Decision Analytics for Business and Policy

Homework 5: Multi-Objective Optimization

Due Date: Oct 11, 11:59am ET **Submission:** Canvas

Problem Description: Nutrition Policy.

The World Health Organization (WHO) has issued new child nutrition guidelines, which provides minimal daily intake in 10 critical nutrients (shown in Table 1). It now aims to provide diet recommendations for a developing country to implement these guidelines. The country has provided a list of 1,007 food items (e.g., "eggplant", "cheddar") organized into 6 food groups (vegetables, fruits, grains, dairy, proteins, oils). The diet will determine a daily quantity (in g) of each food item, subject to the following constraints:

Nutrient	WHO Recommendation
Protein	20 g/d
Calcium	400 mg/d
Iron	7 mg/d
Zinc	6.5 mg/d
Copper	0.57 mg/d
Vitamin C	20 mg/d
Thiamine	0.7 mg/d
Riboflavin	1.1 mg/d
Folate	0.050 mg/d
Vitamin B12	0.0005 mg/d

Table 1: WHO recommendations for each nutrient.

- Each nutrient needs to be provided in quantities at least equal to the WHO guidelines.
- The relative energy intake stemming from each food group needs to fall between the minimum and maximum values reported in Table 2 in order to comply with local supply-side resource availability. For instance, the total energy intake from vegetables can fall between 1.1% and 25.1%, the one from fruits can fall between 0.3% and 8.7%, etc.

The WHO has two major objectives: (i) minimizing the daily energy intake by each child, and (ii) minimizing the total daily cost of the diet. In this problem, you will develop and implement a bi-objective optimization model to support the design of this diet.

You have access to the following data files:

- `Pb1_composition.csv`: A matrix of size 1,007 by 10 that indicates the amount of each nutrient in each food item (in mg per 100 grams of food). The nutrients are given in the order shown in Table 1.
- `Pb1_energy.csv`: A vector of size 1,007 that indicates the amount of energy in each food item (in Kcal per 100 grams of food).
- `Pb1_mapping.csv`: A matrix of size 1,007 by 6 with binary indicators that maps the food items into the six food groups (vegetables, fruits, grains, dairy, proteins, oils, in that order).
- `Pb1_price.csv`: A vector of size 1,007 that indicates the price of each food item (in $ per 100 grams of food).

Food Groups	**Minimum Proportion**	**Maximum Proportion**
Vegetables	1.1%	25.1%
Fruits	0.3%	8.7%
Grains	35%	75%
Dairy	4%	33%
Proteins	0.9%	10.2%
Oils	0.5%	8.5%

Table 2: Minimum and maximum proportion of daily energy intake for each food group.

Questions:

1. Formulate a bi-objective model that optimizes the design of the diet, subject to the constraints described above. Write out each of the two objectives, and develop a weighted approach to model the bi-objective problem. [40%]
2. Implement the weighted approach computationally. Please describe how you used Python for this problem. Provide the code in your submission. Vary the weight(s) considered and plot the resulting Pareto-optimal frontier between the diet cost (on the x-axis) and the daily energy intake (on the y-axis). [40%]
3. Suppose that the WHO imposes that the cost of the diet must not exceed $0.5 per day. Obtain the optimal diet that minimizes the energy intake using the goal programming approach. [20%]