Middle East Technical University Civil Engineering Department

Advanced Analysis Techniques in Structural Engineering

Project Report

‘Implementation of New Dynamic Analysis Algorithm’

‘Bathe Method’

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1. Introduction

The governing equation of equilibrium in linear dynamic analysis response of finite elements system is

$$\mathrm{M\ddot{U}} + \mathrm{C\dot{U}} + \mathrm{KU} = R \tag{1.1}$$

where M, mass matrix, C, damping matrix, and K, stiffness matrix; R is the vector of external applied load; and U, displacement, $\dot{\mathrm{U}}$, velocity, and $\ddot{\mathrm{U}}$, acceleration vectors.

Eqns. (1.1) is second order differential equation, and the solution to the equation with standard procedures can be expensive and time consuming if the sizes of the matrices are large. Therefore, there are practical solution methods. These practical methods are separated to two methods of solution which are, direct integration and mode superposition. In direct integration method, components of Eqns. (1.1) are integrated by numerically. While integrating, no transformation is carrying out and form of the components in Eqns. (1.1) remains same. Therefore, these methods are called as 'Direct Integration Methods'.

In this project, one of the direct integrations, which is Bathe method, is surveyed, and step-by-step solution algorithm is mentioned. Lastly, one demonstrative example is solved.

2. Bathe Method

Bathe method is implicit time integration procedures. It uses two sub-steps for each time integration step Δt. The Newmark trapezoidal rule in the first sub-step and the 3-point Euler backward method in second sub-step are employed.

In the first sub-step, it is assumed that displacement, velocity, and acceleration at the time t is known, and displacement, velocity, and acceleration at the time $\mathrm{t} + \gamma\Delta\mathrm{t}$ are wanted to be known. Scheme of first sub-step is like Figure 1.

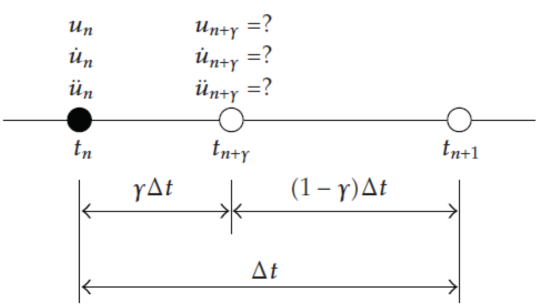


Figure 1:Scheme of First Sub-step

Consider $t+\gamma\Delta t$ is between time t and $t+\Delta t$, and for the first sub-step following velocity and displacement equations of trapezoidal rule are used:

$$^{t+\gamma\Delta t}\dot{U} = {}^{t}\dot{U} + \frac{^{t}\ddot{U} + {}^{t+\gamma\Delta t}\ddot{U}}{2}\gamma\Delta t$$

$$^{t+\gamma\Delta t}U = {}^{t}U + \frac{^{t}\dot{U} + {}^{t+\gamma\Delta t}\dot{U}}{2}\gamma\Delta t$$

Then, Newton-Raphson iteration is employed and following governing equation for $t+\gamma\Delta t$ is obtained:

$$\left({}^{t+\gamma\Delta t}K^{(i-1)} + \frac{2}{\gamma\Delta t}C + \frac{4}{\gamma^2\Delta t^2}M\right) * \Delta U^{(i)}$$
$$= {}^{t+\gamma\Delta t}R + M\left(\frac{4}{\gamma^2\Delta t^2}\,{}^{t}U + \frac{4}{\gamma\Delta t}\,{}^{t}\dot{U} + {}^{t}\ddot{U}\right) + C\left(\frac{2}{\gamma\Delta t}\,{}^{t}U + {}^{t}\dot{U}\right)$$

where $^{t+\gamma\Delta t}U^{(i)} = {}^{t+\gamma\Delta t}U^{(i-1)} + \Delta U^{(i)}$

In second sub-step, displacement, velocity, and acceleration at the times t and $t+\gamma\Delta t$ are known, and displacement, velocity, and acceleration at the time $t+\Delta t$ are intended. Scheme of second sub-step is like Figure 2.

