

## **Ebola Outbreak Preparedness: Allegheny County, Pennsylvania**

**December 6, 2021**

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## **I. Background/literature review**

Ebola virus disease (EVD) (sometimes called Ebola Hemorrhagic Fever) is a highly contagious disease caused by infection with an Ebola virus.<sup>1</sup> The U.S. Centers for Disease Control and Prevention (CDC) categorizes Ebola virus as a Category A select agent. This group includes high priority agents that pose a risk to national security because they can be easily disseminated or transmitted from person to person; result in high mortality rates and have the potential for major public health impact; might cause public panic and social disruption; and require special action for public health preparedness.

The Ervebo vaccine (rVSV-ZEBOV) has been shown to be effective in protecting people from the species Zaire ebolavirus, and is recommended by the Strategic Advisory Group of Experts on Immunization as part of a broader set of Ebola outbreak response tools.<sup>2</sup>

Inmazeb and Ebanga are FDA approved monoclonal antibodies for the treatment of Zaire ebolavirus. Both showed greatly improved survival rates in a randomized control trial<sup>3</sup>.

## **II. Problem statement**

This project considers an Ebola outbreak in Allegheny County, Pennsylvania. The goal is to set up Points of Dispense (PODs) at selected candidate sites, send shipments of vaccines and antibiotics from the Allegheny County Central Warehouse to PODs, and assign medical personnel to PODs, all while constraining the budget available for these efforts and availability of these resources, with an objective to minimize the average distance travelled by individuals to get vaccinated. This project also considers the alternative objective of minimizing the maximum distance travelled by an individual to get vaccinated.

## **III. Assumptions**

### **1) Vaccine and Antibiotic supply is unlimited**

The vaccines and antibiotics have been procured by the federal government, awaiting a formal request for the quantity required by the Allegheny County Department of Health. The quantity delivered to the county is unconstrained, given that the vaccine is not currently commercially marketed in the U.S. but is currently stockpiled in the Strategic National Stockpile and is made available through CDC for pre-exposure vaccination.<sup>4</sup>

### **2) The county administers a vaccine or antibiotic to every person who needs one**

Although, in reality, not everyone will choose to receive the vaccine or antibiotics, we formulate our solution to administer these to the entire affected population so that no one who wants such treatment is unable to receive it. Moreover, based on literature review<sup>5</sup>, we have assumed economies of scale would have an important role to play here. This concept is leveraged to alter the vaccine administration efficiency of a doctor based on whether each PoD site has been classified as a small, medium or large.

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<sup>1</sup> Ebola Virus Disease - WHO - <<https://www.who.int/news-room/fact-sheets/detail/ebola-virus-disease>>

<sup>2</sup> Ebola Virus Disease - WHO - <<https://www.who.int/news-room/fact-sheets/detail/ebola-virus-disease>>

<sup>3</sup> Centers for Disease Control and Prevention - <<https://www.cdc.gov/vhf/ebola/treatment/index.html>>

<sup>4</sup> Centers for Disease Control and Prevention - <<https://www.cdc.gov/vhf/ebola/clinicians/vaccine/index.html>>

<sup>5</sup> The National Center for Biotechnology Information - <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5888956/>

3) Although the supply of medical professionals is limited, all 1370 can be applied to one site.

This is one of our strongest assumptions because funneling 137,000 people through one high school in a day sounds like a stretch. But, we decided that in the case of an emergency, presiding authorities would be resourceful and use parking lots, sports fields, and other

4) Medical professionals can be assigned to a different site each day.

To optimally avail the services of limited medical professionals available in Allegheny County, assignment of medical professionals to each site is being done on a daily basis. This flexibility was added in the model to minimize idle time for the doctors and ensure their availability at sites where they are needed the most.

#### **IV. Mathematical formulation** *(please refer to Appendix II)*

We formulated preparations for an Ebola outbreak in two ways:

1. The county anticipates significant danger of a mass Ebola outbreak and plans to vaccinate its entire population to prevent the outbreak from occurring. This is the “Vaccine-only” model
2. An outbreak has occurred, affecting some proportion of neighborhoods in Alleghany County. In response the county plans to administer antibiotics to people from the “infected” neighborhoods and vaccines to people from the “uninfected” neighborhoods. This is the “Vaccine and Antibiotic” model.

In both of these formulations we consider multiple objectives:

- Minimizing total distance traveled to acquire vaccines or antibiotics
- Minimizing cost for administering vaccines and antibiotics
- Minimizing the maximum distance traveled for any person attempting to acquire vaccines or antibiotics

We found in our Vaccine-only model that minimizing the total distance traveled also minimizes the maximum distance traveled.

##### Vaccine-only Model

The full formulation for the Vaccine-only model is included in the appendix. The major differences between our model and a classic facility location problem are: (1) the detailed costs we included in the total cost calculation, (2) the innovative calculation of daily inventory, and (3) the incorporation of economies of scale in the model. The complexity of the model significantly increased its run-time in Gurobi which influenced us to mathematically calculate optimal solutions where possible. The parameters for each cost is included in the data summary section.

