Growth, Income Distribution and Policy Implications of Automation*

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November 2021

Abstract

We study the distributional consequences of automation in a model with two kinds of agents — workers, who supply labor, and entrepreneurs, who own capital. We assume that production involves tasks that can be done by either capital or labor with varying productivity. We conceptualize automation as a shift in the relative productivity of capital at certain tasks that reduces the set of tasks done by labor. We contrast this with "traditional technical progress", which is an increase in capital productivity at tasks previously done by capital. We derive a simple condition that governs whether labor share goes to zero in the long run, for given tax rates. We then characterize the distributional consequences of a shift in technology, using a tractable case that allows us to cleanly distinguish between automation and traditional technological progress. Finally, we endogenize the tax rate by computing the political economy equilibrium under majority voting, where the government has access to a capital tax and a transfer to workers (a "universal basic income"). We give conditions for zero or positive capital taxation in the steady state, and conditions under which workers prefer that the labor share go to zero and they derive income wholly from the UBI.

Keywords: Automation, labor share, economic growth, majority voting

JEL Classification: E32; F34; F41; F44; F47; H63

*We thank XXX. All errors remaining are ours.

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1 Introduction

Technical progress has been the driving force of economic growth since the dawn of agriculture, and particularly since the Industrial Revolution ushered in the era of modern growth. This economic growth has been accompanied by replacement of human labor, first by animals and then by machines. While this process has doubtless harmed some individual workers, historically the overall effect has been for technological development to boost the incomes of workers and owners of capital in tandem, as evidenced by the roughly constant share of labor in national income over time. In recent years, however, there has been a growing perception that the current episode of technical progress (which we call automation) is qualitatively different, with much more drastic implications for growth and income distribution. As this automation is substituting for human labor in a broader manner than previous episodes of technical change (which we call traditional technical progress), it is likely to both accelerate economic growth (by replacing scarce labor with reproducible capital), and decrease the labor share of national income. Since most households derive most of their income from labor, whereas capital ownership is concentrated, this will likely lead to growing inequality and political instability. Some experts, such as Brynjolfsson and McAfee (2014) and Ford (2015), forecast these trends to continue and even accelerate in the coming years

Public discussion of these trends has included consideration of a wide range of fiscal policy tools to ameliorate these effects. Some commentators have called for an expansion of safety net and income assistance programs, or even for the institution of widespread unconditional monetary transfers to all citizens, sometimes called a "universal basic income" (UBI).[1] The idea of UBI, the most novel of the policies being discussed, has garnered increasing public support, rising from 12 percent ten years ago to 48 percent in 2018 in a poll of the American public CNBC (2018). Moreover, governments throughout the world have started piloting projects to understand how individuals respond to such a public policy tool and its broader economic consequences Forbes (2015); Wired (2017).

In this paper, we explore the implications of automation for economic growth, income distribution, and redistributive transfers similar to UBI in a task-based model of production. We derive a simple and parsimonious aggregate representation of the production process in which the task-level distribution of relative efficiency of capital and labor is succinctly captured by two variables: the share of tasks done by capital, which we call α, and the

[1] While the idea of unconditional cash transfers has come to the fore recently, it is an idea with a long history in economics. Milton Friedman famously advocated replacing our current system of means-tested welfare programs and progressive income taxation with a "negative income tax" system— a scheme whereby households would pay a flat tax rate, and would receive an unconditional transfer from the governmentFriedman (1966).

average productivity of capital across tasks done by capital, A. Moreover, this representation intuitively captures the distinction between automation and traditional technological progress — changes in the distribution of relative efficiency of capital that lead to an increase in α represent automation, while technical change that increases A is traditional technical progress.[2]

We use this aggregate representation to derive a simple and intuitive condition that is both necessary and sufficient for technical progress to ultimately result in zero labor share for a given level of capital taxation. Specifically, all that matters is what happens to the average productivity of capital, A, as the share of tasks done by capital, α, approaches 1. We call this variable $A(1)$. Then as long as $A(1)$ is above a threshold value r^*, we find that production of robots always "pays for itself", and the economy will grow through the accumulation of capital alone, with the labor share asymptotically approaching zero.[3] We also show that, while traditional technical progress can lead to sustained economic growth when tasks are gross substitutes ($\sigma > 1$, where σ is the elasticity of substitution between tasks), full automation (defined as strictly positive capital productivity for all tasks) is a precondition for sustained economic growth when task substitutability is low ($\sigma < 1$).

We next study the consequences of technical progress in a particularly tractable case that allows us to cleanly distinguish between traditional technical progress and automation. In a steady state, both traditional technical progress and automation lead to a decline in the labor share of income when task substitutability is high ($\sigma > 1$), but automation results in a decline in the labor share of income even when task substitutability is low ($\sigma < 1$). Moreover, short-run distributional outcomes for the labor share are always worse for automation than for traditional technical progress. These results accord with the perception of automation outlined above.

The above results are all derived for a given level of taxation and transfers to workers. The final step of our analysis is to endogenize capital taxation and study the likely political consequences of automation. We assume that policy is set by majority voting and workers are in the majority. Thus, the political economy equilibrium is equivalent to the solution of a Ramsey Planner seeking to maximize the welfare of workers with given policy tools. These policy tools include a uniform capital income tax and a transfer to workers (intended to capture the idea of a UBI). This setup is closely related to a well-known problem in optimal capital taxation (e.g. famously studied by Judd (1985) and Chamley (1986), and more recently by Straub and Werning (2020)). Our model differs in that we allow for

[2]Of course, most technological changes will shift both variables, and thus this representation allows a decomposition of a particular episode of technological progress into a labor-substituting "automation" component, and a labor-complementing "traditional technical progress" component.

[3]Whether it reaches zero will depend on workers' preference for leisure and the tax rate.

relatively impatient workers and the possibility of continuous growth (due to automation). We show that positive taxation of capital may arise when workers are impatient (relative to entrepreneurs), even when entrepreneurs' coefficient of relative risk aversion is less than 1. Moreover, we derive simple expressions for capital taxes with log utility which extends to the case where continuous growth is possible. We show that capital taxation for redistributive purposes is constant as technology progresses up to the point that continuous growth becomes possible (at this tax rate). Above this point, further technological progress increases the rate of growth and leads to a lower optimal tax rate.

Related Literature. We use task-based modeling of production and model automation as a process of replacing human labor with capital in particular tasks as put forward by Acemoglu and Restrepo (2018a) and Acemoglu and Restrepo (2018b). We find this modeling of automation very intuitive: the basic dynamics of substituting capital for labor due to automation operates at the level of the tasks that are undertaken for the production of goods. Moreover, with this modeling of automation in view, the task-based representation of production becomes a natural choice because the traditional division of all the tasks performed in the productive sector into distinct goods is neither central nor necessary. Furthermore, disregarding good-based labeling of tasks and instead focusing on their factor-usage allows a sharper analysis of the questions that interest the literature on automation.

The task-based model of production and automation has been used to explore a number of important issues related to automation. For example, Acemoglu and Restrepo (2017) apply the task-based framework in a structural model to examine the causes of the recent decline in manufacturing employment. They find that a significant share has been due to automation with a more limited role played by globalization.

Despite the abundance of empirical evidence on the role of automation in income distribution, and the likely importance of a continuation of these trends, there has been relatively little theoretical analysis of the consequences of labor-substituting technological change, particularly in a political economy framework. A few papers have examined the consequences of automation within the framework of standard growth models. In particular, our work vis-a-vis growth and income distribution implications is related to Aghion et al. (2017), who consider how automation and artificial intelligence (AI) alter standard results in models of economic growth. An early example is Zeira (1998), who argues that different incentives to engage in automation may play a role in explaining international differences in productivity. A few theoretical papers have recently analyzed the distributional consequences of automation. Korinek and Stiglitz (2017) analyze the consequences of automation for income inequality. Guerreiro et al. (2017) examine optimal taxation of labor-substituting capital with consideration of distributional implications, though they do not analyze this in a political economy framework.

