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1.

a) $(2^n+n^3) \in O(4^n)$ ✓

$(2^n+n^3) \leq c.4^n$ for $n \geq n_0$ and $c>0$

If we choose $n_0=3$ and $c=3$

$2^3+3^3 \leq 3.4^3 \Longrightarrow 35 \leq 192$ for $n \geq 3$ (True)

b) $\sqrt{10n^2+7n+3} \in \Omega(n)$ ✓

$\sqrt{10n^2+7n+3} \geq c.n$ for $n \geq n_0$ and $c>0$

If we choose $n_0=3$ and $c=3$

$\left(\sqrt{10.3^2+7.3+3}\right)^2 \geq (3.3)^2 \Longrightarrow 114 \geq 81$ for $n \geq 3$ (True)

c) $n^2+n \in o(n^2)$ X

For this expression, expression's left hand side should grow slower than expression's right hand side.

n^2+n growth rate is n^2. Because n is low order term.

growth rate of n^2 is n^2.

Therefore, $n^2+n \in o(n^2)$ expression is false.

d) $3\log_2^2 n \in \Theta(\log_2 n^2)$ X

For this expression, expression's left hand side should grow same or faster than expression's right hand side

$3\log_2^2 n$ growth rate is $\log_2(\log_2 n)$. Because 3 is constant.

$\log_2 n^2 = 2\log_2 n$

$2\log_2 n$ growth rate $\log_2 n$. Because 2 is constant.

Therefore, $3\log_2^2 n \in \Theta(\log_2 n^2)$ expression is false.

e) $(n^3+1)^6 \in O(n^3)$ X

For this expression, expression's left hand side should grow same or slower than expression's right hand side.

$(n^3+1)^6$ growth rate $\Rightarrow (n^3)^6 = n^{18}$

n^3 growth rate $\Rightarrow n^3$

Therefore, $(n^3+1)^6 \in O(n^3)$ expression is false.

2)

a) $2n \log (n+2)^2 + (n+2)^2 \log \frac{n}{2}$

growth rate of $2n.\log (n+2)^2 \Rightarrow n.\log n$

growth rate of $(n+2)^2.\log \frac{n}{2} \Rightarrow n^2.\log n$

$$\downarrow$$

$$(n^2+4n+4).\log \frac{n}{2}$$

Θ notation $\Rightarrow 2n \log (n+2)^2 + (n+2)^2 . \log \frac{n}{2} \in \Theta(n^2 \log n)$

b) $0{,}001n^4 + 3n^3 + 1$

growth rate of this expression is n^4. Because $0{,}001$ is constant and $3n^3+1$ is low order term.

Θ notation $\Rightarrow 0{,}001n^4 + 3n^3 + 1 \in \Theta(n^4)$

3)

a) $\log n$, $n^{\log n}$, $n^{1,5}$

☆ $\lim\limits_{n \to \infty} \frac{\log n}{n^{\log n}} = \frac{\infty}{\infty}$

$$\frac{(\log n)'}{(n^{\log n})'} = \lim_{n \to \infty} \frac{n \cdot \frac{1}{n}}{2n^{\log n} \cdot \log n}$$

$$= \lim_{n \to \infty} \frac{1}{2 \cdot n^{\log n} \cdot \log n} = \frac{1}{\infty} = 0$$

Therefore, $n^{\log n} > \log n$

☆ $\lim\limits_{n \to \infty} \frac{\log n}{n^{1,5}} = \frac{\infty}{\infty}$

$$\frac{(\log n)'}{(n^{1,5})'} = \lim_{n \to \infty} \frac{\frac{1}{n}}{\frac{3}{2}\sqrt{n}} = \frac{1}{\frac{3}{2} n\sqrt{n}} = 0$$