##### Vaccine and Antibiotic Model

The full formulation for the Vaccine and Antibiotic model is also in the appendix. This problem is modeled as stochastic with two stages. In the first stage, the model decides whether or not to open a POD and whether that POD will be used for administering antibiotics. If a site is used for antibiotics, it cannot be used for vaccinations so that we do not unintentionally infect uninfected people by mingling them with infected people. We used a creative solution to add this constraint to our model while preserving linearity.

In the second stage, the model decides which zip code to assign to which site, and otherwise makes the same decisions as the previous model. An additional parameter is the breakout status of a neighborhood. This will determine whether a person needs to receive the antibiotic or the vaccine. We randomly generated 97 breakout scenarios using a clustering algorithm as described in the data summary section.

## V. Data summary

| Category                | Data Description   | Sources  |
|-------------------------|--|--|
| Cost Components         | <b>o</b> : first-stage-fixed-cost (opening cost)                                       | The following <a href="#">link</a> offered insight into potential opening cost for POD sites, assumed to be \$20,000.  |
|                         | <b>f</b> : second-stage fixed cost   | The following <a href="#">link</a> assisted in rationalizing the estimated one-time-cost (\$8,000) to utilize a POD after the outbreak.  |
|                         | <b>h</b> : per unit holding cost   | This would be the cost of storing a vaccine or an antibiotic in a freezer. A freezer would generally cost around \$12,500 as per the following <a href="#">link</a> . Assuming each freezer can hold a total of 1000 vaccines or antibiotics, the per unit holding cost would come out to be $\$12,500 / 1000 = \$12.5$ .  |
|                         | <b>r</b> : medical professional's wage   | According to the following <a href="#">link</a> , the average hourly pay for Medical Assistants is listed as \$17.23 per hour. Assuming a medical professional works for 12 hours a day, the total cost for one medical professional per day would come out to be $\$17.23 * 12 = \$206.76$  |
|                         | <b>v</b> : Per unit cost of transporting vaccines from sourcing site to any POD site j | The estimate for per unit cost of transporting vaccines from the sourcing site to the relevant POD site was \$50, based on the cost breakdown shown in the following <a href="#">link</a> .  |
| Distance Estimation     | <b>z</b> : Number of zip codes in Allegheny County                                     | We use the following <a href="#">link</a> , to find the total number of zip codes with their relevant populations, in Allegheny County.  |
|                         | <b>D<sub>97x47</sub></b>   | D <sub>97x47</sub> represents the distance to travel from each of 97 neighborhoods to each of 47 POD sites. To find the distance between a neighborhood (zip code, i) and a POD site (zip code, j), we used the Manhattan distance measure to find the distance between two coordinates. The coordinates were retrieved from the following <a href="#">link</a> .  |
| Population of Zip codes | <b>P<sub>i</sub></b> : population of zip code i  | 97 x 1 Matrix based on the following <a href="#">link</a> , where each value would represent the population of that zip code.  |
| Economies of Scale      | <b>e<sub>s</sub></b> : 72, <b>e<sub>m</sub></b> : 85, <b>e<sub>L</sub></b> : 100       | According to the following <a href="#">research papers</a> , we found that economies of scale are achieved when there is mass drug administration and that the baseline rate of administering vaccines or antibiotics would be 72. We get this number by assuming that a medical professional will work for 12 hours per day, and it would take 10 minutes to administer a vaccine to a person. As a result, we find that a medical professional |

|   |   |   |
|---|---|---|
|   |   | would be able to administer 72 vaccines per day. We have assumed the values for $e_M$ and $e_L$ .   |
| <b>Parameters for Supply constraints</b>                  | <b>C:</b> the number of available medical professionals   | According to the following <a href="#">link</a> , a physician sees 891 patients. The total population of Allegheny County is 1.22 million. So, the total number of medical professionals available would be $1,220,000/891 = 1370$  |
| <b>Clustering-based Approach (Dropped from the model)</b> | <b><math>B_{Z \times 10,000}</math>:</b> Whether zip code $i$ has an outbreak in a scenario (represented by 10,000) | We initialized each county to have a random infection rate between 0 and 0.5. We then randomly selected Counties and simulated them as being 'starting points of the infection'. Next, we found the 5 closest neighborhoods which would have the highest chance of an infection and this being-prone-to-being-infected-ness was simulated by essentially doubling their infection rate. We then added these neighbors to a set. We simulated this approach for each of the neighbors of neighbors set until the infection rate for all neighborhoods in this set had been doubled (contagiousness). We then used a threshold of 0.8 to say that if the infection rate in that neighborhood was greater than the threshold, we would consider that neighborhood to be infected, otherwise not. Simulating this randomly for each of the neighborhoods, we were able to generate many different scenarios including one that considered each neighborhood to be infected. |

## VI. Implementation

### Vaccine and Antibiotic Model

Unfortunately, due to the complexity of this model, we were unable to implement it in Gurobi. The model was not infeasible, but the run times were so large that we would need to dedicate a computer to run the model over multiple days to find a solution.

