it gives higher score and consider that score as the best score that Lasso regression will achieve in this case study. Grid Search was used only to optimize the best performing model which is the random forest in this study to save computational cost. Grid search is used to estimate the number of decision trees or estimators to be used.

3.1 Used Libraries

- Pandas
- Numpy
- Matplotlib
- Seaborn
- Scikit-Learn
- mlxtend

4 Results

4.0.1 Reading the data

[170]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

[171]:
```
data = pd.read_csv('arranged_mathData.csv')
```

[172]:
```
data.shape
```

[172]:
```
(395, 33)
```

[173]:
```
data.head()
```

[173]:
```
  school sex  age address famsize Pstatus  Medu  Fedu     Mjob      Fjob  ...  \
0     GP   F   18       U     GT3       A     4     4  at_home   teacher  ...
1     GP   F   17       U     GT3       T     1     1  at_home     other  ...
2     GP   F   15       U     LE3       T     1     1  at_home     other  ...
3     GP   F   15       U     GT3       T     4     2   health  services  ...
4     GP   F   16       U     GT3       T     3     3    other     other  ...

  famrel freetime  goout  Dalc  Walc health absences  G1  G2  G3
0      4        3      4     1     1      3        6   5   6   6
1      5        3      3     1     1      3        4   5   5   6
2      4        3      2     2     3      3       10   7   8  10
3      3        2      2     1     1      5        2  15  14  15
4      4        3      2     1     2      5        4   6  10  10

[5 rows x 33 columns]
```

[174]:
```
data.Fjob.unique()
```

[174]:
```
array(['teacher', 'other', 'services', 'health', 'at_home'], dtype=object)
```

Columns Names: The following pandas index shows all the columns names in the dataset.

[175]:
```
data.columns
```

[175]:
```
Index(['school', 'sex', 'age', 'address', 'famsize', 'Pstatus', 'Medu', 'Fedu',
       'Mjob', 'Fjob', 'reason', 'guardian', 'traveltime', 'studytime',
       'failures', 'schoolsup', 'famsup', 'paid', 'activities', 'nursery',
       'higher', 'internet', 'romantic', 'famrel', 'freetime', 'goout', 'Dalc',
       'Walc', 'health', 'absences', 'G1', 'G2', 'G3'],
      dtype='object')
```

Checking if there is any data is null:

[176]:
```
data.isnull().values.any()
```

[176]:
```
False
```

Statistics of raw data:

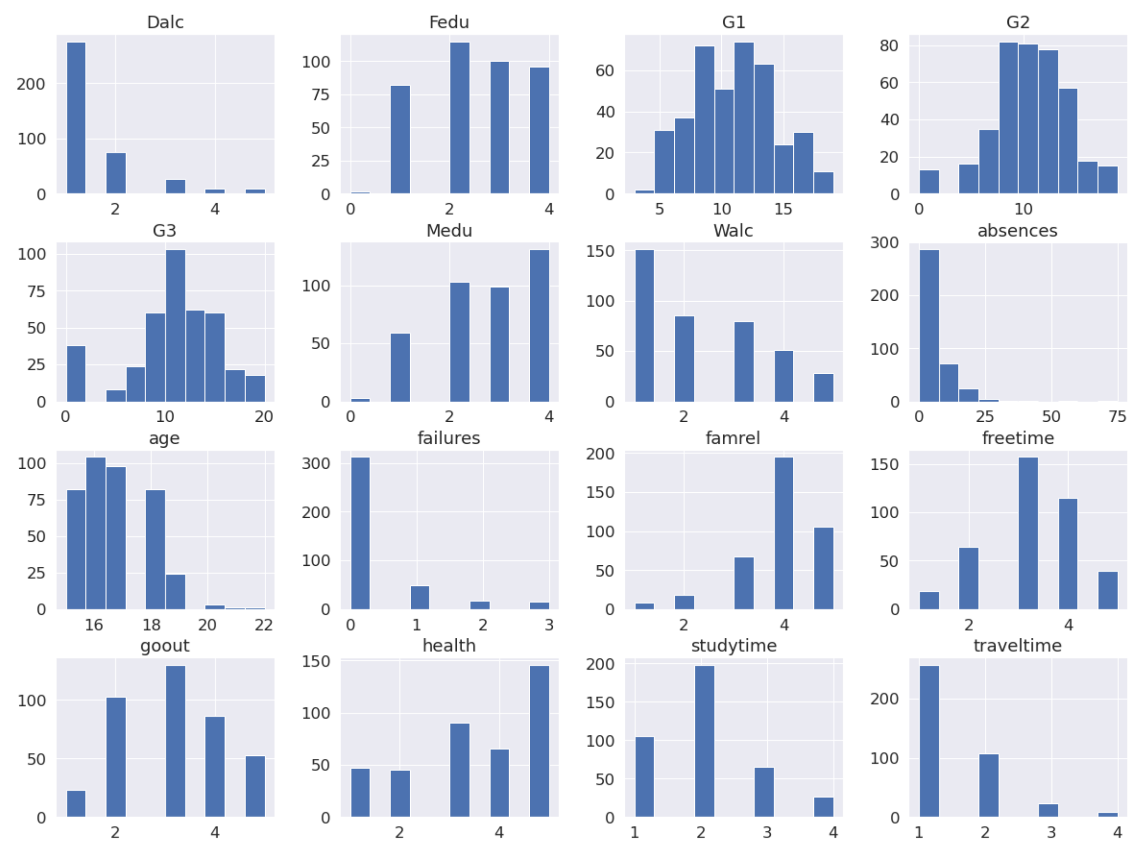
[177]:
```
data.describe()
```

[177]:
```
              age        Medu        Fedu  traveltime   studytime    failures  \
count  395.000000  395.000000  395.000000  395.000000  395.000000  395.000000
mean    16.696203    2.749367    2.521519    1.448101    2.035443    0.334177
std      1.276043    1.094735    1.088201    0.697505    0.839240    0.743651
min     15.000000    0.000000    0.000000    1.000000    1.000000    0.000000
25%     16.000000    2.000000    2.000000    1.000000    1.000000    0.000000
50%     17.000000    3.000000    2.000000    1.000000    2.000000    0.000000
75%     18.000000    4.000000    3.000000    2.000000    2.000000    0.000000
max     22.000000    4.000000    4.000000    4.000000    4.000000    3.000000

           famrel    freetime       goout        Dalc        Walc      health  \
count  395.000000  395.000000  395.000000  395.000000  395.000000  395.000000
mean     3.944304    3.235443    3.108861    1.481013    2.291139    3.554430
std      0.896659    0.998862    1.113278    0.890741    1.287897    1.390303
min      1.000000    1.000000    1.000000    1.000000    1.000000    1.000000
25%      4.000000    3.000000    2.000000    1.000000    1.000000    3.000000
50%      4.000000    3.000000    3.000000    1.000000    2.000000    4.000000
75%      5.000000    4.000000    4.000000    2.000000    3.000000    5.000000
max      5.000000    5.000000    5.000000    5.000000    5.000000    5.000000

         absences          G1          G2          G3
count  395.000000  395.000000  395.000000  395.000000
mean     5.708861   10.908861   10.713924   10.415190
std      8.003096    3.319195    3.761505    4.581443
min      0.000000    3.000000    0.000000    0.000000
25%      0.000000    8.000000    9.000000    8.000000
50%      4.000000   11.000000   11.000000   11.000000
75%      8.000000   13.000000   13.000000   14.000000
max     75.000000   19.000000   19.000000   20.000000
```

Histogram of the raw data: Figure 1 shows the histogram of the raw numeric data. Categorical features like Fjob and Mjob are not shown here and that will be encoded later. From the figure 1 , the numeric data is relatively skewed.

[178]:
```
sns.set(font_scale=1.5)
data.hist(figsize=(20, 15))
plt.suptitle('Figure 1: Histograms of raw data', fontsize=20)
plt.show()
```

Figure 1: Histograms of raw data

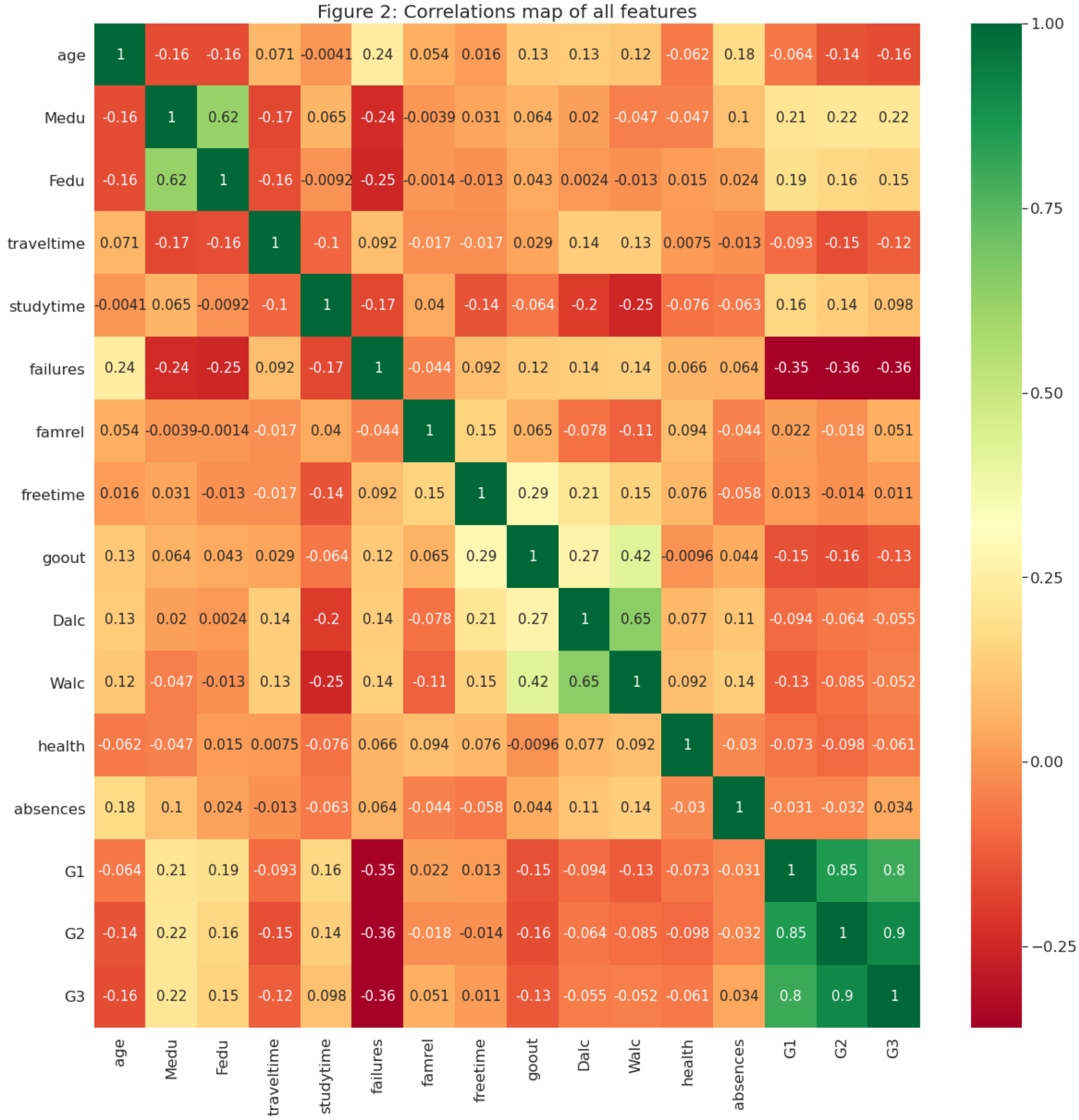


Correlation matrix for all the features on raw data: Correlation matrix (Figure 2) was use to get overview patterns between the raw data features, It's obvious that G3 is highly correlated with G1 and G2 features.

