Econometrics III 47-813
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ASSIGNMENT 5 (Kang and Miller, 2022)

Overview. This assignment concerns the structural estimation of a procurement model with adverse selection, risk-averse sellers, and endogenous competition, based on Kang and Miller (2022, *Review of Economic Studies*). You will: (i) trace the chain from the buyer's optimal menu design to the sequential estimation procedure, (ii) confront identification challenges arising from unobserved heterogeneity and the gap between nonparametric identification and parametric estimation, and (iii) explore how simplifying either the information structure or seller preferences changes the model, its key equations, and the estimation procedure. The final question asks you to design and implement a Monte Carlo experiment that evaluates the consequences of parametric restrictions on risk preferences.

Instructions. Work in groups of about three. Each group should submit a single, self-contained report together with documented code for Question 5. Hand written work will not be graded. Poor grammar, unclear expression, and lack of precision, will be graded as if I have very limited expertise in this area. All questions carry equal weight unless otherwise stated.

Model. A buyer (procurement agency) awards a project to one of $n+1$ sellers, where n additional sellers arrive via a Poisson search process with intensity λ. Each seller is privately either low-cost ($k = 1$) with probability π or high-cost ($k = 0$). The buyer designs a menu of contracts $\{p_{jn}, q_{jn}(s)\}$ consisting of a base price and a price adjustment that depends on contractible outcomes s. After the project is completed, outcomes s are realized and total payment $p_{jn} + q_{jn}(s)$ is made. We use the notation of the paper throughout.

Question 1. The paper solves three interrelated problems: menu design, competition, and search. This question asks you to study the key equations for each and trace how they connect.

(a) **Primitives** (equations 3.1–3.3, Assumption 3.1). Read the cost structure in (3.2) and the seller's payoff in (3.3).

   (i) Decompose the seller's total cost c_k and classify each component by who observes it and when. What feature of the model gives the buyer leverage to distinguish seller types through contract design?

   (ii) The function $\psi(\cdot)$ satisfies three conditions: $\psi(0) = 0$, $\psi'(0) = 1$, $\psi'' \leq 0$. Interpret each. Then consider a hypothetical contract that offers a seller random payment $\tilde{r}$ with $E[\tilde{r}] = 0$. What is the seller's valuation of this lottery, and what does this imply for contract design?

(iii) Assumption 3.1 imposes a joint ordering on initial costs and expected cost variation. Construct an example of a cost structure that satisfies $\gamma_1 < \gamma_0$ but violates the second inequality. What would go wrong with the screening argument in Theorem 3.1?

(b) **Optimal menu** (Theorem 3.1, equations 3.9–3.11). The buyer designs a menu of contracts, one per seller type.

(i) Examine equation (3.9). The price adjustment distortion $r(s)$ varies with the contractible outcome s. What property of s determines the sign and magnitude of $r(s)$? Trace the economic logic: how does the structure of $r(s)$ make the high-cost contract unattractive to a low-cost seller?

(ii) Read equations (3.10) and (3.11) together. Both are base prices, but they respond differently to the number of bidders n. Explain why, by identifying which equilibrium condition pins down each price. What happens to the gap between them as n grows?

(c) **Competition and search** (Corollary 3.2, equations 3.14–3.17). The buyer's expected payment $T(n)$ depends on the number of bidders.

(i) Corollary 3.2 decomposes $T(n)$ into two components. Interpret each and determine their relative magnitudes. How does the buyer's ability to screen through contract design interact with the value of attracting additional bidders?

(ii) The buyer solicits if $\eta \leq \Omega(\pi)$. The parameter η bundles several distinct economic forces. What are they, and why can't the model disentangle them? What does the empirical finding of small average η tell us — and what does it leave unresolved?

(iii) If π is close to 1, trace what happens to the information rent, the screening distortion, the value of competition, and the solicitation decision. Does the model predict that high-π projects are more or less likely to receive a single bid?

Question 2. This question asks you to work through the sequential identification strategy and its key challenges.

(a) **Risk preferences via the ODE** (Section 4.4.2, equations 4.1–4.2, Lemma 4.3). The first-order condition for the optimal price adjustment yields equation (4.1). Differentiating with respect to l produces the ODE (4.2).