### Vaccine-only Model

#### 1. Minimize the Total Distance

In the formulation for minimizing total distance, we phased our model building into several stages to give decision makers a holistic sense of the information they needed to ensure preparedness. First, we computed the minimum cost to vaccinate the entire population. We found we were able to determine the minimal cost using mathematical computation. Given our cost formulation, and the assumption that there is no cap on the number of medical professionals assigned to a site, the cheapest way to administer the vaccines is to choose one site and hire the maximum number of medical professionals available until the entire inventory is depleted. On the final day, however we only hire the number of professionals necessary to administer the remaining vaccines. We found that it would take 8 days to vaccinate the entire population. Under this strategy, we computed the minimum cost to vaccinate the entire population to be \$97,633,920.83.

Next, we approached the problem of minimizing total distance while keeping the cost at the minimum computed above. With the understanding that the minimal cost solution is limited to only utilizing one site, we found the site that minimizes total distance traveled. Using matrix multiplication, the minimal total distance was calculated to be 14,255,248.29 km, with an average distance for one person equal to 13.67 km. The maximum distance travelled by any one person turned out to be 33.36 km in this scenario.

Following this, we implemented a linear programming model to minimize the distance, irrespective of the cost. This model yielded a solution of 1,080,260.97 km, with an average distance per person of 1.04 km and a maximum distance of 14.66 km. Finally, conditional on this minimum total distance, we ran another linear programming model to minimize the total cost. This model gave us a solution of \$98,677,054.97, with 39 sites utilized. This is a significant improvement, decreasing average travel distance from 13.67 km to 1.04 km in exchange for a budget increase of \$1,043,134.14 (+1.1%).

## **2. Minimize the Maximum Distance**

For the objective to minimize the maximum distance travelled by an individual to their assigned site, we first ran a linear programming model, with an unconstrained budget. This model yielded a minimized maximum distance of 14.66 km, which was achieved by utilizing all 47 candidate PoD sites. Next, we minimized the maximum distance while constraining the cost to the minimum possible cost (i.e. using only one site). This max distance was calculated to be 30.52 km, using site # 29.

Next, we ran a linear programming model to minimize the cost while holding the maximum distance at the minimum found above. The solution used only 6 sites and achieved the minimum max-distance at a cost of \$97,758,033.33. Again, this is a significant improvement with the maximum distance decreasing from 30.52 km to 14.66 km in exchange for a budget increase of \$124,112.50 (+0.1%).