[179]:
```
#get correlations of each features in dataset
corrmat = data.corr()
top_corr_features = corrmat.index
plt.figure(figsize=(20,20))
plt.title('Figure 2: Correlations map of all features', fontsize=20)
#plot heat map
```

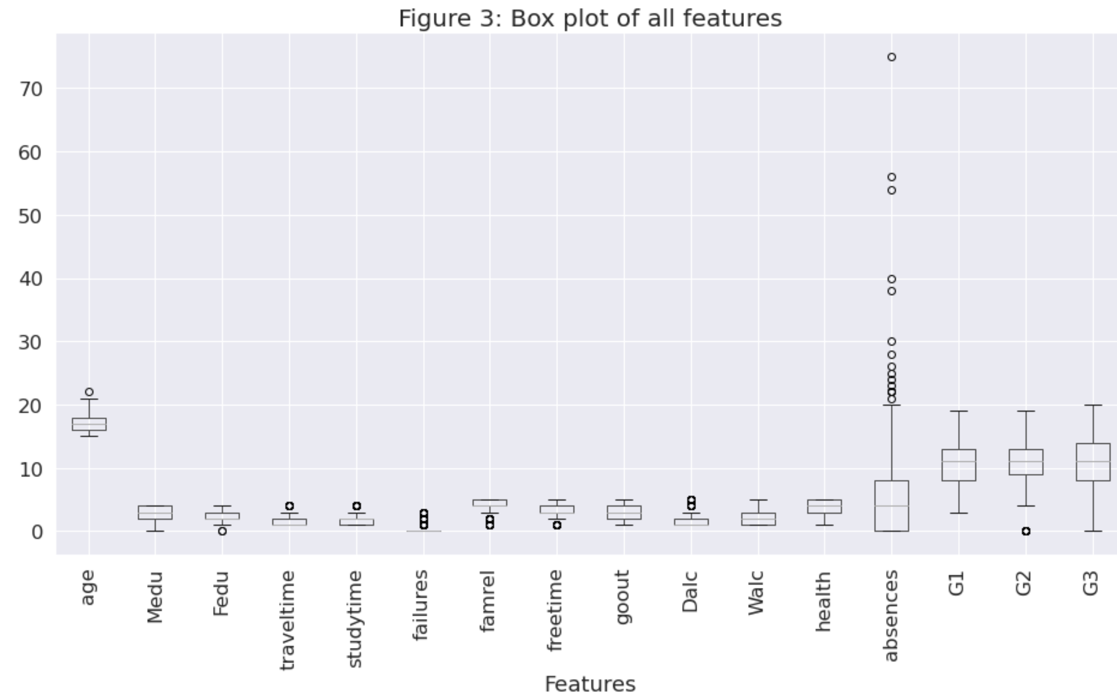
```python
g = sns.heatmap(data[top_corr_features].corr(),
                annot=True,
                cmap="RdYlGn",
                annot_kws={"size": 15})
```

Figure 2: Correlations map of all features



Box Plot of raw data: Figure 3 displays the data quartiles is used to watch the data outliers, absences features shows values outside the inter-quartile range. However, the outliers were not removed from the data because it still gives r^2 score of more than 0.8 for most regression algorithms as will be shown later in the report.

[180]:
```
plt.figure(figsize=(16,8))
plt.xlabel('Features')
plt.title('Figure 3: Box plot of all features', fontsize=20)
data.boxplot()
plt.xticks(rotation='vertical')
plt.show()
```



[181]:
```
# this dictionary is used to save the R2 scores from all regressions
saved_results_r2 = {}
```

4.0.2 Conversion of categorical features into dummy variables

Through data analysis, we noticed that the dataset includes 13 binary features (i.e. sex, internet) that were converted into zeros and ones for simplification of the data. Other features that include more than 2 unique values like Mjob used one-hot-encoding. One-hot-encoding will assign a numeric value to categorical features. For example, Fjob features has 5 unique values, Those values are teacher, other, services, health and at_home. The value 'teacher' will be encoded as [1, 0, 0, 0, 0] while the value 'other' is encoded as [0, 1, 0, 0, 0]. The newly converted data matrix now has 46 columns.

[182]:
```
#Dumify Bool Values and Take only One Column For them
dummy_data =pd.get_dummies(data,columns=['school', 'sex', 'address', 'famsize',
                                      'Pstatus','schoolsup','famsup', 'paid',
                                      'activities','nursery', 'higher',␣
 ↪'internet',
```

```
                                        'romantic'],drop_first=True)
#Dumify Categorical Values
dummy_data =pd.get_dummies(dummy_data,columns=['Mjob', 'Fjob', 'reason',␣
 ↪'guardian'])
```

[183]:
```
dummy_data.shape
```

[183]: (395, 46)

4.0.3 Columns names:

The following pandas index shows all the columns names in the dataset after converting the raw catgorical variables into dummy variables

[184]:
```
dummy_data.columns
```

[184]:
```
Index(['age', 'Medu', 'Fedu', 'traveltime', 'studytime', 'failures', 'famrel',
       'freetime', 'goout', 'Dalc', 'Walc', 'health', 'absences', 'G1', 'G2',
       'G3', 'school_MS', 'sex_M', 'address_U', 'famsize_LE3', 'Pstatus_T',
       'schoolsup_yes', 'famsup_yes', 'paid_yes', 'activities_yes',
       'nursery_yes', 'higher_yes', 'internet_yes', 'romantic_yes',
       'Mjob_at_home', 'Mjob_health', 'Mjob_other', 'Mjob_services',
       'Mjob_teacher', 'Fjob_at_home', 'Fjob_health', 'Fjob_other',
       'Fjob_services', 'Fjob_teacher', 'reason_course', 'reason_home',
       'reason_other', 'reason_reputation', 'guardian_father',
       'guardian_mother', 'guardian_other'],
      dtype='object')
```

4.1 Splitting the data matrix into variables matrix and output vector

The target variable (y) is set to be the column G3, all other columns are considered as the X matrix. As explained in the data, The target variable G3 has a strong correlation with G2 and G1. It was not possible to predict G3 without G2 and G1.

[185]:
```
X = dummy_data.loc[:, dummy_data.columns != 'G3']
y = dummy_data['G3']
```

[186]:
```
X.shape, y.shape
```

[186]: ((395, 45), (395,))

4.2 Train Test Split

The data has 395 obsevations, and was split the data with 70% train and 30%.

[187]:

```
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.30,␣
 ↪random_state=42)
```

4.3 Standardization

Standardization the data prior to Principal Components Analysis (PCA) so to make the other components contribute in variance. The reason for this is because PCA seeks to maximize the variance of each component. The training data is used to fit the scaler object and the use the parameters of scaling from train data to scale the test data as follows.

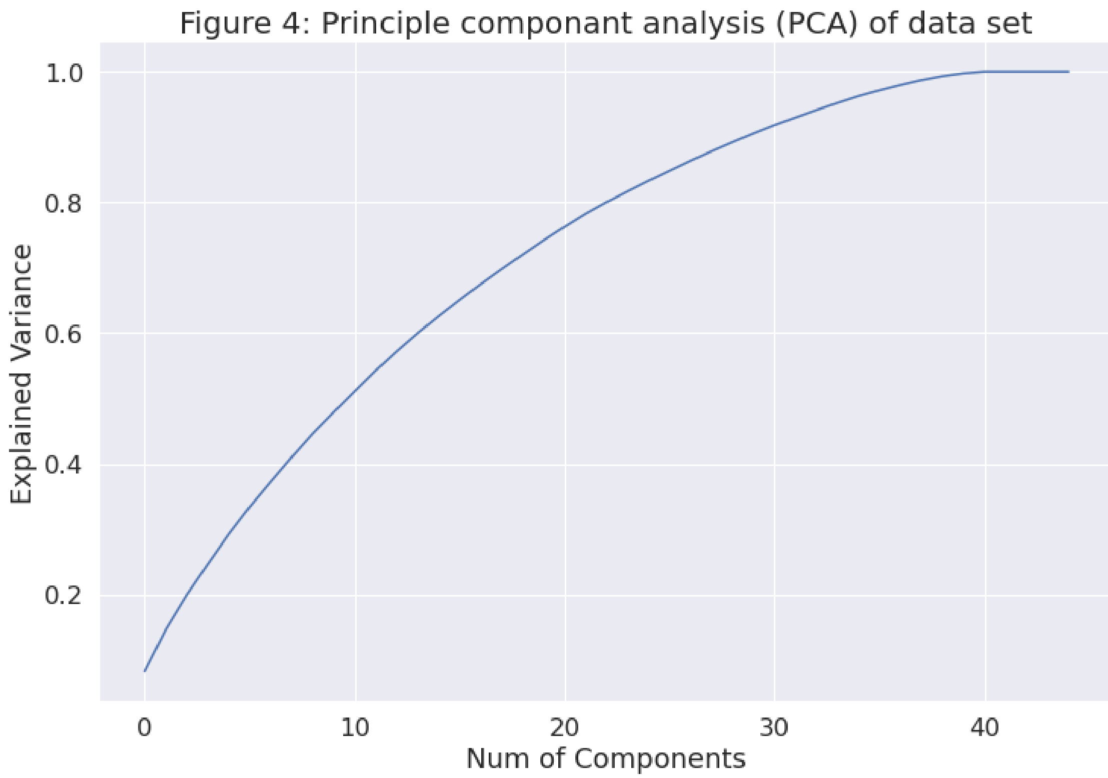
[247]:
```
from sklearn.preprocessing import StandardScaler
sc = StandardScaler()
sc.fit(X_train)
X_train_std = sc.transform(X_train)
X_test_std = sc.transform(X_test)
```

4.4 Principal Components Analysis (PCA)

The matrix X contains 45 columns, so we build instance of the PCA object with 45 components to estimate the explained variance for each component. from the graph, we noticed that the curve flattens at 40 components, which means that 40 components would be enough to explain the variance in the data. However, we didn't use PCA in the modeling, we run the modeling and feature selection on the original standardized data because the original data tends to gives higher scores.