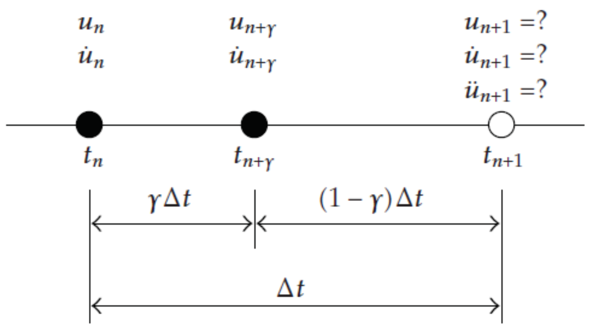


Figure 2:Scheme of Second Sub-step

For second sub-step the equations of the three-point Euler backward method:

$$^{t+\Delta t}\dot{U} = \frac{1-\gamma}{\gamma\Delta t}\,{}^{t}U + \frac{-1}{(1-\gamma)\gamma\Delta t}\,{}^{t+\gamma\Delta t}U + \frac{2-\gamma}{(1-\gamma)\Delta t}\,{}^{t+\Delta t}U$$

$$^{t+\Delta t}\ddot{U} = \frac{1-\gamma}{\gamma\Delta t}\,{}^{t}\dot{U} + \frac{-1}{(1-\gamma)\gamma\Delta t}\,{}^{t+\gamma\Delta t}\dot{U} + \frac{2-\gamma}{(1-\gamma)\Delta t}\,{}^{t+\Delta t}\dot{U}$$

Afterwards, Newton-Raphson is employed similarly as first sub-step and then the governing equation to find the solution at time $t + \Delta t$ is obtained as follows:

$$\left({}^{t+\Delta t}K^{(i-1)} + \frac{2-\gamma}{(1-\gamma)\Delta t}C + \left(\frac{2-\gamma}{(1-\gamma)\Delta t}\right)^2 M\right) * \Delta U^{(i)}$$
$$= {}^{t+\Delta t}R + M\left(\frac{1}{(1-\gamma)\gamma\Delta t}\frac{2-\gamma}{(1-\gamma)\Delta t}{}^{t+\gamma\Delta t}U - \frac{1-\gamma}{\gamma\Delta t}\frac{2-\gamma}{(1-\gamma)\Delta t}{}^{t}U + \frac{1}{(1-\gamma)\gamma\Delta t}{}^{t+\gamma\Delta t}\dot{U} - \frac{1-\gamma}{\gamma\Delta t}{}^{t}\dot{U}\right) + C\left(\frac{1}{(1-\gamma)\gamma\Delta t}{}^{t+\gamma\Delta t}U - \frac{1-\gamma}{\gamma\Delta t}{}^{t}U\right)$$

where ${}^{t+\Delta t}U^{(i)} = {}^{t+\Delta t}U^{(i-1)} + \Delta U^{(i)}$

3. Step-by-Step Solution Algorithm

A. Initial calculations:

1. Constructing stiffness matrix, K, mass matrix, M, and damping matrix, C
2. Entering initial displacement, ${}^{0}U$, velocity, ${}^{0}\dot{U}$ and acceleration, ${}^{0}\ddot{U}$.
3. Choosing time step Δt and parameter γ
4. Calculating integration constants:

$$a_0 = \frac{4}{\gamma^2\Delta t^2};\ a_1 = \frac{2}{\gamma\Delta t}; a_2 = \left(\frac{2-\gamma}{(1-\gamma)\Delta t}\right)^2; a_3 = \frac{2-\gamma}{(1-\gamma)\Delta t}; a_4 = \frac{4}{\gamma\Delta t};$$

$$a_5 = \frac{1}{(1-\gamma)\gamma\Delta t}\frac{2-\gamma}{(1-\gamma)\Delta t}; a_6 = -\frac{1-\gamma}{\gamma\Delta t}\frac{2-\gamma}{(1-\gamma)\Delta t}; a_7 = -\frac{1-\gamma}{\gamma\Delta t};$$

5. Forming effective stiffness matrices $\hat{K}_1$ and $\hat{K}_2$:

$$\hat{K}_1 = K + a_0 M + a_1 C;$$

$$\hat{K}_2 = K + a_2 M + a_3 C$$

B. Iterative time steps:

First sub-step

1. Calculating effective loads at time $t + \gamma\Delta t$:

$${}^{t+\gamma\Delta t}\hat{R} = {}^{t+\gamma\Delta t}R + M\left(a_0\,{}^{t}U + a_4\,{}^{t}\dot{U} + {}^{t}\ddot{U}\right) + C(a_1\,{}^{t}U + {}^{t}\dot{U})$$

