

AACHENER VERFAHRENSTECHNIK

Mass Transfer in and at Membranes

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Outline

- Mass Transfer in Membranes
 - The Pore Model
 - The Solution-Diffusion Model
 - Pore Models for Gas Transport
- Mass Transfer at Membranes
 - Physics of Mass Transfer at Membranes
 - Effect of Concentration Polarization on Membrane Performance
 - Countermeasures

Objective of Mass Transfer Modeling

Operating parameters

$$(\Delta p, c_F, T, v)$$

Membrane properties

e.g. solubility, diffusivity, membrane thickness and structure, etc.

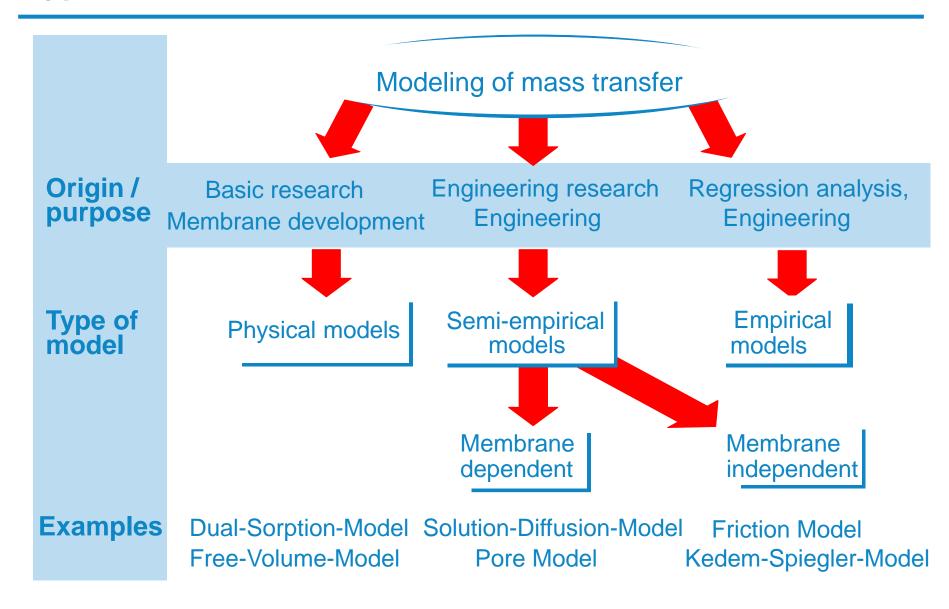
Properties of the fluid system

e.g. dynamic viscosity, condensation temperatures of components

Permeate flux and

Rejection coefficient R

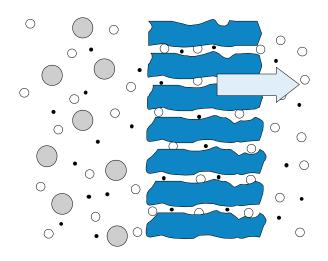
Types of Models



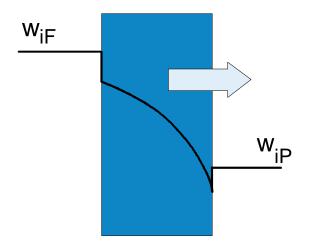
Models of Membranes (Semi-empirical)

Pore Membrane

Solution-Diffusion-Membrane



Ultrafiltration Microfiltration Dialysis

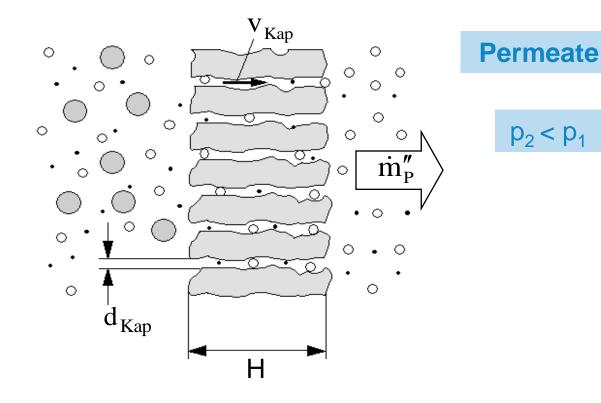


Reverse Osmosis Pervaporation Gas permeation

Model of a Pore Membrane

Feed

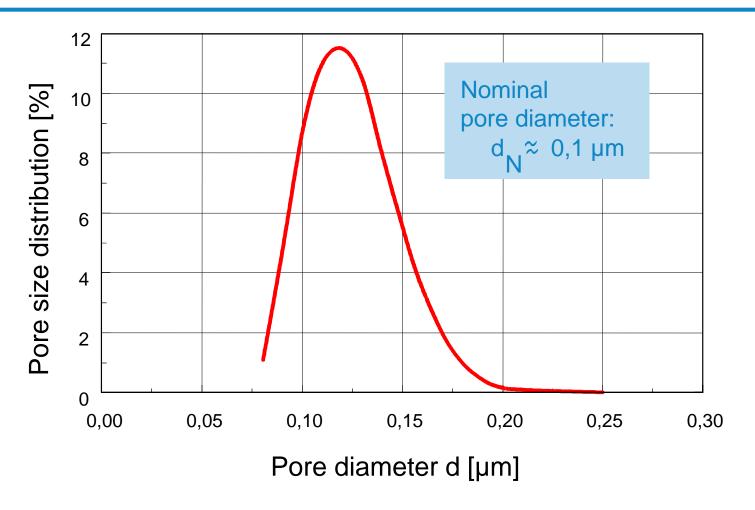
 p_1



$$V_{P} = \frac{\dot{m}_{P}''}{\rho_{P}} = A \cdot \Delta p$$

- Linear increase of permeate flux with increasing pressure
- Permeability constant A depending on respective membrane

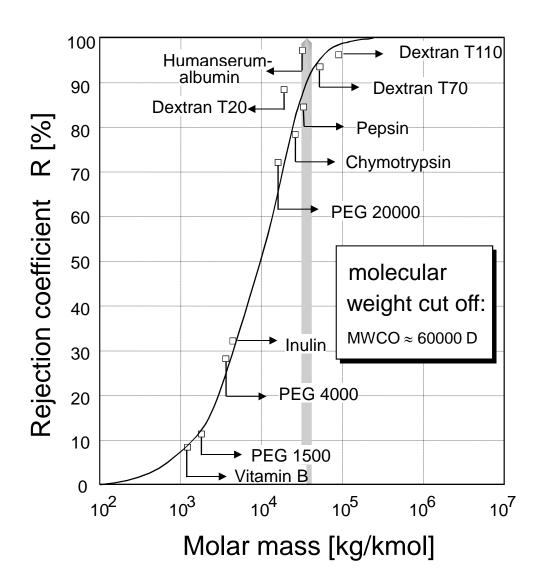
Pore Size Distribution (MF-Membrane)



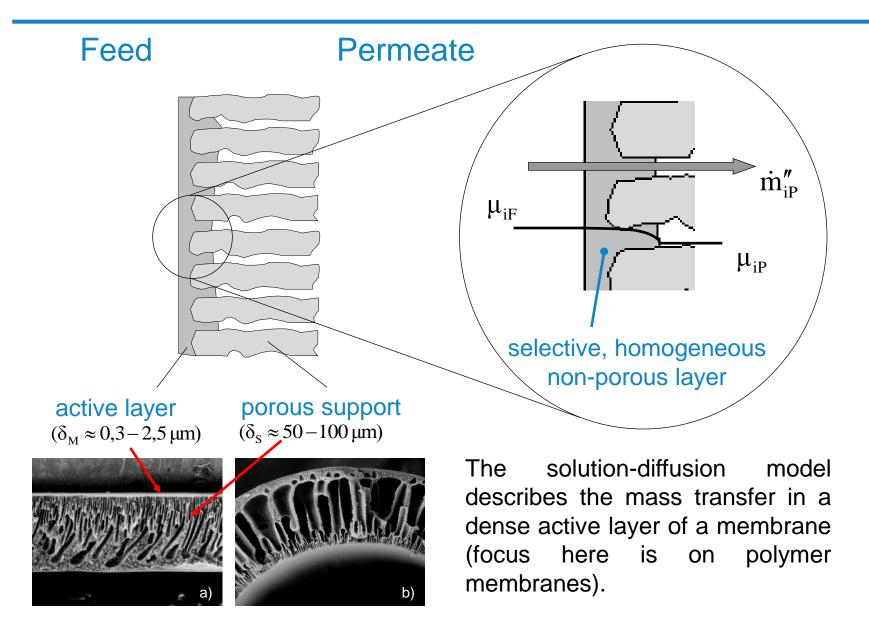
Selectivity of the membrane Separation curve

(experimentally determinable)

Separation Curve (Pore membrane)



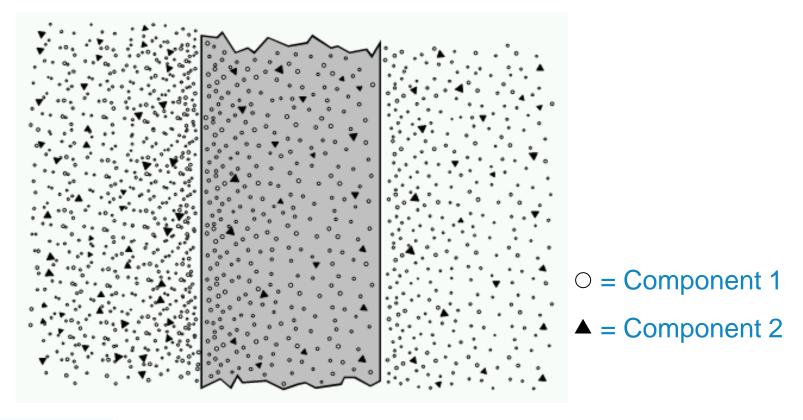
Model of a Solution-Diffusion Membrane



Model of a Solution-Diffusion Membrane

- The solution diffusion model is based on the following assumptions:
 - the membrane is a continuum,
 - equilibrium of the chemical potentials at the membrane surfaces on the feed/retentate and permeate side,
 - no interaction between the different substances permeating through the membrane
- The polymer membrane is considered as a liquid which absorbs the permeating substances.
- The substances diffuse through the membrane according to the driving force.

General Equation of the Solution-Diffusion Model



$$\dot{n}_{k}'' = c_{kM} \cdot V$$
 Flux = concentration * velocity

$$\dot{n}_{k}'' = -c_{kM} \cdot b_{kM} \cdot \frac{\partial \mu_{kM}}{\partial z}$$
 Flux = concentration * mobility * driving force

Diffusion Coefficient – Simplification

Flux = concentration * mobility * driving force

$$\dot{n}_{k}'' = -c_{kM} \cdot b_{kM} \cdot \frac{\partial \mu_{kM}}{\partial z}$$

$$\dot{n}_{k}'' = -c_{kM} \cdot \frac{D_{kM,0}}{RT} \cdot \frac{\partial \mu_{kM}}{\partial z}$$

$$D_{k0} = \Re \cdot T \cdot b_k \quad \begin{array}{ll} \text{Nernst-Einstein} \\ \text{equation} \end{array}$$

Thermodynamic diffusion coefficient

Alternatively: Fick's first law

$$\dot{n}_{k}'' = -c_{ges} \cdot O_{kM} \cdot \frac{\partial X_{kM}}{\partial Z}$$

Counter diffusion coefficient for binary mixtures

 $k \leftrightarrow \text{component}$

 $M \leftrightarrow \text{membrane}$

Simple Solution-Diffusion Model

Application: Reverse Osmosis of Seawater

$$\dot{m}_{w}^{"} = A^{*} \cdot (\Delta p - \Delta \pi_{w})$$
 Water flux

$$\dot{m}_{S}'' = B^* \cdot (w_{S,F} - w_{S,P})$$
 Salt flux

$$R = 1 - \frac{W_{S,P}}{W_{S,F}}$$
 Rejection coefficient

$$\Delta \pi_w = b_w \cdot R \cdot w_{sF}$$
 Osmotic pressure difference

- → Water flux dependent on feed pressue
- → Salt flux dependent on feed salt concentration

Simple Solution-Diffusion Model – Rejection

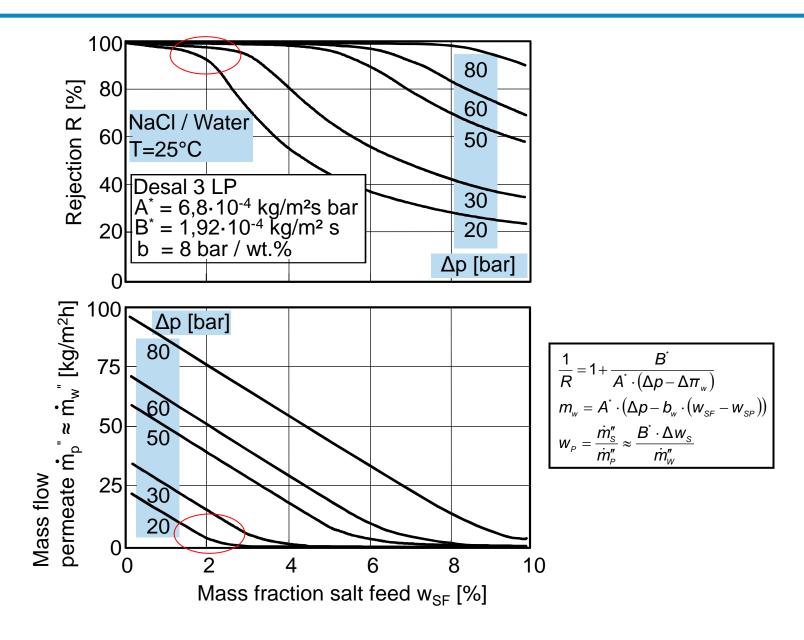
$$W_{S,P} = \frac{\dot{m}_{S}''}{\dot{m}_{S}'' + \dot{m}_{W}''} \approx \frac{\dot{m}_{S}''}{\dot{m}_{W}''} \implies \frac{W_{S,P}}{W_{S,F}} = \frac{B^{*} \cdot (W_{S,F} - W_{S,P})}{\dot{m}_{W}'' \cdot W_{S,F}} \implies \frac{W_{S,P}}{W_{S,F}} = \frac{B^{*} / \dot{m}_{W}''}{1 + B^{*} / \dot{m}_{W}''}$$

$$R = 1 - \frac{W_{S,P}}{W_{S,F}} \qquad R = \left[1 + \frac{B^*}{A^* \cdot (\Delta p - \Delta \pi_w)}\right]^{-1}$$

$$\Delta \pi_{w} = b_{w} \cdot R \cdot w_{SF} \qquad R = \left[1 + \frac{B^{*}}{A^{*} \cdot (\Delta p - b_{w} \cdot R \cdot w_{SF})}\right]^{-1}$$

 b_w = salt dependent constant, e.g. NaCl: b_w = 8 bar / wt.%

Simple Solution-Diffusion Model – Rejection and Flux



Reverse Osmosis of Diluted Salt Solutions

Summary

$$\dot{m}_{\scriptscriptstyle W}'' = A^* \cdot (\Delta p - \Delta \pi_{\scriptscriptstyle W})$$

$$\dot{m}_{S}'' = B * \cdot (w_{S,F} - w_{S,P})$$

water flux
salt flux

Simple solutiondiffusion model

Useful for diluted salt solutions; deficits for the characterisation of organic – aqueous systems.



Extended solution-diffusion model

Extended Solution-Diffusion Model

Application: Reverse Osmosis of solvent mixtures

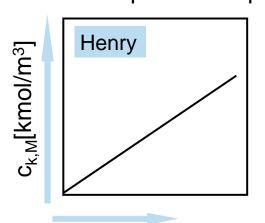
$$\dot{m}_{w}^{\prime\prime} = A^* \cdot c_{w,M} \cdot (\Delta p - \Delta \pi_{w})$$
 Water flux

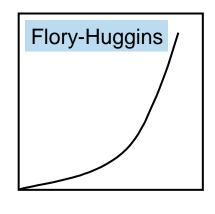
$$\dot{m}_i^{\prime\prime} = B^* \cdot c_{i,M} \cdot (\Delta p - \Delta \pi_i)$$
 Flux of solvent i, e.g. organic solvent

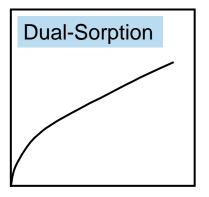
- → Permeate flux / separation performance dependent on permability A* / B* and on concentration c of the diverse solvents in the membrane.
- → Competitive sorption of the solvents into the free space of the membane material.

Sorption models

Sorption models are used to describe equilibrium concentrations of a component k in the membrane as a function of the state variables pressure, temperature and phase composition.







Activity a_k [-] or partial pressure p_k [bar]

Henry:

$$c_{k,M} = S_k \cdot a_k$$

$$oldsymbol{c}_{k,M} = oldsymbol{S}_k \cdot oldsymbol{a}_k$$
 $oldsymbol{c}_{k,M} = oldsymbol{S}_k \cdot oldsymbol{p}_k$

Only for slight concentrations $c_{k,M}$

Dual-Sorption:

$$\boldsymbol{c}_{k} = \boldsymbol{c}_{k}^{H} + \boldsymbol{c}_{k}^{L} = \boldsymbol{S}_{k} \cdot \boldsymbol{p}_{k} + \boldsymbol{c}_{k}^{0L} \cdot \frac{\boldsymbol{b}_{k} \boldsymbol{p}_{k}}{1 + \boldsymbol{b}_{k} \boldsymbol{p}_{k}}$$

Henry-Sorption

Langmuir correlation

Sorption models – Example

Dual-Sorption-Model:

$$\boldsymbol{c}_{k} = \boldsymbol{c}_{k}^{H} + \boldsymbol{c}_{k}^{L} = \boldsymbol{S}_{k} \cdot \boldsymbol{p}_{k} + \boldsymbol{c}_{k}^{0L} \cdot \frac{\boldsymbol{b}_{k} \boldsymbol{p}_{k}}{1 + \boldsymbol{b}_{k} \boldsymbol{p}_{k}}$$

neglecting Henry-sorption:

$$\frac{c_{iM}}{c_{ges,M}} = \frac{k_1 \cdot w_{iF}}{1 + k_1 \cdot w_{iF}}$$

with:
$$c_{ges,M} = c_{iM} + c_{wM} = const.$$

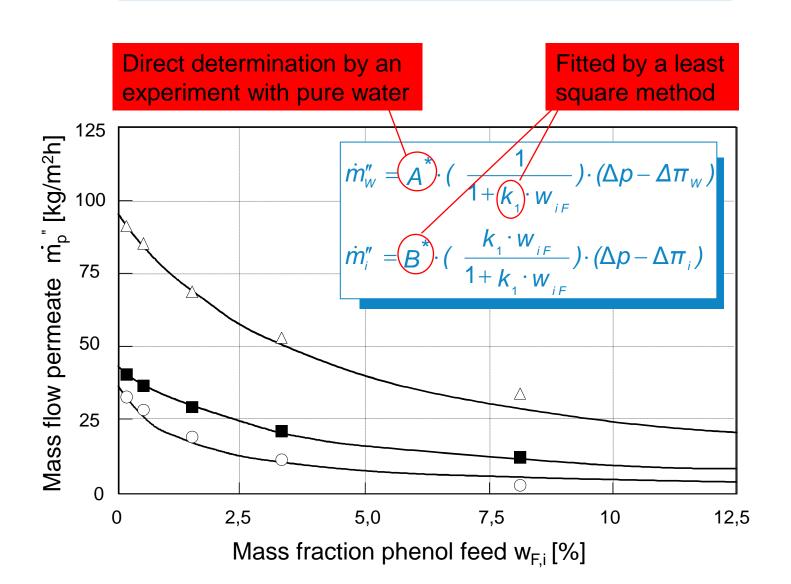


Result:
$$\dot{m}_{W}'' = A^* \cdot (\frac{1}{1 + k_1 \cdot w_{iF}}) \cdot (\Delta p - \Delta \pi_{W})$$

$$\dot{m}_{i}'' = B^{*} \cdot \left(\frac{k_{1} \cdot W_{iF}}{1 + k_{1} \cdot W_{iF}} \right) \cdot \left(\Delta p - \Delta \pi_{i} \right)$$

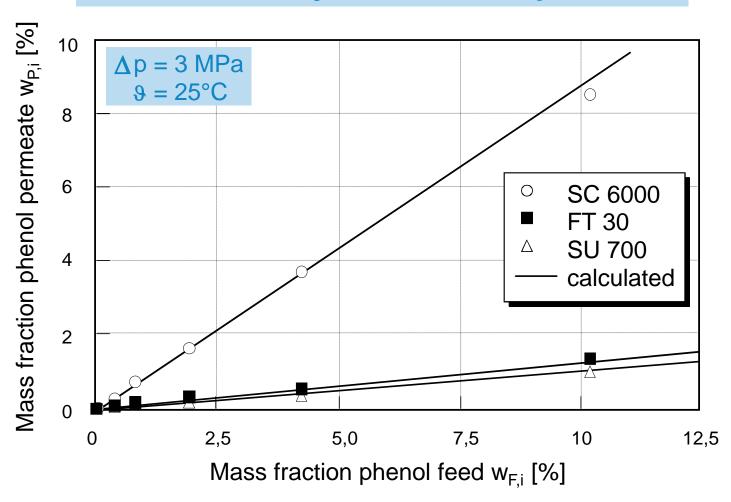
Extended Solution-Diffusion Model – Example

Substance system: water / phenol



Extended Solution-Diffusion Model – Example

Substance system: water / phenol



Solution-Diffusion Model for Permanent Gases

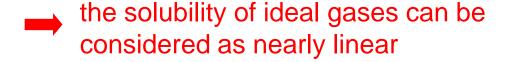
In case of ideal gas behaviour (permanent gases) the flux of a species k is proportional to the partial pressure difference between feed and permeate:

$$n_k = Q_k \cdot (x_k \cdot p_f - y_k \cdot p_p)$$

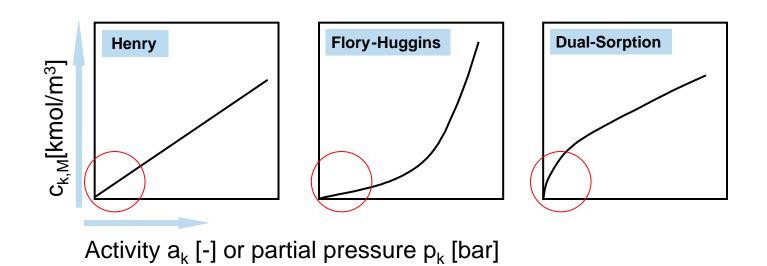
- measurements show no relevant pressure dependence on the permeability for ideal gases
- for highly non-ideal gases or vapours pressure dependence has to be taken into account
- the solubility coefficient Q_k follows a simple Henry approach for ideal gases

Permeation of Permanent Gases

Gas	Solubility Coefficent [cm³ (STP) / cm³ bar]		
H_2	0,0375		
N_2	0,075		
O_2	0,1125		
CO ₂	0,9		
C_2H_6	2,3		







Summary

Solution-Diffusion Model

Reverse Osmosis (simple SDM)

$$\dot{m}_{\scriptscriptstyle W}'' = A^* \cdot (\Delta p - \Delta \pi_{\scriptscriptstyle W})$$
 water $\dot{m}_{\scriptscriptstyle S}'' = B^* \cdot (w_{\scriptscriptstyle S,F} - w_{\scriptscriptstyle S,P})$ salt

Reverse Osmosis (Extended SDM)

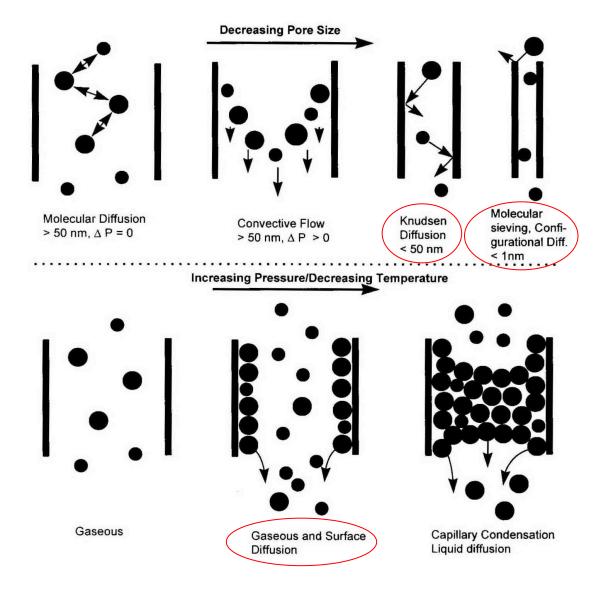
$$\dot{m}_{w}'' = A^{*} \cdot \left(\frac{1}{1 + k_{1} \cdot w_{i,F}}\right) \cdot \left(\Delta p - \Delta \pi_{w}\right) \quad \text{water}$$

$$\dot{m}_{i}'' = B^{*} \cdot \left(\frac{k_{1} \cdot w_{i,F}}{1 + k_{1} \cdot w_{i,F}}\right) \cdot \left(\Delta p - \Delta \pi_{i}\right) \quad \text{organic}$$

Gas permeation

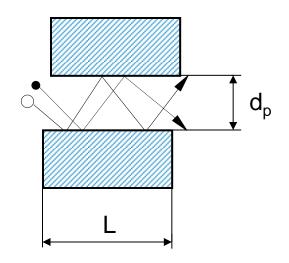
$$\dot{n}_{k}'' = Q_{k} \cdot (x_{k} p_{F} - y_{k} p_{P})$$

Transport Regimes in Porous Media



Knudsen Diffusion

Gas transport in capillaries: Knudsen diffusion



Gas - wall collisions must dominate

Driving force: p_k

$$Kn = \frac{Free path \lambda}{Pore diameter} >> C$$

$$\lambda >> d_{P}$$

$$d_{gas} = \frac{10}{32} \cdot \left(\frac{M \cdot R \cdot T}{\pi \cdot N_A^2 \cdot \eta^2} \right)^{\frac{1}{4}}$$
 (e.g.: H₂ = 2,7 Å)

$$\lambda = \frac{R \cdot T}{\sqrt{2} \cdot \pi \cdot d_{gas}^2 \cdot P \cdot N_A}$$
 (e.g.: H₂ = 1195 Å)

Knudsen Diffusion

Transport in capillaries: Knudsen diffusion

$$V \sim M^{-1/2}, D_{Kn} \sim T^{1/2}, \rho \sim T^{-1}$$

based on the kinetic gas theory [2]

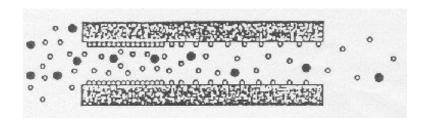
$$\dot{n}_{k}'' = \frac{4 \cdot d_{P}}{3 \cdot \sqrt{2 \cdot \pi \cdot R \cdot T \cdot M_{k}}} \cdot \frac{\Delta p_{k}}{L} \sim \frac{\Delta p_{k}}{L \cdot \sqrt{M_{k} \cdot T}}$$

$$\alpha_{ij} = \lim_{\mathcal{D}_{D} \to 0} S_{ij} = \left(\frac{M_{i}}{M_{i}}\right)^{\frac{1}{2}}$$
 e.g.: N₂ / O₂ = 1,07

[2] Atkins P.W., Physical Chemistry

Surface Diffusion

Transport in adsorbing capillaries: Surface diffusion



> flux due to surface diffusion can be added to the diffusive and convective flux in the pore

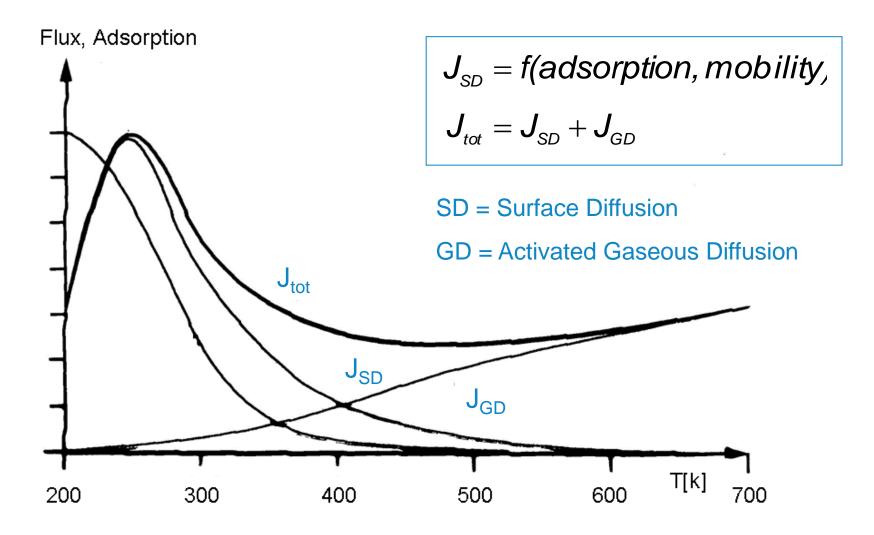
$$\dot{n}_{SD}'' = S_{(V)} \cdot D_{SD} \cdot \frac{\partial C_{S}}{\partial Z}$$

 C_s = surface concentration [mol/m²]

 $S_{(V)} = spec. surface [m^2/m^3]$

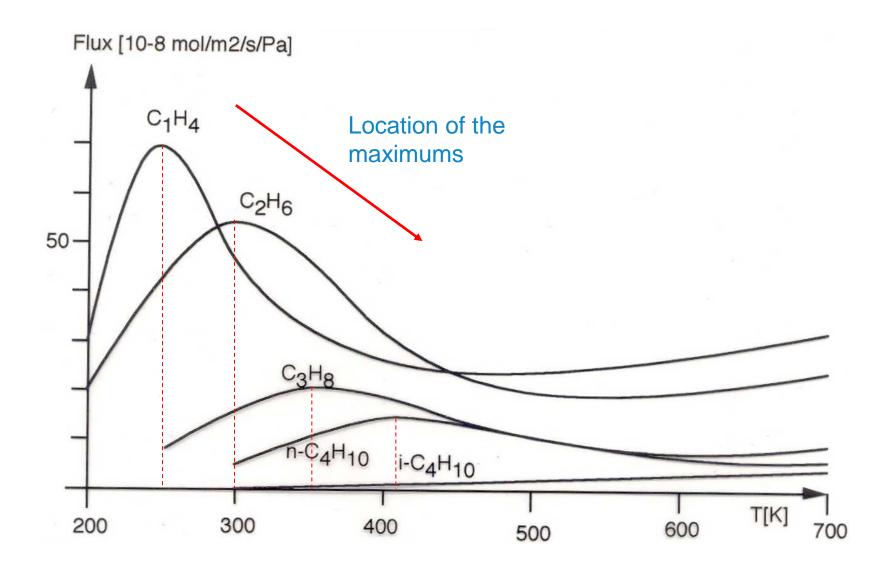
 D_{SD} = surface diffusion coefficient [m²/s]

Surface Diffusion + Activated Gaseous Diffusion



Diffusion and adsorption are strongly temperature dependent.

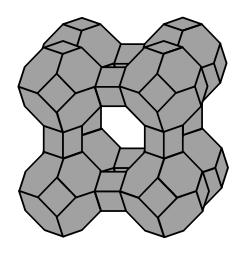
Surface Diffusion + Activated Gaseous <u>Diffusion</u>



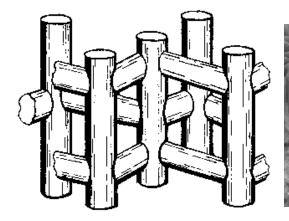
Molecular Sieves (Zeolites)

Zeolites:

- Strictly regular porous crystal structures
- Inorganic, polar membranes



A-Typ Zeolite



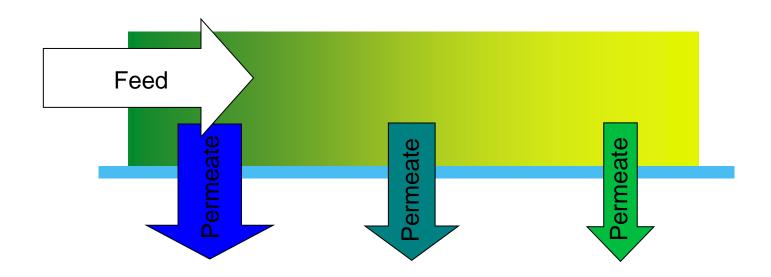
ZSM-5, e.g. Silicalite-Zeolite

Тур	Na-A Typ	ZSM-5	
SiO ₂ /Al ₂ O ₃	2	30 bis ∞	
d _p [Å]	(4,1)	(5,5)	

Mass Transfer at Membranes

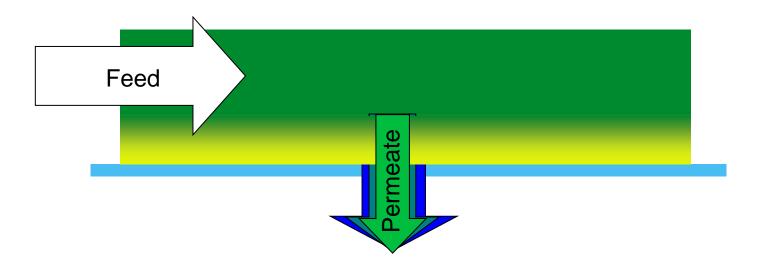
General Idea: Concentration Along The Module

- Two component mixture:
 - Blue: Low retention, Yellow: High retention
- Along the main direction of flow in the feed:
 - Different feed composition,
 - Different permeate quality (Driving force changes),
 - Different amount of permeate per unit area.



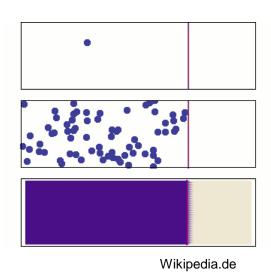
General Idea: Concentration Polarisation

- Two component mixture:
 Blue: Low retention, Yellow: High retention
- Flux over the membrane relatively small,
- Backdiffusion of yellow component very slow:
 - Feed composition almost constant in direction of flow,
 - Yellow component is enriched at the membrane
 ⇒ Concentration Polarisation



Diffusion

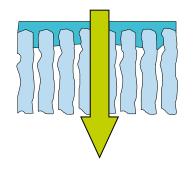
- Diffusion: net transport of molecules from region of higher concentration to one of lower concentration by random molecular motion.
- Diffusion increases entropy of system
 ⇒ Spontaneous and irreversible process.

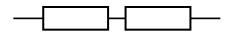


- Diffusion depends on mobility:
 - Diffusion in gases is fast: D_{ii} ≈ 10⁻⁵ m²/s
 - Diffusion in liquids is slow: D_{ii} ≈ 10⁻⁹ m²/s
 - Diffusion in solids is very slow and only possible if vacancy defects exist (and the temperature is high): D_{ij} < 10⁻¹² m²/s

Membrane Resistances

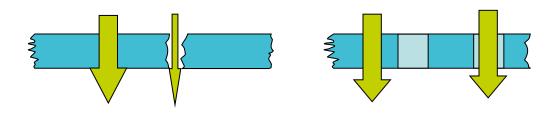
Serial Resistance

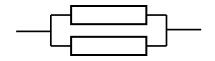


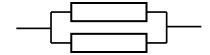


$$R_{tot} = R_{AL} + R_{Supp}$$

Parallel Resistance







$$\frac{1}{R_{tot}} = \frac{1}{R_{AL}} + \frac{1}{R_{pin\,hole}}$$

Collection: Effects Reducing the Driving Force

Needed:

Resistance of the active layer.

Possible other influences:

- Concentration polarisation in the feed.
- Pressure loss in porous support layer.
- Concentration polarisation in porous support and in the permeate.
- Fouling, Scaling, Pore blocking.
- Temperature polarisation.

Different Control Mechanisms for Mass Transfer

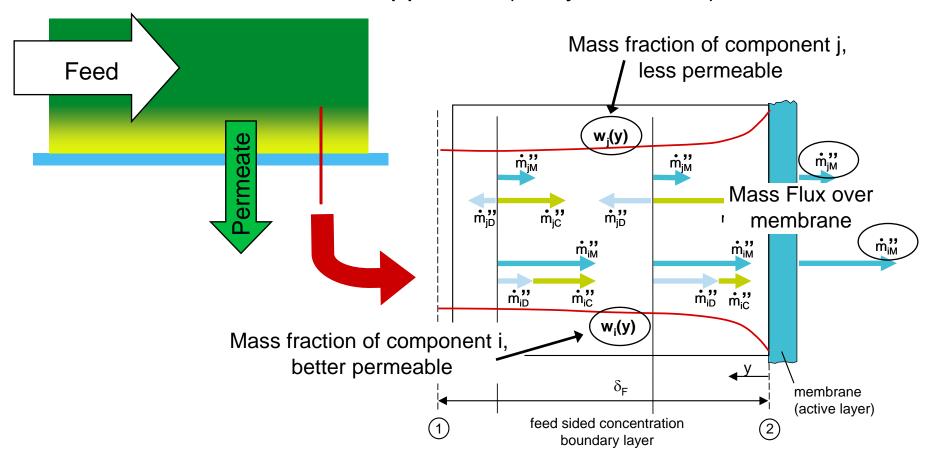
- When to take which factors into account?
- Membrane controlled mass transfer, if
 - Solution-Diffusion-Membrane with relatively small flux,
 - Small concentration polarisation.
- Surface layer controlled mass transfer, if
 - High transmembrane flux,
 - High concentration polarisation,
 - Low diffusivity of less permeable component j.

Concentration Polarisation

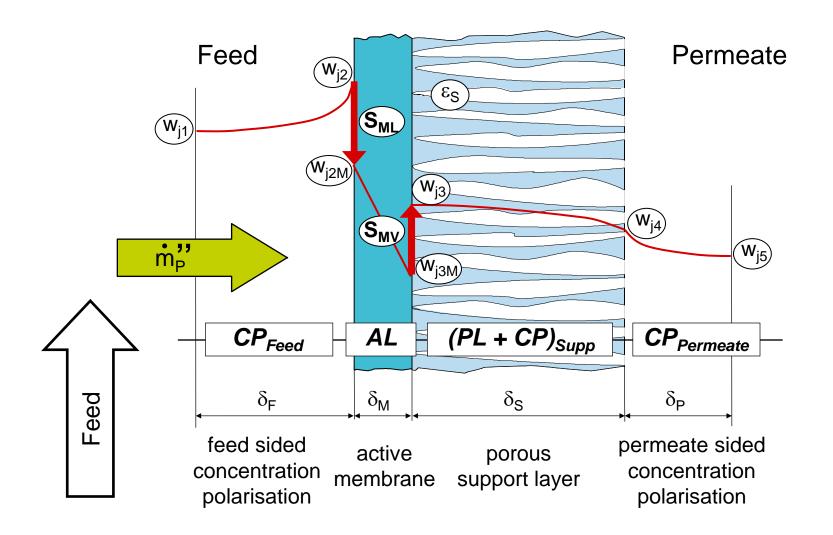
WANTED:

Model to calculate mass transfer effects.

⇒ One dimensional approach (Easy to handle)!



Serial Resistance and Feed Sided Concentration Polarisation



Feed-Sided Concentration Polarisation

- Assumptions:
 - Steady state condition.
 - Concentration gradient only orthogonal to membrane.
 - Fickian diffusion only.
 - No chemical conversion or precipitation.

$$0 = \underbrace{\frac{\partial}{\partial y} w_j(y) \, \dot{m}_{tot}''}_{\text{Convection}} - \underbrace{\frac{\partial}{\partial y} \rho_F \, D_{ij} \, \frac{\partial \, w_j(y)}{\partial \, y}}_{\text{Fickian Diffusion}}.$$

Constant mass flux:

$$\dot{m}_{tot}^{"} = -\dot{m}_{P}^{"} = -(\dot{m}_{iP}^{"} + \dot{m}_{jP}^{"})$$

 Constant mass fraction of component j in locally produced permeate:

$$w_j^* = \frac{\dot{m}_{jP}^{"}}{\dot{m}_{jP}^{"} + \dot{m}_{iP}^{"}} = const. \implies \frac{\partial w_j^*}{\partial y} = 0.$$

Mass Balance for Less Permeable Component *j*

Mass conservation leads to:

$$\dot{m}_P'' w_j(y) + \rho_F D_{ij} \frac{\partial w_j(y)}{\partial y} = \dot{m}_P'' w_j^*. \tag{1}$$

Boundary conditions:

$$w_j(y) = w_{j2},$$
 for $y = 0,$ $y = 0,$ $y = \delta_F.$

Substitution:

$$ilde{w}_j(y) = w_j(y) - w_j^*, \quad ext{and} \quad rac{\partial \, ilde{w}_j(y)}{\partial \, y} = rac{\partial \, w_j(y)}{\partial \, y}.$$

By inserting this in (1) one gets the following differential equation:

$$-\frac{\dot{m}_P''}{\rho_F D_{ij}} \, \tilde{w}_j = \frac{\partial \, \tilde{w}_j}{\partial \, y}.$$

Integration of the Mass Balance

Solve the differential equation via separation of variables:

$$-\frac{\dot{m}_P''}{\rho_F D_{ij}} \partial y = \frac{\partial \tilde{w}_j}{\tilde{w}_j}.$$

Integration leads to:

$$-\frac{\dot{m}_{P}''}{\rho_{F} D_{ij}} \int_{0}^{\delta_{F}} \partial y = \int_{\tilde{w}_{j}(y=0)}^{\tilde{w}_{j}(y=\delta_{F})} \frac{\partial \tilde{w}_{j}}{\tilde{w}_{j}} = \int_{w_{j2}-w_{j}^{*}}^{w_{j1}-w_{j}^{*}} \frac{\partial \tilde{w}_{j}}{\tilde{w}_{j}} = \ln(\tilde{w}_{j}) \Big|_{w_{j2}-w_{j}^{*}}^{w_{j1}-w_{j}^{*}}.$$

One gets:

Using the exponential function gives the final result:

$$e^{-\frac{\dot{m}_P''}{\rho_F k_F}} = \frac{w_{j1} - w_j^*}{w_{j2} - w_j^*}.$$

Dimensionless Groups

- Sherwood number: $Sh = rac{k \, d_{hyd}}{D_{ij}}$.
 - Characterises mass transfer (from fluid flow through specified boundary layer.)
 - Function of fluid and geometry properties. $\rho : \text{density,} \\ v : \text{flux velocity,}$
- Reynolds number: $Re=rac{
 ho\,v\,d_{hyd}}{\eta}$. $d_{hyd}=rac{4\,A}{U} \qquad \text{hydraulic diameter,} \\ =d \qquad \qquad \text{diameter of tube,} \\ =2\,h \qquad \text{channel with } h\ll b$
 - Characterises hydrodynamics.
 - "inertia / viscosity"
 - Function of fluid properties, flow conditions and geometry.
 - Key parameter to determine whether flow is laminar or turbulent.
 - $Re \approx 2000$ is typical limit for laminar flow

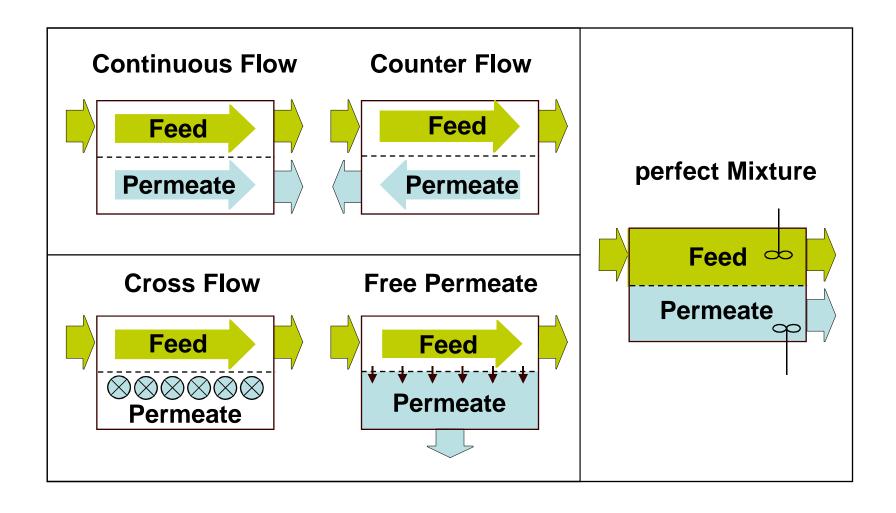
effective mass transfer coeff.

Dimensionless Groups

- Schmidt number: $Sc = \frac{\eta}{\rho D_{ij}}$.
 - Important interpretation is the relative thickness of velocity and concentration boundary layer.
 - Sc = 1: Boundary layers of equal thickness,
 - Sc > 1: Velocity boundary layer thinner (Momentum transfer more rapid than mass transfer)
- Peclet number: $Pe = \frac{\dot{m}_P'' d_{hyd}}{\rho D_{ij}}$.
 - Characterises the nature of mass transport.
 - "convection / diffusion"
- Example for the use of dimensionless groups:

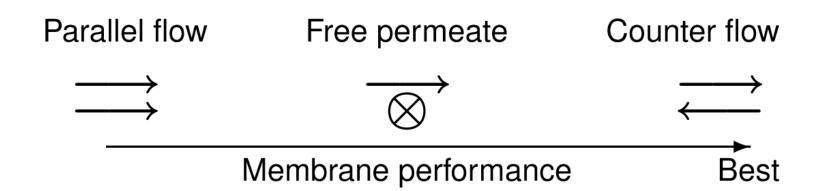
$$\frac{w_{j1} - w_j^*}{w_{j2} - w_j^*} = e^{-\frac{\dot{m}_P''}{\rho_F k_F}} = e^{-\frac{Pe D_{ij}}{k_F d_{hyd}}} = e^{-\frac{Pe}{Sh}}$$

Different Flow Arrangements

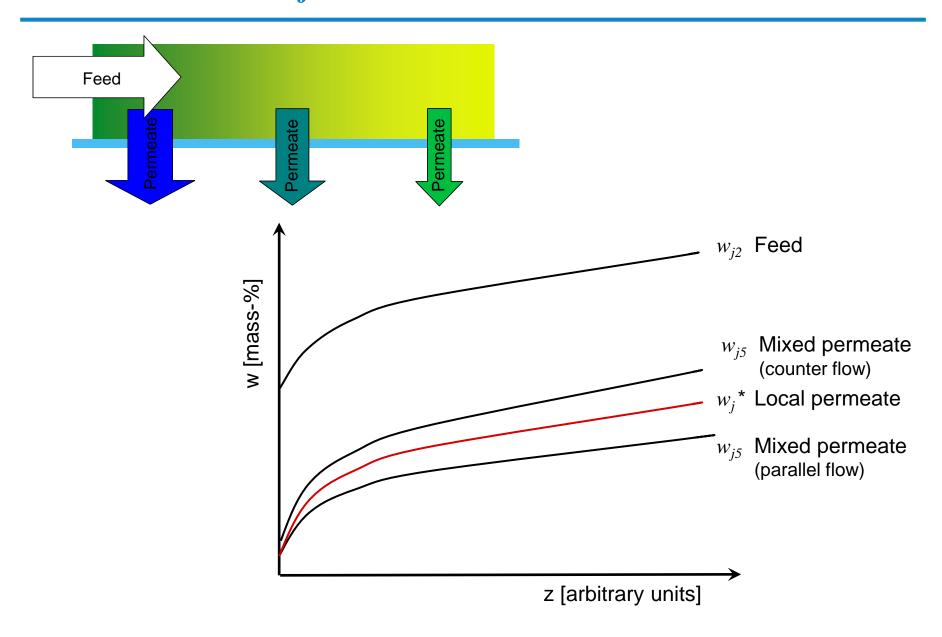


Conduction of Flow and Membrane Performance

- Conduction of flow influences selectivity and flux.
- Influence of conduction of flow varies for different Peclet numbers:
- Pe >> 1: Blocking of diffusion
 Almost) No influence of conduction of flow.
- Pe << 1: High diffusivity
 ⇒ Maximum influence of conduction of flow:



Concentration of j in the Permeate



Design Techniques To Enhance Membrane Performance

- Main aim of all design techniques:
 Reduction of the thickness of the boundary layer.
- Possible measures: Higher turbulence by
 - increased flow velocity,
 - additional installations,
 - generation of multiphase flow.
- Disadvantages:
 - increased energy demand,
 - higher investment costs.

Design Techniques (cont'd)

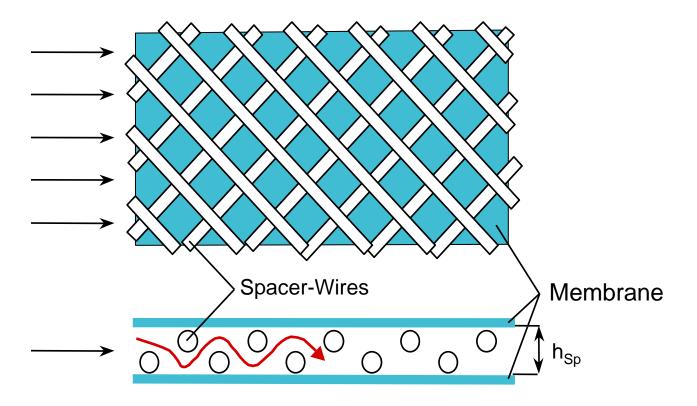
Design Technique	Effect
Feed channel spacer	turbulent flow
Multiphase flow	better mixing esp. in capillary tubes
Rotor above membrane	higher shear stress at membrane surface
Back flushing/ pulsing	removal of cake layers
Vibration of membrane	higher shear stress at membrane surface
Curved flow channel	Dean vortices
Pulsed feed flow	unsteady flow generates eddies
Rotation of membrane	higher shear stress at membrane and Taylor vortices

Feed Channel Spacers

- Most common technique for flux enhancement!
- Spacer are net-like structures with a tickness of 1 to 2 mm.
- Role of spacers:
 - Definition of channels,
 - Increased mass transfer,
 - Greater flux,
 - But also increased pressure loss.
- The optimal design of spacers is current research topic at our institute.
- Membrane processes need optimal spacer design to be efficient.

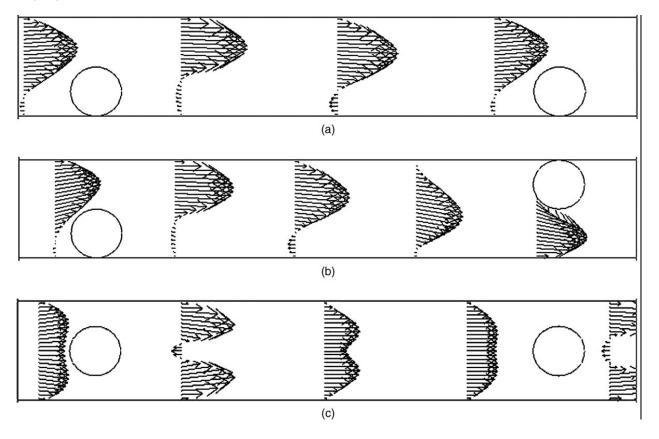
Flow Through A Spacer Filled Channel

Diamond spacers are the most common spacer type:



Flow Through A Spacer Filled Channel (cont'd)

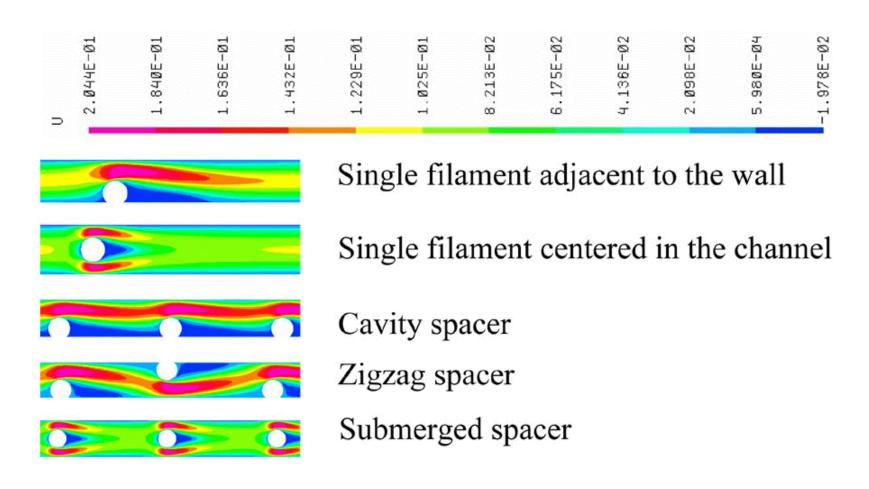
 Velocity vector diagramms for short spacer filled sections of channels:



(a) cavity, (b) zigzag, (c) submerged spacer configurations [A. Subramani et al. / Journal of Membrane Science 277 (2006) 7-17]

Flow Through A Spacer Filled Channel (cont'd)

CFD simulation for different spacer structures:



Flux Dependency Of Particle Deposition

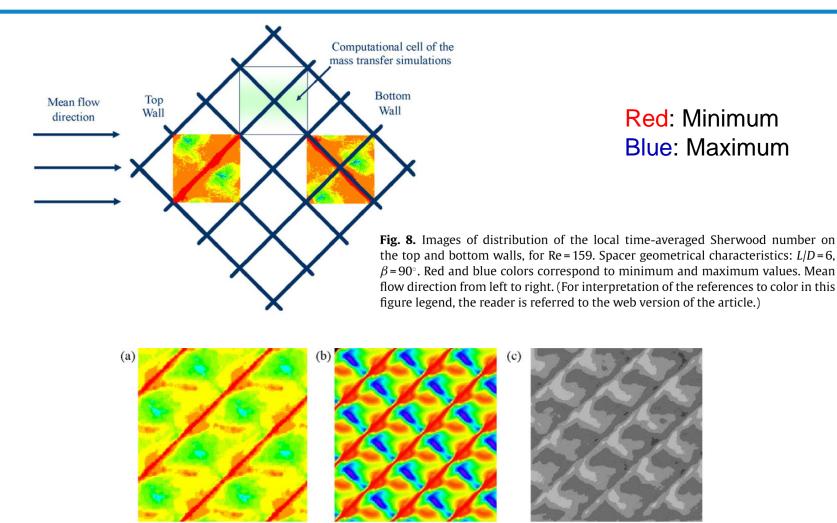
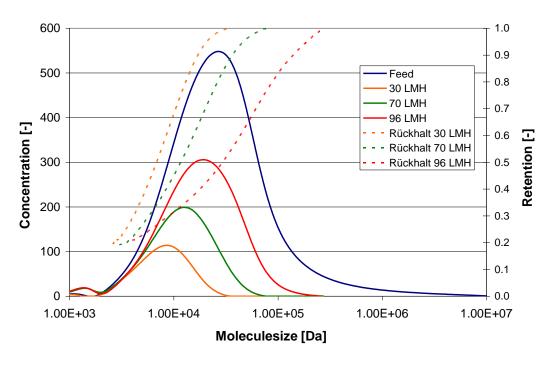


Fig. 13. (a) Predicted local time-averaged mass transfer coefficient pattern. Sc = 25, Re = 90. Parameter values: L/D = 8, $\beta = 90^{\circ}$. (b) Predicted local time-averaged shear stress pattern. Re = 90. Parameter values: L/D = 8, $\beta = 90^{\circ}$. (c) Fouling pattern of an RO membrane with spacer geometrical characteristics: $L/D \sim 9$, $\beta = 90^{\circ}$. Mean flow direction from left to right.

C.P. Koutsou, S.G. Yiantsios, A.J. Karabelas, Journal of Membrane Science 326 (2009) 234–251

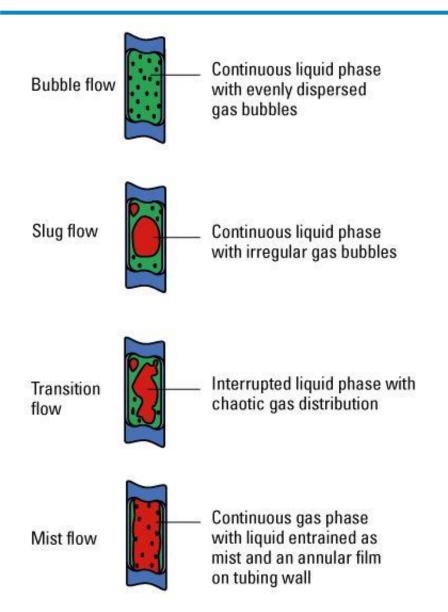
Sharper separation with a spacer

Dextran Retention (Gel-Permeations-Chromatography)



- Narrower separation than net spacer
- Molecular Weight Cut-off (90%) shifted

Flow Regimes of Multiphase Flow



 Characterisation of different flow regimes via gasphase proportion ε:

 ϵ < 0.2 Bubble flow, 0.2 < ϵ < 0.9 Slug flow,

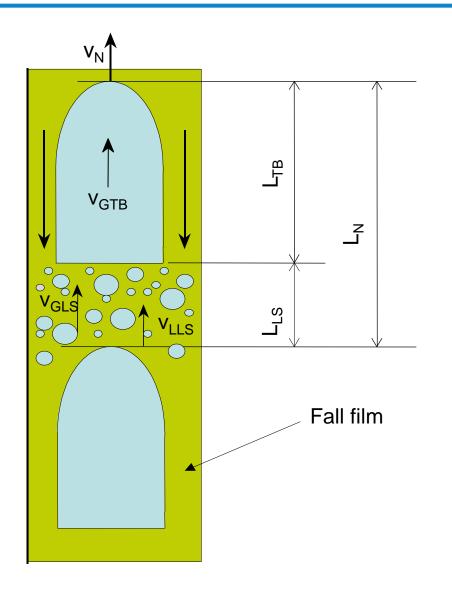
 $0.9 < \varepsilon$ Mist flow.

http://www.glossary.oilfield.slb.com

Creation Of Multiphase Flow

- In narrow capillary tubes only laminar flow.
- Use gas bubbles to create multiphase flow.
- Best results with "slug flow":
 - Alternaetly large "rocket shaped" bubbles: Taylor bubbles
 - and fluid sections with only small bubbles.
- This leads to turbulent flow and enhances the shear rate on the membrane.

Gas sparging in tubular membranes



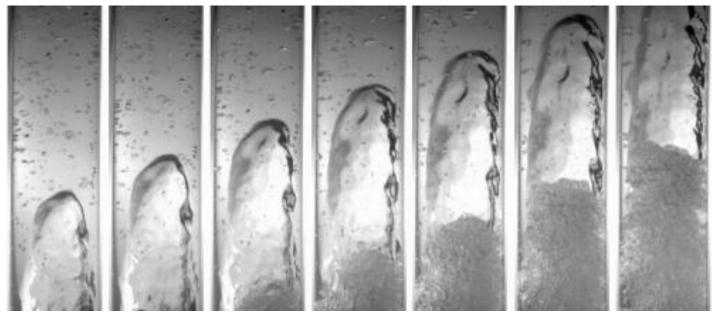
 Influence of "slug flow" on effective mass transfer coefficient k is given by:

$$k = 1.62 \left(\frac{d\,\dot{\gamma}\,D}{d_{hyd}\,L}\right)^{0.33}$$

With:

 $egin{array}{lll} d &=& {
m tube\ diameter}, \ \dot{\gamma} &=& {
m local\ shear\ rate}, \ D &=& {
m Diffusion\ coefficient}, \ d_{hyd} &=& {
m hydraulic\ diameter}, \ L &=& {
m tube\ length}. \end{array}$

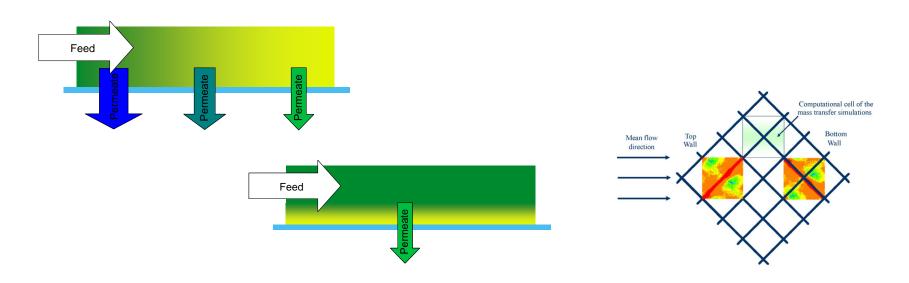
Gas sparging in tubular membranes



Quelle: www.iahrmedialibrary.net

Summary

- Collection of Effects Reducing the Driving Force
- Influence of Feed-sided Concentration Polarisation
- Dimensionless Groups to Describe the Boundary Layer
- Pressure Loss in the Porous Support Layer
- Design Techniques to Enhance Membrane Performance





AACHENER VERFAHRENSTECHNIK

Thank you for your attention!

Any questions left?