[189]:
```
from sklearn.decomposition import PCA
pca = PCA(n_components=45)
pca.fit(X_train_std)
cum_var = pca.explained_variance_ratio_.cumsum()
```

[190]:
```
plt.figure(figsize=(12,8))
plt.plot(cum_var)
plt.xlabel('Num of Components')
plt.ylabel('Explained Variance')
plt.title('Figure 4: Principle componant analysis (PCA) of data set',␣
 ↪fontsize=20)
plt.show()
```



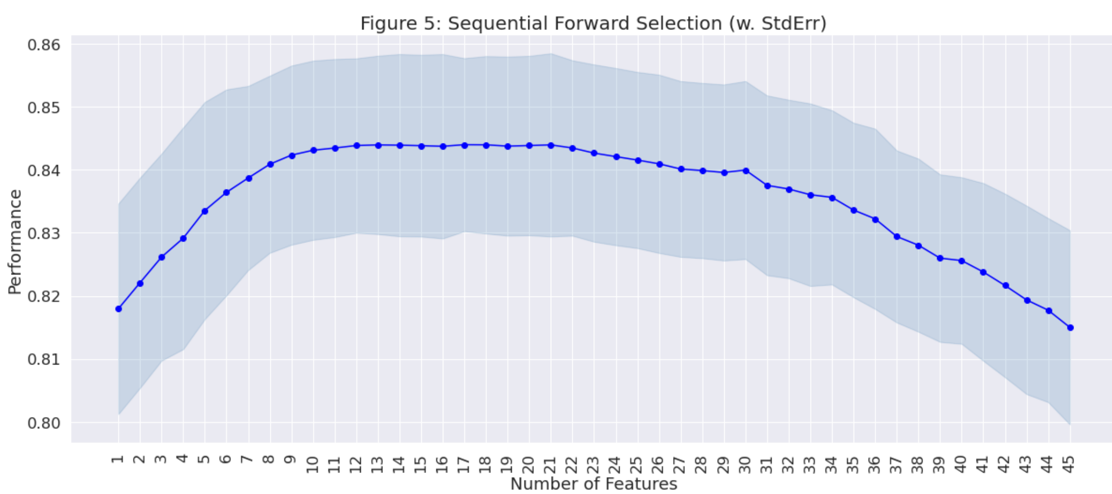
4.5 Sequential Feature Selection

4.5.1 Forward Sequential Feature Selection

Mlextend library is used in sequential feature selection. It was decided to stick to only 21 features, after 21 features, the R^2 score declines again (fig. 5), The calculation was done using linear regression.

[191]:
```
from mlxtend.feature_selection import SequentialFeatureSelector as SFS
from mlxtend.plotting import plot_sequential_feature_selection as plot_sfs
from sklearn.linear_model import LinearRegression

# build linear regression
lr = LinearRegression()

# Initialise Sequential Feature Selector
sfs1 = SFS(lr,
           k_features=45,
           forward=True,
           floating=False,
           verbose=0,
           scoring='r2',
           cv=10)
```

```
# Fit models
sfs1 = sfs1.fit(X_train_std, y_train)

# This dictionary contains results from all compuations
metric_dict = sfs1.get_metric_dict()

# Get indices of best features
k_ind = sfs1.k_feature_idx_

# Get names of best features using numpy arrays
feature_names = X.columns

# Plot the score vs the feature index
fig = plot_sfs(metric_dict, kind='std_err', figsize=(20, 8))
plt.title('Figure 5: Sequential Forward Selection (w. StdErr)', fontsize=20)
plt.xticks(rotation='vertical')
plt.show()
```



By the following, we take only the 21 features that are only need from the train data .

[192]:
```
# Initialise Sequential Feature Selector
sfs1 = SFS(lr,
           k_features=21,
           forward=True,
           floating=False,
           verbose=0,
           scoring='r2',
           cv=10)

# Fit models
```

```
sfs1 = sfs1.fit(X_train_std, y_train)

# This dictionary contains results from all compuations
metric_dict = sfs1.get_metric_dict()

# Get indices of best features
k_ind = sfs1.k_feature_idx_

# Get names of best features using numpy arrays
feature_names = X.columns
```

The next cell is a list of the features names of selected features based on forward sequential selection

[193]:
```
forward_selected_features_names = [dummy_data.columns[col] for col in k_ind]
print('Forward selected features names', forward_selected_features_names)
```

```
Forward selected features names ['age', 'famrel', 'goout', 'Dalc', 'absences',
'G1', 'G2', 'school_MS', 'Pstatus_T', 'schoolsup_yes', 'famsup_yes', 'paid_yes',
'higher_yes', 'internet_yes', 'Mjob_teacher', 'Fjob_other', 'Fjob_teacher',
'reason_course', 'reason_home', 'reason_other', 'guardian_father']
```

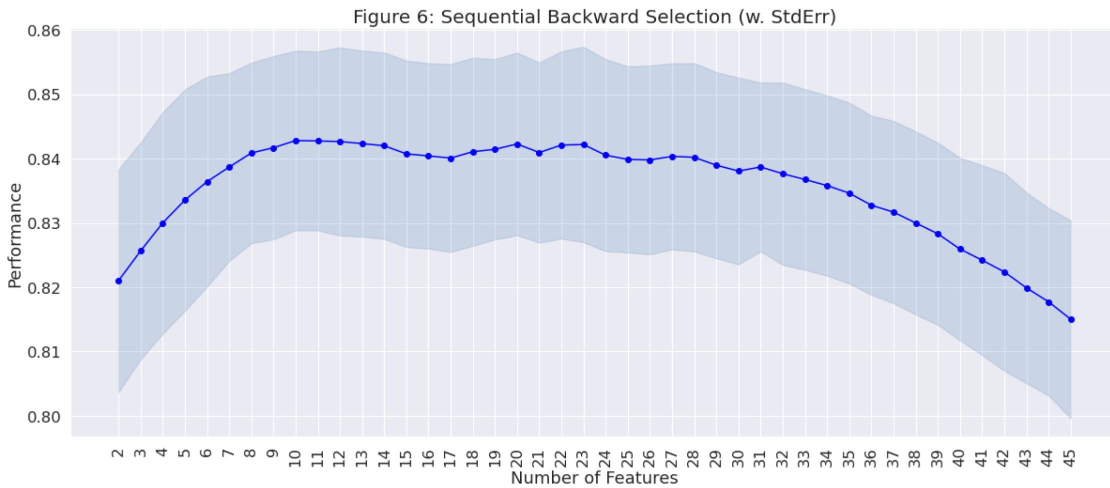
[194]:
```
X_test_forward_selected_features = X_test_std[:, k_ind]
X_train_forward_selected_features = X_train_std[:, k_ind]
```

4.5.2 Backward Features Selection

In backward selection features, 10 features gives optimal scores (fig. 6).

[195]:
```
# build linear regression
lr = LinearRegression()

# Initialise Sequential Feature Selector
sfs1 = SFS(lr,
           k_features=(2,45),
           forward=False,
           floating=False,
           verbose=0,
           scoring='r2',
           cv=10)

# Fit models
sfs1 = sfs1.fit(X_train_std, y_train)

# This dictionary contains results from all compuations
metric_dict = sfs1.get_metric_dict()

# Get indices of best features
```

```
k_ind = sfs1.k_feature_idx_

# Get names of best features using numpy arrays
feature_names = X.columns

# Plot the score vs the feature index
fig = plot_sfs(metric_dict, kind='std_err', figsize=(20, 8))
plt.title('Figure 6: Sequential Backward Selection (w. StdErr)', fontsize=20)
plt.xticks(rotation='vertical')
plt.show()
```



Now in backward selection features, only 10 features were selected

[196]:
```
# build linear regression
lr = LinearRegression()

# Initialise Sequential Feature Selector
sfs1 = SFS(lr,
           k_features=10,
           forward=False,
           floating=False,
           verbose=0,
           scoring='r2',
           cv=10)

# Fit models
sfs1 = sfs1.fit(X_train_std, y_train)

# This dictionary contains results from all compuations
metric_dict = sfs1.get_metric_dict()
```

```
# Get indices of best features
k_ind = sfs1.k_feature_idx_
```

[197]:
```
X_test_backward_selected_features = X_test_std[:, k_ind]
X_train_backward_selected_features = X_train_std[:, k_ind]
```

The next cell is a list of the features names of selected features based on backward sequential selection

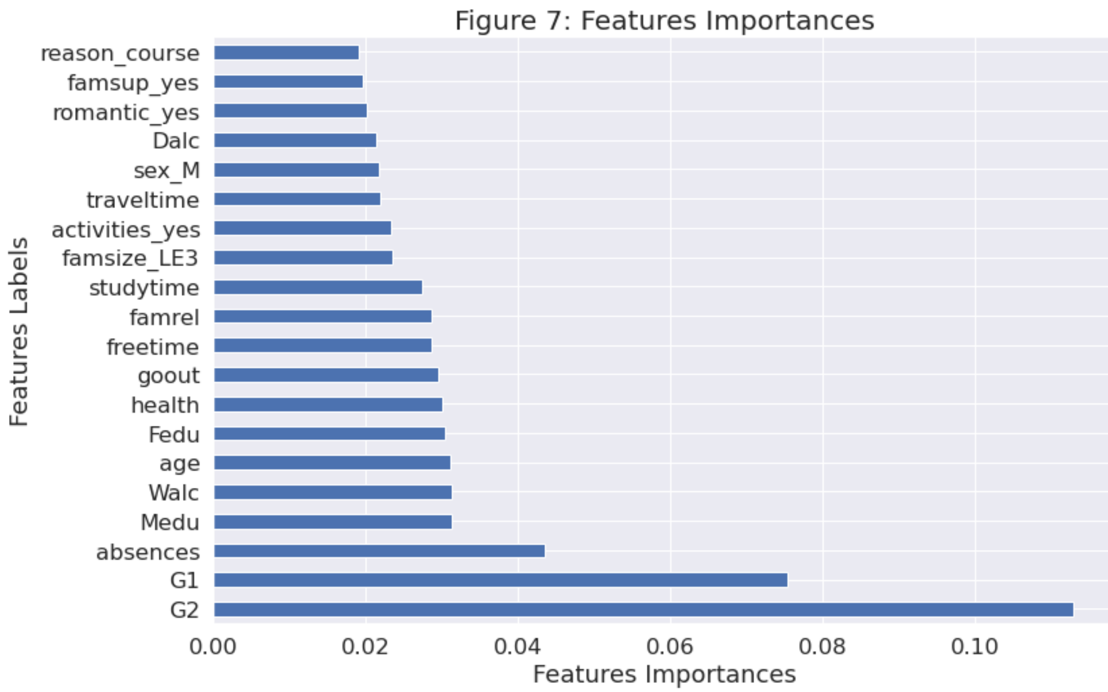
[198]:
```
backward_selected_features_names = [dummy_data.columns[col] for col in k_ind]
print('Backward selected features names', backward_selected_features_names)
```

```
Backward selected features names ['famrel', 'absences', 'G1', 'G2', 'Pstatus_T',
'paid_yes', 'internet_yes', 'Mjob_services', 'Fjob_health', 'reason_course']
```

4.5.3 Using ExtraTreesRegressor to plot features importances

ExtraTreesRegressor was applied to find the features importance. The most important features are presented in figure 7. G2, G1, absences, age and health features are among the top most important features that can be used to predict the final grade (G3) of the student.

