

# Matrix inverse for separated real and imaginary

Tuesday, May 18, 2021 12:08 PM

## Summary

For a complex matrix  $A + iB$ , where  $A$  and  $B$  are real matrices, we can calculate the inverse matrix  $C + iD$  as:

$$C = (A + BA^{-1}B)^{-1}$$

$$D = -A^{-1}BC$$

## Derivation

Given complex matrix  $X$  and its inverse  $Y$  as:

$$X = A + iB$$

$$Y = C + iD$$

$$Y = X^{-1}$$

Substituting in the inverse definition:

$$XY = I$$

$$(A + iB)(C + iD) = I$$

$$(AC - BD) + i * (AD + BC) = I$$

Separating real and imaginary parts, we get two equations

$$AC - BD = I$$

$$AD + BC = 0$$

Using the second equation

$$AD = -BC$$

$$D = -A^{-1}BC$$

Substituting in the first equation

$$AC - B(-A^{-1}BC) = I$$

$$AC + BA^{-1}BC = I$$

$$(A + BA^{-1}B)C = I$$

$$C = (A + BA^{-1}B)^{-1}$$

Once we have  $C$ , we can calculate  $D$ .