### **Analysis & Recommendations** *(please refer to Appendix I)*

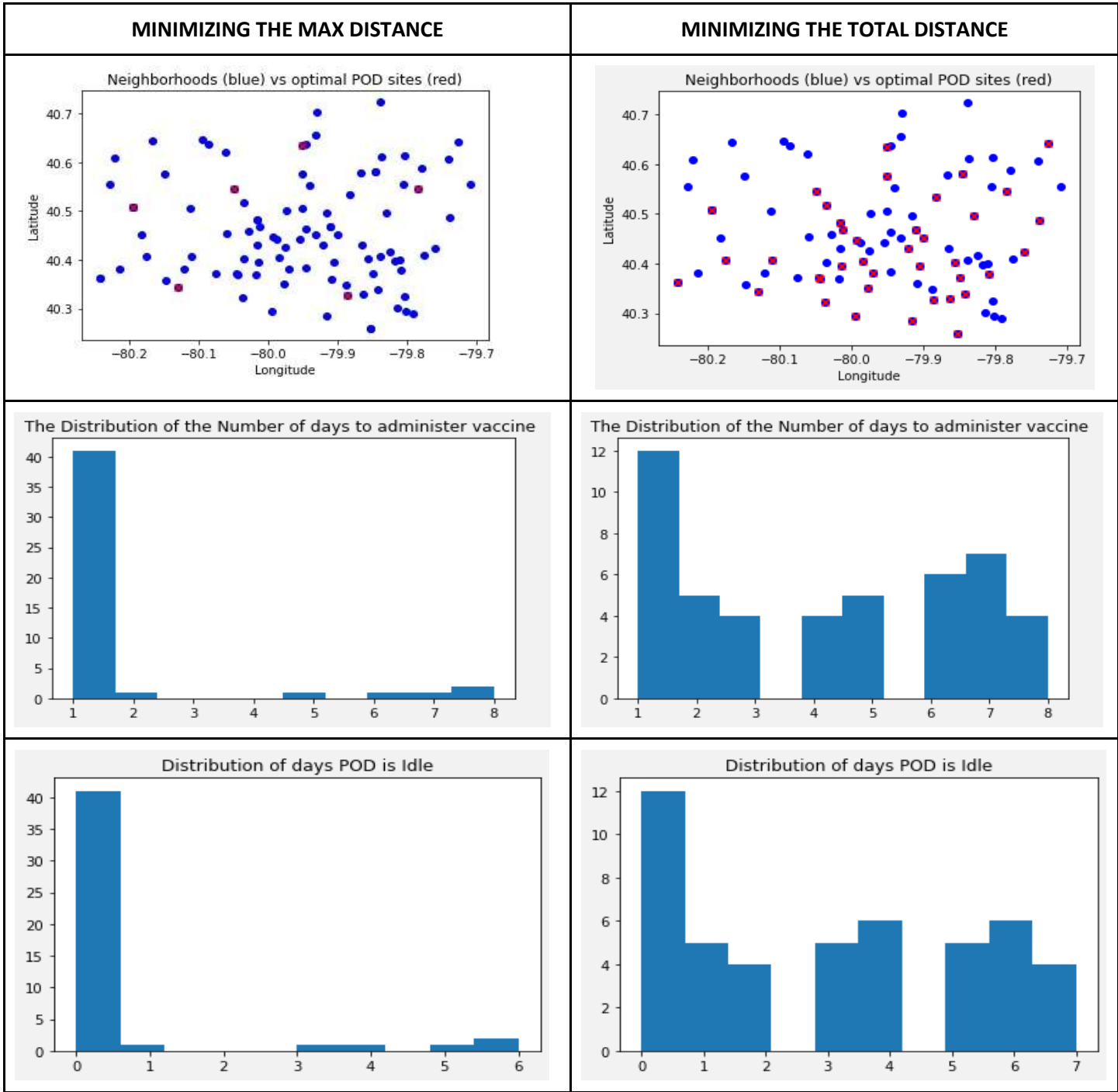
Given that minimizing average total distance in the vaccine-only model also minimized maximum distance traveled, we recommend Alleghany County open 39 sites according to this model's optimal solution. This solution provides significant benefit in decreasing average distance traveled (from 13.67 km to 1.04 km) with a marginal relative increase in cost (+1.1%). It does however have potential drawbacks which we address, providing suggestions to lessen their magnitude.

The most surprising result of all the models was the way the total-distance, vaccine-only model assigned medical professionals. We assumed that there would be an association between the scale-size of a POD and its relative location. That is, large-scale sites would be centrally located and small-scale sites would be opened near the fringes to decrease the distance those people would need to travel. Surprisingly, however, only one of the 39 optimal locations is medium-scale and the rest are large-scale. We also assumed that a site would assign a fixed number of medical professionals at a site and leave them there until the entire inventory is depleted. However, as seen below, 12 sites behave this way but the rest have at least 1 day of idle time. The model is assigning every available medical professional every day, but where that medical professional is assigned changes from day to day. Therefore the model is able to deplete the costly-to-hold inventory as fast as it would in the one-site solution, but across many sites, significantly reducing the average distance traveled as described in the previous section. This model provides added benefit in not only where to open sites and which neighborhoods to assign to each site,