[199]:
```
from sklearn.ensemble import ExtraTreesClassifier
import matplotlib.pyplot as plt
model = ExtraTreesClassifier(n_estimators=100)
model.fit(X,y)
#plot graph of feature importances for better visualization
plt.figure(figsize=(12, 8))
plt.title('Figure 7: Features Importances', fontsize=20)
plt.ylabel('Features Labels')
plt.xlabel('Features Importances')
feat_importances = pd.Series(model.feature_importances_, index=X.columns)
feat_importances.nlargest(20).plot(kind='barh')
plt.show()
```



4.6 Regression Algorithms:

4.6.1 Partial Least Square Regression (PLS)

Figure 8 shows the validation Curve for PLS regression on Forward Selected Features. From the fig. 8, the number of regression components to be used is 6 (at the flatted point of the validation of the accuracy).

IV.14.1.1. Partial Least Square Regression with Forward selected features data

[248]:
```
from sklearn.cross_decomposition import PLSRegression
from sklearn.model_selection import cross_val_score
from sklearn.metrics import mean_squared_error
from sklearn.metrics import make_scorer

pls = PLSRegression(n_components=6)

pls_r2 = cross_val_score(pls, X_train_forward_selected_features, y_train, cv=10,␣
 ↪scoring='r2')
pls_mse = cross_val_score(pls, X_train_forward_selected_features, y_train,␣
 ↪cv=10, scoring= make_scorer(mean_squared_error))
```

[249]:
```
print('Validation R2 average for using PLS on forward selected features '+␣
 ↪str(np.mean(pls_r2)))
```

```
Validation R2 average for using PLS on forward selected features
0.8429343620491305
```

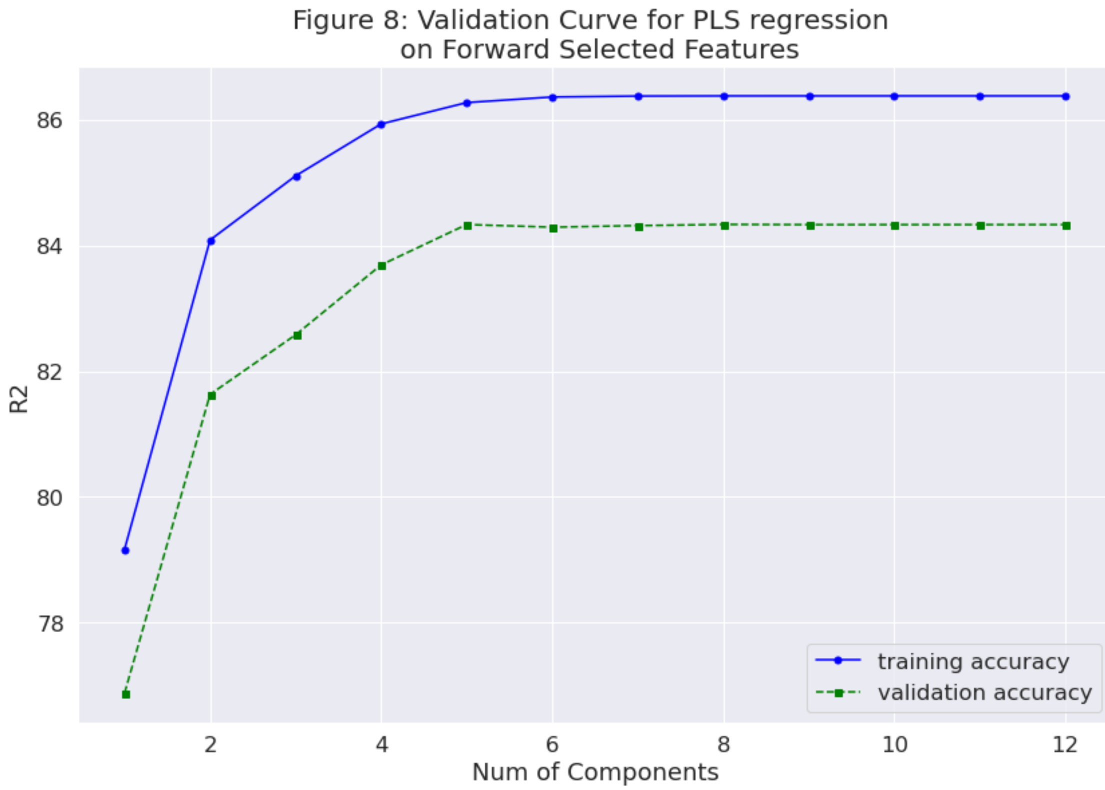
[251]:
```
print('Validation MSE average for using PLS on forward selected features '+␣
↪str(np.mean(pls_mse)))
```

```
Validation MSE average for using PLS on forward selected features
3.1107850800464387
```

PLS Regression components were decided to be 6 based on the following plot, Using PLS regression as an estimator and get the best validation accuracy. From the plot, after number of components reaches 6, the validation and training curve flatten.

[252]:
```
from sklearn.model_selection import validation_curve

pls = PLSRegression()

# Validation curve for parameter estimation
param_range = list(range(1,13))

train_scores, test_scores = validation_curve(
                estimator=pls,
                X=X_train_forward_selected_features,
                y=y_train,
                param_name='n_components',
                param_range=param_range,
                cv=10)

# Calculate validation curves for training and test sets
train_mean = np.mean(train_scores, axis=1)*100
train_std = np.std(train_scores, axis=1)*100
test_mean = np.mean(test_scores, axis=1)*100
test_std = np.std(test_scores, axis=1)*100

plt.figure(figsize=(12, 8))

plt.plot(param_range, train_mean,
         color='blue', marker='o',
         markersize=5, label='training accuracy')

plt.plot(param_range, test_mean,
         color='green', linestyle='--',
         marker='s', markersize=5,
         label='validation accuracy')

plt.legend()
plt.xlabel('Num of Components')
plt.ylabel('R2')
```

```
plt.tight_layout()
plt.title("Figure 8: Validation Curve for PLS regression \n on Forward Selected␣
↪Features", fontsize=20)
plt.show()
```

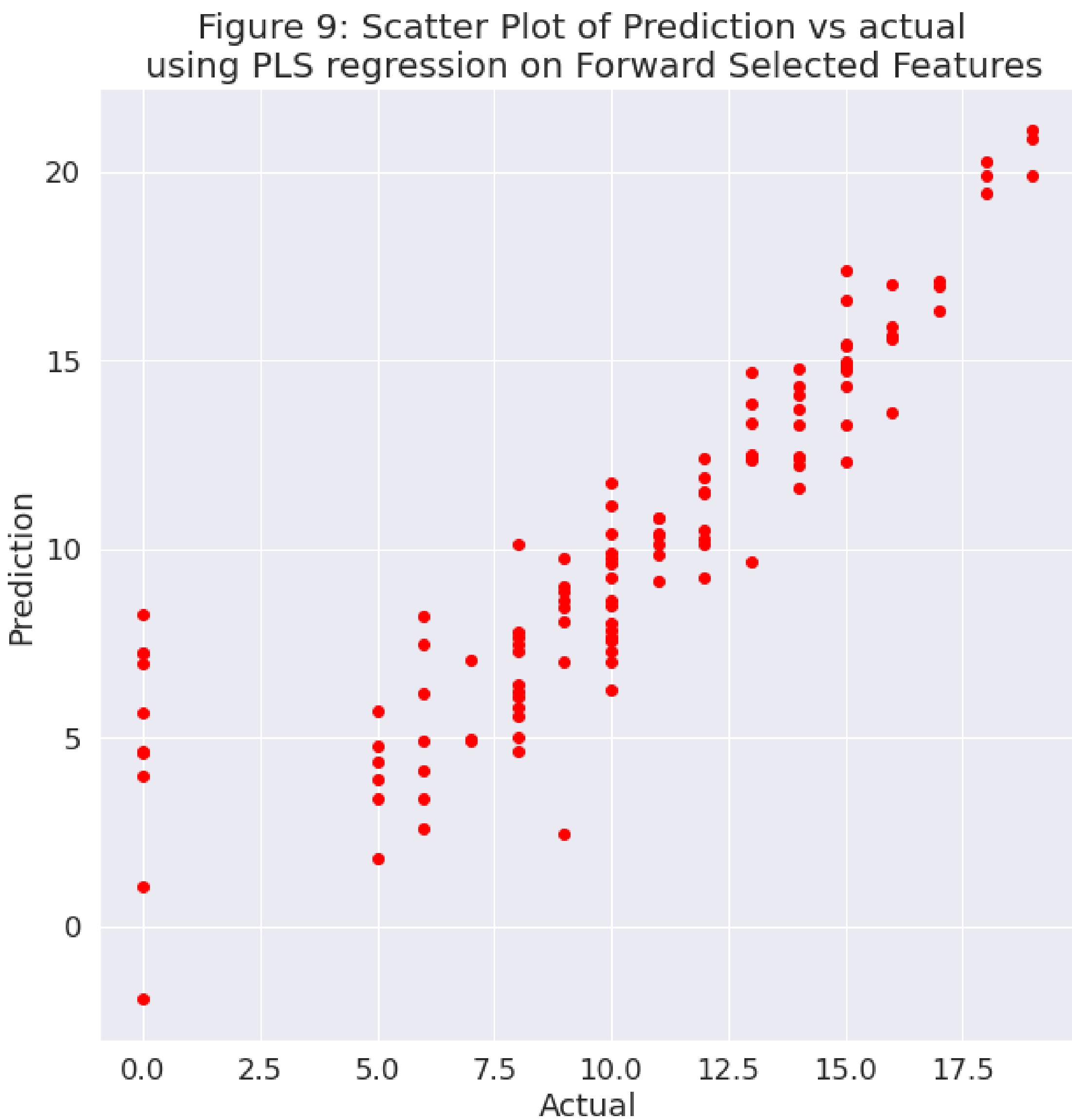


Training of PLS: We train the model on the X_train_forward_selected_features,

[253]:
```
pls.fit(X_train_forward_selected_features, y_train)
y_pred = pls.predict(X_test_forward_selected_features)
```

Scatter Graphs for Prediction vs Actual using PLS on Forward Selected features: Fig.9 shows that the prediction based on the model and the real data are relatively closer. The R^2 score is 0.766.

[255]:
```
plt.figure(figsize=(10, 10))
plt.scatter(y_test, y_pred, color = "red")
plt.title("Figure 9: Scatter Plot of Prediction vs actual \n using PLS␣
↪regression on Forward Selected Features", fontsize=20)
plt.xlabel("Actual")
plt.ylabel("Prediction")
plt.show()
```



[256]:
```
print(pls.score(X_test_forward_selected_features, y_test))
```

```
0.7587037084655428
```

Partial Least Square Regression with backward selected features data: Number of components is decided to be 5 based on the optimal R^2 score on training and validation data as shown in fig.10.

[257]:
```
pls = PLSRegression(n_components=5)

pls_r2 = cross_val_score(pls, X_train_backward_selected_features, y_train,␣
  ↪cv=10, scoring='r2')
```

```
pls_mse = cross_val_score(pls, X_train_backward_selected_features, y_train,␣
↪cv=10, scoring= make_scorer(mean_squared_error))
```

[258]:
```
print('Validation R2 averge for using PLS on backward selected features '+str(np.
↪mean(pls_r2)))
print('Validation MSE average for using PLS on backward selected features '+␣
↪str(np.mean(pls_mse)))
```

```
Validation R2 averge for using PLS on backward selected features
0.8429651279757728
Validation MSE average for using PLS on backward selected features
3.1091512827784813
```

PLS gives relatively higher R^2 scores on test data when we use the forward selected features comparing to using backward selected features as training data.

