Homework 5
Submission by: Mohammad Mirza (mmmirza)

## Question 1)

Decision Variable:

- $x_i$: daily quantity in grams of each food item i

Sets:

- i = 1, …, 1007 food items i.e. eggplant, cheddar etc.
- j = 1, …, 6 food groups i.e. vegetables, fruits, grains, dairy, proteins, oils.
- n = 1, …, 10 nutrients i.e. protein, calcium, iron, zinc, copper, vitamin C, thiamine, riboflavin, folate, vitamin B12.

Parameters:
- $r_n$: minimum WHO daily requirement for nutrient n in grams
  r = [20, 0.4, 0.007, 0.0065, 0.00057, 0.02, 0.0007, 0.0011, 0.00005, 0.0000005] ← Defined in Code
- $c_{in}$: amount of nutrient n in food item i in milligrams/ 100 grams of food *[composition]*
- $e_i$: amount of energy in food item i in Kcal/ 100 grams of food *[energy]*
- $m_{ij}$ = 1 if food item i belongs to food group j, 0 otherwise *[mapping]*
- $l_j$: lower bound for the fraction of energy intake in the diet from food group j
  $l = [1.1, 0.3, 35, 4, 0.9, 0.5]$ ← Defined in Code
- $u_j$: upper bound for the fraction of energy intake in the diet from food group j
  $u = [25.1, 8.7, 75, 33, 10.2, 8.5]$ ← Defined in Code
- $p_i$: price of food item i in $/ 100 grams of food *[price]*

*Wherever needed, parameters were converted to grams (÷ 1000), & analysis done per gram (÷ 100)*

Objective Function (daily energy intake by a child):

- Min $\sum_{i=1}^{1007} \left( x_i * \frac{e_i}{100} \right)$

Objective Function (total daily cost of the diet):

- Min $\sum_{i=1}^{1007} \left( x_i * \frac{p_i}{100} \right)$

Weighted Objective Function:

- $(\propto)$ * Min $\sum_{i=1}^{1007} \left( x_i * \frac{e_i}{100} \right)$ + (1 - $\propto$) * Min $\sum_{i=1}^{1007} \left( x_i * \frac{p_i}{100} \right)$

Constraints:

- Nutrient Requirements (LHS and RHS in grams):
  $\sum_{i=1}^{1007} x_i * \frac{c_{in}}{100*1000} \geq r_n, \forall n = 1, …, 10$

- Relative Energy Restriction:

$$l_j \leq \sum_{i=1}^{1007}\left[(x_i * \frac{e_i}{100} * m_{ij})\right] \div \sum_{i=1}^{1007}\left[(x_i * \frac{e_i}{100})\right] * 100 \leq u_j, \forall j = 1, \dots, 6$$

➔ Defined Linearly in Code as follows:

- $$\sum_{i=1}^{1007}\left[(x_i * \frac{e_i}{100} * m_{ij})\right] \geq l_j * \sum_{i=1}^{1007}\left[(x_i * \frac{e_i}{100})\right] \div 100, \ \forall j = 1, \dots, 6$$
- $$\sum_{i=1}^{1007}\left[(x_i * \frac{e_i}{100} * m_{ij})\right] \leq u_j * \sum_{i=1}^{1007}\left[(x_i * \frac{e_i}{100})\right] \div 100, \forall j = 1, \dots, 6$$

- Non-negativity constraint:
  $$x_i \geq 0, \forall i = 1, \dots, 1007$$

**Question 2)**

Python was used to implement the weight-based approach to multi-objective optimization by varying the weights assigned to each of the two segregated objectives in the problem i.e. minimizing daily energy consumption and minimizing daily dietary costs. The objective function, which was a weighted sum of each of the two objectives i.e. $\propto(Z_1) + 1 - \propto (Z_2)$ is optimized for each iteration i.e. weight/ alpha-value, thereby giving a minimum dietary cost and daily energy intake every time. With the results stored in a 14 x 2 matrix, we then plotted the pareto efficient frontier below for the two objectives below, where each point reflects a solution corresponding to a specific alpha/ weight value.



Daily Energy Intake v/s Diet Cost
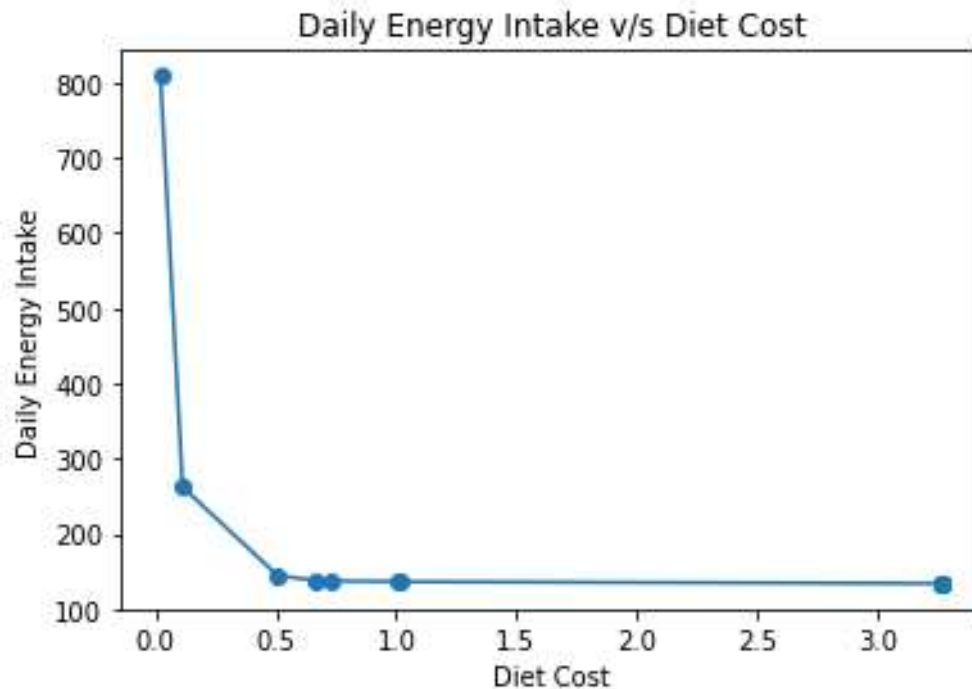
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**Question 3)**

The optimal diet with a daily diet cost of $0.5 and energy intake of 145.044 Kcal is as follows:

```
Solved in 15 iterations and 0.01 seconds
Optimal objective   1.450442861e+02
Food Item 46 is required in the diet in the quantity of 9.53 grams
Food Item 217 is required in the diet in the quantity of 3.59 grams
Food Item 241 is required in the diet in the quantity of 6.32 grams
Food Item 352 is required in the diet in the quantity of 22.36 grams
Food Item 404 is required in the diet in the quantity of 0.68 grams
Food Item 432 is required in the diet in the quantity of 12.56 grams
Food Item 482 is required in the diet in the quantity of 1.66 grams
Food Item 618 is required in the diet in the quantity of 3.11 grams
Food Item 727 is required in the diet in the quantity of 3.44 grams
Food Item 988 is required in the diet in the quantity of 12.09 grams
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