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ASSIGNMENT 5

Overview. The goal of this assignment is to replicate and extend the static principal–agent model with moral hazard studied in Margiotta and Miller (2000), and in the accompanying lecture slides on optimal contracting under moral hazard.
You will: (i) derive the optimal wage contract using Kuhn–Tucker conditions, (ii) work through the identification logic, (iii) estimate a parametric version of the model, (iv) compute three measures of the cost of moral hazard, and (v) conduct counterfactual exercises.

Instructions. Work in groups of about three. Each group should submit a single, self-contained report and code (well documented) as an appendix. All questions carry equal weight unless otherwise stated.

Model A risk–neutral principal hires a risk–averse agent for one period. The agent chooses unobservable effort $\ell \in \{0, 1\}$:

$$\ell = 1 \text{ (working)}, \qquad \ell = 0 \text{ (shirking)}.$$

Output $x \in \mathbb{R}_+$ is observable and stochastic.

Preferences The agent has CARA preferences with risk aversion $\gamma > 0$:

$$U(w, \ell) = -\exp(-\gamma w) \times \begin{cases} \alpha & \text{if } \ell = 1 \text{ (working)}, \\ \beta & \text{if } \ell = 0 \text{ (shirking)}, \end{cases} \tag{1}$$

where $\alpha > \beta$ captures that shirking yields higher utility (less disutility) than working. The outside option (rejecting the contract) yields utility normalized to -1.

Technology Output is distributed as

$$x \sim \begin{cases} f(x) & \text{if } \ell = 1 \text{ (working)}, \\ f(x)g(x) & \text{if } \ell = 0 \text{ (shirking)}, \end{cases} \tag{2}$$

where f is a density on $\mathbb{R}_+$ and $g(x) \geq 0$ is a likelihood ratio satisfying

$$\int g(x)f(x)\,dx = 1, \qquad \mathbb{E}[xg(x)] < \mathbb{E}[x],$$

so that expected output is lower when the agent shirks.

Principal's problem The principal offers a wage contract $w(\cdot)$ contingent on x and chooses $w(\cdot)$ to maximize expected profit:

$$\max_{w(\cdot)} \mathbb{E}[x - w(x)] \tag{3}$$

subject to

(i) Individual rationality (IR) for working: the agent must prefer accepting and working to the outside option:

$$\alpha\, \mathbb{E}\big[\exp(-\gamma w(x))\big] \;\leq\; 1. \tag{4}$$

(ii) Incentive compatibility (IC): working must weakly dominate shirking:

$$\alpha\, \mathbb{E}\big[\exp(-\gamma w(x))\big] \;\leq\; \beta\, \mathbb{E}\big[\exp(-\gamma w(x)) g(x)\big]. \tag{5}$$

Expectations are taken with respect to $f(x)$.

Question 1. Let $v(x) = \exp(-\gamma w(x))$. Write the principal's problem in $v(\cdot)$ and derive the FOC using KKT multipliers for IR and IC. Show that the optimal $v(x)$ satisfies

$$v(x) = \frac{1}{\alpha(1 + \eta(\alpha/\beta - g(x)))}$$

and obtain the corresponding wage function $w^*(x)$.

Question 2. Explain why the parameters $(\alpha, \beta, g(\cdot))$ are not identified without knowing γ in the static nonparametric model. Assuming γ known, use the binding IR–IC conditions to show:

$$\alpha = (\mathbb{E}[v(x)])^{-1}, \qquad \beta = (\mathbb{E}[v(x)g(x)])^{-1},$$

and derive the expression for $g(x)$ from the optimality condition.

Question 3. Assume that under high effort ($\ell = 1$), output has a truncated normal distribution with lower truncation at ψ, mean μ_w and variance σ^2:

$$X \mid \ell = 1 \sim \mathrm{TN}(\mu_w, \sigma^2; \psi).$$

Under shirking ($\ell = 0$), output has the same variance and truncation point but a different mean μ_s:

$$X \mid \ell = 0 \sim \mathrm{TN}(\mu_s, \sigma^2; \psi).$$

Let $f_w(x)$ and $f_s(x)$ denote these two densities, and define

$$f(x) = f_w(x), \qquad g(x) = \frac{f_s(x)}{f_w(x)},$$

so that $f(x)g(x)$ is truncated normal with cutoff ψ, mean μ_s and variance σ^2.

Suppose observed wages satisfy

$$w_i = w^*(x_i; \theta, \eta) + \varepsilon_i, \qquad \varepsilon_i \sim N(0, \sigma_w^2),$$

where $w^*(x; \theta, \eta)$ is the optimal contract from Question 1 and η is chosen so that IR and IC bind. Let

$$\theta = (\mu_w, \sigma, \mu_s, \gamma, \alpha, \beta, \sigma_w^2).$$

Using the sample $\{(x_i, w_i)\}_{i=1}^N$:

(i) Write down the likelihood for θ under this parametric specification and explain how $\eta(\theta)$ is determined by the binding IR and IC conditions.

(ii) Estimate $\hat{\theta}$, report parameter estimates and standard errors, and verify that IR and IC approximately bind at $\hat{\theta}$ using sample analogs.

(iii) Repeat the exercise without parametric assumptions on f and g (e.g. estimate f and g nonparametrically from the data). Briefly comment on how this affects identification and the estimates of (γ, α, β).

Question 4. Using p.12 definitions in slides 3, compute the following with your estimated parameters:

(1) Risk–compensation component Δ_1.

(2) Disutility-of-effort component Δ_2.

(3) Output-loss component Δ_3.

Question 5. Holding all else fixed, recompute the optimal contract and summarize $(\Delta_1, \Delta_2, \Delta_3)$ under:

(1) Higher risk aversion: $\gamma \to 2\gamma$.

(2) Lower truncation Point: $\psi \to \psi/2$.

(3) Higher output volatility: $\sigma \to 2\sigma$.