Training PLS on backward selected features

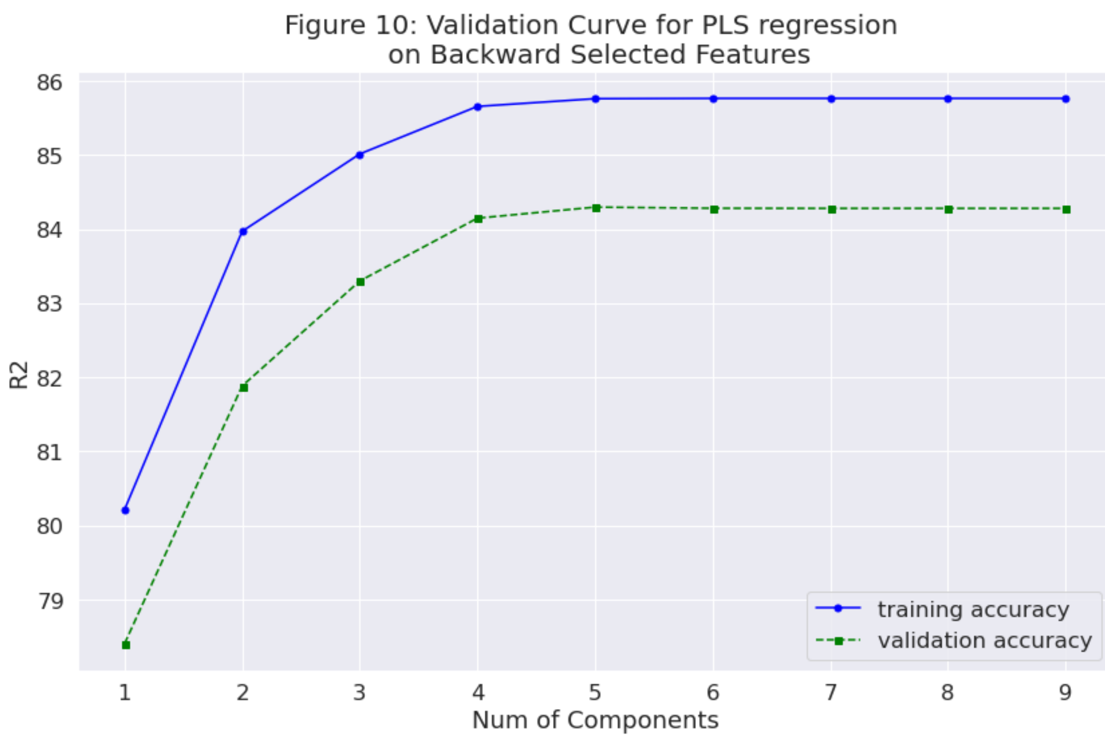
[259]:
```
pls.fit(X_train_backward_selected_features, y_train)
y_pred = pls.predict(X_test_backward_selected_features)
```

[260]:
```
score = pls.score(X_test_backward_selected_features, y_test)
print('Testing PLS score on backward selected features '+ str(score))
#Save the score to the result dictionary
saved_results_r2['PLS'] = score
```

```
Testing PLS score on backward selected features 0.7812965448665099
```

[261]:
```
from sklearn.model_selection import validation_curve

pls = PLSRegression()

# Validation curve for parameter estimation
param_range = list(range(1,10))

train_scores, test_scores = validation_curve(
                estimator=pls,
                X=X_train_backward_selected_features,
                y=y_train,
                param_name='n_components',
                param_range=param_range,
                cv=10)

# Calculate validation curves for training and test sets
train_mean = np.mean(train_scores, axis=1)*100
train_std = np.std(train_scores, axis=1)*100
test_mean = np.mean(test_scores, axis=1)*100
test_std = np.std(test_scores, axis=1)*100
```

```
plt.figure(figsize=(12, 8))

plt.plot(param_range, train_mean,
         color='blue', marker='o',
         markersize=5, label='training accuracy')

plt.plot(param_range, test_mean,
         color='green', linestyle='--',
         marker='s', markersize=5,
         label='validation accuracy')

plt.legend()
plt.xlabel('Num of Components')
plt.ylabel('R2')
plt.title("Figure 10: Validation Curve for PLS regression \n on Backward␣
↪Selected Features", fontsize=20)
plt.tight_layout()
plt.show()
```



4.6.2 Ridge Regression

Ridge Regression on forward selected features

```
[212]: from sklearn.linear_model import Ridge
ridge = Ridge(alpha=0.01)
ridge_r2 = cross_val_score(ridge, X_train_forward_selected_features, y_train,␣
 ↪cv=10, scoring='r2')
ridge_mse = cross_val_score(ridge, X_train_forward_selected_features, y_train,␣
 ↪cv=10, scoring= make_scorer(mean_squared_error))
```

```
[262]: print('Validation R2 average for using Ridge regression on forward selected␣
 ↪features '+ str(np.mean(ridge_r2)))
print('Validation MSE average for using Ridge regression on forward selected␣
 ↪features '+ str(np.mean(ridge_mse)))
```

```
Validation R2 average for using Ridge regression on forward selected features
0.8428034737508195
Validation MSE average for using Ridge regression on forward selected features
3.1169902020826985
```

Ridge Regression on backward selected features

```
[263]: ridge = Ridge(alpha=0.01)
ridge_r2 = cross_val_score(ridge, X_train_backward_selected_features, y_train,␣
 ↪cv=10, scoring='r2')
ridge_mse = cross_val_score(ridge, X_train_backward_selected_features, y_train,␣
 ↪cv=10, scoring= make_scorer(mean_squared_error))
```

```
[265]: print('Validation R2 average for using Ridge regression on backward selected␣
 ↪features '+ str(np.mean(ridge_r2)))
print('Validation MSE average for using Ridge regression on backward selected␣
 ↪features '+ str(np.mean(ridge_mse)))
```

```
Validation R2 average for using Ridge regression on backward selected features
0.8428034737508195
Validation MSE average for using Ridge regression on backward selected features
3.1169902020826985
```

Using ridge regression on backward selected features gives slightly higher R^2 than using ridge on forward selected features. So, we used it for training ridge regression.

```
[266]: ridge = Ridge(alpha=0.01)
ridge.fit(X_train_forward_selected_features, y_train)
score = ridge.score(X_test_forward_selected_features, y_test)
print('Best ridge regression score on test data ' + str(score))
#Save the score to the results dictionary
saved_results_r2['Ridge'] = score
```

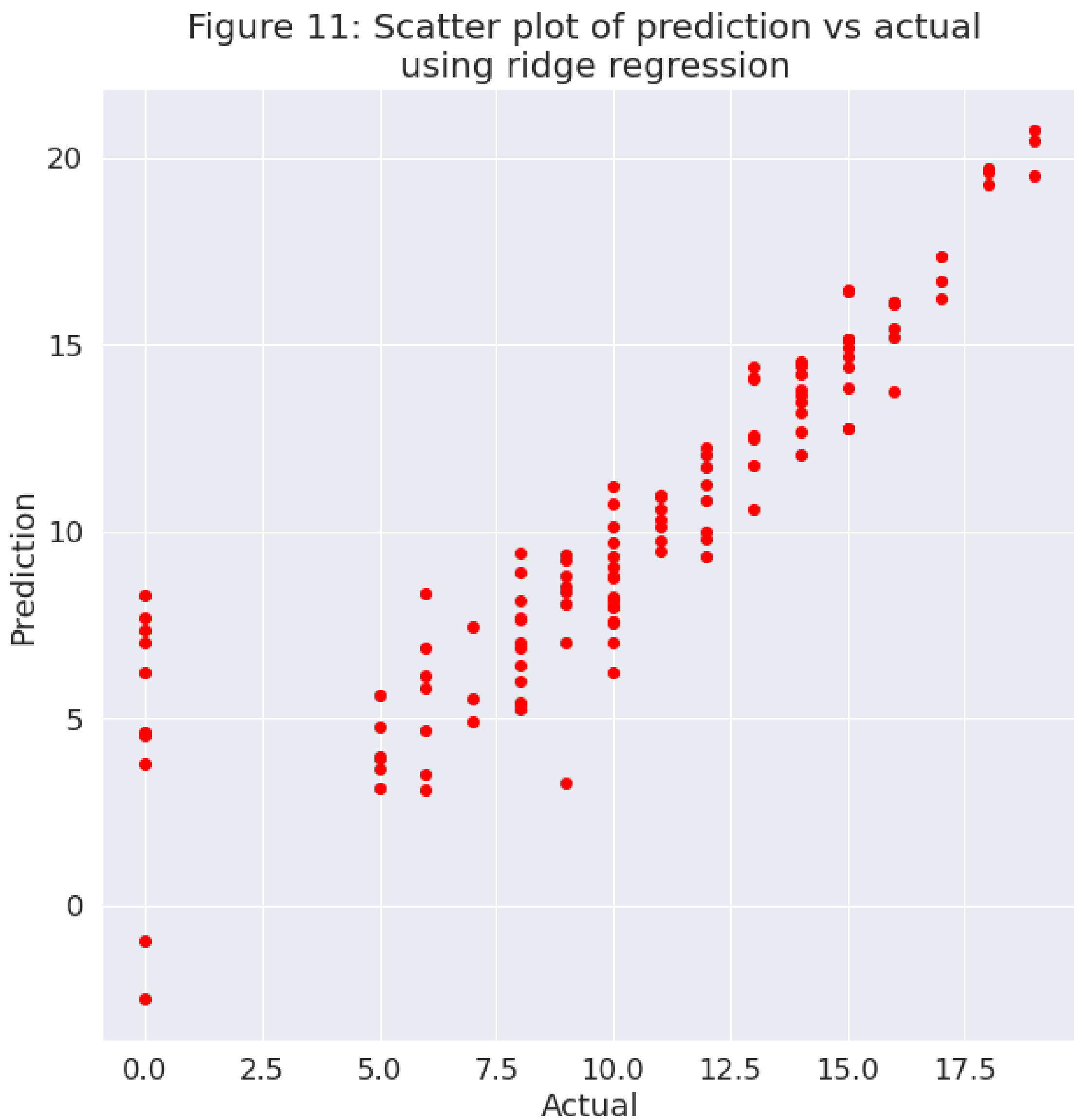
```
Best ridge regression score on test data 0.7774667805064169
```

As shown in the next cell, There are no zero coefficient when using ridge regression

[267]:
```
ridge.coef_
```

[267]:
```
array([-0.14111955,  0.33449974,  0.17707001, -0.13700221,  0.43326351,
        0.7400074 ,  3.55746441,  0.15941667,  0.27988831,  0.09799072,
        0.11757277, -0.23307826, -0.06814845, -0.16910114, -0.06146422,
       -0.18238672,  0.09192528, -0.22388877,  0.14852941,  0.01944424,
        0.15308966])
```