(i) Examine the ODE (4.2) and its initial conditions. What variation in the observed data traces out the function ψ? For a fixed π, what moves along the ODE, and what determines how far along it the data can reach?

(ii) Compare the identification result in Section 4.4.2 with the estimation procedure in Section 5.2.3. What is gained and what is lost in the passage from one to the other? Under what circumstances would the two approaches yield materially different estimates?

(iii) Suppose you wanted to estimate ψ using the ODE directly. Trace the chain of objects that must be estimated beforehand, and assess the feasibility of each step given the sample described in Section 2.

(b) **Seller costs** (equations 4.5–4.6, Lemma 4.4). The initial costs $\gamma_0(\pi)$ and $\gamma_1(\pi)$ are not directly observed. They must be recovered from equilibrium prices.

(i) Read equations (4.5) and (4.6). What observable variation drives the identification of $\gamma_1(\pi)$, and why does the structure of (3.10) make this variation informative? Consider a dataset in which every contract has exactly one bidder. What goes wrong?

(ii) Equations (4.5) and (4.6) both involve ψ. What does this imply about the ordering of the estimation steps? Trace what would change if sellers were risk neutral.

(iii) In Assignment 4, $\alpha_{jkt}(h)$ is likewise a structural primitive recovered from equilibrium outcomes. What source of variation plays an analogous role there, and where does the analogy break down?

Question 3. Risk-neutral sellers. Suppose $\psi(r) = r$ for all r, so that sellers evaluate uncertain payments at their expected value. The following parts ask you to check, equation by equation, what changes and what does not.

(a) **Optimal contract.** Substitute $\psi(r) = r$ into the high-cost contract distortion (3.9). Show that the high-cost price adjustment $q_0(s) = c(s) + r(s)$ simplifies. Does the buyer still screen seller types, or does screening become unnecessary? Examine what happens to the low-cost contract.

(b) **Base prices.** Substitute $\psi(r) = r$ into the base price equations (3.10) and (3.11). How do p_n and $\bar{p}$ simplify? Show that the information rent in (3.10) now depends only on $\gamma_0 - \gamma_1$ and n, with no role for the contract outcome distributions f_0, f_1.

(c) **Expected payment and competition.** Compute Γ from Corollary 3.2 when $\psi(r) = r$. What is its sign? What does this imply about the marginal value of competition relative to the case with risk-averse sellers? Does the buyer search more or less intensively?

(d) **Equations that do not change.** Verify that the following are unaffected: the buyer's search decision structure (3.16)–(3.17), the Poisson arrival assumption, and the sequential structure of the game (menu design after search). Explain briefly why each is preserved.

(e) **Identification and estimation.** Consider the five-step sequential estimation procedure (Section 5).

(i) Step 1 estimates $f_0(s)$, $f_1(s)$, and $c(s)$. Is this step still needed? What feature of the data still requires $f_k(s)$?

(ii) Step 2 estimates ψ via the extremum estimator. Is this step needed when ψ is known? What happens to the ODE (4.2)?

(iii) Step 3 recovers the distribution of π. Does the method in Section 5.2.2 simplify? Can π be recovered more directly from base prices?

(iv) Steps 4 and 5 estimate costs and search costs. Do these steps simplify?

Question 4. Monte Carlo experiment. The paper identifies ψ nonparametrically via the ODE (4.2) but estimates it using a parametric CARA specification. This question asks you to design and implement a Monte Carlo experiment that evaluates the consequences of this gap.

(a) **Data generating process.** Construct a simplified DGP based on the model:

(i) Let s be scalar (a single cost-change outcome). Specify $f_0(s)$ and $f_1(s)$ as known distributions with $l(s) = f_1(s)/f_0(s)$ that spans a nontrivial range. Propose specific distributional choices and justify them.

(ii) Specify $\gamma_0(\pi)$, $\gamma_1(\pi)$, and $c(s)$. Choose parameter values that generate realistic cost magnitudes.

(iii) For the true ψ, consider two specifications:

* DGP A: CARA, $\psi_A(r) = [1 - \exp(-ar)]/a$ for some $a > 0$.
* DGP B: DARA (decreasing absolute risk aversion), for example $\psi_B(r) = \log(1 + ar)/a$.

For each, verify $\psi(0) = 0$, $\psi'(0) = 1$, $\psi'' < 0$. Plot both functions and discuss how they differ for large $|r|$.

(iv) For each draw, generate $\pi_i \sim \text{Beta}(\alpha, \beta)$, then generate (n_i, k_i, s_i) from the equilibrium: $n_i \sim \text{Poisson}(\lambda(\pi_i))$, k_i from the winning probability $(1 - \pi_i)^{n_i}$, and $s_i \sim f_{k_i}$. Compute the equilibrium prices p_i and $q_i(s_i)$ using the true model equations (3.9)–(3.11). Add measurement error if desired.

(b) **Estimation methods.** Implement two estimators for ψ:

(i) **Parametric (paper's method).** Assume $\psi(r) = [1 - \exp(-\hat{a}r)]/\hat{a}$ and estimate $\hat{a}$ by minimizing the sum of squared residuals between observed and model-predicted prices, as in Section 5.2.3.

(ii) **Sieve (ODE-based).** Approximate ψ using a B-spline or polynomial sieve $\psi(r; \alpha) = \sum_{j=1}^{J} \alpha_j B_j(r)$ subject to $\psi(0) = 0$, $\psi'(0) = 1$, $\psi'' \leq 0$. Use the ODE (4.2) to construct moment conditions:

$$\psi''(r) = g(l, r, \psi'(r))$$

where g is determined by the first-order condition (4.1). Estimate α by minimizing the integrated ODE residual over the observed support of (l, r) pairs from high-cost contracts.

(c) **Experiment 1: Baseline recovery.** Generate $R = 500$ datasets of size $N = 7{,}000$ (matching the paper) from DGP A (CARA is true). For each replication, estimate ψ using both methods. Report:

  (i) Bias and RMSE of the CARA parameter $\hat{a}$ under the parametric method.

  (ii) Integrated squared error $\int[\hat{\psi}(r) - \psi_0(r)]^2\,dr$ for both methods.

  (iii) Is the parametric method more efficient when correctly specified? By how much?

(d) **Experiment 2: Misspecification.** Repeat (c) using DGP B (DARA is true, CARA is imposed).

  (i) Document the bias in $\hat{\psi}$ under the parametric method. Where in the domain of r is the bias largest?

  (ii) Compute the downstream bias in γ_0, γ_1, Δ, and the counterfactual "switch to auction" prediction. Present these as a fraction of the true values.

  (iii) Does the sieve method eliminate the bias? At what cost in terms of variance?

(e) **Experiment 3: Finite-sample sensitivity.** Fix DGP B and vary the number of high-cost contracts $N_0 \in \{50, 100, 264, 500, 1000\}$ (recall that only 4% of the paper's sample are high-cost).

  (i) How does the RMSE of the sieve estimator change with N_0? At what sample size does it begin to outperform the (misspecified) parametric estimator in terms of MSE?

  (ii) How should the number of sieve basis functions J be chosen as a function of N_0? Implement a simple cross-validation or information criterion procedure and report the selected J for each N_0.

(f) **Discussion.**

  (i) Based on your results, evaluate the paper's decision to use a parametric ψ. Under what conditions is this choice defensible?

  (ii) Propose a specification test that could be applied to the real data to detect whether the CARA assumption is violated. How would you construct the test statistic from the ODE residuals?

  (iii) What additional data or institutional features would most improve the non-parametric estimation of ψ?

Deliverables. Submit a report that:

(i) Answers each question with appropriate derivations and economic reasoning.

(ii) References specific equations, theorems, tables, and figures from the paper.

(iii) Draws connections to the methods from Assignments 1–4 where relevant.

(iv) For Question 5: includes documented Python code, figures showing $\hat{\psi}$ recovery, and tables summarizing Monte Carlo results.

(v) Discusses limitations and possible extensions.

References.

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Gayle, G.-L., Golan, L., and Miller, R.A. (2015). Promotion, Turnover, and Compensation in the Executive Labor Market. *Econometrica*, 83(6):2293–2369.

Rothschild, M. and Stiglitz, J. (1976). Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. *The Quarterly Journal of Economics*, 90(4):629–649.

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