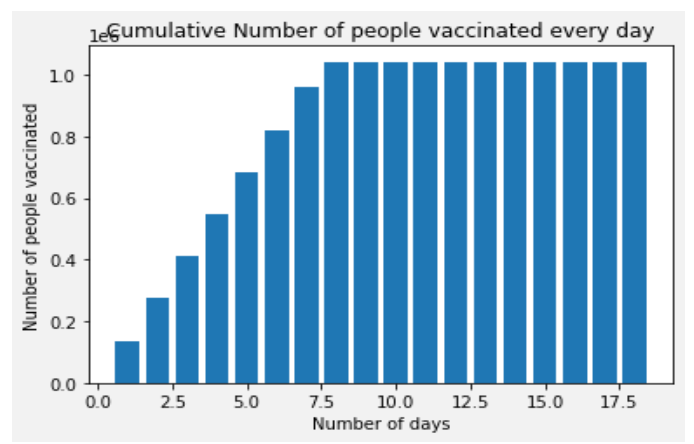
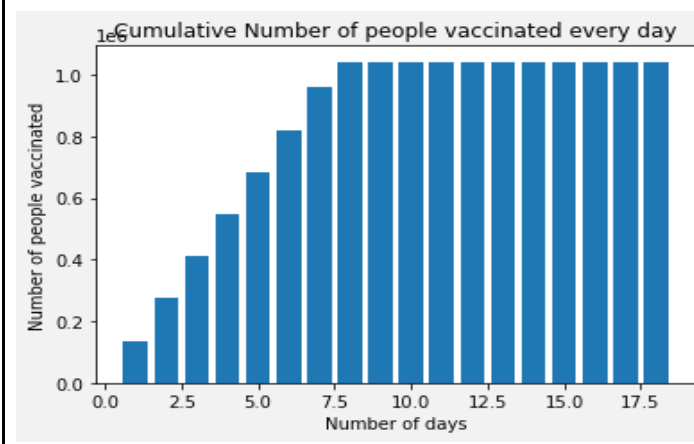
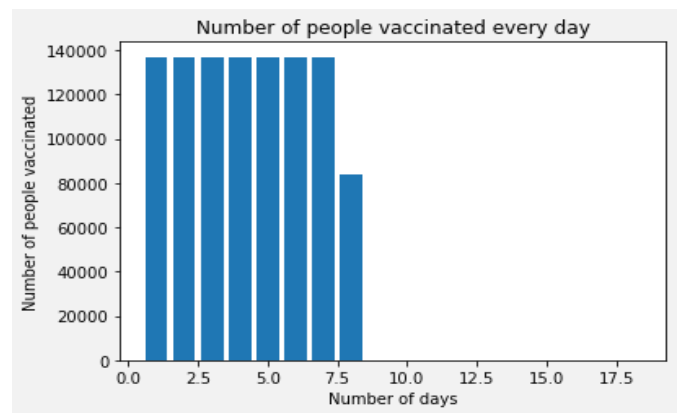
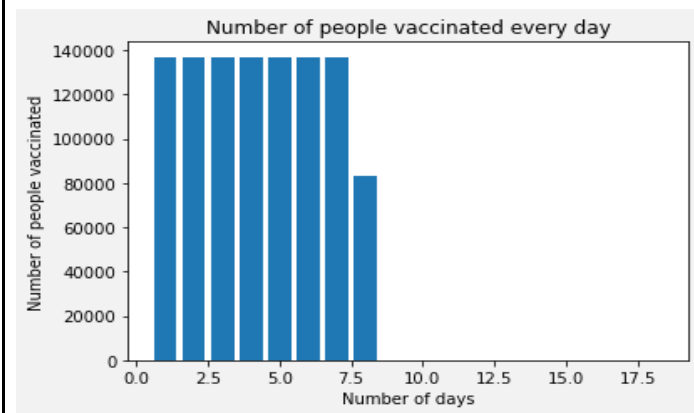
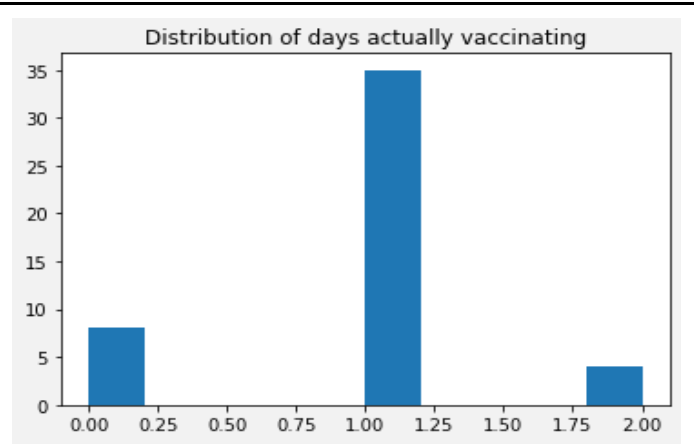
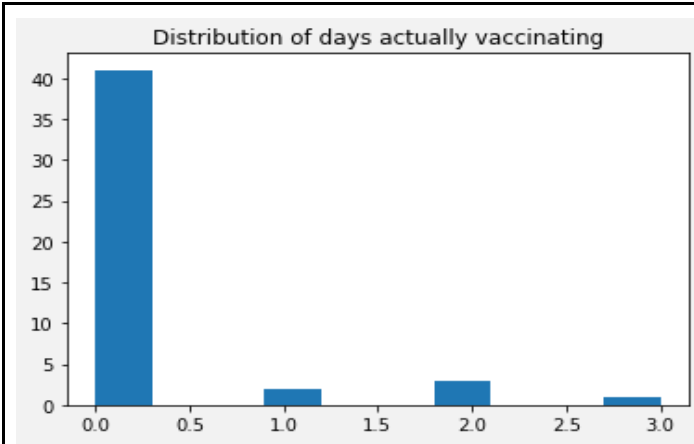
but also in where to send medical professionals each day to dramatically decrease the total cost of distribution while maintaining minimum average distance traveled.

Although our model increases efficiency, it may be unpopular to some. Many people have to wait to be vaccinated, some up to 8 days, and it's not random who must wait; it's assigned by neighborhood. Although the only purpose of this allocation is to minimize costs, it may be perceived as favoritism for certain neighborhoods. Therefore, in application, the county should weigh the time it makes vaccines available to each neighborhood with the associated cost. Alternatively, the county could seek ways to increase the number of available medical professionals which would shorten the wait times for all in line for a vaccine. Finally, the 8-day wait is the *worst-case-scenario*, given that it is unlikely everyone will travel to get the vaccine.