As discussed above, our analysis of the political economy equilibrium under majority voting is closely related to a strand of the literature on optimal capital taxation. Judd (1985) and Chamley (1986) famously found that zero capital taxation is optimal in the long run. Lansing (1999) pointed out that this result does not hold for log utility, a result explained by Reinhorn (2019). More recently, Straub and Werning (2020) studied this problem and concluded that the original Chamley-Judd result only holds when entrepreneurs have an intertemporal elasticity of substitution below 1. Our analysis differs in two respects — first that we allow workers to be relatively impatient compared to entrepreneurs, and secondly (and more importantly) that we allow for the possibility of continuous growth through capital accumulation arising from automation.

Organization The paper is organized as follows: Section 2 lays out the model. Section 3 outlines the long-run growth implications of automation and contrasts it with that of traditional technical progress. Section 4 explores the implications of an episode of technical change for the income distribution in a particularly tractable case. In Section 5, we work out the implications of automation for taxation from a political economy perspective.

2 The Model

The model is set in continuous time with an infinite horizon and periods $t \geq 0$. There are four types of agents in the economy: workers who supply labor and consume their income every period; entrepreneurs, who own capital; firms, which rent capital and hire labor and produce goods; and the government which levies various kinds of taxes.

2.1 The Workers

There is a unit measure of workers with lifetime preferences

$$\int_0^\infty e^{-\gamma t} U\left(C_t^w, L_t\right) dt, \tag{1}$$

where C_t^w is consumption and $L_t \geq 0$ is labor supply. The worker households supply L_t units of labor to firms, for which they receive an after-tax wage rate of $\left(1-\tau_t^\ell\right) w_t$. They also receive a transfer from the government, T_t^w. They consume all of their income instantly, so their consumption is

$$C_t^w = \left(1-\tau_t^\ell\right) w_t L_t + T_t^w. \tag{2}$$

We assume that workers are not able to own capital. Therefore they exhibit hand-to-mouth consumption behavior by assumption. Their only non-trivial decision is labor supply, which satisfies the condition

$$\left(1-\tau_t^{\ell}\right) w_t U_C\left(C_t^w, L_t\right) \leq -U_L\left(C_t^w, L_t\right), \tag{3}$$

which holds with equality when $L_t > 0$.

A particularly useful utility function is log in consumption and leisure:

$$U\left(C, L\right) = \log\left(C\right) + \phi \log\left(1-L\right),$$

in which case labor supply satisfies

$$L_t = \frac{1}{1+\phi} \cdot \max\left(0, 1 - \frac{\phi T_t^w}{\left(1-\tau_t^{\ell}\right) w_t}\right).$$

The condition for zero labor supply is

$$\frac{T_t^w}{\left(1-\tau_t^{\ell}\right) w_t} \geq 1.$$

In other words, if the transfer is sufficiently large relative to the after-tax wage, the household does not supply any labor.

2.2 The Entrepreneurs

There is also a unit measure[4] of entrepreneurs who have preferences given by

$$\int_0^{\infty} e^{-\rho t} U\left(C_t^e\right). \tag{4}$$

Entrepreneurs each own capital k_t, which they rent to firms at rental rate r_t. Capital depreciates at rate δ, and capital income is subject to a tax of τ_t^k gross of depreciation. Therefore, an entrepreneur's budget constraint is given by

$$\dot{k} + c_t^e = r_t^k k_t + T_t^e, \tag{5}$$

[4]The model could be modified so that the relative population of workers and entrepreneurs differ. However, a unit measure of each is the simplest.

where

$$r_t^k \equiv \left(1 - \tau_t^k\right) r_t - \delta \tag{6}$$

is the after-tax gross return on capital, and T_t^e is a transfer entrepreneurs receive from the government. Their consumption behavior satisfies the maximization condition

$$u'(c_t^e) = \lambda_t, \tag{7}$$

the co-state equation on capital

$$\dot{\lambda}_t = \lambda_t(\rho - r_t^k), \tag{8}$$

and the transversality condition on capital

$$\lim_{t\to\infty} e^{-\rho t}\lambda_t k_t = 0. \tag{9}$$

2.3 The Firms

The firms choose an optimal production plan for given wage w_t and rental rate of capital r_t^k. The production technology is constant-returns-to-scale (CRS) and is given by

$$Y_t = \left[\int_0^1 \left(y_t\left(i\right)\right)^{1-\frac{1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}, \tag{10}$$

where $y_t\left(i\right)$ is the contribution of task i to overall output, Y_t. There is a unit measure of tasks indexed by $i \in [0, 1]$. Each task can be performed by labor or capital. The output of task i satisfies:

$$y_t\left(i\right) = a_t\left(i\right) k_t\left(i\right) + \gamma_t\left(i\right) \ell_t\left(i\right) \tag{11}$$

where $a_t\left(i\right)$ denotes the productivity of capital in task i, and $\gamma_t\left(i\right)$ denotes the productivity of human labor in task i. We assume that $a_t\left(i\right), \gamma_t\left(i\right) \geq 0$ for all i, and that tasks are ordered such that $a_t\left(i\right)/\gamma_t\left(i\right)$ is weakly decreasing in i. We further require that $k_t\left(i\right), \ell_t\left(i\right) \geq 0$.

We adopt a task-based aggregation of the overall production process of the economy. This choice is more appropriate for our purposes as our focus is on automation and the process of automation is associated with substitution between capital and labor at the level of tasks, rather than goods. Therefore, tasks represent an intuitive disaggregated unit for the specification of the production technology.

2.4 An Aggregate Representation of the Production Process

The production technology specified in (10) depends on choices of capital and labor for each task i, as can be seen from (11). It is possible to simplify this representation using ideas from duality theory. Formally, we set up a firm's optimization problem and use its optimality conditions to simplify the production function.

We can write the firm optimization problem formally as

$$\max_{\ell_t(i),k_t(i)} \left\{ \left[\int_0^1 \left(a_t\left(i\right) k_t\left(i\right) + \gamma_t\left(i\right) \ell_t\left(i\right) \right)^{1-\frac{1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} - w_t \int_0^1 \ell_t\left(i\right) di - r_t \int_0^1 k_t\left(i\right) di \right\}, \quad (12)$$

subject to non-negativity constraints for factor inputs, $k_t\left(i\right)$ and $\ell_t\left(i\right)$. This yields two optimality conditions,

$$w_t \geq \gamma_t\left(i\right) \cdot \left(\frac{Y_t}{y_t\left(i\right)} \right)^{\frac{1}{\sigma}} \quad (13)$$

and

$$r_t \geq a_t\left(i\right) \cdot \left(\frac{Y_t}{y_t\left(i\right)} \right)^{\frac{1}{\sigma}}, \quad (14)$$

which hold with equality for positive use of labor or capital respectively in the performance of task i.

With linear substitution between capital and labor in the performance of a given task, it will be performed by capital only if $a_t\left(i\right)/\gamma_t\left(i\right) > r_t/w_t$, and by labor only if $a_t\left(i\right)/\gamma_t\left(i\right) > r_t/w_t$. If $a_t\left(i\right)/\gamma_t\left(i\right) = r_t/w_t$, then the task may be performed by both capital or labor. This will often pin down a unique production plan, but if $a_t\left(i\right)/\gamma_t\left(i\right) = r_t/w_t$ for all tasks in an interval, then there are many equivalent production plans. To avoid this, we adapt the convention that (1) all tasks are done purely by capital or purely by labor, and (2) if $i < j$, then it is never the case that i is done by labor and j by capital. We can then set α_t such that tasks $i < \alpha_t$ are done only by capital, and tasks $i > \alpha_t$ are done only by labor. Therefore, α_t is the share of tasks done by capital and it will satisfy:

$$\begin{cases} \frac{a_t(i)}{\gamma_t(i)} \geq \frac{r_t}{w_t} & \text{for } i < \alpha_t \\ \frac{a_t(i)}{\gamma_t(i)} \leq \frac{r_t}{w_t} & \text{for } i > \alpha_t. \end{cases} \quad (15)$$

In the event that $a_t\left(i\right)/\gamma_t\left(i\right)$ is strictly decreasing (at least in the neighborhood of α_t), α_t will be uniquely defined by

$$\frac{a_t\left(\alpha_t\right)}{\gamma_t\left(\alpha_t\right)} = \frac{r_t}{w_t}. \quad (16)$$

Otherwise, α_t will be determined together with factor demands.