[268]:
```
y_pred = ridge.predict(X_test_forward_selected_features)
plt.figure(figsize=(10, 10))
plt.scatter(y_test, y_pred, color = "red")
plt.title("Figure 11: Scatter plot of prediction vs actual \n using ridge␣
 ↪regression", fontsize=20)
plt.xlabel("Actual")
plt.ylabel("Prediction")
plt.show()
```



4.6.3 Lasso Regression

Lasso Regression on forward selected features

[269]:
```
from sklearn.linear_model import Lasso

lasso = Lasso(alpha=0.1, max_iter = 10000)
lasso_r2 = cross_val_score(lasso, X_train_forward_selected_features, y_train,␣
↪cv=10, scoring='r2')
lasso_mse = cross_val_score(lasso, X_train_forward_selected_features, y_train,␣
↪cv=10, scoring= make_scorer(mean_squared_error))
```

[270]:
```
print('Validation R2 average for using Lasso regression on forward selected␣
↪features '+ str(np.mean(lasso_r2)))
print('Validation MSE average for using Lasso regression on forward selected␣
↪features '+ str(np.mean(lasso_mse)))
```

```
Validation R2 average for using Lasso regression on forward selected features
0.8375819873020628
Validation MSE average for using Lasso regression on forward selected features
3.2118833572809917
```

Lasso Regression on backward selected features

[271]:
```
lasso = Lasso(alpha=0.1, max_iter = 10000)
lasso_r2 = cross_val_score(lasso, X_train_backward_selected_features, y_train,␣
↪cv=10, scoring='r2')
lasso_mse = cross_val_score(lasso, X_train_backward_selected_features, y_train,␣
↪cv=10, scoring= make_scorer(mean_squared_error))
```

[272]:
```
print('Validation R2 average for using Lasso regression on backward selected␣
↪features '+ str(np.mean(lasso_r2)))
print('Validation MSE average for using Lasso regression on backward selected␣
↪features '+ str(np.mean(lasso_mse)))
```

```
Validation R2 average for using Lasso regression on backward selected features
0.8369102223774684
Validation MSE average for using Lasso regression on backward selected features
3.2247756130766363
```

Using Lasso with forward selected features gives slightly better R^2 score than backward selected features. So, we used forward for training Lasso regression

[273]:
```
lasso.fit(X_train_forward_selected_features, y_train)
score = lasso.score(X_test_forward_selected_features, y_test)
print('Best lasso regression score on test data ' + str(score))

#Save the score to the results dictionary
saved_results_r2['Lasso'] = score
```

```
Best lasso regression score on test data 0.7999199578554859
```

Lasso shows 7 zeros coefficients and the features that were assigned zero coefficents are as in the following, We called those features that are not used in modeling 'muted features'.

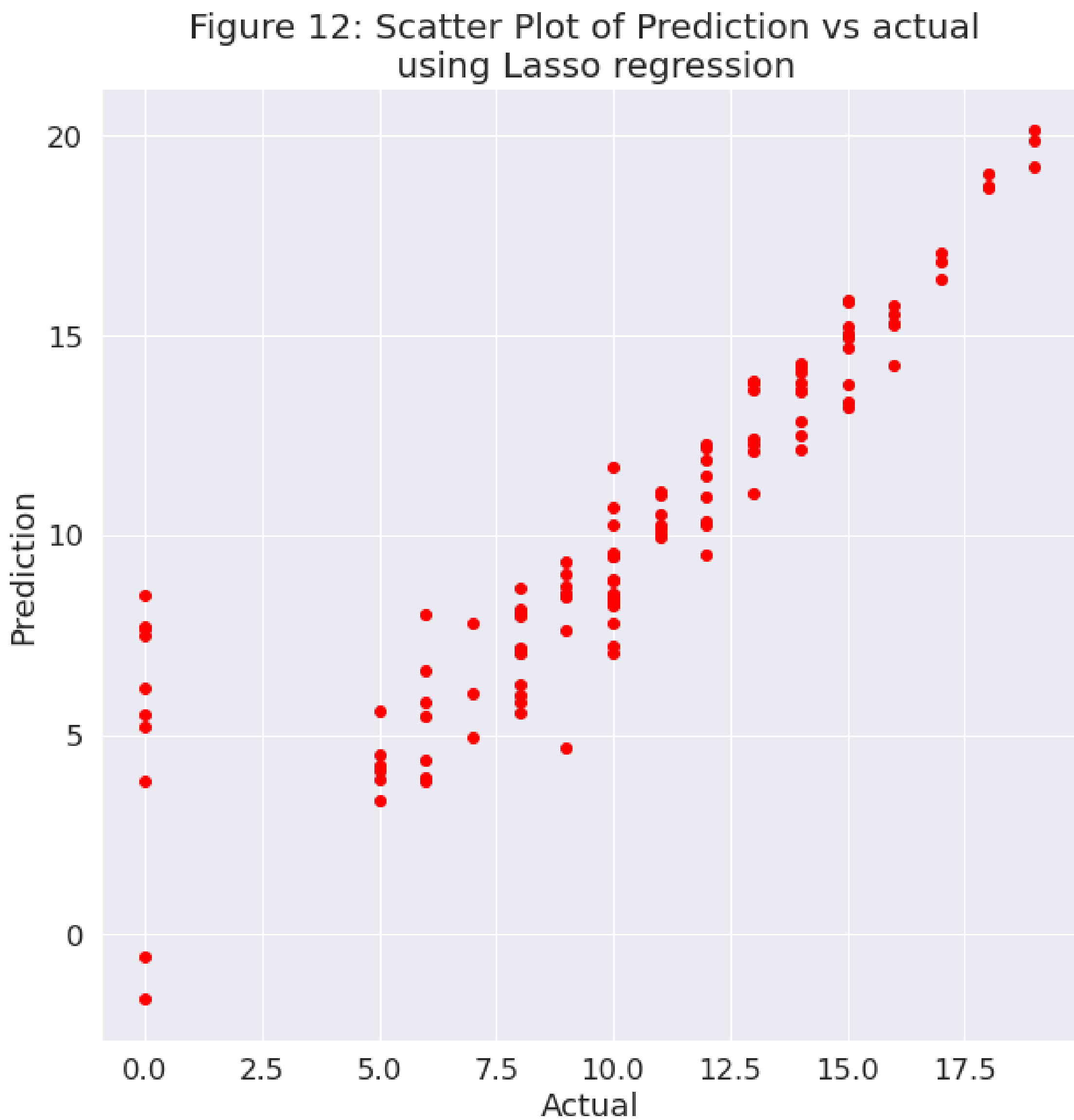
[274]:
```
print('Lasso Coefficents', lasso.coef_)
#lasso_muted_features = [forward_selected_features_names[coef] if coef == 0 for␣
↪coef in lasso.coef_]
lasso_muted_features = []
idx = 0
for coef in lasso.coef_:
```

```
    if coef == 0:
        lasso_muted_features.append(forward_selected_features_names[idx])
    idx += 1
print('Lasso muted features', lasso_muted_features)
```

```
Lasso Coefficents [-8.52570396e-02  2.36553327e-01  1.35120155e-02
-0.00000000e+00
  2.44425075e-01  5.78351093e-01  3.55575070e+00  0.00000000e+00
  1.33833672e-01  3.12220539e-03  2.67811139e-02 -1.11699354e-01
 -0.00000000e+00 -1.14033333e-01 -0.00000000e+00 -9.88139268e-02
  0.00000000e+00 -1.67068654e-01  0.00000000e+00  0.00000000e+00
  1.14892767e-01]
Lasso muted features ['Dalc', 'school_MS', 'higher_yes', 'Mjob_teacher',
'Fjob_teacher', 'reason_home', 'reason_other']
```

[275]:

```
y_pred = lasso.predict(X_test_forward_selected_features)
plt.figure(figsize=(10, 10))
plt.scatter(y_test, y_pred, color = "red")
plt.title("Figure 12: Scatter Plot of Prediction vs actual \n using Lasso␣
 ↪regression", fontsize=20)
plt.xlabel("Actual")
plt.ylabel("Prediction")
plt.show()
```



4.6.4 Elastic Net Regression

Elastic Net regression using forward selected features

[226]:
```
from sklearn.linear_model import ElasticNet
elastic = ElasticNet(alpha=0.01, l1_ratio=0.5, max_iter = 100)
elastic_r2 = cross_val_score(elastic, X_train_forward_selected_features,␣
 ↪y_train, cv=10, scoring='r2')
elastic_mse = cross_val_score(elastic, X_train_forward_selected_features,␣
 ↪y_train, cv=10, scoring= make_scorer(mean_squared_error))
```

[276]:
```
print('Validation R2 average for using Elastic net regression on forward␣
 ↪selected features '+ str(np.mean(elastic_r2)))
print('Validation MSE average for using Elastic net regression on forward␣
 ↪selected features '+ str(np.mean(elastic_mse)))
```

```
Validation R2 average for using Elastic net regression on forward selected
features 0.8427158609243568
Validation MSE average for using Elastic net regression on forward selected
features 3.11791854928766
```

Elastic Net regression using forward selected features

[277]:
```
elastic_r2 = cross_val_score(elastic, X_train_backward_selected_features,␣
 ↪y_train, cv=10, scoring='r2')
elastic_mse = cross_val_score(elastic, X_train_backward_selected_features,␣
 ↪y_train, cv=10, scoring= make_scorer(mean_squared_error))
```

[278]:
```
print('Validation R2 average for using Elastic net regression on backward␣
 ↪selected features '+ str(np.mean(elastic_r2)))
print('Validation MSE average for using Elastic net regression on backward␣
 ↪selected features '+ str(np.mean(elastic_mse)))
```

```
Validation R2 average for using Elastic net regression on backward selected
features 0.8427158609243568
Validation MSE average for using Elastic net regression on backward selected
features 3.11791854928766
```

Elastic net regression gives slightly higher score on forward selected features. So, forward selected features were used for training Elastic net

Training Elastic Net regression on forward selected features

[279]:
```
elastic.fit(X_train_forward_selected_features, y_train)
score = elastic.score(X_test_forward_selected_features, y_test)
print('Best elastic net regression score on test data ' + str(score))

#Save the score to the results dictionary
saved_results_r2['Elastic Net'] = score
```

```
Best elastic net regression score on test data 0.7796918612142189
```

