Middle East Technical University Civil Engineering Department

Structural Reliability

Term Project

# Title: Review of An Overview of Probabilistic Seismic Hazard Analysis

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# Table of Contents

## 1. Introduction

The structural earthquake engineers are responsible that a structure should maintain itself within desired level of performance under ground shaking. While, they are designing building against seismic events, the main three problems are event itself (when, where, and how earthquake occurs.), the ground motion intensity, and the effects on structure. Structural effects can be solved by proper structural analysis ways. However, when earthquake is considered, location, magnitude of earthquake and intensity of shaking are major uncertainties. These uncertainties are estimated by Probabilistic Seismic Hazard Analysis (PSHA). PSHA use collected data from past earthquakes to quantify future events. In second part of the project, reasons why PSHA is preferred against deterministic approach is explained. In the third part of the project, PSHA procedure is clarified.

## 2. Reasons Why Probabilistic Seismic Hazard Analysis is Preferred

There are two concepts for seismic hazard analysis, probabilistic seismic hazard analysis (PSHA)and deterministic seismic hazard analysis (DSHA). If you consider worst case condition for earthquake, this consideration is DSHA. DSHA makes calculation quick and conceptually easy, but DSHA includes some conceptual problems.

### 2.1. Variability in the design event

Let observe the site which is close to two faults as Figure 1. Fault A is 10 km away from the site and maximum magnitude of earthquake is measured as 6.5. Fault B is 20 km away from the site and maximum magnitude of earthquake is measured as 7.5. In this case, which of the fault would be chosen as worst case is? Closest fault or fault with higher maximum magnitude of earthquake.

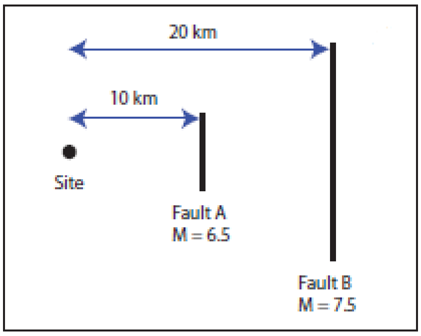


*Figure 1: Map view of an illustrative site*

Additionally, median predicted response spectra for two events with maximum magnitude of earthquakes in the site is as Figure 2. As seen in the figure, Fault A with low magnitude and closely site creates larger spectral acceleration for lower periods, but Fault B with high magnitude and far distance creates higher spectral acceleration for higher periods. The example shows that there is not a single "worst-case" event for predicted median spectral acceleration.

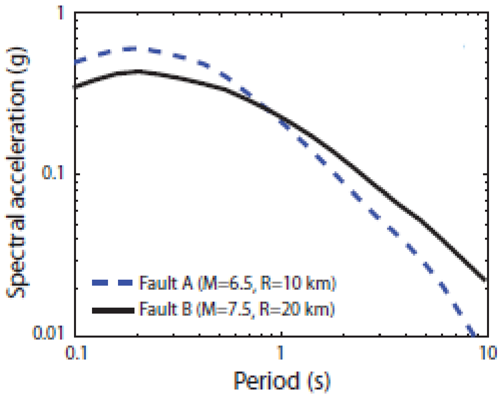


*Figure 2: Predicted median response spectra for two events in the site (prediction obtained from the model of Campbell & Bozorgnia (2008))*

### 2.2. Variability of ground motion intensity

The median response spectra in Figure 2. is predicted by stochastic model derived from recorded ground motions. The recorded ground motion includes very large amount of data. Data is scattered as Figure 3. The Figure shows spectral acceleration values at 1 second that observed in a past earthquake (1999 Chi-Chi, Taiwan). The Figure is drawn in logarithmic scale and mean prediction of scatter exhibited by normal distribution. It is seen that there is not a certain line type of solution so that the predicted line can be estimated from probabilistic approaches.

