Decision Analytics for Business and Policy

Homework 2: Linear Optimization

Due Date: Sep 15, 11:59am ET **Submission:** Canvas

In this homework, you will formulate and implement a linear optimization model. Please pay attention to the following guidelines:

- When asked to "formulate" an optimization model: define decision variables, the objective function, and the constraints of your problem mathematically.
- When asked to "implement" the model: implement the model in code and solve it computationally.
- Please state clearly any modeling or implementation assumptions you made in the write-up.

Problem Description: Integrated Manufacturing and Inventory Planning.

In this problem, you aim to solve two subproblems together: the manufacturing problem, which involves determining which products to make, given limited resource availability and limited demand, and the inventory planning problem, which involves sequencing the production decisions over time to balance the objectives of maximizing operating profits and of minimizing holding costs. Specifically, you aim to optimize the weekly production of a facility over a one-year period (i.e., 52 weeks). The facility manufactures 100 different product types from 10 materials that come in limited supply each week (materials have to be utilized on a weekly basis and cannot be carried over from one week to the next). Each product type generates a fixed per-unit profit that does not vary over time. In addition, you incur a holding cost at the end of every week, based on the number of (finished goods) items held in inventory. The per-unit holding cost varies over time, but is identical across the 100 product types. You know customer demand for each product type, and cannot sell more products than the demand value in any given week. Of course, if your materials supply is too low, then you will not be able to satisfy all the demand. You aim to maximize the net profitability (including the sales profits, minus the holding costs), given demand and supply information. All quantities can be fractional.

You have access to the following data files:

- `Pb1_requirements.csv`: A matrix of size 100 by 10 that indicates the number of units of each of the 10 materials required to manufacture one unit of each of the 100 product types.
- `Pb1_availability.csv`: A matrix of size 10 by 52 that indicates the number of units of each of the 10 materials that can be used in the manufacturing process during each of the 52 weeks.
- `Pb1_demand.csv`: A matrix of size 100 by 52 that indicates the demand (in number of units) for each of the 100 product types during each of the 52 weeks.
- `Pb1_unitprofit.csv`: A vector of size 100 that indicates the per-unit profit contribution of each of the 100 product types (in $).
- `Pb1_holdingcost.csv`: A vector of size 52 that indicates the per-unit holding cost during each of the 52 weeks (in $).

Questions:

1. Formulate a linear programming model that optimizes the production of the facility to maximize net profitability. Please specify your decision variables, your objective function and your constraints carefully. [25%]

2. Implement your model computationally. Please show your work. How long does the model take to get the optimal solution? [25%]

3. Report the total sales profit, and the total holding cost. (Hint for visualization: not every product is manufactured/stored, so you do not have to plot 100 products) [15%]

4. Represent graphically the number of units manufactured and the number of units stored in inventory, for each product type and each week. Interpret and explain your results. [20%]

5. In week 30, how many of the 10 materials were fully utilized? For how many of the 100 product types was demand fully satisfied? Comment briefly. [15%]