For tasks done by capital ($i \leq \alpha_t$), the optimal production of task i satisfies

$$r_t = \left(\frac{Y_t}{a_t\left(i\right) \cdot k_t\left(i\right)} \right)^{\frac{1}{\sigma}} a_t\left(i\right). \tag{17}$$

Thus $\left(a_t\left(i\right)\right)^{1-\sigma} k_t$ is the same across all $i \leq \alpha_t$ and aggregate inverse capital demand, given unit measure of entrepreneurs, satisfies:

$$r_t = \left(A_t\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\alpha_t Y_t}{K_t} \right)^{\frac{1}{\sigma}}. \tag{18}$$

where

$$K_t \equiv \int_0^{\alpha_t} k_t\left(i\right) di \tag{19}$$

is aggregate capital and

$$A_t(\alpha_t) \equiv \left[\frac{1}{\alpha_t} \int_0^{\alpha_t} \left(a_t\left(i\right)\right)^{\sigma-1} di \right]^{\frac{1}{\sigma-1}} \tag{20}$$

is the productivity of capital. Analogously, inverse labor demand, given a unit measure of workers, satisfies

$$w_t = \left(\Gamma_t\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\left(1-\alpha_t\right) Y_t}{L_t} \right)^{\frac{1}{\sigma}}, \tag{21}$$

where

$$L_t \equiv \int_{\alpha_t}^{1} \ell_t\left(i\right) di \tag{22}$$

is aggregate labor and

$$\Gamma_t(\alpha_t) \equiv \left[\frac{1}{1-\alpha_t} \int_{\alpha_t}^{1} \left(\gamma_t\left(i\right)\right)^{\sigma-1} di \right]^{\frac{1}{\sigma-1}} \tag{23}$$

is labor productivity.

The expressions above allow us to derive the following expression for aggregate production as a function of aggregate capital and labor:

$$Y_t \equiv F(K_t, L_t) = \left[\alpha_t^{\frac{1}{\sigma}} \left(A_t(\alpha_t) K_t\right)^{1-\frac{1}{\sigma}} + \left(1-\alpha_t\right)^{\frac{1}{\sigma}} \left(\Gamma_t(\alpha_t) L_t\right)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{24}$$

where $\{A_t(\alpha_t), \Gamma_t(\alpha_t), \alpha_t\}$ are defined as above and

$$F_L(K_t, L_t) = w_t \tag{25}$$

$$F_K(K_t, L_t) = r_t. \tag{26}$$

Thus, the production function has the fairly standard CES representation, except for the presence of α_t, the share of tasks performed by the capital. That is, we have a simple and parsimonious aggregate representation of the production process in which the task-level distribution of relative efficiency of capital and labor is succinctly captured by a single parameter identifying the marginal task that separates the tasks that are done by capital and labor respectively.

This, however, has some important implications about changes in the productivity of capital. In the standard case, it shows up just as a change in A (which in that case is independent of α) and, if $\sigma < 1$, it implies labor-complementing technical progress. In our case, an increase in capital's productivity also increases α and thus will necessarily have a labor-substituting effect. The relative distribution of these two (labor-complementing and labor-saving) effects, among other things, depends on the distribution of the technical progress across the tasks. More broadly speaking, this parameter/marginal task (α) quite intuitively captures the distinction between automation versus traditional technological progress. A change in the relative efficiency of capital and labor that shows up as a change in this parameter represents automation, and the orthogonal component represents traditional technical progress.

2.5 Government

The government sets taxes on capital income and labor income. It uses this income to pay for transfers to workers and entrepreneurs, and also to finance expenditures, G_t, which otherwise do not enter the model. The government budget constraint, therefore, is

$$\tau_t^\ell w_t L_t + \tau_t^k r_t K_t = T_t^w + T_t^e + G_t \tag{27}$$

In the political economy equilibrium considered in the paper, the government's tax rates are decided based on majority voting. Although, for simplicity, we assumed both entrepreneurs and workers have unit measure, we assume, on the margin and consistent with fact, workers are the majority and, hence, tax rates are chosen to maximize a worker's welfare.

3 Economic Growth: Steady-state versus sustained growth

In this section, following the literature on automation, we explore the implications for growth of the task-based CRS production specification (with perfect substitutability between capital and labor at task level). Specifically, we are interested in when the model will exhibit a steady-state versus continuous growth. Since our focus here is on long-run growth implications, we focus on the stationary version of the model. To induce stationarity, we make a few additional assumptions. First, we suppose that the capital and labor productivity of each task is time-invariant, *i.e.*,

$$a_t(i) = a(i) \quad \text{and} \quad \gamma_t(i) = \gamma(i). \tag{28}$$

Further, suppose that government policies are also time-invariant with zero government spending, constant tax rates, and zero transfers to entrepreneurs, so that:

$$G_t = 0, \qquad \tau_t^\ell = \tau^\ell, \qquad \tau = \tau^k, \quad \text{and} \quad T_t^e = 0. \tag{29}$$

Setting the government's spending and transfers to entrepreneurs to zero not only somewhat simplifies the discussion that follows immediately, but also is consistent with the spirit of the subsequent politico-economic analysis where the focus is on the use of government policy for redistribution from entrepreneurs to workers.

Under these assumptions, the model equations can be written as follows: The government budget constraint implies that transfers to workers satisfy

$$T_t^w = \tau^\ell w_t L_t + \tau^k r_t K_t, \tag{30}$$

which when substituted into (2) gives worker's consumption as

$$C_t^w = w_t L_t + \tau^k r_t K_t, \tag{31}$$

whereas entrepreneur consumption, using (19), satisfies

$$\dot{K}_t + c_t^e = r_t^k K_t, \tag{32}$$

and the other equations of the model are unchanged, except for time-invariance implied by (28) and (29).

3.1 The existence of a steady state and the critical interest rate, r^*

When does a steady state exist? A steady-state implies constant consumption of entrepreneurs, $c_t^e = c^e$. Then, the co-state equation on capital and the entrepreneurs' maximization condition, combined with the definition of r_t^k, requires that

$$r = \frac{1}{1-\tau^k}(\rho+\delta) \equiv r^* > 0 \tag{33}$$

Thus the steady-state r, which we call the critical interest rate and denote as r^*, does not depend on technology (besides the depreciation rate). It depends only on the entrepreneurs' time discount rate and the tax rate on capital (plus the depreciation rate).

Thus, while a steady state requires that $r = r^*$, if instead it is always the case that $r_t > r^*$, then the entrepreneurs' consumption grows continuously, *i.e.*,:

$$\dot{c}_t^e > 0$$

More generally, there is now continuous growth through capital accumulation. Further, as capital grows, the relative role of labor becomes small, and the economy approaches an AK model. In particular, the labor share of income approaches zero in this case and is zero along the corresponding balanced growth path. This leads us to our first proposition:

Proposition 1. *The economy has sustained economic growth if and only if*

$$A(1) > r^* > 0$$

where $A(1)$ is defined by (20), which, in particular, implies $A(1) = 0$ when $\sigma < 1$ and $a(i) = 0$ for a positive measure of tasks.