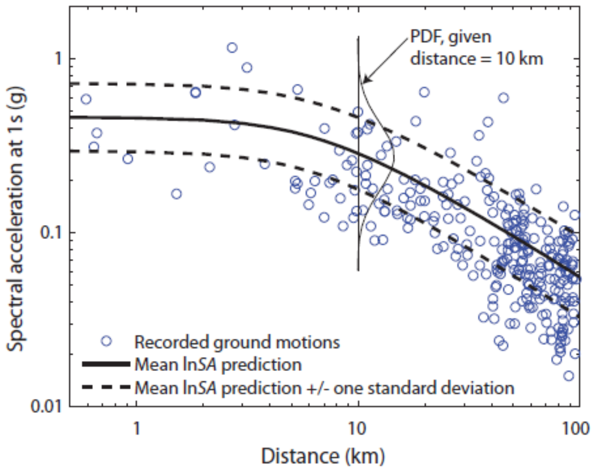


*Figure 3:Observed Spectral acceleration values from the 1999 Chi-Chi, Taiwan Earthquake, illustrating variability in ground motion intensity. The predicted distribution comes from the model of Campbell & Bozorgnia (2008)*

### 2.3. Can we use a deterministic approach, given these uncertainties?

Given few reasons, it is obviously seen that there isn't a "worst-case" condition. Also, when complex continental is considered, it is impossible to decide a "worst-case" condition. Therefore, consideration of all events is necessary to make reliable design.

## 3. Probabilistic Seismic Hazard Analysis Calculations

Worst case ground motion intensity is necessary in DSHA. However, all possible earthquake events and resulting ground motion, associated with probabilities of occurrence, are investigated to find the level of ground motion intensity in PSHA. 5 steps of PSHA are as follows:

1. Identify all earthquake sources, which can cause damage to the site
2. Characterize the distribution of earthquake magnitudes.
3. Characterize the distribution of source-to-site distances associated with potential earthquakes.
4. Predict the resulting distribution of ground motion intensity as a function of earthquake magnitude, distance, etc.
5. Combine uncertainties in earthquake size, location, and ground motion intensity, using a calculation known as the total probability theorem

These steps are explained detailly below.

## 3.1. Identify earthquake sources

In deterministic approach, a single worst-case sources is considered. Instead of that, all earthquake sources, which can cause damage to site, are examined in probabilistic approach. These sources are usually faults, observed from past earthquakes. Type of faults can be line source or area source, which are seen in Figure 4. All possible earthquake sources are defined and then distribution of magnitudes and source-to-site distances associated with earthquakes from each source.

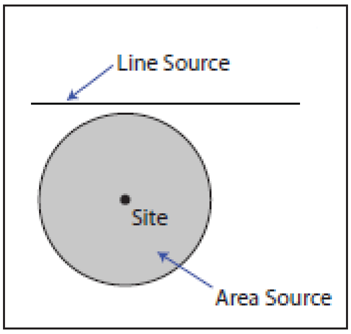


*Figure 4: Earthquake Sources*

## 3.2. Identify earthquake magnitudes

Gutenberg & Richer (1944) studied observations of earthquake magnitudes. The number of earthquakes in a region greater than a given size generally follows a particular distribution

$$\log_{10} \lambda_m = a - bm$$

where $\lambda_m$ is the rate of earthquakes with magnitudes greater than m, and a, b are constants. The equation is called as the Gutenberg-Richter recurrence law. Figure 5. shows typical distribution of observed earthquake magnitudes with Gutenberg-Richter recurrence laws fit to the observations. a and b constants in the equation are estimated by using statistic of past historical observations.