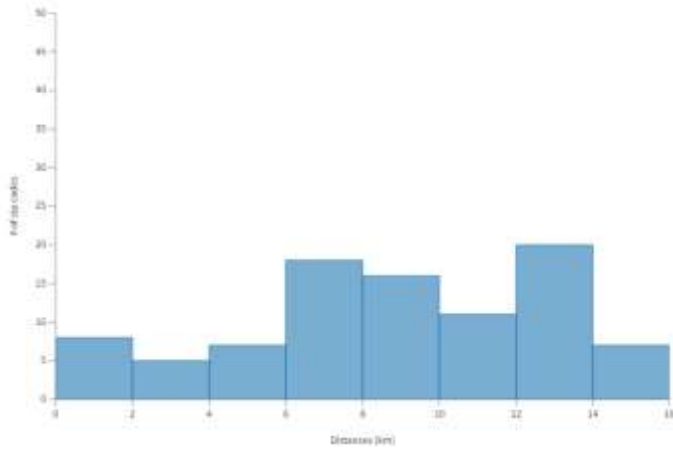
APPENDIX I



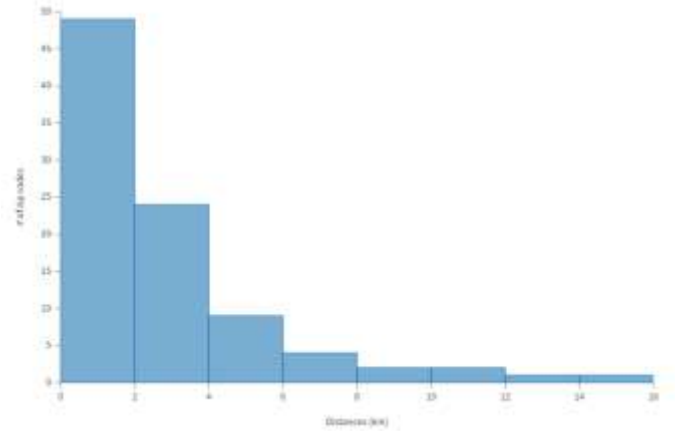




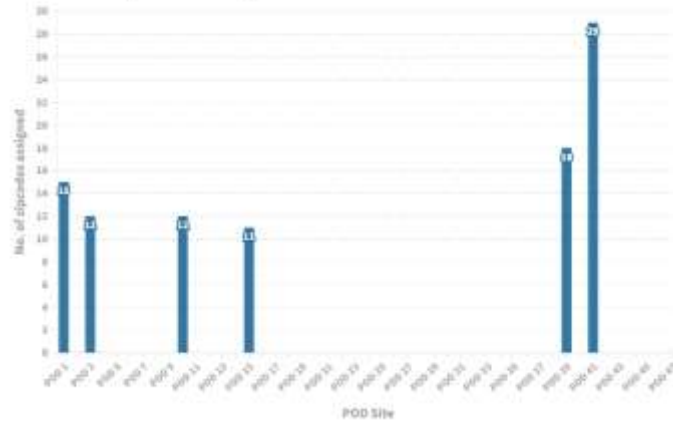
Distribution of travel distances to PODs by zip codes.



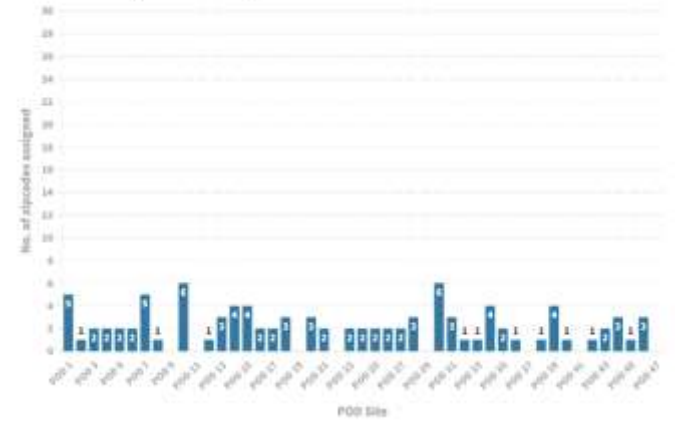
Distribution of travel distances to PODs by zip codes.



Number of zip codes assigned to selected POD sites



Number of zip codes assigned to selected POD sites



# Appendix 2

## Vaccine-only Model

In this setup, the breakout has not yet occurred and we want to get everyone vaccinated to prevent major breakout. Our goals are minimizing cost and minimizing distance traveled. We assume everyone will come get a vaccine. This is unlikely to be the case, but at least we will be prepared to give vaccines to as many as want them.