Proof. First note that, if $r_t > r^*$, then from the entrepreneur's maximization condition and co-state equations (7, and 8), entrepreneurial consumption c_t^e is strictly increasing in t, which (from the budget constraint) also implies strictly increasing capital over time, i.e. continuous growth. Thus it is sufficient to prove the claim to show that $r_t \geq A(1)$. To show that $r_t \geq A(1)$, suppose that $L_t = 0$. In this case, from the aggregate representation of the production function, production satisfies:

$$Y_t = A(1) \cdot K_t$$

and the marginal product of capital satisfies:

$$r_t = A(1)$$

Then the proposition holds as long as r_t is increasing in L_t. This we prove in the appendix. $\square$

Interestingly, Proposition 1 shows that the possibility of continuous growth does not depend at all on initial conditions (such as the initial level of capital), but only on relative values of $A(1)$, the return to capital in the limiting case of full automation, and r^*. While $A(1)$ is solely a function of the production technology, recall that r^* depends on both the patience (discount rate) of entrepreneurs and government policy (tax rate on capital, τ^k). In general, the level of capital or growth rate of an economy depends on the tax rate on capital. However, Proposition 1 says more. In particular, it shows that the response of government policy to automation may potentially determine whether such a change leads to sustained growth or merely a steady state with higher capital.

To examine this interplay of policy and technical change, we propose a political economy model in the next section. However, before that, we now elaborate on the implications of Proposition 1.

1. If $a(i) > r^*$ for all i, then $A(1) > r^*$ and there is sustained growth. The intuition is simple. If capital is sufficiently productive at all tasks, sustained growth will occur.

2. If $\sigma < 1$ and $a(i) = 0$ for a positive measure of tasks, then $A(1) = 0 < r^*$ and no long-run growth is possible. Note that a positive measure is necessary. It is not sufficient for $a(i) = 0$ just at $i = 1$.[5] For example, for linear productivity $a(i) = b(1-i)$, we have $A(1) = \sigma^{\frac{1}{1-\sigma}} b$.
   The intuition is as follows. First note that, despite generic perfect substitutability between capital and labor at task level, labor is a necessary factor for the tasks in which capital has zero productivity. When $\sigma < 1$, as each task is necessary, this indispensibility of labor at the task level shows up at the aggregate level if capital has zero productivity in a positive measure of tasks. Moreover, labor commands a positive share of income that is bounded away from zero and constrains the benefit of continued capital accumulation, and hence, sustained economic growth. This is related to Baumol's cost disease (Baumol and Bowen, 1966).

3. If $\sigma > 1$, a sufficient condition for sustained growth is that there exists m such that for all $i \in [0, m]$ we have:
$$a(i) \geq m^{\frac{1}{1-\sigma}} r^*$$

[5] Given, our assumption of $a(i)/\gamma(i)$ being weakly decreasing, it is not possible for a set of measure zero where $a(i) = 0$ to contain more than a single point, $i = 1$.

This inequality can be satisfied even if $a(i) = 0$ for a positive measure of tasks. While labor is still necessary for the tasks with zero capital productivity, those tasks themselves are not necessary when $\sigma < 1$. Rather, the tasks are gross substitutes and sufficiently high productivity on a subset of tasks is enough to ensure sustained growth. The level of productivity necessary depends on the size of the subset m and the degree of substitutability across tasks σ. As $\sigma \to \infty$, the necessary interval of productive tasks approaches 0 — intuitively, if tasks are perfect substitutes, only one task with high productivity is needed, since only one task is necessary in production. If tasks are only slight substitutes, there will need to be either a very high productivity of the subset of tasks, or a measure of tasks close to 1.

4 Implications of the Nature of Technical Progress

How is an increase in the efficiency of capital/machines related to economic growth and the distribution of income in our model? This section examines this issue in detail. Specifically, we analyze how the improvement in the ability of capital to perform each of the tasks—as represented by the $a(i)$ curve—affects economic growth and the labor share of income.

To clearly contrast the impact of automation from traditional technical progress, we start with a stylized, but very useful, form of the $a(i)$ curve where

$$a(i) = \begin{cases} a & \text{for } 0 \leq i \leq \bar{\alpha} \\ 0 & \text{for } \bar{\alpha} < i \leq 1 \end{cases} \tag{34}$$

along with fixing $\gamma(i) = 1$, $i \in [0, 1]$. Thus, the capital can perform a fraction $\bar{\alpha}$ of the tasks (for $i \in [0, \bar{\alpha}]$) at the same level of efficiency (relative to labor, $a(i)/\gamma(i)$) denoted by efficiency parameter a. However, it cannot do other tasks, $i \in (\bar{\alpha}, 1]$ at all.

In this stylized representation, traditional technical progress—which we construe as increased efficiency in performing tasks currently being perform by capital—can be viewed as an increase in parameter, a. In contrast, automation—which is considered to be the ability of capital to do tasks that earlier could only be done by labor—naturally can be thought of as an increase in $\bar{\alpha}$. Thus, in this task-based production representation, we can think of traditional technical progress as progress on the *intensive* margin (ability to better accomplish the tasks that can be already done) whereas automation can be thought of as progress on the *extensive* margin (ability to do tasks that could not be done earlier).[6] Finally, to high-

[6] Technical progress that simultaneously combines both the intensive and the extensive margin is also

light the distinction between the nature of technical progress outlined above, without loss of generality, we assume $r^* < A(1)$. Thus, as per Proposition 1, technical progress results in transition (in the long run) from one steady to another steady state.

We now highlight some of the implications of the proposed stylized representation of $a(i)$ in (34) along with the assumption of $\gamma(i) = 1$ and begin with the following lemma:

Lemma 1. *Given the $a(i)$ and $\gamma(i)$ as above and given a value of L, there exists a $\bar{K}(a, \bar{\alpha}) = \frac{\bar{\alpha}}{1-\bar{\alpha}} \frac{L}{a}$ such that for $0 \leq K \leq \bar{K}(a, \bar{\alpha})$, the production function has following linear representation*

$$Y(K, L) = aK + L. \tag{35}$$

In particular, $\bar{K}(a, \bar{\alpha})$ is the level of capital stock for which all tasks from 0 to $\bar{\alpha}$ are done using capital alone (and remaining using only labor).

Proof. See Appendix. □

For the positive values of K for which Lemma 1 is applicable, despite capital's equal efficiency in doing all tasks 0 to $\bar{\alpha}$, the stock of capital is not enough to do all these tasks solely using capital. Moreover, as $a(i)/\gamma(i) = r/w$, for all these tasks the production plan is not unique, as mentioned earlier. It is useful to focus on two alternative production plans. The first alternative, is to follow the convention of Section 3.4, which implies that there exists an $\alpha < \bar{\alpha}$ and all tasks with $0 \leq i \leq \alpha$ are done solely using capital and tasks with $\alpha < i \leq \bar{\alpha}$ (as well as those with $\bar{\alpha} < i \leq 1$) are done only with labor. The second alternative assumes all tasks with $0 \leq i \leq \bar{\alpha}$ are done using both capital and labor in a fixed proportion.

The linear representation of the production function in Lemma 1, which implies no diminishing returns to either factor of production, has strong resemblance to a result in the literature on international trade related to the "cone of diversification" for production in a small-open economy (see Dixit and Norman (1980), Bhagwati et al. (1998), and Grossman and Helpman (1991)). For example, in a two-good, two-factor economy with goods prices fixed in international markets, zero profit conditions for the production of the two goods imply fixed values of the two factor prices. Thus, within the cone of diversification, the aggregate economy displays no diminishing returns to changes in (relative) quantity (or endowment) of factors just as our linear representation in (35). The changes in relative endowments are absorbed by changes in relative production of the two goods, without setting off a change in relative factor prices or diminishing returns to scale. A similar mechanism is operational in our setup as changes in K in the range specified by Lemma 1 merely rearrange

considered in future. Moreover, to contrast the growth and distributional implications of two types of technical progress, for now, we focus on long-run outcomes. Issues of transition are secondary to our purpose are pursued in a later section.

the production of tasks between capital and labor as changes in K change relative quantities of capital and labor. This is feasible since both capital and labor are equally cost-effective in doing all tasks from 0 to $\bar{\alpha}$.