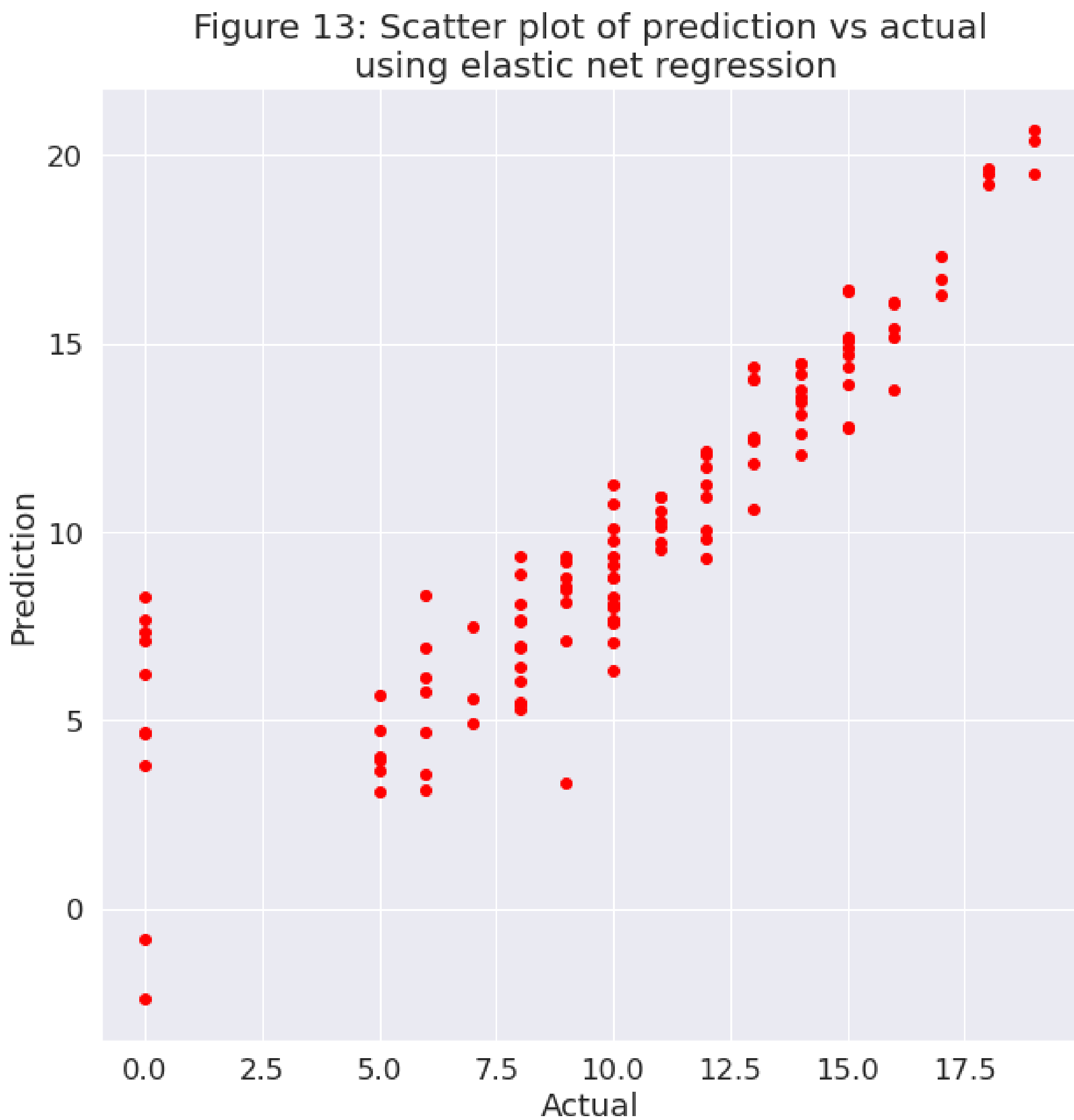
Elastic Net shows only one zero coefficient

[280]:
```
print('Elastic Net Coefficents', elastic.coef_)
#lasso_muted_features = [forward_selected_features_names[coef] if coef == 0 for␣
 ↪coef in lasso.coef_]
elastic_muted_features = []
idx = 0
for coef in elastic.coef_:
```

```
    if coef == 0:
        elastic_muted_features.append(forward_selected_features_names[idx])
    idx += 1
print('Elastic Net muted features', elastic_muted_features)
```

```
Elastic Net Coefficents [-0.14082278  0.32740024  0.16179784 -0.1214087
0.41792485  0.76938178
  3.50507217  0.14865198  0.2735164   0.08970137  0.11526739 -0.22534169
 -0.05672475 -0.17019116 -0.05874692 -0.17603062  0.05659844 -0.24104478
  0.12504654 -0.          0.15271677]
Elastic Net muted features ['reason_other']
```

[281]:
```
y_pred = elastic.predict(X_test_forward_selected_features)
plt.figure(figsize=(10, 10))
plt.scatter(y_test, y_pred, color = "red")
plt.title(" Figure 13: Scatter plot of prediction vs actual \n using elastic net␣
 ↪regression", fontsize=20)
plt.xlabel("Actual")
plt.ylabel("Prediction")
plt.show()
```



4.6.5 Random Forest Regression

Random Forest Regressor on forward selected features

[282]:
```
from sklearn.ensemble import RandomForestRegressor

forest = RandomForestRegressor(n_estimators=400,
                               criterion='mse',
                               random_state=1,
                               n_jobs=-1)
forest_r2 = cross_val_score(forest, X_train_forward_selected_features, y_train,␣
 ↪cv=20, scoring='r2')
```

```
forest_mse = cross_val_score(forest, X_train_forward_selected_features, y_train,␣
↪cv=20, scoring= make_scorer(mean_squared_error))
```

We estimate the number of estimators hyper-parameter by using grid search that gives the best validation score. Number of estimators from 100 to 1500 were pre-selected for the GridSearchCV function.

[283]:
```
from sklearn.model_selection import GridSearchCV

estimator = RandomForestRegressor()
para_grids = {
            "n_estimators" : [100, 200, 300, 400 ,500, 1000, 1500]
        }
grid = GridSearchCV(estimator, para_grids, scoring = 'r2',cv=10,n_jobs=-1,␣
↪iid=False)
grid.fit(X_train_std, y_train)
print (grid.best_score_, grid.best_params_)
```

```
0.8838558191007291 {'n_estimators': 400}
```

[284]:
```
print('Validation R2 average for using Random Forest regression on forward␣
↪selected features '+ str(np.mean(forest_r2)))
print('Validation MSE average for using Random Forest regression regression on␣
↪forward selected features '+ str(np.mean(forest_mse)))
```

```
Validation R2 average for using Random Forest regression on forward selected
features 0.8926458381084181
Validation MSE average for using Random Forest regression regression on forward
selected features 1.9760714680631875
```

Training the Random Forest Regressor on forward selected features

[285]:
```
forest.fit(X_train_forward_selected_features, y_train)
score = forest.score(X_test_forward_selected_features, y_test)
print('Best Random Forest regression score on test data ' + str(score))

#Save the score to the results dictionary
saved_results_r2['Random Forest'] = score
```

```
Best Random Forest regression score on test data 0.8453352799096014
```

Scatter plot of the actual data vs the predicted data

[293]:
```
y_pred = forest.predict(X_test_forward_selected_features)
plt.figure(figsize=(10, 10))
plt.scatter(y_test, y_pred, color = "red")
plt.title("Figure 14: Scatter Plot of Prediction vs actual \n using Random␣
↪Forest Regression", fontsize=20)
plt.xlabel("Actual")
```

```
plt.ylabel("Prediction")
plt.show()
```

Using random forest regression with KBest selected features Below we have used select k best for feature selection. The scores of the features are shown in the score table. Looking at the table, we see that 22 columns have a score greater than zero. So we decided to use the variables with score of at least 1.

[287]:
```
from sklearn.feature_selection import SelectKBest, f_regression
selector = SelectKBest(f_regression, k=22).fit(X_train_std, y_train)
kbest_features_train = selector.transform(X_train_std)
kbest_features_test = selector.transform(X_test_std)
```

```
print(kbest_features_train.shape)
```

(276, 22)

[288]:
```
#Calculate the score of features
selected_features_df = pd.DataFrame({'Feature':list(X_train.columns),
                                      'Scores':selector.scores_})
selected_features_df.sort_values(by='Scores', ascending=False)
```

[288]:

	Feature	Scores
14	G2	1310.693812
13	G1	440.445682
5	failures	44.124162
25	higher_yes	16.595315
1	Medu	9.461505
22	paid_yes	7.715219
27	romantic_yes	5.119667
29	Mjob_health	5.063262
0	age	4.792613
8	goout	4.728930
41	reason_reputation	3.427594
26	internet_yes	3.329182
38	reason_course	3.080103
4	studytime	3.016719
28	Mjob_at_home	2.934689
2	Fedu	2.751941
11	health	2.148822
9	Dalc	1.950665
3	traveltime	1.466995
19	Pstatus_T	1.423250
40	reason_other	1.042527
12	absences	1.000620
17	address_U	0.952532
30	Mjob_other	0.798771
32	Mjob_teacher	0.734784
20	schoolsup_yes	0.611683
16	sex_M	0.575303
44	guardian_other	0.564390
43	guardian_mother	0.482776
6	famrel	0.472103
37	Fjob_teacher	0.471535
10	Walc	0.444683
39	reason_home	0.422118
7	freetime	0.358124
21	famsup_yes	0.331425
18	famsize_LE3	0.275379
34	Fjob_health	0.270579

```
33        Fjob_at_home     0.111409
31        Mjob_services    0.076220
42     guardian_father    0.069556
35          Fjob_other     0.049117
23      activities_yes     0.044006
36       Fjob_services     0.042992
24         nursery_yes     0.038869
15           school_MS     0.000035
```

We train random forest regression on the KBest selected features and calculating the R^2 score. Random forest gives the best score we measured in this study which is 0.871 on test data and 0.879 on validation data.

[240]:
```
rf = RandomForestRegressor(n_estimators = 400, random_state = 42)
forest_r2 = cross_val_score(forest, kbest_features_train, y_train, cv=20,␣
 ↪scoring='r2')
forest_mse = cross_val_score(forest, kbest_features_train, y_train, cv=20,␣
 ↪scoring= make_scorer(mean_squared_error))
```

[289]:
```
print('Validation R2 average for using Random Forest regression on KBest␣
 ↪selected features '+ str(np.mean(forest_r2)))
print('Validation MSE average for using Random Forest regression regression on␣
 ↪KBest selected features '+ str(np.mean(forest_mse)))
```