### Minimizing Total Distance

**Decision Variables:**

$A_{97 \times 47}$  : Binary, whether or not to assign zip-code i to site j

$S_{47 \times 1}$  : Binary, whether or not to open a small site (less than 10 medical professionals)

$M_{47 \times 1}$  : Binary, whether or not to open a medium site (10 to 20 medical professionals)

$L_{47 \times 1}$  : Binary, whether or not to open a large site (more than 20 medical professionals)

$U_{47 \times 25}$  : Binary, whether or not to utilize a site each day

$X_{47 \times 25}$  : Integer, the number of medical professionals for each point of dispense for every time period

$I_{47 \times 25}$  : Integer, the amount of inventory to hold every day out of 25 days for each of 47 sites

**Parameters:**

$D_{97 \times 47}$  : Travel distance from zipcode i to site j

$P_{97 \times 1}$  : population of each zipcode

$e_{3 \times 1}$  : efficiencies per medical professional for each dispense size (Small, Medium, Large)

$o$  : Fixed cost (opening cost)

$v$  : Variable cost

$f$  : Daily fixed cost

$r$  : medical professional wage

$h$  : Per unit daily holding cost

$K$  : A constant serving as the budget constraint

$C$  : The number of available medical professionals

**Helping Functions**

Total people served by a site =  $\sum_i A_{ij} p_i$

Total Population =  $\sum_{i=1}^{97} p_i$

Total Cost =  $\sum_{j=1}^{47} o(S_j + M_j + L_j) + \sum_{j=1}^{47} \sum_{i=1}^{97} v A_{ij} p_i + \sum_{j=1}^{47} \sum_{t=1}^{25} (r X_{jt} + h I_{jt} + f U_{jt})$

The total cost is a combination of variable and fixed costs

### Formulation

Minimize *Total Distance*

subject to

$Total\ Cost \leq K$

$\sum_{j=1}^{47} A_{ij} = 1, \forall i \in \{1, 2, \dots, 97\}$

Each neighborhood must be assigned to one and only one POD (Point of Dispense)

$A_{ij} \leq S_j + M_j + L_j, \forall i \in \{1, 2, \dots, 97\}, \forall j \in \{1, 2, \dots, 47\}$

A neighborhood can only be assigned to a POD if the POD is opened

$S_j + M_j + L_j \leq 1$

If we open a site, it can only be one of the following site sizes: Small, Medium, or Large scale

$10M_j + 20L_j - 20(1 - U_{jt}) \leq X_{jt} \leq 10S_j + 20M_j + 1,000L_j$

The number of medial professionals assigned to a site determines its scale-size. X must be within the appropriate range for the desired scale-size.

$\sum_{t=1}^{25} e_S X_{jt} \geq \sum_{i=1}^{97} A_{ij} p_i - (1 - S_j)(\text{Total Population})$

$\sum_{t=1}^{25} e_M X_{jt} \geq \sum_{i=1}^{97} A_{ij} p_i - (1 - M_j)(\text{Total Population})$

$\sum_{t=1}^{25} e_L X_{jt} \geq \sum_{i=1}^{97} A_{ij} p_i$

Enough medical professionals must be assigned to a site over the time period to administer all of the vaccines. The number of medical professionals need depends on the scale-size of the site. The least restrictive of these three is the third constraint, which we make general to provide a minimum lower bound on X not provided elsewhere.

$\sum_{j=1}^{47} X_{jt} \leq C$

Every day, no more medical professionals than the total available can be hired across the sites

$X_{jt} \leq C U_{jt}$

A site can only be assigned medical professionals if it is "used" that day

$U_{jt} \leq S_j + M_j + L_j$

A site can only be used if it has been opened

$I_{jt} + (1 - S_j)(\text{Total Population}) \geq \sum_i A_{ij} p_i - \sum_1^t e_S X_{jt}$

$I_{jt} + (1 - M_j)(\text{Total Population}) \geq \sum_i A_{ij} p_i - \sum_1^t e_M X_{jt}$

$I_{jt} + (1 - L_j)(\text{Total Population}) \geq \sum_i A_{ij} p_i - \sum_1^t e_L X_{jt}$

$I_{jt} - (1 - S_j)(\text{Total Population}) \leq \sum_i A_{ij} p_i - \sum_1^t e_S X_{jt}$

$I_{jt} - (1 - M_j)(\text{Total Population}) \leq \sum_i A_{ij} p_i - \sum_1^t e_M X_{jt}$

$I_{jt} - (1 - L_j)(\text{Total Population}) \leq \sum_i A_{ij} p_i - \sum_1^t e_L X_{jt}$

The daily inventory is determined by the total number of vaccinations administered by that point, which is dependent on the scale-size of the site

$X_{jt}, I_{jt} \geq 0$ , integer

$S_j, M_j, L_j, A_{ij}, U_{jt} \geq 0$ , binary

### Minimize Max Distance

This formulation uses the same model as above with the following changes

**New Objective:**

Minimize  $z$

**New Constraint:**

$z \geq A_{ij} D_{ij}, \forall i, j$

# Vaccine and Antibiotic Model

In this setup, a breakout has occurred. Some in the population need treatment. The rest we will give vaccinations to. To prevent further outbreak, we do not want to provide prevention and treatment in the same locations. In this problem we use multi-objective programming to balance the distance infected and uninfected people have to travel while staying within budget.