Lemma 1 also says that if there is no capital in the economy (this can happen in equilibrium, as discussed below), the production function reduces to being linear in labor, $Y = L$, which is very intuitive as the production function has constant returns to scale and there is only one factor of production (labor) being used in all tasks. However, our production function has a very interesting "bang-bang" outcome for capital in the long run as the economy emerges from the no-capital phase (say, due to an increase in a) as summarized in the following lemma:

Lemma 2. *In steady state, (i) if $a < r^*$, then $\alpha = 0$ and equilibrium capital stock $K = 0$ implying $Y(K, L) = Y(L) = L$, (ii) if $a = r^*$, then there is a continuum of steady states with $\alpha \in [0, \bar{\alpha}]$ and equilibrium capital stock $K = \bar{K}(r^*, \alpha)$ so that $Y(K, L) = Y(\bar{K}, L) = a\bar{K}(r^*, \alpha) + L$, and (iii) if $a > r^*$, then $\alpha = \bar{\alpha}$ and equilibrium capital stock $K = \hat{K}(a, \bar{\alpha}, r^*) = \frac{\bar{\alpha}}{(1-\bar{\alpha})^{-\frac{1}{\sigma-1}}((\frac{r^*}{a})^{\sigma-1}-\bar{\alpha})^{\frac{\sigma}{\sigma-1}}}\frac{L}{a} > \bar{K}(r^*, \bar{\alpha})$ with $Y(K, L) = \left((\bar{\alpha})^{\frac{1}{\sigma}} (aK)^{\frac{\sigma-1}{\sigma}} + (1-\bar{\alpha})^{\frac{1}{\sigma}} L^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$ so the production function exhibits the usual diminishing returns in both factors.*

Proof. The results follow easily from the linear representation in Lemma 1. (i) follows from the fact that the return on capital (a) is below the return that the entrepreneurs want (r^*); (ii) arises because in the knife edge case of $a = r^*$ any value of $0 \le K \le \bar{K}(r^*, \bar{\alpha})$ is consistent with the entrepreneurs' Euler equation in the steady state because with a linear production function there are no diminishing returns to capital accumulation in that range of K; and (iii) is the case where Lemma 1 is not applicable so the argument in (ii) about no diminishing returns does not apply. For the derivation of the expression for $\hat{K}$, see Appendix. Note that $\hat{K}(r^*, \bar{\alpha}, r^*) = \bar{K}(r^*, \bar{\alpha})$. □

Lemmas 1 and 2 are also useful for examining the role of traditional technical progress versus automation to which we turn next. As we focus initially on comparative statics, Lemma 2 tells us that it is most instructive to model traditional technical progress as an increase in a upwards from r^* which will result in an increase in K (but keep α fixed at $\bar{\alpha}$) and will be accompanied by diminishing returns. In contrast, automation, modeled as an increase in $\bar{\alpha}$, will increase $\bar{K}(a, \bar{\alpha})$ and, depending on the initial value of K, involves a transition period in which capital accumulation occurs without experiencing diminishing returns.

4.1 Growth Implications of Automation

Traditional technical progress, as mentioned above, is an increase in parameter a of the $a(i)$, which implies a uniform increase in the efficiency of capital (relative to labor) in doing the tasks it already does (*i.e.,* tasks with $i \in [0, \bar{\alpha}]$). In contrast, automation is an increase in parameter $\bar{\alpha}$ of the $a(i)$, which implies an increase in fraction of tasks that capital can do without an increase in the efficiency of capital (relative to labor) in doing the tasks it already does (*i.e.,* with $i \in [0, \bar{\alpha}]$). We thus have the following easy corollary of Proposition 1:

Corollary 1. *Full automation, defined as the ability of capital to perform all tasks (except for a measure zero of tasks), is a precondition for sustained economic growth when task substitutability is low ($\sigma < 1$).*

Proof. Follows immediately from remark 3 following Proposition 1. □

The key observation here is that when $\sigma < 1$, automation (*i.e.*, increase in $\bar{\alpha}$) is a necessary condition for sustained growth as long as $\bar{\alpha} < 1$. In the absence of automation, traditional technical progress is unable to deliver sustained economic growth. Every episode of automation (increases in $\bar{\alpha}$) gives an additional boost to growth from traditional technical progress. Only when there is full automation, *i.e.*, $\bar{\alpha} = 1$, can traditional technical progress lead to sustained economic growth.

4.2 Traditional Technical Progress and Income Distribution

Traditional technical progress in our setup is modeled as an increase in parameter a of the $a(i)$, which implies a uniform increase in the efficiency of capital (relative to labor) in doing the tasks it already does (*i.e.,* tasks with $i \in [0, \bar{\alpha}]$). To evaluate the effect of traditional technical progress on the income distribution, we focus on the comparative statics of changes in a. The interesting and relevant range for a is r^* and higher, as capital is not used in production below r^*. As Lemma 2 shows, technical progress does not operates on the extensive margin as the range of tasks that are performed by capital remains unchanged. Formally, from Lemmas 2 we have:

Proposition 2. *Traditional technical progress, modeled as an increase in a, increases wages, whereas the labor share of income (i) increases for $\sigma < 1$; (ii) remains constant for $\sigma = 1$ (the Cobb-Douglas case); and (iii) decreases for $\sigma > 1$. In particular,*

$$\frac{dw}{da} = \frac{\bar{\alpha}}{\left(\frac{r^*}{a}\right)^{\sigma-1} - \bar{\alpha}} \frac{w}{a} > 0 \tag{36}$$

and

$$\frac{d(wL/Y)}{da} = \frac{\bar{\alpha}(1-\sigma)}{a}\left(\frac{r^*}{a}\right)^{1-\sigma} \gtreqless 0 \qquad as \qquad \sigma \lesseqgtr 1. \tag{37}$$

Proof. One can easily show that the equilibrium wage is given by

$$w = \left(\frac{1-\bar{\alpha}}{\left(\frac{r^*}{a}\right)^{\sigma-1} - \bar{\alpha}}\right)^{\frac{1}{\sigma-1}} \left(\frac{r^*}{a}\right) \tag{38}$$

which yields

$$\frac{dw}{d\left(r^*/a\right)} \frac{r^*/a}{w} = -\left[\frac{\bar{\alpha}}{\left(\frac{r^*}{a}\right)^{\sigma-1} - \bar{\alpha}}\right] < 0 \tag{39}$$

which then implies (36). The denominator in (36) (as well as (38)) is positive. For $\sigma < 1$, it is positive as $a > r^*$ so that $\left(\frac{r^*}{a}\right)^{\sigma-1} > 1$, and therefore it is definitely greater than $\bar{\alpha}$. The denominator is positive even when $\sigma > 1$ and the condition is the same as for the existence of the steady state.

Moreover, total output can be written as

$$Y = (1-\bar{\alpha})^{-1} w^{\sigma} L \tag{40}$$

so that the elasticity of output with respect to the wage is less than 1 and hence the labor share of income increases with technical progress. Specifically, the labor share is given by

$$\frac{wL}{Y} = 1 - \bar{\alpha}\left(\frac{r^*}{a}\right)^{1-\sigma} \tag{41}$$

which implies (37). $\square$

Note that for $a > r^*$ (and $A(1) < r^*$), the aggregate production function has the standard Constant-Elasticity-of-Substitution (CES) form (see Lemma 2), implying that capital and labor are gross (or Edgeworth) complements. Thus, standard results from production theory show that an increase in a will lead to an increase in the wage, w, and the marginal product of labor. However, the change in the labor share of income crucially depends on σ, which controls the degree of substitutability between capital and labor. As is well known, when $\sigma = 1$, the opposing substitution effect and the Edgeworth complementarity exactly cancel each other and the labor share of income/output is constant. However, for $\sigma > 1$ the substitution effect dominates causing the labor share to decline despite the increase in wages, whereas the reverse is true for $\sigma < 1$. Thus, the distributive implications of traditional technical progress are as expected and we now turn to the case of automation.