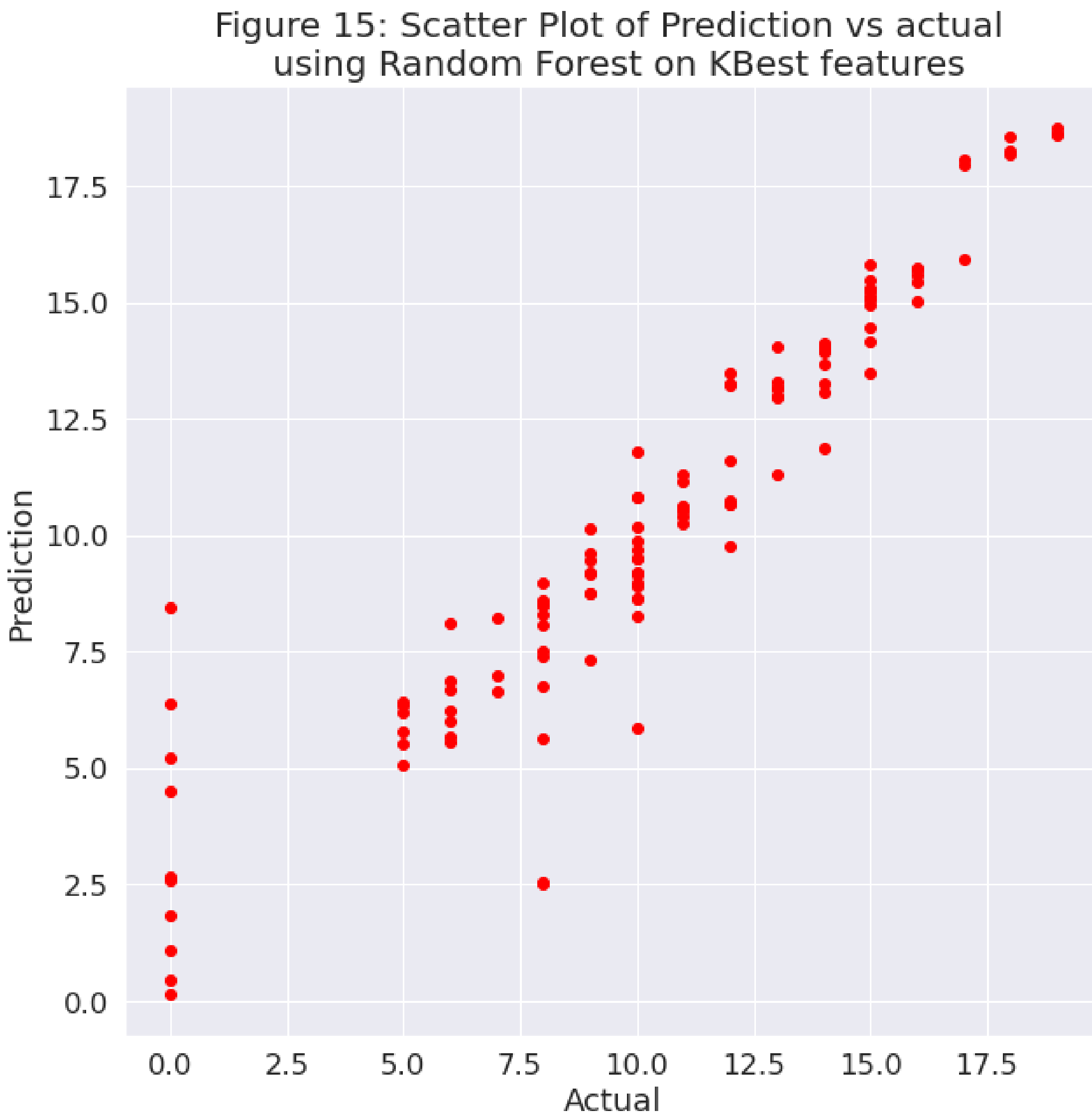
```
Validation R2 average for using Random Forest regression on KBest selected
features 0.8926458381084181
Validation MSE average for using Random Forest regression regression on KBest
selected features 1.9760714680631875
```

[290]:
```
#Training the random forest on Kbest features
rf.fit(kbest_features_train, y_train)
y_pred = rf.predict(kbest_features_test)
score = rf.score(kbest_features_test, y_test)
print('Random Forest regression score on test data with KBest selected features␣
 ↪' + str(score))

#Save the score to the results dictionary
saved_results_r2['Random Forest with KBest'] = score
```

```
Random Forest regression score on test data with KBest selected features
0.8710430256105279
```

[291]:
```
y_pred = rf.predict(kbest_features_test)
plt.figure(figsize=(10, 10))
plt.scatter(y_test, y_pred, color = "red")
plt.xlabel("Actual")
plt.ylabel("Prediction")
```

```
plt.title("Figure 15: Scatter Plot of Prediction vs actual \n using Random␣
↪Forest on KBest features", fontsize=20)
plt.show()
```



By combining and comparing the results of all the used regression methods, Random Forest Regression on KBest features gives the best R^2 score as shown in fig.16.

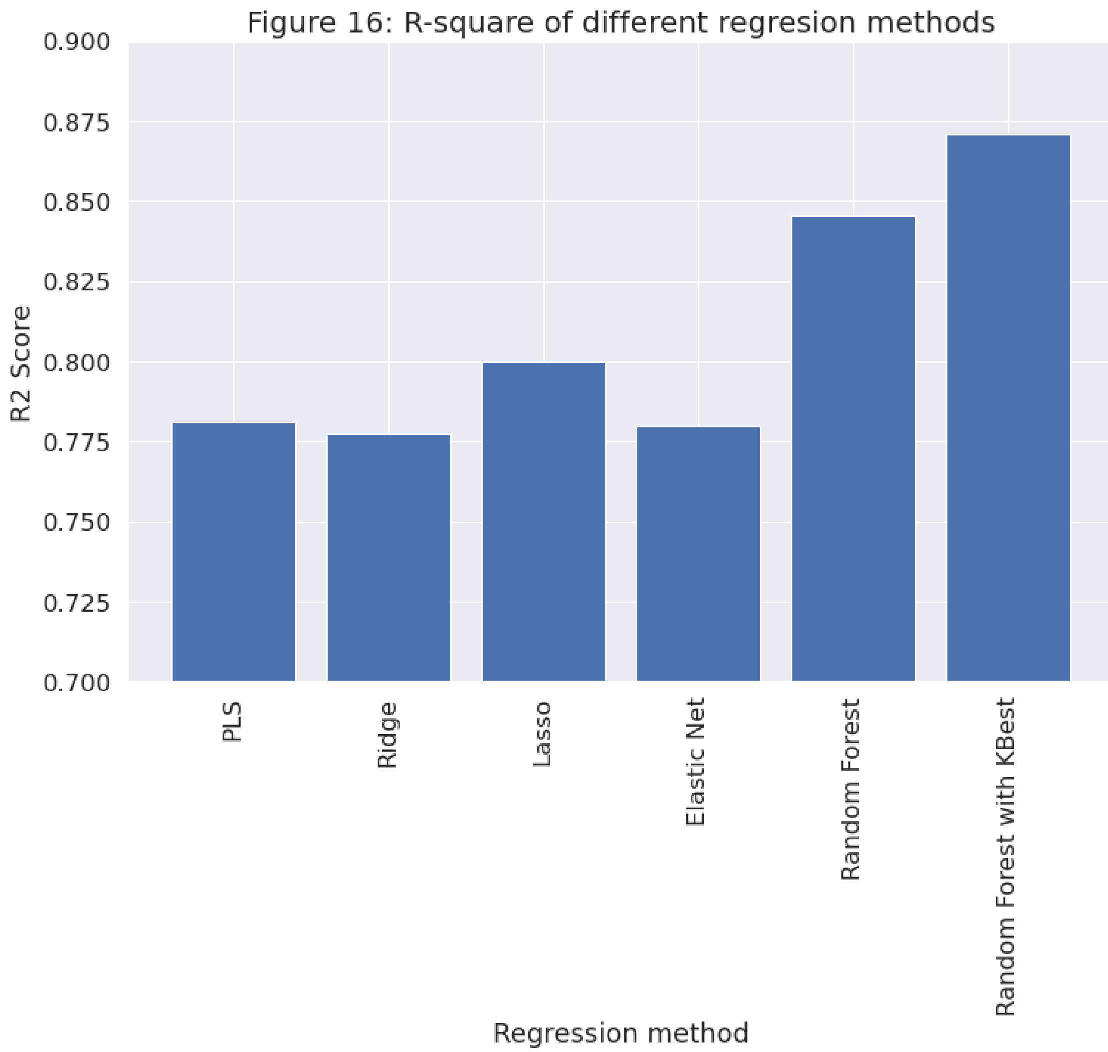
5 Disscusion

The outcomes show that at all regression models, the predictions and the observations are close in the mark ranging from 5 to 20. This outcome is similar with Cortez and Silva, 2008. This study shows that G1, G2, failures, age and health are amongst the most important independent variables to predict the grades of students which is similar to the study conducted by Cortez and Silva, 2008.

6 Conclusion

From the above scores, Random Forest gives the best predictions with R^2 of 0.87 on test data as shown in fig.16. We can get into the conclusion that student's performance is greatly affected by past grades (G1 & G2). Also, features like (age, health, etc.) affected the student's performance. More research is needed to predict the G1 and G2 grades based on those features. Further research can be done like tuning the hyper-parameters using grid search for other regressions like elastic net.

[292]:
```
lists = saved_results_r2.items()
plt.figure(figsize=(12, 8))
x, y = zip(*lists)
plt.title("Figure 16: R-square of different regresion methods", fontsize=20)
plt.xlabel('Regression method')
plt.ylim(0.7, 0.9)
plt.ylabel('R2 Score')
plt.bar(x, y)
plt.xticks(rotation='vertical')
plt.show()
```



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8 Appendix 1: Varibles in datasets

1- school - student's school (binary: "GP" - Gabriel Pereira or "MS" - Mousinho da Silveira)

2- sex - student's sex (binary: "F" - female or "M" - male)

3- age - student's age (numeric: from 15 to 22)

4- address - student's home address type (binary: "U" - urban or "R" - rural)

5- famsize - family size (binary: "LE3" - less or equal to 3 or "GT3" - greater than 3)

6- Pstatus - parent's cohabitation status (binary: "T" - living together or "A" - apart)

7- Medu - mother's education (numeric: 0 - none, 1 - primary education (4th grade), 2 – 5th to 9th grade, 3 – secondary education or 4 – higher education)

8- Fedu - father's education (numeric: 0 - none, 1 - primary education (4th grade), 2 – 5th to 9th grade, 3 – secondary education or 4 – higher education)

9- Mjob - mother's job (nominal: "teacher", "health" care related, civil "services" (e.g. administrative or police), "at_home" or "other")

10- Fjob - father's job (nominal: "teacher", "health" care related, civil "services" (e.g. administrative or police), "at_home" or "other")

11- reason - reason to choose this school (nominal: close to "home", school "reputation", "course" preference or "other")

12- guardian - student's guardian (nominal: "mother", "father" or "other")

13- traveltime - home to school travel time (numeric: 1 - <15 min., 2 - 15 to 30 min., 3 - 30 min. to 1 hour, or 4 - >1 hour)

14- studytime - weekly study time (numeric: 1 - <2 hours, 2 - 2 to 5 hours, 3 - 5 to 10 hours, or 4 - >10 hours)

15- failures - number of past class failures (numeric: n if 1<=n<3, else 4)

16- schoolsup - extra educational support (binary: yes or no)

17- famsup - family educational support (binary: yes or no)

18- paid - extra paid classes within the course subject (Math or Portuguese) (binary: yes or no)

19- activities - extra-curricular activities (binary: yes or no)

20- nursery - attended nursery school (binary: yes or no)

21- higher - wants to take higher education (binary: yes or no)

22- internet - Internet access at home (binary: yes or no)

23- romantic - with a romantic relationship (binary: yes or no)

24- famrel - quality of family relationships (numeric: from 1 - very bad to 5 - excellent)

25- freetime - free time after school (numeric: from 1 - very low to 5 - very high)

26- goout - going out with friends (numeric: from 1 - very low to 5 - very high)

27- Dalc - workday alcohol consumption (numeric: from 1 - very low to 5 - very high)

28- Walc - weekend alcohol consumption (numeric: from 1 - very low to 5 - very high)

29- health - current health status (numeric: from 1 - very bad to 5 - very good)

30- absences - number of school absences (numeric: from 0 to 93)

31- G1 - first period grade (numeric: from 0 to 20)

32- G2 - second period grade (numeric: from 0 to 20)

33- G3 - final grade (numeric: from 0 to 20, target feature)