## Minimizing Average Total Distance

### Decision Variables:

*First-stage:*

$y_{47x1}$  : Binary, wether or not to open a point of dispense at each site

$T_{47x1}$  : Binary, whether to use a site for therapeutics (0 means use for vaccination if y = 1)

*Second stage:*

$A_{97x47x97}$  : Binary, whether or not to assign zip-code i to site j

$S_{47x97}$  : Binary, wether or not to assign less than 10 medical professionals to a site

$M_{47x97}$  : Binary, wether or not to assign between 10 and 20 medical professionals to a site

$L_{47x97}$  : Binary, wether or not to assign more than 20 medical professionals to a site

$U_{47x25x97}$  : Binary, whether or not to utilize a site each day

$X_{47x25x97}$  : Integer, the number of medical professionals for each point of dispense for every time period for every scenario

$I_{47x25x97}$  : Integer, the amount of inventory to hold every day out of 25 days for each of 47 sites in 97 scenarios

### Parameters:

$D_{97x47}$  : Travel distance from zipcode i to site j

$p_{97x1}$  : population of each zipcode

$e_{3x2}$  : efficiencies per medical professional for each dispense size (Small, Medium, Large) and for each treatment type

$o$  : First stage fixed cost (opening cost)

$v$  : Second stage variable cost

$f$  : Second stage daily fixed cost

$r$  : medical professional wage

$h$  : Per unit daily holding cost

$K$  : A constant serving as the budget constraint

$B_{97x97}$  : Binary, whether the population for a zipcode has an outbreak for each of the 97 scenarios

$C$  : The number of available medical professionals

### Helping Functions

Average Total Distance to get therapeutics =  $z_1 = \sum_{k=1}^{97} \sum_{i=1}^{97} \sum_{j=1}^{47} \frac{1}{97} p_i B_{ik} D_{ij} A_{ijk}$

Average Total Distance to get vaccination =  $z_2 = \sum_{k=1}^{97} \sum_{i=1}^{97} \sum_{j=1}^{47} \frac{1}{97} p_i (1 - B_{ik}) D_{ij} A_{ijk}$

Average Total Cost =  $\sum_{j=1}^{47} o y_j + \sum_{k=1}^{97} \sum_{j=1}^{47} \sum_{i=1}^{97} \frac{1}{97} v A_{ijk} p_i + \sum_{k=1}^{97} \sum_{j=1}^{47} \sum_{t=1}^{25} \frac{1}{97} (r X_{jtk} + h I_{jtk} + f U_{jtk})$

## Formulation

Minimize  $\alpha z_1 + (1 - \alpha) z_2$

subject to

*Average Total Cost*  $\leq K$

$\sum_{j=1}^{47} A_{ijk} = 1, \forall i \in \{1, 2, \dots, 97\}$

$A_{ijk} \leq y_j, \forall i \in \{1, 2, \dots, 97\}, \forall j \in \{1, 2, \dots, 47\}$

$A_{ijk} + (B_i - T_j) \leq 1$

$A_{ijk} + (T_j - B_i) \leq 1$

Zipcode  $i$  can only be assigned to site  $j$  if breakout matches therapeutics value

$S_{jk} + M_{jk} + L_{jk} \leq y_j$

$10M_{jk} + 20L_{jk} - 20(1 - U_{jtk}) \leq X_{jtk} \leq 10S_{jk} + 20M_{jk} + 1,000L_{jk}$

$\sum_{t=1}^{25} e_S X_{jtk} \geq \sum_{i=1}^{97} A_{ijk} p_i - (1 - S_{jk})(\text{Total Population})$

$\sum_{t=1}^{25} e_M X_{jtk} \geq \sum_{i=1}^{97} A_{ijk} p_i - (1 - M_{jk})(\text{Total Population})$

$\sum_{t=1}^{25} e_L X_{jtk} \geq \sum_{i=1}^{97} A_{ijk} p_i$

$\sum_{j=1}^{47} X_{jtk} \leq C$

$X_{jtk} \leq C U_{jtk}$

$U_{jtk} \leq S_{jk} + M_{jk} + L_{jk}$

$I_{jtk} + (1 - S_{jk})(\text{Total Population}) \geq \sum_i A_{ijk} p_i - \sum_1^t e_S X_{jtk}$

$I_{jtk} + (1 - M_{jk})(\text{Total Population}) \geq \sum_i A_{ijk} p_i - \sum_1^t e_M X_{jtk}$

$I_{jtk} + (1 - L_{jk})(\text{Total Population}) \geq \sum_i A_{ijk} p_i - \sum_1^t e_L X_{jtk}$

$X_{jtk}, I_{jtk} \geq 0$ , integer

$y_j, T_j, S_{jk}, M_{jk}, L_{jk}, A_{ijk}, U_{jtk} \geq 0$ , binary