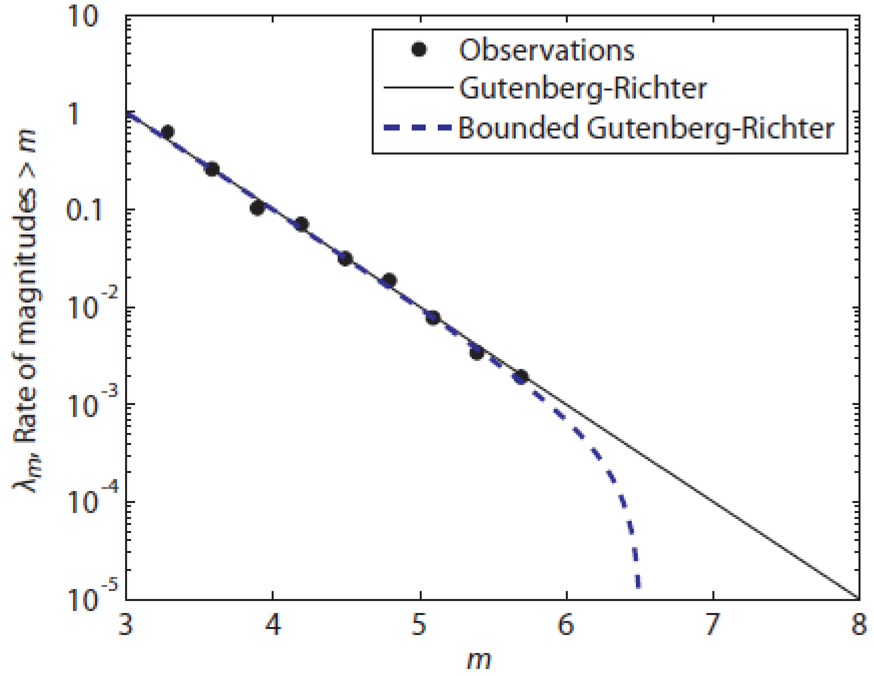


*Figure 5: Typical distribution of observed earthquake magnitudes, along with Gutenberg-Richter and bounded Gutenberg-Richter laws*

By using Gutenberg-Richter recurrence laws, cumulative distribution function (CDF) can be calculated for larger than some minimum magnitude $m_{min}$.

$$F_M(m) = P(M \leq m | M > m_{min})$$

$$F_M(m) = \frac{Rate\ of earthquake\ with\ m_{min} < M \leq m}{Rate\ of earthquake\ with\ m_{min} < M}$$

$$F_M(m) = \frac{\lambda_{m_{min}} - \lambda_m}{\lambda_{m_{min}}}$$

$$F_M(m) = \frac{10^{a-bm_{min}} - 10^{a-bm}}{10^{a-bm_{min}}}$$

$$F_M(m) = 1 - 10^{-b(m-m_{min})}$$

where $F_M(m)$ denotes the cumulative distribution function for M. Probability density function (PDF) for M can be found from $F_M(m)$.

$$f_M(m) = \frac{d}{dm} F_M(m)$$

$$f_M(m) = \frac{d}{dm}\left[1 - 10^{-b(m-m_{min})}\right]$$

$$f_M(m) = bln(10)10^{-b(m-m_{min})}\ for\ m > m_{min}$$

where $f_M(m)$ denotes the probability density function of M. For the pdf PDF and CDF are considered without upper limit. If upper limit is considered CDF and PDF are obtained respectively as follows:

$$F_M(m) = \frac{1-10^{-b(m-m_{min})}}{1-10^{-b(m_{max}-m_{min})}}\ for\ m_{min} < m < m_{max}$$

$$f_M(m) = \frac{bln(2.10)10^{-b(m-m_{min})}}{1-10^{-b(m_{max}-m_{min})}}\ for\ m_{min} < m < m_{max}$$

where $m_{max}$ is the maximum earthquake that a given source can produce. This limited magnitude distribution is called as a bounded Gutenberg-Richter recurrence law. Also, Figure 5. shows Gutenberg-Richter recurrence law. Lastly, if discrete set of magnitude is necessary, continuous CDF can convert to discrete set of magnitude.

### 3.3. Identify earthquake distance

Distance from earthquake center to effected site is essential to model the distribution to predict ground shaking at the site. Probability of occurring earthquake on the fault is assumed equal. Therefore, the distribution of source-to-site distances is identified by using the geometry of source. As an example, line source and area source are shown.

Example: Area Source

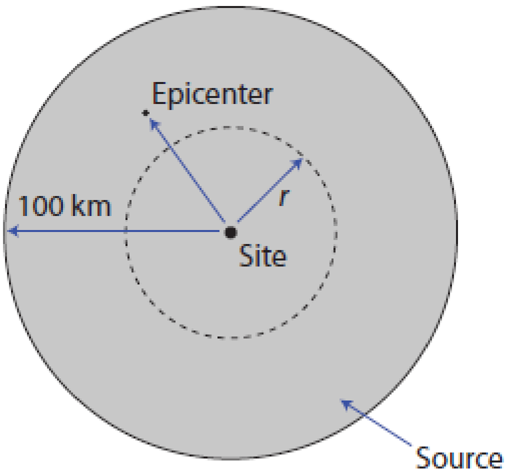


*Figure 6: Area Source*

Area source is seen in Figure 6. Earthquake can occur in range 0 – 100 km range from the site. Probability of occurrence earthquake in anywhere on fault is equal. Cumulative distribution function (CDF) is as follows:

$$F_R(r) = P(R \leq r)$$

$$F_R(r) = \frac{area\ of\ circle\ with\ radius\ r}{area\ of\ circle\ with\ radius\ 100}$$

$$F_R(r) = \frac{\pi r^2}{\pi 100^2}$$

$$F_R(r) = \frac{r^2}{10000}$$

For all range, CDF is as follows:

$$F_R(r) = \begin{Bmatrix} 0 & if\ r < 0 \\ \dfrac{r^2}{10000} & if\ 0 \leq r \leq 100 \\ 1 & if\ r \geq 100 \end{Bmatrix}$$

Probability density function (PDF) is found by taking derivative of CDF:

$$f_R(r) = \frac{d}{dr} F_R(r) = \begin{Bmatrix} \dfrac{r}{5000} & if\ 0 \leq r \leq 100 \\ 1 & otherwise \end{Bmatrix}$$

Graphs of PDF and CDF are as follows:

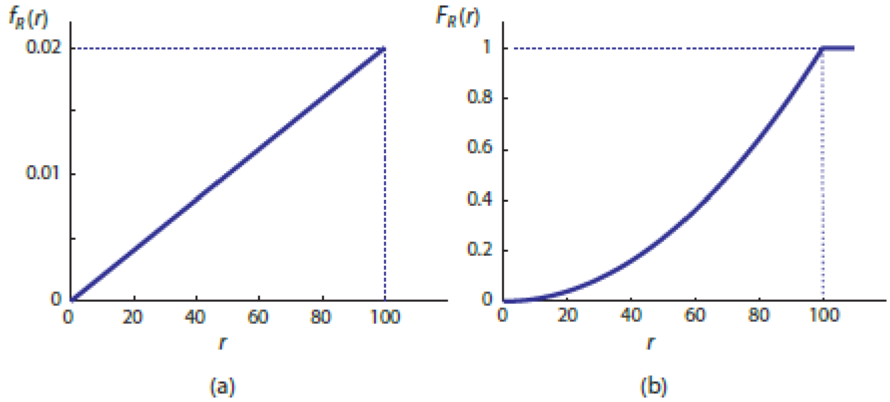


*Figure 7: PDF and CDF of the source-to-site distance for future earthquakes*

<u>Example: Line Source</u>

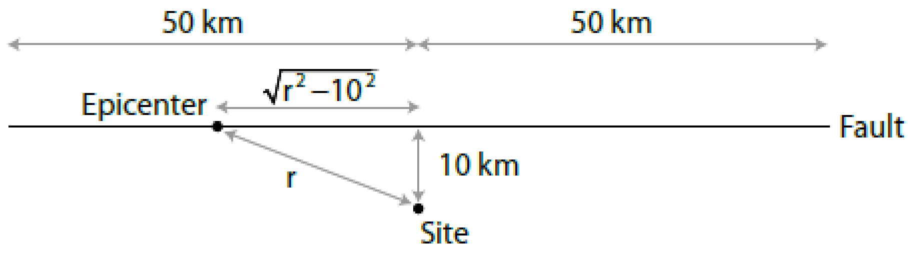


*Figure 8: Line Source*

Line source is seen in Figure 8. Fault length is 100 km and closest length between fault and site is 10 km. Probability of occurrence earthquake in anywhere on fault is equal. Cumulative distribution function (CDF) is as follows:

$$F_R(r) = P(R \leq r)$$

$$F_R(r) = \frac{length\ of\ fault\ within\ distance\ r}{total\ length\ of\ fault}$$

$$F_R(r) = \frac{2\sqrt{r^2 - 10^2}}{100}$$

For all range, CDF is as follows:

$$F_R(r) = \begin{Bmatrix} 0 & if\ r < 10 \\ \dfrac{2\sqrt{r^2 - 10^2}}{100} & if\ 10 \leq r \leq 51 \\ 1 & if\ r \geq 51 \end{Bmatrix}$$

Probability density function (PDF) is found by taking derivative of CDF:

$$f_R(r) = \frac{d}{dr} F_R(r) = \begin{Bmatrix} \dfrac{r}{50\sqrt{r^2 - 100}} & if\ 0 \leq r \leq 100 \\ 1 & otherwise \end{Bmatrix}$$

Graphs of PDF and CDF are as follows:

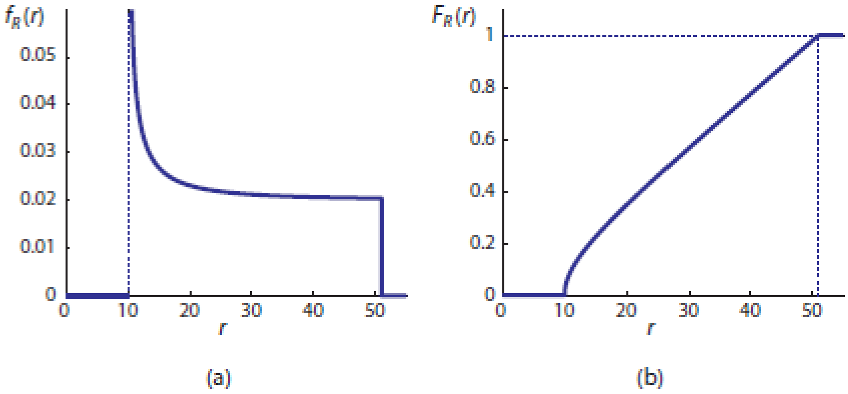


*Figure 9: PDF and CDF of the source-to-site distance for future earthquakes*

### 3.4. Ground motion intensity

Predictions of ground motion model are generally developed by statistical regression on observations from large libraries of observed ground motion intensities.

Prediction models of intensity measure are following form:

$$lnIM = \overline{lnIM}(M, R, \theta) + \sigma(M, R, \theta). \varepsilon$$

where lnIM is the natural log of the ground motion intensity measure of interest, $\overline{lnIM}(M, R, \theta)$ is mean of the model, and $\sigma(M, R, \theta)$ is standard deviation of model. M is magnitude, R is distance and $\theta$ is referred other parameters. $\varepsilon$ is a standard normal random variable that represents the observed variability in lnIM.

The predictive model for the mean of log peak ground acceleration (PGA) by Cornell et al. (1979) is as follows:

$$\overline{\ln PGA} = -0.152 + 0.859\, M - 1.803 \ln(R + 25)$$

The standard deviation of lnPGA was 0.57 in this model. From the natural logarithm of PGA, probability of exceedance any PGA level with mean and standard deviation is:

$$P(PGA > x|m, r) = 1 - \phi\left(\frac{\ln x - \overline{\ln PGA}}{\sigma_{\ln PGA}}\right)$$

where $\phi()$ is the standard normal cumulative distribution function.

Figure 10. shows visual view of ground motion prediction by probability of exceedance any PGA level. The figure shows PGA predictions for a magnitude 6.5 earthquake as a function of distance. Also, probability of PGA greater 1g at distance 3, 10 and 30 km is shown.