4.3 Automation and Income Distribution

As mentioned earlier, automation is an increase in parameter $\bar{\alpha}$ of the $a(i)$, which implies an increase in the fraction of tasks that capital can do without an accompanying increase in the efficiency of capital (relative to labor) in doing the tasks it already does (*i.e.,* with $i \in [0, \bar{\alpha}]$). Again, for the steady-state (comparative static) analysis, Lemma 2 now shows that when $a > r^*$, automation operates only on the extensive margin as the range of tasks performed by capital increases with an increase in $\bar{\alpha}$.

While, across steady states, we are still dealing with traditional CES production functions, the standard results from production theory are not directly intuitively usable to assess the impact of changes in $\bar{\alpha}$. However, we have the following easy result from Lemma 2 and equations (38) and (41):

Proposition 3. *Automation, that is an increase in $\bar{\alpha}$, increases wages but decreases labor share. In particular,*

$$\frac{dw}{d\bar{\alpha}}\frac{\bar{\alpha}}{w} = -\frac{\bar{\alpha}}{1-\bar{\alpha}}\left[\frac{\frac{\left(\frac{r^*}{a}\right)^{\sigma-1}-1}{1-\sigma}}{\left(\frac{r^*}{a}\right)^{\sigma-1}-\bar{\alpha}}\right] > 0 \tag{42}$$

and

$$\frac{d(wL/Y)}{d\bar{\alpha}} = -\left(\frac{r^*}{a}\right)^{\sigma-1} < 0 \tag{43}$$

Proof. Follows directly from (38) and (41). □

First, an important implication of Propostion 3 is that the effect of automation, unlike that of traditonal technical progress outlined Proposition 2, does not depend on the elasticity of substitution σ. Second, these effects are adverse both for wages and for labor's income share. Thus, in a sense, automation is bad news for workers both from an absolute (*i.e.*, consumption level) and a relative (*i.e.*, consumption relative to entrepreneurs) perspective—quite generally and irrespective of task substitutability governed by σ. Third, the qualitative effects of automation compared to traditional technical progress are worse for $\sigma < 1$ than for $\sigma > 1$.

For $\sigma < 1$, an increase in a not only increases wages, but more importantly, increases the labor share of income. An increase in $\bar{\alpha}$ also increases wages. However, it decreases the labor share of income, making workers unambiguously worse off relative to entrepreneurs. This is a very vital difference that speaks to the current debate centered around automation and associated policy proposals like universal basic income. The key to this difference lies intuitively in the fact that the effect of automation manifests as substitution *within* tasks (or at the task level), whereas the effect of traditional technical progress (a) operates *between* tasks (or across tasks at the aggregate level). Thus, the distributional effect of automation are mainly driven by perfect substitutability of capital and labor at the task level whereas the effect of traditional technical progress arises from "poor" substitutability (*i.e.*, $\sigma < 1$) across tasks, or capital and labor being allocated to different tasks. When $\sigma > 1$, both automation and traditional technical progress increase wages and reduce the labor share of income. We, however, conjecture that for an appropriately defined 'equivalent' increase in the two will show automation to be worse than traditional technical progress from a distributional perspective as the underlying mechanism is same irrespective of the value of σ.

4.4 Automation and Dynamics of the Wage and Labor Share of Income

We can shed further light on how automation changes wages and the labor share of income over time. Specifically, let us consider a one-time automation represented by a one-time increase in $\bar{\alpha}$ from $\bar{\alpha}_o$ to $\bar{\alpha}_1 > \bar{\alpha}_o$ for some fixed $a > r^*$ (and implicitly fixed L for ease of exposition) starting in a steady-state. Moreover, to have a more realistic assessment of the dynamics, the increase in $\bar{\alpha}$ is allowed to occur gradually. In particular, $\bar{\alpha}$ follows the following exogenous process:

$$\dot{\bar{\alpha}} = \theta(\bar{\alpha}_1 - \bar{\alpha}) \tag{44}$$

with $\bar{\alpha} = \bar{\alpha}_0$ initially.

The results are presented for values of σ below and above 1. In each case, we choose parameter values that are very standard in the macro literature. We calibrate the model so that the initial steady state is typical in terms of tax rates, ratio of consumption to output, labor income share, and ratio of capital to output, among other variables. For both values of σ we set $\rho = .04$ which gives an annual real interest rate of 4% and set $\tau^k = \tau^l$ to .2. For ϕ we choose a value of 2, which implies $l = .33$. Similarly, $\delta = .1$ corresponds to 10% depreciation rate for capital. All of these values are standard in the literature. The parameter a for labor productivity is set to 1 and, recall, $\gamma_i = 1$.

In order to calibrate the model, the only parameter that needs to be changed with σ is $\bar{\alpha}_0$. For first choice of $\sigma = .8$, it requires setting $\bar{\alpha} = .5$ and for the other choice of $\sigma = 1.2$, $\bar{\alpha}_0$ needs to be set to .25. The calibrated steady state is very similar in both cases with a labor share of income of about 64.6%. Consumption is about 80% of private sector income, and the capital-to-output ratio is about 2.02.

Finally, to have similar increases in output for both cases of σ we allow for an identical increase in $\bar{\alpha}$ of .10. Moreover, θ is set to .1 which implies the time path for $\bar{\alpha}$ as shown in Figure 1 (for the case with $\sigma = .8$) so that the impact of automation lasts for a couple of decades–a typical duration for a general purpose technology.

Figure 1: Time Path for $\bar{\alpha}$ for $\sigma = .8$

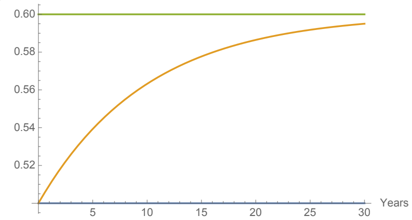
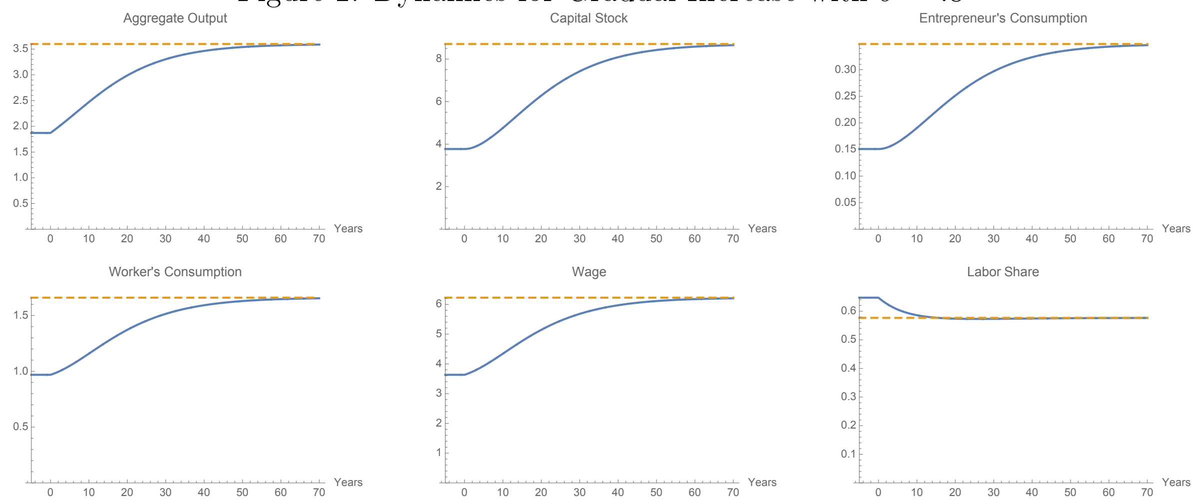
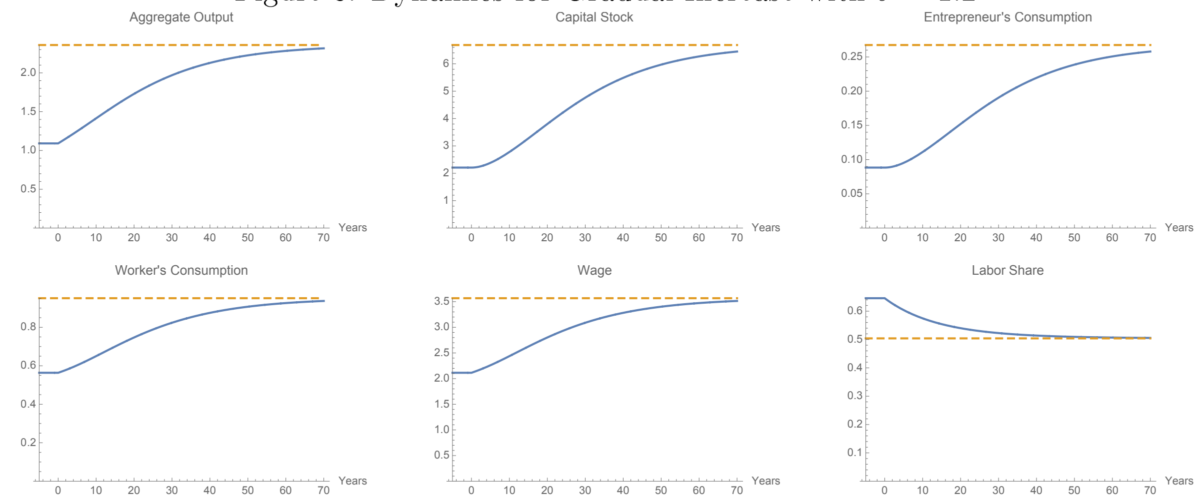


Figure 2: Dynamics for Gradual Increase with $\sigma = .8$



The results for the transitional dynamics are shown in Figure 2 for $\sigma = .8$ and in Figure 3 for $\sigma = 1.2$. Before turning to the dynamics, it is instructive to look at changes across steady states in the two cases. The precise increase in output for σ equal to .8 and 1.2 is 92% and 116%. The labor share of income falls in both cases but, as shown earlier, it falls less for $\sigma = .8$ where is goes down from 64.7% to 57.7% whereas the decline is twice as much from 64.6% to 50.4% for $\sigma = 1.2$. This shows up in relative changes in the consumption of workers and entrepreneurs, which increase by 71.3% and 130.8% for low σ respectively compared to an increase of 68.9% and 202.7% for high σ. In particular, notice that the increase in workers' consumption in itself is very similar for both values of σ. It is just that entrepreneurs gains much more when σ is high. As labor supply does not change across steady states, the wages mirror changes in workers' consumption rising by 71.3% and 68.9% respectively for the low and high σ scenarios.

Figure 3: Dynamics for Gradual Increase with $\sigma = 1.2$



The transition itself is monotonic for all the macro variables as seen in Figures 2 and 3. Additionally, it takes more time for the macro variables to transition compared to the exogenous change in $\bar{\alpha}$ due to the endogenous dynamics of the capital accumulation. However, the transition is more protracted for a high σ of 1.2 than for a σ of .8. This is intuitive since with high σ there is greater capital accumulation by the entrepreneurs which extends the duration of the transition. In fact, the capital-to-output ratio has to increase from 2.02 to 2.83 for $\sigma = 1.2$ compared to the half-as-much increase from 2.02 to 2.42 for $\sigma = .8$.

5 Government Policy under Majority Voting

To examine the interplay of policy and technical change alluded to earlier, we endogenize government policy by considering a political economy model with majority voting, where worker households are in the majority.

5.1 Statement and Solution of the Planning Problem

Consider the model outlined previously in Section 2. Recall that we set the government's spending and transfers to entrepreneurs to zero. This is consistent with the spirit of our politico-economic analysis, which focuses on the use of government policy for redistribution from entrepreneurs to workers. We simplify the analysis somewhat by focusing on cases in which entrepreneurs have CRRA utility:

$$u(c_e) = \begin{cases} \frac{c_e^{1-\varphi}}{1-\varphi} & \varphi \neq 1 \\ \log(c_e) & \varphi = 1 \end{cases}$$

The policy is set by majority voting with full commitment. Since we assume that workers are in the majority, this amounts to setting policy to maximize the welfare of workers. Thus we can formulate the problem as a Ramsey Planner problem. There is only one state variable in the model, the capital stock, K_t. Thus, the problem of the Ramsey Planner is to set a path of $\{C, L, K\}$ subject to resource and implementability constraints, which we derive next.

We would like to reduce the model to a minimal set of restrictions on the choices of main variables $\{C_w, c_e, L, K\}$. Production, r, and w are defined for a given (K, L) as

$$F(K, L), \quad r(K, L), \quad w(K, L).$$

Now observe that choosing τ^k will then imply a choice for r^k according to (6). We can therefore treat $\{r^k\}$ as a choice variable of the planner. Once the path of $\{r^k\}$ is given, we are left with the two conditions, (8) and (32), which, together with the definition of λ, can be combined into a single implementability condition:

$$\dot{c}_e = \frac{1}{\varphi}\left(\frac{c_e + \dot{K}}{K} - \rho\right) c_e \tag{45}$$

Next, we observe that the planner can (essentially) determine the level of labor supply by altering the tax on labor income, so as to satisfy workers' labor supply condtion, (5). Finally, we observe that the remaining conditions only enter as part of the resource constraint. In particular, combining (31), (30), and (32), we get

$$C_w + c_e + \dot{K} = F(K, L) - \delta K. \tag{46}$$

We can therefore state the Ramsey Planning Problem for the social planner which maximizes a worker's life-time utility under majority voting as

$$\max_{\{C^w, c_e^e, K, L\}} \left\{ \int e^{-\gamma t} U(C_w, L) \right\},$$

subject to (45), (46), and

$$L \geq 0 \tag{47}$$

Relation to Literature on Optimal Capital Taxation. This problem is closely related to a well-studied problem in the theory of optimal capital taxation. The formulation of the problem here is nearly identical to that in Judd (1985), more recently studied by Straub and Werning (2020). The chief difference is the possibility of relatively impatient workers ($\gamma > \rho$), and of continuous growth through automation. Therefore the results coincide when $\rho = \gamma$ and a steady state is reached.[7]

5.2 Solving the Ramsey Planning Problem

The Hamiltonian for the Ramsey Planning problem is

$$\mathcal{H} = U\left(C_w, L\right) + \mu \dot{K} + \lambda \left[F\left(K, L\right) - \delta K - \dot{K} - c_e - C_w\right] + \frac{\xi}{\varphi}\left(\frac{c_e + \dot{K}}{K} - \rho\right) c_e + \mu_L L$$

The choice $\left(C_w, L, \dot{K}\right)$ and the state variables are (K, c_e). This yields optimality conditions which we may write as follows:

$$\lambda = U_C^w \tag{48}$$

$$-U_L^w \geq \lambda F_L \tag{49}$$

$$\lambda = \mu + \frac{\xi}{\varphi}\left(\frac{c_e}{K}\right) \tag{50}$$

$$\gamma\mu - \dot{\mu} = \lambda\left(F_K - \delta\right) - \frac{\xi}{\varphi}\left(\frac{c_e + \dot{K}}{K^2}\right) c_e \tag{51}$$

$$\gamma\xi - \dot{\xi} = \frac{\xi}{\varphi}\left(\frac{c_e + \dot{K}}{K} - \rho\right) - \mu \tag{52}$$

In addition, the implementability condition (45) and aggregate resource constraint (46) must hold. These seven equations, along with with complementary slackness conditions on L, determine the four choice variables of the Ramsey problem and the three Lagrange multipliers. Further, since $c_e(0)$ is chosen as a jump variable, we have $\xi(0) = 0$.

Labor Taxation. We begin the implications for labor taxation, which hold generally, and then turn capital taxation, where the results vary depending, among other things, on the nature of the long-run equilibrium (steady state versus balanced growth) and the relative patience of the the workers and entrepreneurs.

[7] We also formulate the problem in continuous time, whereas they use discrete time, but this should not qualitatively alter any result.

The PEE implies zero taxation of labor. The result is unsurprising from an intuitive point of view. Given that there is no government spending and workers have all the political power, the sole of purpose of taxation is to transfer funds from entrepreneurs to workers. This should not rely on (distortionary) labor taxation. To see this clearly, one needs to combine (48) and (49) and set $\mu_L = 0$ as labor taxation is relevant only when there is positive labor supply. This gives

$$-U_L\left(C_w^w, L\right) = U_C\left(C_w, L\right) \cdot F_L\left(K, L\right) \tag{53}$$

which on comparison with (3) with equality, noting (25), implies zero labor taxation.

Finite Interior Steady States. Now suppose that a finite interior steady state is reached. This implies that continuous automation does not occur, i.e. $A(1) < r^*$ in the long run.[8] In this case the problem coincides with the case considered in Judd (1985) and Straub and Werning (2020), with only a minor difference due to the possibility of $\gamma > \rho$.

Suppose further that $\gamma = \rho$. Then (45) implies $c_e = \rho K$, and (50) plus (51) implies:

$$F_K\left(K^*, L^*\right) = \rho + \delta$$

Comparing with the entrepreneurs' Euler equation (8) with constant c_e, this implies $\tau^k = 0$, i.e. zero capital taxation in the long run. This is the famous Chamley-Judd result, derived separately by Chamley (1986) and Judd (1985), though our formulation most closely matches the latter.

This benchmark result has recently been challenged by Straub and Werning (2020). Focusing on the case with $\rho = \gamma$, they show that a finite interior steady state does not exist when $\varphi > 1$, and instead the economy converges to positive capital taxation and zero consumption by entrepreneurs in the long run.[9] This is due to the powerful effect of far future capital tax rates on current investment decisions by entrepreneurs.

Given this, and because a balanced growth path does not exist for $\varphi \neq 1$, we will focus on the case of $\varphi = 1$ for the remainder of this section.

[8] Since r^* depends on the capital tax rate, which is an endogenous object, we cannot immediately say whether this case holds. We will discuss this question in section 5.3.

[9] In earlier work, Lansing (1999) observed that the Chamley-Judd result failed to hold with log utility, and Reinhorn (2019) showed that this is because the Lagrange multiplier does not converge, although a finite steady state does exist in this case.

5.3 Log Utility

Suppose that $\varphi = 1$, i.e. entrepreneurs have log utility. In this case, we can solve for the entrepreneurs' behavior exactly:

$$c_e = \rho K \tag{54}$$

Substituting $\varphi = 1$ and $c_e = \rho K$ into (50) – (52), we obtain:

$$\lambda = \mu + \xi \rho \tag{55}$$

$$\gamma \mu - \dot{\mu} = \lambda \left(F_K - \delta \right) - \xi \left(\rho + \frac{\dot{K}}{K} \right) \rho \tag{56}$$

$$\gamma \xi - \dot{\xi} = \xi \left(\frac{\dot{K}}{K} \right) - \mu \tag{57}$$

We can then use 55 to eliminate μ from the other expressions, and combine to eliminate $\dot{\xi}$. If we do this, all other ξ terms will cancel, and we are left with the single optimality condition:

$$-\frac{\dot{\lambda}}{\lambda} = F_K \left(K, L \right) - \delta - \rho - \gamma \tag{58}$$

Tax Rate. We can now derive a simple expression for the capital tax τ^k. The entrepreneurs' Euler equation, together with the expression for r^k, implies:

$$\frac{\dot{c}_e}{c_e} = \left(1 - \tau^k \right) F_K \left(K, L \right) - \delta - \rho$$

Combining this with (58), we obtain:

$$\tau^k = \frac{\gamma - \dot{\lambda}/\lambda - \dot{K}/K}{F_K \left(K, L \right)} = \frac{\gamma - \dot{\lambda}/\lambda - \dot{K}/K}{\gamma - \dot{\lambda}/\lambda + \delta + \rho} \tag{59}$$

Steady State. Suppose that we reach a steady state. Then steady state capital will satisfy:

$$F_K(K, L) = \rho + \delta + \gamma$$

and steady state taxation will be:

$$\tau^k_{ss} = \frac{\gamma}{\gamma + \delta + \rho}$$

Note that there will always be a positive tax rate in steady state, and steady state capital is decreasing, and steady state tax rate increasing, in worker impatience γ.

Balanced Growth Next we consider the possibility of continuous growth through the accumulation of capital. Here we make a further simplifying assumption: suppose that worker utility is separable in consumption and labor, and is CRRA in consumption, i.e.:

$$U(C_w, L) = \frac{C_w^{1-\psi}}{1-\psi} - h(L)$$

Suppose we reach a point after which $L = 0$ and the economy enjoys continuous growth. Suppose that a balanced growth path exists, such that the variables $\{C_w, c_e, K\}$ grow at a constant rate g. Now, on any balanced growth path, we have $L = 0$ and the production function is AK with[10]

$$Y = A_1 K \tag{60}$$

where $A_1 = A(1)$. The marginal product of capital is likewise

$$F_K(K, 0) = A_1 \tag{61}$$

Further, we can place an upper bound on the growth rate on the balanced growth path. From the economy's resource constraint, and $C_w \geq 0$ plus $c_e = \rho K$, we find that:

$$g \leq A_1 - \delta - \rho \equiv \bar{g}$$

Next we use (58) plus the fact that $-\dot{\lambda}/\lambda = \psi g$ to derive an expression for g:

$$g = \frac{A_1 - \delta - \rho - \gamma}{\psi} \tag{62}$$

Then a balanced growth path exists if g so defined satisfies (1) $g > 0$ and (2) $g < \bar{g}$. The first condition requires:

$$\bar{g} > \gamma$$

while the second requires:

$$\psi > 1 - \gamma/\bar{g}$$

If a balanced growth path exists, the capital tax along it will be:

$$\tau_{bg}^k = \frac{(\psi - 1)\,\bar{g} + \gamma}{\psi A_1} \tag{63}$$

Note that the tax rate will always be positive given our assumption on ψ.

[10] Note that this will not be the case along the transition to the balanced growth path.

We can compare this growth rate to the optimal growth rule with a representative agent (i.e. that owns capital and supplies labor). In this case we have:

$$g = A_1 - \delta - \rho$$

which corresponds to zero capital taxation. Thus growth will be a bit slower in this case due to positive capital taxation.

We can summarize our results for the log case using the following proposition:

Proposition 4. *Suppose that entrepreneurs have log utility in consumption. Then:*

(i) If $A_1 < \rho + \gamma + \delta$, then the economy will reach a steady state with positive labor supply $L > 0$, at which capital satisfies:

$$F_K(K, L) = \rho + \delta + \gamma$$

and the capital tax rate satisfies:

$$\tau^k_{ss} = \frac{\gamma}{\rho + \delta + \gamma}$$

(ii) If $A_1 > \rho + \gamma + \delta$ and workers have CRRA utility in consumption with CRRA parameter ψ, which satisfies:

$$\psi > 1 - \frac{\gamma}{\bar{g}}$$

where

$$\bar{g} \equiv A_1 - \delta - \rho$$

Then a balanced growth path exists, with growth rate:

$$g = \frac{\bar{g} - \gamma}{\psi} > 0$$

and capital tax rate:

$$\tau^k_{bg} = \frac{(\psi - 1)\bar{g} + \gamma}{\psi A_1} > 0$$

and

$$\tau^k_{bg} < \tau^k_{ss}.$$

6 Conclusion

Recent trends in automation have re-ignited the debate about economic growth and, especially, the distribution implications of technical progress. There has also been suggestions of possible policy responses to this recent wave of technical change. In this paper, we have analyzed all three issues in a task-based model of production that is well suited to studying automation as a vehicle of technical change. We show that automation opens the door for sustained growth without the necessity of sustained technical progress, ultimately driven solely by capital accumulation. When this happens, the labor share of income goes to zero in the long run and worker's consumption is supported by a tax on capital income. