Submission: Mohammad Mirza
Collaborators: Raja Safiullah, Javier Rico, Anum Malik

Question 1)

Decision & Auxiliary Variables:

x_{it}: units of product i to produce in week t
b_{it}: inventory of product i at the beginning of week t
e_{it}: inventory of product i at the end of week t
s_{it}: units of product i to sell in week t

Sets:

i = 1, …, n: products, where n = 100th product
j = 1, …, m: materials where m = 10th material
t = 1, …, T: weeks where T = 52nd week

Parameters:

a_{ij}: amount of material j required by product i
v_{jt}: amount of material j available in week t
π_i: profit per unit of product i
h_t: holding cost per unit in week t
d_{it}: demand for product i in week t

Objective Function:

$$\text{Max} \sum_{t=1}^{T} \sum_{i=1}^{n}(\pi_i * s_{it} - h_t * e_{it})$$

Constraints:

Inventory Dynamics:
b_{i1} = 0, $\forall i = 1, …, n$
$e_{it} = b_{it} + x_{it} - s_{it}$, $\forall i = 1, …, n, \forall t = 1, …, T$
$b_{it} = e_{it-1}$, $\forall t = 2, …, T$

Availability:
$$\sum_{i=1}^{n} a_{ij}x_{it} \leq v_{jt}, \forall j = 1, …, m, \forall t = 1, …, T$$

Demand:
$s_{it} \leq d_{it}$, $\forall i = 1, …, n, \forall t = 1, …, T$

Non-negativity:
$s_{it}, x_{it}, b_{it}, e_{it} \geq 0$, $\forall i = 1, …, n, \forall t = 1, …, T$

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Question 2)

The optimal objective value is $35,936,031.40. The model was solved in 4537 iterations, which took 1.03 seconds. Refer to Python code file for details.
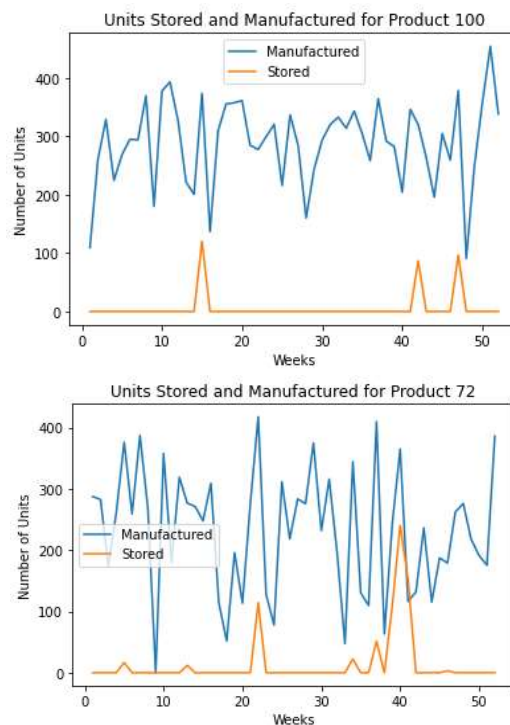
Question 3)

Total Sales Profit: $36,003,210.15
Holding Costs: $67,178.75

Question 4)

Each graph in the Python code represents a product, with a trend line each for units manufactured and stored over the 52 weeks horizon. Similar to the example graph below for Product 100, some products experience weeks when a spike in inventory leads to a fall in manufacturing. However, other products such as Product 72 as seen below, see a concomitant rise in both inventory and manufacturing, which is then compensated by a fall in manufacturing in the following week.

Question 5)

In Week 30, 9 of the 10 material types were fully utilized based on Gurobi's optimal solution.
In Week 30, 19 of the 100 products types had fully satisfied demand on Gurobi's optimal solution.

Since 9 of the materials were fully utilized, it reflects that the supply of materials in week 30 was constrained. Therefore, it was only possible to meet the demand for 19 of the 100 products.