2. Solve for displacements at time $t + \gamma\Delta t$:

$$\hat{K}_1\,{}^{t+\gamma\Delta t}U = {}^{t+\gamma\Delta t}\hat{R}$$

3. Calculating velocities and accelerations at time $t + \gamma\Delta t$:

$${}^{t+\gamma\Delta t}\dot{U} = a_1\left({}^{t+\gamma\Delta t}U - {}^{t}U\right) - {}^{t}\dot{U}$$

$${}^{t+\gamma\Delta t}\ddot{U} = a_1\left({}^{t+\gamma\Delta t}\dot{U} - {}^{t}\dot{U}\right) - {}^{t}\ddot{U}$$

Second sub-step

1. Calculating effective loads at time $t + \Delta t$:

$${}^{t+\Delta t}\hat{R} = {}^{t+\Delta t}R + M\left(a_5\,{}^{t+\gamma\Delta t}U + a_6\,{}^{t}U + a_1\,{}^{t+\gamma\Delta t}\dot{U} + a_7\,{}^{t}\dot{U}\right) + C\left(a_1\,{}^{t+\gamma\Delta t}U + a_7\,{}^{t}U\right)$$

2. Solve for displacements at time $t + \Delta t$:

$$\hat{K}_2\,{}^{t+\Delta t}U = {}^{t+\Delta t}\hat{R}$$

3. Calculating velocities and accelerations at time $t + \Delta t$:

$${}^{t+\Delta t}\dot{U} = -a_7\,{}^{t}U - a_1\,{}^{t+\gamma\Delta t}U + a_3\,{}^{t+\Delta t}U$$

$${}^{t+\Delta t}\ddot{U} = -a_7\,{}^{t}\dot{U} - a_1\,{}^{t+\gamma\Delta t}\dot{U} + a_3\,{}^{t+\Delta t}\dot{U}$$

4. Demonstrative Problem

System of single degree of freedom is solved. Stiffness constant, K, is 100 and mass, M, is 1. There is no damping and the force, R is constant 100. Initial displacement and velocity are zero. Displacement, velocity, and acceleration are obtained as following graph:

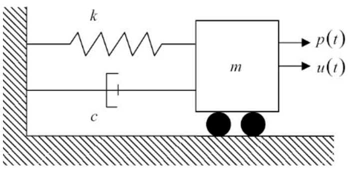
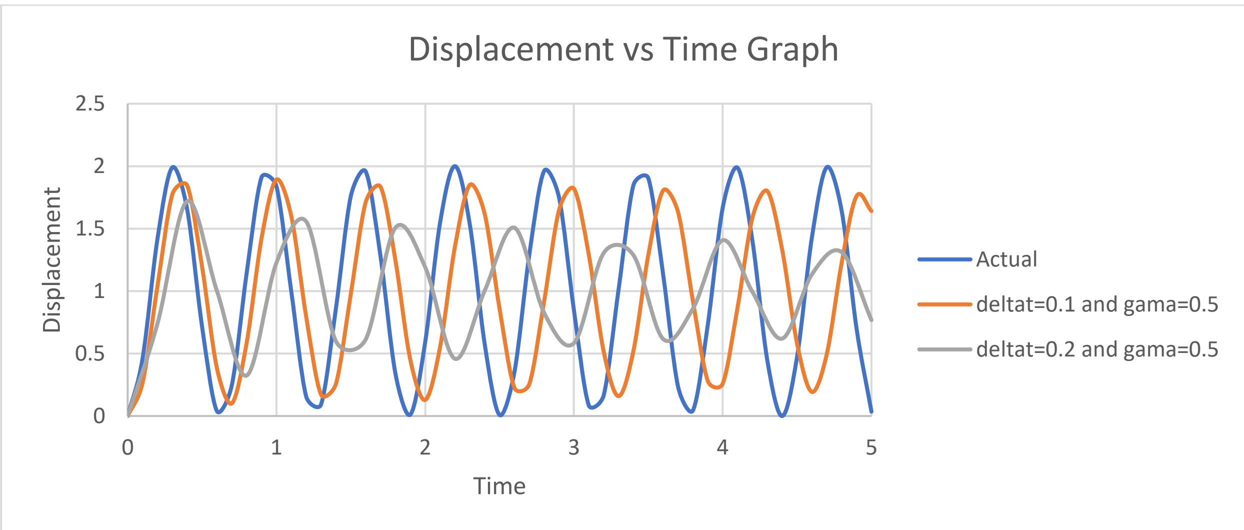
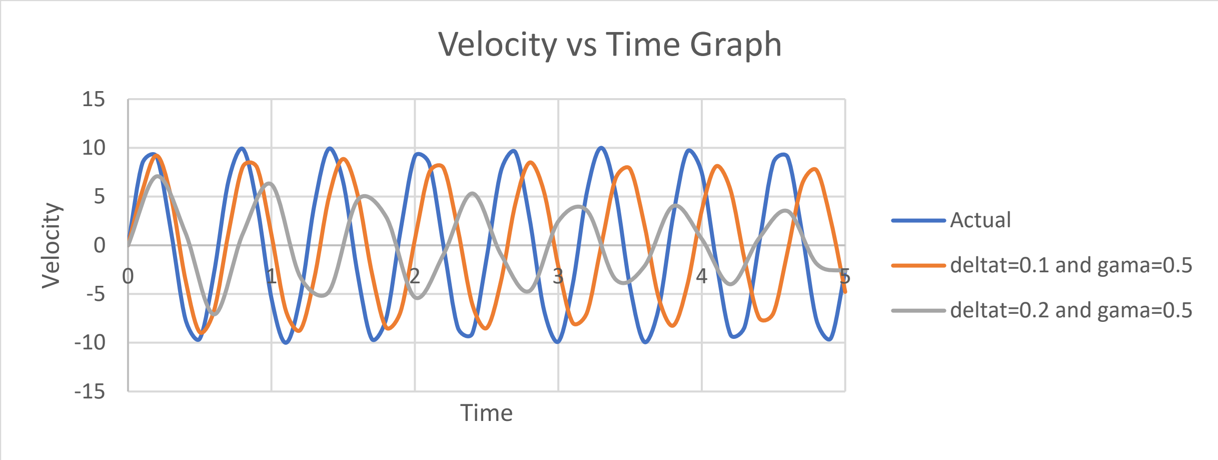
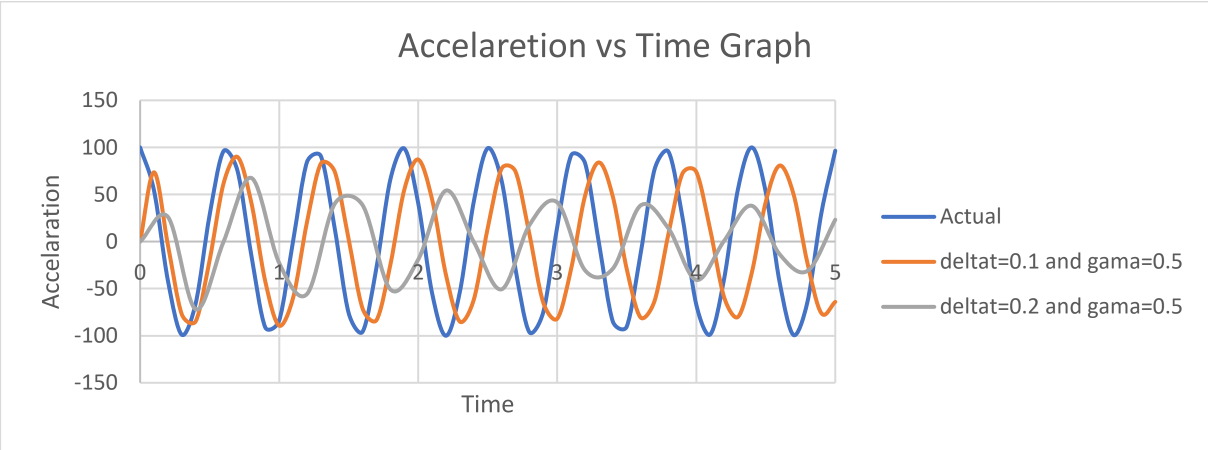


Figure 3:Single Degree of Freedom







5. Discussion

- After making some works on Bathe method. I observed that Bathe method works clearly when γ parameter is equal to 0.5 (or it is in middle length). However, when γ parameter is changed (different from 0.5), there is a divergence problem. So, it needs optimization for stability.
- Also, when Δt get smaller, the accuracy increases as usual.
- According to my research, Bathe method is used in nonlinear dynamics and provides energy conservation and momentum in nonlinear dynamics.

6. Conclusion

Bathe method can work better than other methods in some condition. However, it is not correct for all type of problems. Therefore, while studying time integrated dynamic analysis, to know all methods (Houbolt method, Newmark methods etc…) will help to solve dynamic analysis problems correctly.

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