Effectiveness Correlations for Heat and Moisture Transfer Processes in an Enthalpy Exchanger With Membrane Cores

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The performance correlations for the effectiveness of heat and moisture transfer processes in an enthalpy exchanger with membrane cores are presented. The physical phenomena relevant to the heat and moisture transfer in these devices have been used to develop a novel set of correlations based on the relevant dimensionless parameters. The total enthalpy effectiveness can be calculated from sensible effectiveness, latent effectiveness, and the ratio of latent to sensible energy differences across the unit. Studies show that the sensible effectiveness is a function of NTU, the number of transfer units for heat; while the latent effectiveness is a function of $\mathrm{NTU_L}$*, the number of transfer units for moisture. The relations between* $\mathrm{NTU_L}$ *and NTU are derived and studied with the proper separation of moisture resistance for membranes. This newly developed dimensionless parameter,* $\mathrm{NTU_L}$*, is to summarize the sorption characteristics of membrane material, the exchanger configurations, as well as the operating conditions. A number of experimental results on an enthalpy exchanger with novel hydrophilic membrane cores has been used to valid these correlations.* [DOI: 10.1115/1.1469524]

Keywords: *Dehumidification, Heat Transfer, Moisture, Membrane, Exchanger*

1 Introduction

Mechanical ventilation with heat recovery is often considered as one of the key elements of a low energy residential building in various climates [1]. In the past, researches were only focused on the recovery of sensible heat [2–4] by neglecting the treatment of humidity for ventilation air. These systems usually employ traditional heat exchangers such as fixed plates, sensible heat exchange wheels, heat pipes, and coil run-around loop heat exchangers, of which the performance is easy to predict. However, in recent years, increasing attention has been paid to energy recovery, or enthalpy recovery, in which both the sensible and the latent heat are recovered. The latent load constitutes a large fraction of the total thermal load for an air-conditioned building, and thus it is more significant to recover the latent heat, as well as the sensible heat.

The present techniques for enthalpy recovery rely on the alternate sorption and regeneration of desiccant materials, either in the form of cycling packed beds or rotary wheels. San [5] investigated the heat and mass transfer in a set of two-dimensional cross-flow regenerative beds. The effects of operating conditions and outdoor climate conditions on the enthalpy effectiveness are numerically studied. Simonson and Besant [6] proposed a numerical model for the heat and moisture transfer in energy wheels during sorption, condensation, and frosting conditions, and the sensitivity of condensation and frosting to wheel speed and desiccant type are studied. Simonson and Besant [7,8] also investigated the performances of energy wheels both numerically and experimentally in detail. The fluctuations of sensible and latent effectiveness with various operating temperatures and humidities are discussed in their research.

These studies are very interesting and they are certainly helpful to improve the system efficiency. However, due to the low thermal conductivity of desiccants, the regeneration of desiccants is very slow and energy consuming, which may in return sacrifice the efficiency in energy recovery. Furthermore, cycling beds are bulky and the maintenance of rotating wheels is problematic.

Membrane-based enthalpy exchanger is another alternative for enthalpy recovery. This concept is just like an air-to-air sensible heat exchanger. But in place of traditional metal heat exchange plates, novel hydrophilic membranes are used, through which both heat and moisture are transferred simultaneously. Since the permeation of moisture through the membrane is in a continuous manner, no regeneration is required. Furthermore, it is very easy to construct and implement, therefore it is more promising. It should be noted that membranes have been used in air dehumidification for a long time, see Pan et al. [9]; Asaeda and Du, [10]; Wang et al. [11]; Cha, et al. [12]; to name but a few, and some energy recovery exchangers have been patented by a couple of companies. However, the theoretical studies of hydrophilic membrane-based enthalpy recovery exchangers are still scarce [13,14].

To optimize the membrane system, the effectiveness of enthalpy exchange under various conditions should be calculated. Due to the complex coupling between heat and moisture transfer, there is no simple design methodology available for the membrane-based enthalpy exchanger. Finite difference simulations, which have been used to study the sensitivity of performance to outside conditions [13,15], is time consuming and inconvenient for engineers to estimate and select efficient membrane enthalpy recovery devices that provide greater life cycle cost savings. Therefore it is imperative to develop some simple correlations that could predict the sensible and latent effectiveness by summarizing the couplings between the performance and the sorption characteristics of membrane material, and the operating conditions. This is the objective of this research.

2 Basic Equations

A cross-flow enthalpy exchanger with membrane cores is shown in Fig. 1. Two air streams- the supply and the exhaust flow

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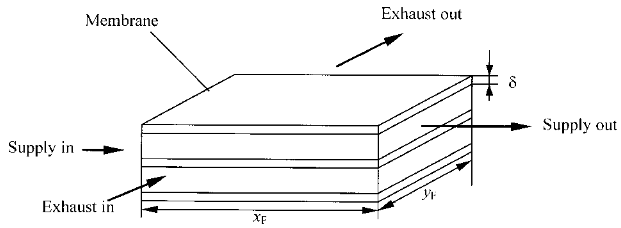

Fig. 1 Schematic of a cross-flow enthalpy exchanger with membrane cores

in thin, parallel, alternating membrane layers, in order to transfer heat and moisture from one air stream to the other. In air conditioning, the supply is usually the outside fresh air and the exhaust is the room air that needs to be discharged to the outside. The enthalpy exchanger is just like a traditional plate-type heat recuperator. The only difference is that hydrophilic membranes are used in place of metal plates. The governing dimensionless equations for simultaneous heat and moisture transfer in enthalpy exchangers, based on the assumptions listed in Table 1, are as follows:

Supply

$$\frac{\partial T_s}{\partial x^*}=2\mathrm{NTU}_s(T_{ms}-T_s) \tag{1}$$

$$\frac{\partial \omega_s}{\partial x^*}=2\mathrm{NTU}_{\mathrm{Ls}}(\omega_{ms}-\omega_s) \tag{2}$$

Exhaust

$$\frac{\partial T_e}{\partial y^*}=2\mathrm{NTU}_c(T_{me}-T_e) \tag{3}$$

$$\frac{\partial \omega_e}{\partial y^*}=2\mathrm{NTU}_{\mathrm{Le}}(\omega_{me}-\omega_e) \tag{4}$$

where

$$x^*=\frac{x}{x_F}, \quad y^*=\frac{y}{y_F},$$

$$\mathrm{NTU}_s=\frac{n_s h_s x_F y_F}{\dot m_s c_{ps}}=\frac{h_s A_{\mathrm{tot}}}{\dot m_s c_{ps}}, \quad \mathrm{NTU}_e=\frac{n_e h_e x_F y_F}{\dot m_c c_{pe}}=\frac{h_e A_{\mathrm{tot}}}{\dot m_c c_{pe}},$$

$$\mathrm{NTU}_{\mathrm{Ls}}=\frac{k_s A_{\mathrm{tot}}}{\dot m_s}, \quad \mathrm{NTU}_{\mathrm{Le}}=\frac{k_e A_{\mathrm{tot}}}{\dot m_e},$$

where T_{ms} and T_{me} are the temperature of membrane in supply side and exhaust side, respectively, ω is moisture uptake in air streams, x and y are coordinates, h is convective heat transfer coefficient, k is convective mass transfer coefficient, $\dot m$ is mass flow rate of dry air, x_F and y_F are lengths of flow channels, n is the number of channels. The subscript "s" means "in supply side". Previous studies found that the temperature difference between the two sides of membrane is very small due to the small thickness of membrane [13]. So it is reasonable to assume that $T_{ms}=T_{me}=0.5(T_s+T_e)$.

The effect of axial fluid conduction is totally negligible for Pe (Peclet number)=100, and is quite small even for Pe=10. In practical applications axial conduction is frequently of considerable significance for laminar flow of liquid metals, which have very low Prandtl numbers. For gases axial conduction can be of importance only at extremely low Reynolds numbers [16]. Generally speaking, assumption (2) is valid when the Peclet number is bigger than 2. The present values of Peclet number in this case are from 11 to 60.

From assumptions (1) and (2), it is seen that heat and moisture transfer is one-dimensional and along the flow direction. However, due to the cross-flow arrangements, the temperature and humidity distributions in the air streams are two-dimensional.

The above dimensionless parameters are the commonly recognized Number of Transfer Units. They give an insight into the characteristics of heat and moisture exchange between fluids and surfaces.

Moisture flow rate through the membrane:

$$\dot m_w=D_{wm}\frac{\theta_{ms}-\theta_{me}}{\delta} \tag{5}$$

where θ_{ms}, θ_{me} are moisture uptake in membrane at two surfaces (kg.kg^{-1}), δ is the membrane thickness, and D_{wm} is the water diffusivity in membrane (kgm^{-1}s^{-1}).

The equilibrium between the membrane and moisture at its surface can be expressed with a general sorption curve as

$$\theta=\frac{w_{\max}}{1-C+C/\phi} \tag{6}$$

where $w_{\max}$ represents the maximum moisture content of the membrane material (i.e., moisture uptake when ϕ=100 percent) and C determines the shape of the curve and the type of sorption.

The parameters of θ, ϕ, and ω can be correlated by ideal gas state equation and psychrometric relations.

The convective heat transfer coefficients are obtained from Nusselt correlations [17] and the mass transfer in boundary layers is often described by Sherwood correlations. By using the well-known Chilton-Colburn Analogy [18]

$$\mathrm{Sh}=\mathrm{Nu}\cdot\mathrm{Le}^{-1/3} \tag{7}$$

We have

$$k=\frac{h}{c_{pa}}\mathrm{Le}^{-1/3} \tag{8}$$

For ventilation air and vapor mixture, which is always near atmospheric states, the Lewis number, Le, varies in the range of 1.19 to 1.22, see Ref. [17].

Analogous to the heat transfer effectiveness commonly used in heat exchanger analysis, the concept of effectiveness can be applied to the heat and moisture transfer processes in a membrane based enthalpy exchanger. For a constant specific heat and heat of phase change, the effectiveness is defined as

Sensible effectiveness

$$\varepsilon_s=\frac{(\dot m c_{pa})_s(T_{si}-T_{so})}{(\dot m c_{pa})_{\min}(T_{si}-T_{ei})} \tag{9}$$

Latent effectiveness

$$\varepsilon_L=\frac{(\dot m c_{pa})_s(\omega_{si}-\omega_{so})}{(\dot m c_{pa})_{\min}(\omega_{si}-\omega_{ei})} \tag{10}$$

Enthalpy transfer effectiveness, i.e., total energy transfer effectiveness

Table 1 Assumptions used in governing equations

1. There is no lateral mixing of the two air streams.
2. Heat conduction and vapor diffusion in the two air streams are negligible compared to energy transport and vapor convection by bulk flow.
3. Adsorption of water vapor and membrane material is in equilibrium adsorption-state.
4. Both the heat conductivity and the water diffusivity in the membrane are constants.
5. Heat and moisture transfer is one-dimensional in membrane.

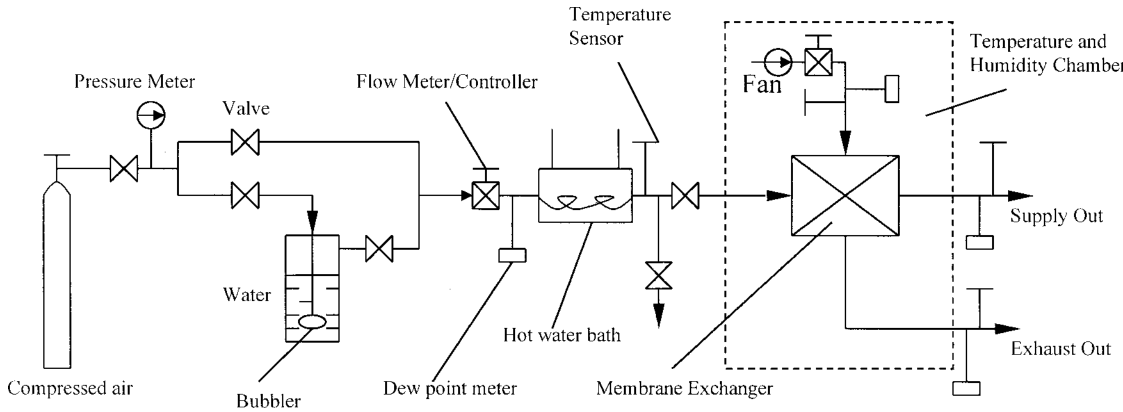

Fig. 2 Schematic of the experimental setup

$$\varepsilon_{\mathrm{tot}}=\frac{\dot{m}_s(H_{si}-H_{so})}{\dot{m}_{\min}(H_{si}-H_{ei})} \tag{11}$$

where H is the specific enthalpy of air, and it is calculated by [7]

$$H=c_{pa}T+\omega(2501+1.86T) \tag{12}$$

where T is in °C.

The third term in Eq. (12) usually has a less than 3 percent effect, thus it can be neglected. Then the enthalpy effectiveness can be further simplified as

$$\varepsilon_{\mathrm{tot}}=\frac{\varepsilon_s+\varepsilon_L H^*}{1+H^*} \tag{13}$$

where

$$H^*=\frac{2501(\omega_{si}-\omega_{ei})}{c_{pa}(T_{si}-T_{ei})}\approx 2501\frac{\Delta\omega}{\Delta T} \tag{14}$$

where H^* is essentially a ratio of latent to sensible energy differences between the inlets of two air streams flowing through the enthalpy exchanger. H^* can in theory vary from $-\infty$ to $+\infty$, but varies typically from -6 to $+6$ for enthalpy recovery in HVAC applications. From above equation, it is clear that the total enthalpy effectiveness is not a simple algebraic average of sensible and latent effectiveness. When $H^*=1$, $\varepsilon_{\mathrm{tot}}=(\varepsilon_s+\varepsilon_L)/2$. As $H^*\to\infty$, $\varepsilon_{\mathrm{tot}}\to\varepsilon_L$; as $H^*\to 0$, $\varepsilon_{\mathrm{tot}}\to\varepsilon_s$; as $H^*\to -1$, $\varepsilon_{\mathrm{tot}}\to\pm\infty$.

3 Experimental

Enthalpy exchange experiments were performed on a cross-flow membrane exchanger with the apparatus shown schematically in Fig. 2. The supply air flows from a compressed air bottle and the exhaust air is supplied from a large humidity and temperature chamber. For the supply air, humidity is adjusted to the desired point by humidifying air through a bubbler immersed in a bottle of water and subsequently mixing it with a dry air stream. The temperature is controlled to the desired point by a hot water bath. The experimental effectiveness is obtained by measuring the temperatures and humidities at the inlets and outlets of two air streams. Temperature is measured by platinum resistance and humidity is measured by chilled-mirror dew point meter. Air mass flow rate is measured and controlled at two values: 0.01 kg/s and 0.05 kg/s. The uncertainties of measurement are: 0.2°C for temperature; 2 percent for humidity; and 5–10 percent for air flow rate. The maximum uncertainties for the tested sensible effectiveness are: 16.8 percent for air mass flow rate of 0.01 kg/s and 12.2 percent for 0.05 kg/s flow rate respectively. The maximum uncertainties for the latent effectiveness are 9.8 percent for air mass flow rate of 0.01 kg/s and 5.2 percent for 0.05 kg/s respectively. A large fraction of the uncertainties came from the uncertainties in air-flow-rate measurement. However, for most of the test results, the uncertainties are less than 5 percent because most of the temperature differences between the two inlets are bigger than 1.5°C and/or the humidity differences are greater than 3.4 g/kg. Before the test, tracer gas is used to detect the cross-over of air streams.

The membranes used are a novel hydrophilic polymer membrane developed and supplied by one of our vendors. It is a modification of poly(vinylchloride) (PVC) membrane by cross-linking it with a selective coating and a 0.02 μm pore diameter porous base material. The final membrane thickness is 20 μm and the composite material has an average degree of polymerization of 480. For the applications of such membranes in heat and moisture transfer, the influences of the microstructure on the performance are reflected by the sorption curve of the material (including shape and sorption potential) and the diffusivity of water in membrane. The sorption curve for the membrane material is measured in a constant temperature and humidity chamber and is shown in Fig. 3. As can be seen, it is a third-type sorption curve that is more sensitive to humidity with increasing humidities. For this material, $C=2.5$, $w_{\max}=0.23$ kg/kg. The effective diffusivity is found to be 2.16×10^{-8} kgm^{-1} s^{-1} from the product manual.

The configurations of the membrane exchanger are: width, 0.5 m; length, 0.5 m; number of membranes for each side, 15; height of flow channels, 5 mm. The exchanger is in cross-flow arrangements, for easy of sealing and construction.

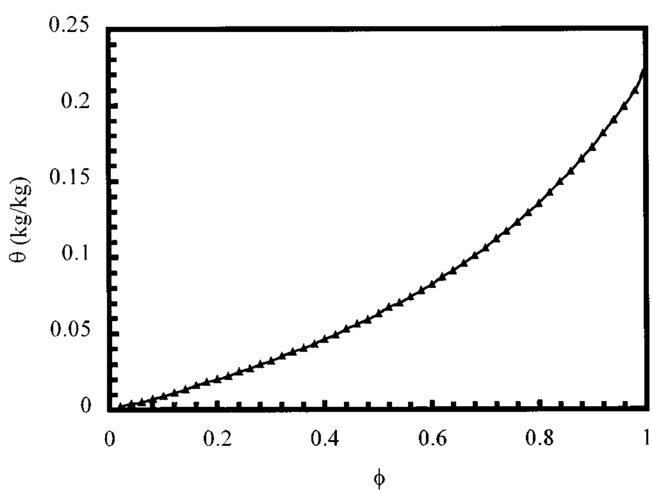

Fig. 3 Sorption curve for the membrane material, $C=2.5$, $w_{\max}=0.22$

4 Development of Effectiveness Correlations

A total number of transfer units is used to reflect the sensible heat transfer in an exchanger. For the membrane exchanger that has equal area on both sides, the total number of transfer units for sensible heat is

$$\mathrm{NTU}=\frac{A_{\mathrm{tot}}U}{(\dot{m}c_{pa})_{\min}}$$

where U is the total heat transfer coefficient. Its general form is

$$U=\left[\frac{1}{h_s}+\frac{\delta}{\lambda}+\frac{1}{h_e}\right]^{-1} \tag{15}$$

The term in the middle is the thermal resistance of membrane, which value is around 0.005 m^2K/kW. Other two terms are convective thermal resistance. Their values are in the order of 40 m^2K/kW, or 8000 times larger than membrane resistance. Therefore, membrane resistance for heat transfer can be neglected.

The sensible effectiveness is a function of two dimensionless parameters, NTU and $R_1=(\dot{m}c_{pa})_{\min}/(\dot{m}c_{pa})_{\max}$, the ratio of minimum to maximum heat capacity rate of two air streams. For unmixed cross flow, it can be expressed as [17]

$$\varepsilon_s=1-\exp\left[\frac{\exp(-\mathrm{NTU}^{0.78}R_1)-1}{\mathrm{NTU}^{-0.22}R_1}\right] \tag{16}$$

This empirical equation is fairly accurate except at the extremes of the variables.

A form similar to Eq. (16) for latent effectiveness may be derived if the moisture resistance for membrane could be clarified and estimated.

The moisture flow rate through the membrane can also be expressed as

$$\dot{m}_W=k_s(\omega_s-\omega_{ms})=k_e(\omega_{me}-\omega_e) \tag{17}$$

$$\theta_{me}=\theta_{ms}+\left.\frac{\partial\theta}{\partial\phi}\right|_{ms}\Delta\phi=\theta_{ms}+\left.\frac{\partial\theta}{\partial\phi}\right|_{ms}(\phi_{me}-\phi_{ms}) \tag{18}$$

Substituting Eq. (18) into Eq. (5), we have

$$\dot{m}_W=\frac{D_{wm}}{\delta}\left(\frac{\partial\theta}{\partial\phi}\right)_{ms}(\phi_{ms}-\phi_{me}) \tag{19}$$

Using Clapeyron equation to represent the saturation vapor pressure and assuming a standard atmospheric pressure of 101325 Pa gives the relation between humidity and relative humidity as

$$\frac{\phi}{\omega}=\frac{e^{5294/(T+273.15)}}{10^6}-1.61\phi \tag{20}$$

where the second term on the right side of the equation will generally have less than a 5 percent effect, thus it can be neglected. Thus the relation between ϕ and ω is expressed by

$$\phi=\frac{e^{5294/(T+273.15)}}{10^6}\omega \tag{21}$$

Substituting Eq. (21) into Eq. (19), we have

$$\dot{m}_W=\frac{D_{wm}}{\delta}\left(\frac{\partial\theta}{\partial\phi}\right)_{ms}\frac{e^{5294/(T+273.15)}}{10^6}(\omega_{ms}-\omega_{me}) \tag{22}$$

From Eq. (17), two equations can be deduced

$$\omega_{ms}=\omega_s-\dot{m}_w/k_s \tag{23}$$

$$\omega_{me}=\omega_c+\dot{m}_w/k_c \tag{24}$$

Substituting above two equations into Eq. (22) to eliminate ω_{ms} and ω_{me} gives

$$\dot{m}_W=\frac{1}{\gamma_{\mathrm{tot}}}(\omega_s-\omega_e) \tag{25}$$

or

$$\dot{m}_W=U_L(\omega_s-\omega_e)$$

where

$$\gamma_{\mathrm{tot}}=U_L^{-1}=\frac{1}{k_s}+\gamma_m+\frac{1}{k_e} \tag{26}$$

and

$$\gamma_m=\frac{\delta}{D_{wm}}\frac{10^6}{e^{(-5294/T)}\left(\frac{\partial\theta}{\partial\phi}\right)_{ms}} \tag{27}$$

It is indicated that γ_{tot}, the total moisture transfer resistance, has an expression similar to thermal resistance where the first and the third term are the convective resistance on the supply side and exhaust side respectively. The middle term, γ_m, is the moisture diffusive resistance in membrane. U_L can be called the total moisture transfer coefficient for the device.

The differentiation of Eq. (6) gives

$$\frac{\partial\theta}{\partial\phi}=\frac{w_{\max}C}{(1-C+C/\phi)^2\phi^2} \tag{28}$$

The Eq. (27) can be further simplified as

$$\gamma_m=\frac{\delta}{D_{wm}}\psi \tag{29}$$

$$\psi=\left.\frac{10^6(1-C+C/\phi)^2\phi^2}{e^{(5294/T)}w_{\max}C}\right|_{ms} \tag{30}$$

where the coefficient of diffusive resistance for membrane, ψ, is co-determined by the operating conditions and the slope of sorption curves of membrane material.

Similar to the definition of total number of transfer units for heat, the total number of transfer units for moisture can be written as

$$\mathrm{NTU}_L=\frac{A_{\mathrm{tot}}U_L}{(\dot{m})_{\min}} \tag{31}$$

The comparison of total transfer units for moisture and sensible heat, assuming equal specific heats for two air streams, gives

$$\beta=\frac{\mathrm{NTU}_L}{\mathrm{NTU}}=\frac{U_Lc_{pa}}{U} \tag{32}$$

Substituting Eqs. (15), (26), (27), into Eq. (32) suggests that

$$\beta=\frac{\left(1+\frac{h_s}{h_e}\right)+\frac{\delta h_s}{\lambda}}{\left(1+\frac{h_s}{h_e}\right)+\frac{\delta}{D_{wm}}\psi k_s} \tag{33}$$

where, $(\delta h_s/\lambda)$ is several orders smaller than other terms, mainly due to the small thickness of membranes, so it can be neglected. The above equation can be further simplified into

$$\beta=\frac{\left(1+\frac{h_s}{h_e}\right)}{\left(1+\frac{h_s}{h_e}\right)+\frac{\gamma_m}{1/k_s}} \tag{34}$$

In most cases, the enthalpy recovery is implemented with balanced flows, i.e., the two air streams have the same flow rates. Therefore, the convective heat transfer coefficients would have the same value on both sides of membrane result from similar fluid fields. Consequently, Eq. (34) can be written in

$$\beta = \frac{1}{1+\alpha} \tag{35}$$

where

$$\alpha = \frac{\gamma_m}{\gamma_c} \tag{36}$$

$$\gamma_c = \frac{2}{k_s} \tag{37}$$

where γ_c is the convective moisture transfer resistance, and α is the ratio of diffusive resistance to convective resistance for membrane. As can be seen, the total number of transfer units for moisture can be estimated from the total number of transfer units for sensible heat, and ratio of diffusive to convective moisture resistance. As $\alpha \to \infty$, $\mathrm{NTU}_L \to 0$, no moisture can be permeated through the membrane. In this case, the "membrane" is like a metal plate. On the other hand, as $\alpha \to 0$, $\mathrm{NTU}_L \to \mathrm{NTU}$, $\epsilon_L = \epsilon_s$. If $\alpha = 1$, $\mathrm{NTU}_L = \mathrm{NTU}/2$. Under the current operating conditions, the values of α vary from 2 to 10 (see Fig. 6), which implies that membrane resistance for moisture transfer cannot be neglected.

Similar to the deduction of Eq. (16) for sensible heat transfer, the correlation for latent effectiveness can be written as

$$\varepsilon_L = 1 - \exp\left[\frac{\exp(-\mathrm{NTU}_L^{0.78} R_2) - 1}{\mathrm{NTU}_L^{-0.22} R_2}\right] \tag{38}$$

$$\mathrm{NTU}_L = \beta \cdot \mathrm{NTU} \tag{39}$$

$$R_2 = \dot{m}_{\min} / \dot{m}_{\max} \tag{40}$$

A more detailed set up of the analogy between Eqs. (38) and (16) can be seen in the appendix. The latent effectiveness correlations for other flow arrangements, such as concurrent flow and counter flow, can also be derived from those corresponding correlations for sensible effectiveness, using the definition of Eq. (35). The value of relative humidity of membrane in supply side, which is determined by latent effectiveness (permeation rate), needs to be known before the calculation of diffusive resistance for membrane γ_m and the diffusive to convective ratio α. Iterations are performed to obtain a converged solution for ϕ_{ms}.

5 Results and Discussion

To demonstrate the suitability of the correlations in predicting the effectiveness, sensible and latent effectiveness are calculated with the proposed correlations and compared with experimental results, as shown in Figs. 4 and 5 respectively. To make more extensive comparisons, simulation results with a finite difference model proposed by Zhang and Jiang [13] are also plotted in the two figures. The uncertainty for the finite difference results is less than 1 percent with a grid size of 2 mm.

From Figs. 4 and 5, it is obvious that both the sensible and the latent effectiveness are properly represented by the correlation from the present study. The largest discrepancies between the predictions by the correlation and the experimental data result from the cases with the smallest air flow rate, where the uncertainties of the experimental data are the biggest. The average errors between the predicted and experimental results are 7.3 percent and 8.6 percent for sensible and latent effectiveness respectively.

For a given exchanger, the sensible effectiveness is a fixed value at the specified flow rates. However the latent effectiveness will be affected by the two important dimensionless factors proposed in this study: the ratio of diffusive to convective resistance (α), and the ratio of total number of transfer units of moisture to that of sensible (β). The values of α and β are in turn affected by the membrane material types and operating inlet conditions. The

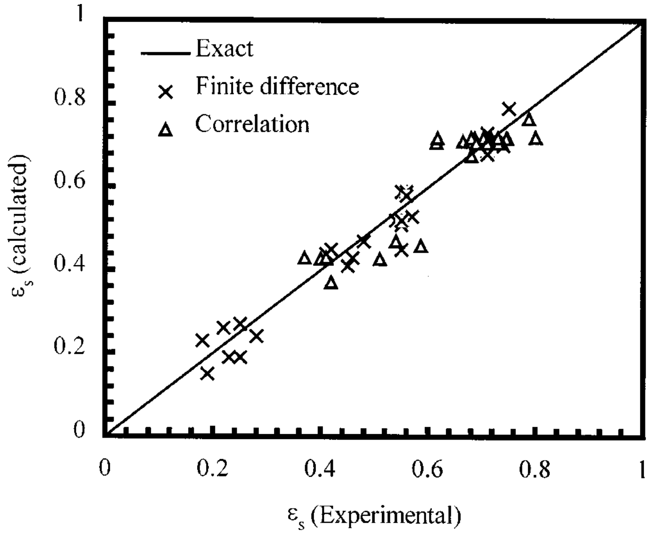

Fig. 4 Calculated and experimental sensible effectiveness for a cross-flow membrane enthalpy exchanger

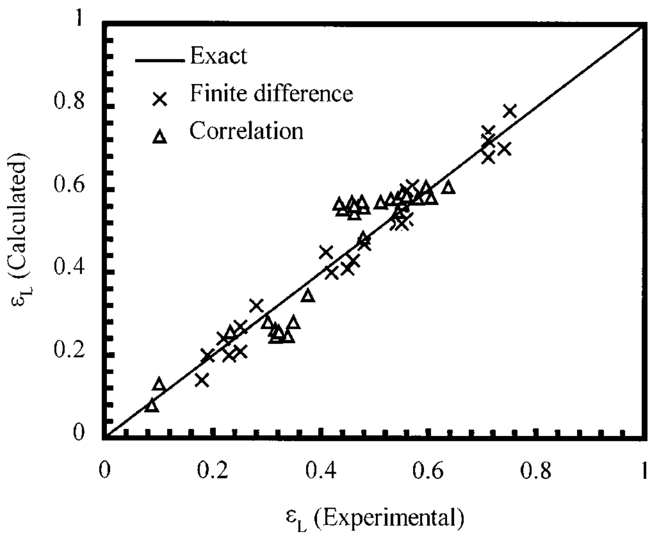

Fig. 5 Calculated and experimental latent effectiveness for a cross-flow membrane enthalpy exchanger

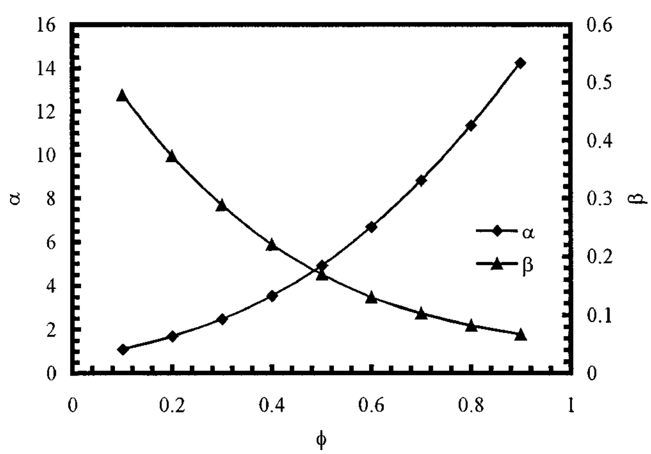

Fig. 6 Variations of α and β with increasing inlet relative humidity for first-type ($C=0.1$) membrane material

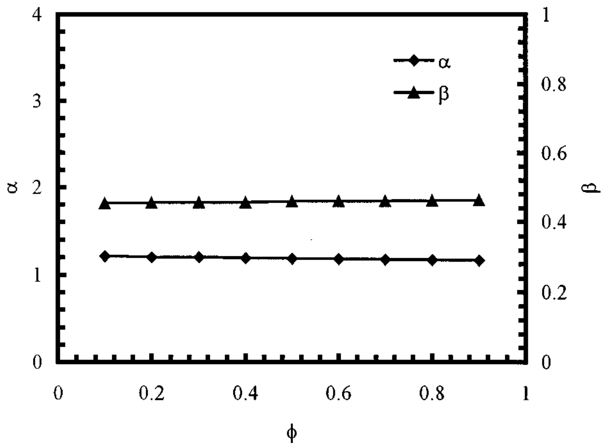

Fig. 7 Variations of α and β with increasing inlet relative humidity for linear type ($C=1.0$) membrane material

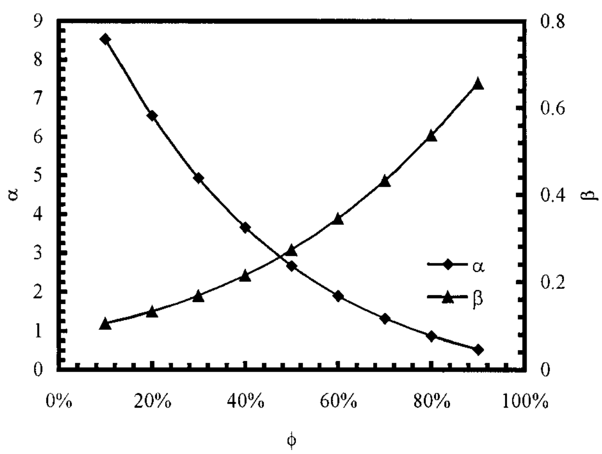

Fig. 8 Variations of α and β with increasing inlet relative humidity for third-type ($C=10$) membrane material

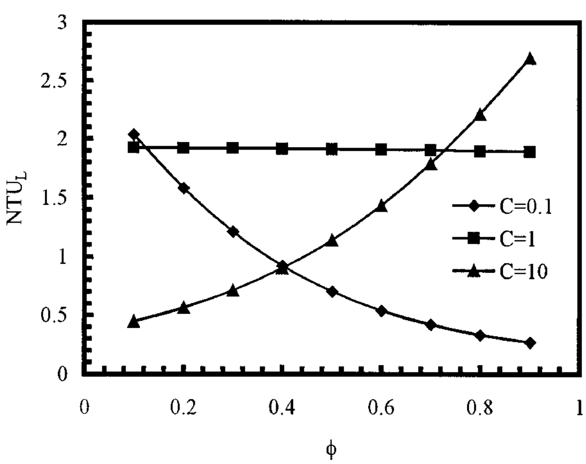

Fig. 9 Variations of NTU_L with relative humidity for different membranes, NTU=4.2

variations of α and β with increasing inlet relative humidities of the supply air are plotted in Figs. 6–8, for three kinds of membrane material that are most often used.

The value of α and β with linear type membrane does not change with inlet relative humidity. However, for first-type membrane material, α increases from around 0.5 at 10 percent RH to

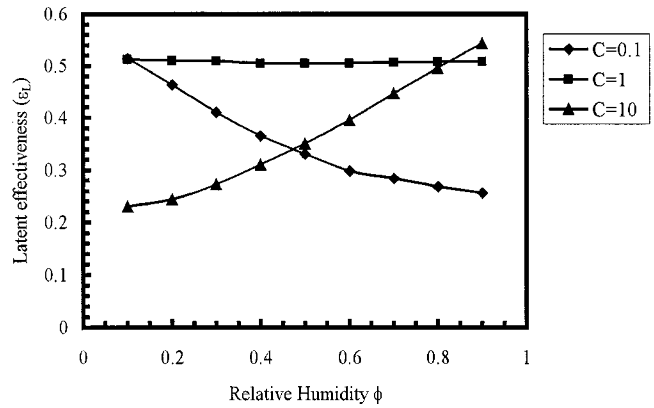

Fig. 10 Latent effectiveness for three types of membranes, NTU=4.2

15 at 90 percent RH, while for third-type membrane, α decreases from 8.6 at 10 percent RH to 0.4 at 90 percent RH. The bigger the α, the larger the β and the latent effectiveness.

Figure 9 demonstrates the variations of NTU_L, when NTU is kept constant, with different inlet humidities. The value of NTU_L decreases and increases for first-type and third-type material, with increasing inlet humidity respectively. The trends of resulting latent effectiveness are the same as those of NTU_L, which can be deduced from Eq. (31), see Fig. 10. For the linear type material, the NTU_L and the latent effectiveness will not change with the outside conditions. The number of transfer units for moisture would keep at 0.45 times of that for sensible heat for this material.

The above discussions suggest that an enthalpy exchanger with linear type membrane cores always performs better than those with other membrane cores. For example, to obtain a latent effectiveness of 0.6 under an inlet humidity of 50 percent, NTU_L should be at least 2.0, which means that the minimum values of NTU required for the exchanger are: 4.44 with linear type; 8 with third-type; and 11.1 with first-type membrane. A smaller NTU usually makes the enthalpy exchanger more compact and cheaper.

6 Conclusions

Enthalpy recovery with new hydrophilic membranes has potentially extensive uses in energy efficient buildings. To evaluate the system performance, the effectiveness correlations are proposed. By separating the moisture resistance through membranes and building up an analogy between the number of transfer units for moisture and that for sensible heat, the latent effectiveness correlation is written in a form very similar to the empirical correlation for sensible effectiveness that can be easily found in many literatures. A comparison with established experimental results shows that the correlations correctly predict the influences of the design variables on the performance of the membrane system. The studies also find that the ratio of diffusive to convective resistance (α) determines the number of transfer units for moisture, with a fixed value of the number of transfer units for sensible heat. The ratio α reflects the couplings between the moisture transfer and the sorption characteristics of membrane material and the operating conditions. For a given number of transfer units for sensible heat, enthalpy exchangers with linear type membrane cores would have the highest transfer units for moisture, which in return results in the highest latent effectiveness.

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Nomenclature

A = transfer area (m^2)
C = constant in sorption curve
c_p = specific heat ($kJkg^{-1}K^{-1}$)
D_{wm} = diffusivity of water in membrane ($kgm^{-1}s^{-1}$)
H = specific enthalpy (kJ/kg)
H^* = ratio of latent to sensible energy differences between the inlets of two air streams
h = convective heat transfer coefficient ($kWm^{-2}K^{-1}$)
k = convective mass transfer coefficient ($kgm^{-2}s^{-1}$)
$\dot{m}$ = mass flow rate of air streams (kg/s)
$\dot{m}_W$ = mass flow rate of moisture flow ($kgm^{-2}s^{-1}$)
NTU = number of transfer units
Nu = Nusselt number
n = number of channels
R = ratio for heat/mass capacity
Sh = Sherwood number
T = temperature (°C)
U = total heat transfer coefficient ($kWm^{-2}K^{-1}$)
U_L = total mass transfer coefficient ($kgm^{-2}s^{-1}$)
$w_{\max}$ = maximum water uptake of desiccant ($kgkg^{-1}$)
x, y = coordinates (m)
x^*, y^* = nondimensional coordinates
x_F = length of supply channel (m)
y_F = length of exhaust channel (m)

Greek Letters

ψ = coefficient of moisture diffusive resistance in membrane (CMDR)
λ = thermal conductivity of membrane ($kWm^{-1}K^{-1}$)
θ = moisture uptake in membrane ($kgkg^{-1}$)
ε = effectiveness
γ = resistance (m^2K/k W for thermal and m^2s/kg for moisture)
ϕ = relative humidity
δ = thickness of membrane (m)
ω = moisture content (kg moisture/kg dry air)
α = ratio of diffusive to convective moisture resistance for membrane
β = ratio of total number of transfer units for moisture to that for sensible heat

Subscripts

a = air
c = convective
e = exhaust
i = inlet
L = latent, moisture
m = membrane, diffusive
s = supply, sensible
o = outlet
tot = total
w = water

Appendix

Deduction of effectiveness correlations

Considering a cross-flow membrane exchanger with only one flow channel. At any point in the exchanger a heat and mass balance for an infinitely small volume *dxdy* can be written from Eq. (15) and Eq. (25) as

$$dq=U(T_s-T_e)dxdy \tag{A1}$$

$$d\dot{m}_W=U_L(\omega_s-\omega_e)dxdy \tag{A2}$$

Equation (*A*1) is a basic heat transfer equation, and Eq. (*A*2) has been widely employed as a mass permeation model through a membrane in chemical industry [19].

Across the elements x_F and y_F units in length the energy and moisture balances yield

$$dq=-\frac{(\dot{m}c_{pa})_s}{y_F}\frac{\partial T_s}{\partial x}dxdy \tag{A3}$$

$$dq=\frac{(\dot{m}c_{pa})_e}{x_F}\frac{\partial T_e}{\partial y}dxdy \tag{A4}$$

$$d\dot{m}_W=-\frac{\dot{m}_s}{y_F}\frac{\partial \omega_s}{\partial x}dxdy \tag{A5}$$

$$d\dot{m}_W=\frac{\dot{m}_e}{x_F}\frac{\partial \omega_e}{\partial y}dxdy \tag{A6}$$

Combining Eqs. (*A*1) and (*A*3) and then Eqs. (*A*1) and (*A*4) gives

$$\frac{Uy_F}{(\dot{m}c_{pa})_s}(T_s-T_e)=-\frac{\partial T_s}{\partial x} \tag{A7}$$

$$\frac{Ux_F}{(\dot{m}c_{pa})_e}(T_s-T_e)=\frac{\partial T_e}{\partial y} \tag{A8}$$

Similarly, combining Eqs. (*A*2) and (*A*5) and then Eqs. (*A*2) and (*A*6) gives

$$\frac{U_Ly_F}{\dot{m}_s}(\omega_s-\omega_e)=-\frac{\partial \omega_s}{\partial x} \tag{A9}$$

$$\frac{U_Lx_F}{\dot{m}_e}(\omega_s-\omega_e)=\frac{\partial \omega_e}{\partial y} \tag{A10}$$

Differentiating Eqs. (*A*7) and (*A*8) with respect to y and x and taking their sum gives

$$\frac{Ux_F}{(\dot{m}c_{pa})_e}\frac{\partial(T_s-T_e)}{\partial x}+\frac{Uy_F}{(\dot{m}c_{pa})_s}\frac{\partial(T_s-T_e)}{\partial y}=-\frac{\partial^2(T_s-T_e)}{\partial x\partial y} \tag{A11}$$

Similarly, differentiating Eqs. (*A*9) and (*A*10) with respect to y and x and taking their sum gives

$$\frac{U_Lx_F}{\dot{m}_e}\frac{\partial(\omega_s-\omega_e)}{\partial x}+\frac{U_Ly_F}{\dot{m}_s}\frac{\partial(\omega_s-\omega_e)}{\partial y}=-\frac{\partial^2(\omega_s-\omega_e)}{\partial x\partial y} \tag{A12}$$

Let dimensionless variables

$$\theta_1=\frac{T_s-T_e}{T_{si}-T_{ei}},\quad \theta_2=\frac{\omega_s-\omega_e}{\omega_{si}-\omega_{ei}},\quad x^*=\frac{x}{x_F},\quad y^*=\frac{y}{y_F}$$

and substituting in Eqs. (*A*11), (A12),

$$\frac{Ux_Fy_F}{(\dot{m}c_{pa})_e}\frac{\partial\theta_1}{\partial x^*}+\frac{Ux_Fy_F}{(\dot{m}c_{pa})_s}\frac{\partial\theta_1}{\partial y^*}=-\frac{\partial^2\theta_1}{\partial x^*\partial y^*} \tag{A13}$$

$$\frac{U_Lx_Fy_F}{\dot{m}_e}\frac{\partial\theta_2}{\partial x^*}+\frac{U_Lx_Fy_F}{\dot{m}_s}\frac{\partial\theta_2}{\partial y^*}=-\frac{\partial^2\theta_2}{\partial x^*\partial y^*} \tag{A14}$$

with

$$\mathrm{NTU}_a=\frac{Ux_Fy_F}{(\dot{m}c_{pa})_e},\quad \mathrm{NTU}_b=\frac{Ux_Fy_F}{(\dot{m}c_{pa})_s},$$

$$\mathrm{NTU}_{La}=\frac{U_Lx_Fy_F}{\dot{m}_e},\quad \mathrm{NTU}_{Lb}=\frac{U_Lx_Fy_F}{\dot{m}_s}$$

Eqs. (*A*13) and (*A*14) become

$$\mathrm{NTU}_a\frac{\partial\theta_1}{\partial x^*}+\mathrm{NTU}_b\frac{\partial\theta_1}{\partial y^*}+\frac{\partial^2\theta_1}{\partial x^*\partial y^*}=0 \tag{A15}$$

$$\mathrm{NTU}_{La}\frac{\partial\theta_2}{\partial x^*}+\mathrm{NTU}_{Lb}\frac{\partial\theta_2}{\partial y^*}+\frac{\partial^2\theta_2}{\partial x^*\partial y^*}=0 \tag{A16}$$

Initial condition: $\theta_1(0,0)=\theta_2(0,0)=1$

Mason [20] obtained a solution for Eq. ($A15$) in the form of an infinite series by employing the Laplace transformation as following:

$$\theta_1(x^*,y^*)=e^{-(\mathrm{NTU}_a)y^*+(\mathrm{NTU}_b)x^*}\sum_{n=0}^{\infty}\left[\frac{(\mathrm{NTU}_a)(\mathrm{NTU}_b)x^*y^*}{(n!)^2}\right]^n \tag{A17}$$

Since Eqs. ($A15$) and ($A16$) are the same form of differential equations. They are identical if NTU_a is replaced by NTU_{La} and NTU_b by NTU_{Lb}. Therefore, the solution to Eq. ($A16$) can be written as

$$\theta_2(x^*,y^*)=e^{-(\mathrm{NTU}_{La})y^*+(\mathrm{NTU}_{Lb})x^*}\sum_{n=0}^{\infty}\left[\frac{(\mathrm{NTU}_{La})(\mathrm{NTU}_{Lb})x^*y^*}{(n!)^2}\right]^n \tag{A18}$$

The overall heat transferred in the exchanger is the integral of Eq. ($A17$)

$$Q=Ux_Fy_F(T_{si}-T_{ei})\int_0^1\int_0^1\theta_1(x^*,y^*)dx^*dy^* \tag{A19}$$

The overall moisture transferred in the exchanger is the integral of Eq. ($A18$)

$$M_w=U_Lx_Fy_F(\omega_{si}-\omega_{ei})\int_0^1\int_0^1\theta_2(x^*,y^*)dx^*dy^* \tag{A20}$$

If we define

$$\mathrm{NTU}=\frac{Ux_Fy_F}{(\dot m c_{pa})_{\min}},\quad \mathrm{NTU}_L=\frac{U_Lx_Fy_F}{(\dot m)_{\min}}$$

$$R_1=\frac{(\dot m c_{pa})_{\min}}{(\dot m c_{pa})_{\max}},\quad R_2=\frac{\dot m_{\min}}{\dot m_{\max}}$$

$$\Omega_1=\int_0^1\int_0^1\theta_1(x^*,y^*)dx^*dy^*,$$

$$\Omega_2=\int_0^1\int_0^1\theta_2(x^*,y^*)dx^*dy^*$$

$$\varepsilon_s=\frac{Q}{(\dot m c_{pa})_{\min}(T_{si}-T_{ei})},\quad \varepsilon_L=\frac{M_W}{\dot m_{\min}(\omega_{si}-\omega_{ei})}$$

then

$$\varepsilon_s=\mathrm{NTU}\Omega_1 \tag{A21}$$

$$\varepsilon_L=\mathrm{NTU}_L\Omega_2 \tag{A22}$$

$$\Omega_1=\frac{1}{(\mathrm{NTU}_a)(\mathrm{NTU}_b)}\sum_{n=0}^{\infty}f(\mathrm{NTU}_a)f(\mathrm{NTU}_b) \tag{A23}$$

$$\Omega_2=\frac{1}{(\mathrm{NTU}_{La})(\mathrm{NTU}_{Lb})}\sum_{n=0}^{\infty}f(\mathrm{NTU}_{La})f(\mathrm{NTU}_{Lb}) \tag{A24}$$

where

$$f(z)=1-e^{-z}\sum_{m=0}^{n}\frac{z^m}{m!} \tag{A25}$$

There are two cases:

If $(\dot m c_{pa})_e=(\dot m c_{pa})_{\min}$, then $\mathrm{NTU}_a=\mathrm{NTU}$, and $\mathrm{NTU}_b=R_1\mathrm{NTU}$

If $(\dot m c_{pa})_e=(\dot m c_{pa})_{\max}$, then $\mathrm{NTU}_a=R_1\mathrm{NTU}$, and $\mathrm{NTU}_b=\mathrm{NTU}$

In both cases, Eq. ($A21$) can be replaced by the relationship

$$\varepsilon_s=\frac{1}{R_1(\mathrm{NTU})}\sum_{n=0}^{\infty}\left[1-e^{-\mathrm{NTU}}\sum_{m=0}^{n}\frac{(\mathrm{NTU})^m}{m!}\right]\times\left\{1-e^{-R_1(\mathrm{NTU})}\sum_{m=0}^{n}\frac{[R_1(\mathrm{NTU})]^m}{m!}\right\} \tag{A26}$$

Similarly, for moisture effectiveness, we have

$$\varepsilon_L=\frac{1}{R_2(\mathrm{NTU}_L)}\sum_{n=0}^{\infty}\left[1-e^{-\mathrm{NTU}_L}\sum_{m=0}^{n}\frac{(\mathrm{NTU}_L)^m}{m!}\right]\times\left\{1-e^{-R_2(\mathrm{NTU}_L)}\sum_{m=0}^{n}\frac{[R_2(\mathrm{NTU}_L)]^m}{m!}\right\} \tag{A27}$$

We can see at this step that the moisture effectiveness has the same form of expression with sensible effectiveness. The only differences are that NTU is replaced by NTU_L and R_1 is replaced by R_2.

For heat transfer, Eq. ($A26$) is too complicated, since it has infinite series. Therefore, Kays and London [21] used following empirical equation to represent the sensible effectiveness as

$$\varepsilon_s=1-\exp\left[\frac{\exp(-R_1(\mathrm{NTU}^{0.78})-1}{R_1\mathrm{NTU}^{-0.22}}\right] \tag{A28}$$

Similarly, Eq. (27), which is similar to Eq. (26), could be approximated with

$$\varepsilon_L=1-\exp\left[\frac{\exp(-R_2\mathrm{NTU}_L^{0.78})-1}{R_2\mathrm{NTU}_L^{-0.22}}\right] \tag{A29}$$

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