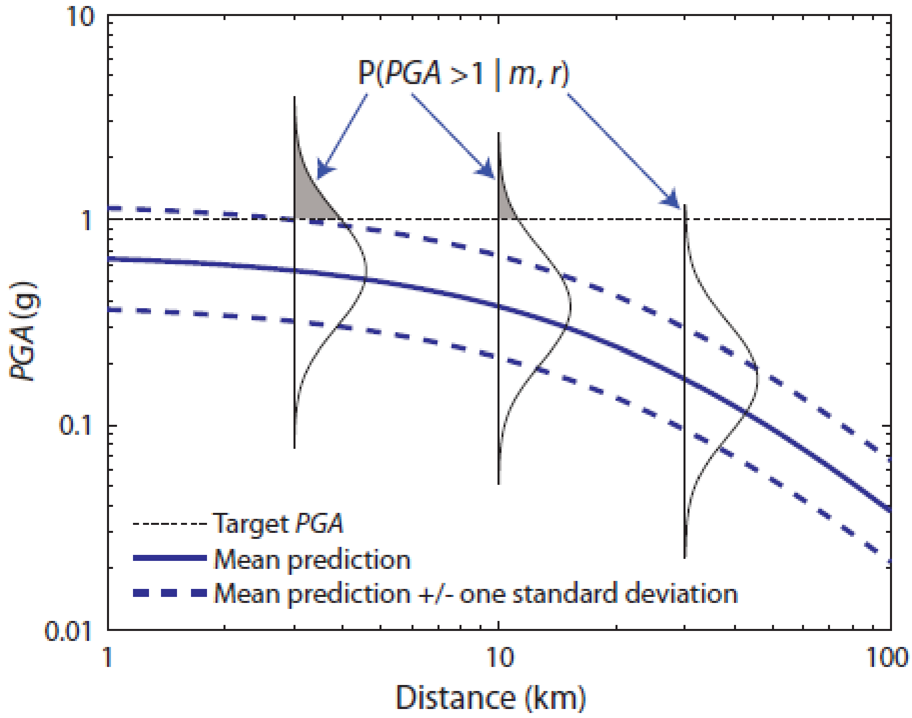


*Figure 10: Graphical view of the ground motion prediction model for a magnitude 6.5 earthquake, and the probability of PGA >1g at several source-to-site distances*

### 3.5. Combine all information

All given information is combined by using PSHA equations. Combination will be made by considering multiple sources towards PSHA equations.

First, combined probability of exceeding an IM intensity level x is by using total probability theorem as follows:

$$P(IM > x) = \int_{m_{min}}^{m_{max}} \int_{0}^{r_{max}} P(IM > x|m,r) f_M(m) f_R(r) dr\, dm$$

where $P(IM > x|m,r)$ comes from the ground motion model, $f_M(m)$ is PDFs for magnitude and $f_R(r)$ is PDFs for distance. The above equation doesn't give how often earthquake occur on the source of interest. That can be found by simple modification rather to find probability of IM>x. It is as follows:

$$\lambda(IM > x) = \lambda(M > m_{min}) \int_{m_{min}}^{m_{max}} \int_{0}^{r_{max}} P(IM > x|m,r) f_M(m) f_R(r) dr\, dm$$

where $\lambda(M > m_{min})$ is the rate of occurrence of earthquakes greater than $m_{min}$ from the source, and $\lambda(IM > x)$ is the rate of intensity measure greater than x.

Lastly, multiple sources are considered and following form is obtained:

$$\lambda(IM > x) = \sum_{i=1}^{n_{sources}} \lambda(M_i > m_{min}) \int_{m_{min}}^{m_{max}} \int_{0}^{r_{max}} P(IM > x|m,r) f_M(m) f_R(r) dr\, dm$$

When integral in the last equation is converted to summation, discrete PSHA is obtained.

$$\lambda(IM > x) = \sum_{i=1}^{n_{sources}} \lambda(M_i > m_{min}) \sum_{j=1}^{n_M} \sum_{k=1}^{n_R} P\big(IM > x|m_j\, , r_k\big) P\big(M_i = m_j\big) P(P_i = r_k\, )$$

where the range of possible $M_i$ and $R_i$ are discretized into number of magnitude ($n_M$) and number of distance ($n_R$).

Before getting result, past earthquakes records are required. After processing the data, required knowledge is provided for PSHA. Last two equation gives us rate of intensity measure greater than x. The equations are integrated from rates of occurrence of earthquakes, the possible magnitudes and distances of those earthquakes, and the distribution of ground shaking intensity due to those earthquakes. Intensity measure is necessary to prevent destructive results.

## 4. Conclusion

In this project, I aimed to make a brief introduction to Probabilistic Seismic Hazard Analysis (PSHA). Why do we use PSHA in design is briefly explained. Afterwards, 5 steps of PSHA procedures are shown. While explaining procedure, I tried to emphasize mathematical background especially. At the end of the section, rate of intensity measure for IM>x is obtained.

## References

Baker, J. W. (2013). Introduction to Probabilistic Seismic Hazard Analysis. *White Paper Version 2.0.1*, 79 pp.