Therefore, $n^{1,5} > \log n$

☆ $\lim\limits_{n \to \infty} \frac{n^{\log n}}{n^{1,5}} = \frac{\infty}{\infty}$

$$= \lim_{n \to \infty} n^{\log n - 1,5} = \infty$$

Therefore, $n^{\log n} > n^{1,5}$

<u>Orders of growth</u>

$$\boxed{n^{\log n} > n^{1,5} > \log n}$$

b) $n!,\ 2^n,\ n^2$

☆ Stirling's formula: $n! \cong \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$

$$\lim_{n\to\infty} \frac{n!}{2^n} = \lim_{n\to\infty} \frac{\sqrt{2\pi n}\cdot\left(\frac{n}{e}\right)^n}{2^n} = \frac{\infty}{\infty}$$

$$\lim_{n\to\infty} \sqrt{2\pi n}\cdot\left(\frac{n}{2e}\right)^n = \infty$$

Therefore, $n! > 2^n$

☆ $$\lim_{n\to\infty} \frac{2^n}{n^2} = \frac{\infty}{\infty}$$

$$\frac{(2^n)'}{(n^2)'} = \frac{2^n.\log 2}{2n} = \frac{\log 2}{2}.\lim_{n\to\infty} \frac{2^n}{n} = \frac{\infty}{\infty}$$

$$\frac{(2^n)'}{(n)'} = \frac{2^n.\log 2}{1} = \log 2.\lim_{n\to\infty} 2^n = \infty$$

Therefore, $2^n > n^2$

Orders of growth

$\boxed{n! > 2^n > n^2}$

c) $n\log n,\ \sqrt{n}$

$$\lim_{n\to\infty} \frac{n\log n}{\sqrt{n}} = \frac{\infty}{\infty} = \frac{\cancel{\sqrt{n}}.\sqrt{n}.\log n}{\cancel{\sqrt{n}}} = \lim_{n\to\infty} \sqrt{n}\log n = \infty$$

Therefore, orders of growth $\boxed{n\log n > \sqrt{n}}$

d) $n.2^n,\ 3^n$

$$\lim_{n\to\infty} \frac{n.2^n}{3^n} = \frac{\infty}{\infty} = \lim_{n\to\infty} n.\left(\frac{2}{3}\right)^n = \infty$$

Therefore, orders of growth $\boxed{n.2^n > 3^n}$

e) $\sqrt{n+10}$, n^3

$$\lim_{n\to\infty} \frac{n^3}{\sqrt{n+10}} \Longrightarrow \lim_{n\to\infty} \frac{n^3}{\sqrt{n}} = \lim_{n\to\infty} n^{3-1/2} = \lim_{n\to\infty} n^{5/2} = \infty$$

Therefore, orders of growth $\boxed{n^3 > \sqrt{n+10}}$

4)

a) Basic operation is $B[i,j] != B[j,i]$ comparison.

b) $$\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} [(n-1)-(i+1)+1]$$

$$= \sum_{i=0}^{n-2} (n-1-i) = (n-1)+(n-2)+(n-3)+\text{-----}+1 = \boxed{\frac{n.(n-1)}{2} \text{ times}}$$

c) $$\sum_{i=0}^{n-2} (n-1-i) = \frac{n.(n-1)}{2} = \frac{n^2-n}{2}$$ $\longrightarrow$ If we remove constant and low order term, time complexity is $\boxed{O(n^2)}$

5)

a) Basic operation is $C[i,j] = C[i,j] + A[i,k] * B[k,j]$

b) $$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1 \Rightarrow \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n \Longrightarrow \sum_{i=0}^{n-1} n^2 \Longrightarrow n^3 \text{ times}$$

c) Time complexity is $O(n^3)$

6)

Let's assume that arr is an unordered array which includes [0,1,-----,n-1]

Also, let's assume that desiredNumber is an integer which is multiplication of pairs in unordered array.

Pseudocode

```
print_pairs (arr[0,1,-----,n-1], desiredNumber)
for i=0 to n-1 do
    for j=i+1 to n-1 do
        if arr[i] * arr[j]== desiredNumber
            print{arr[i], arr[j]}
```

$$\text{Time Complexity} = \sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} 1 = \frac{n(n-1)}{2} = \frac{n^2-n}{2}$$

If we remove constant and lower order term, time complexity is $\boxed{O(n^2)}$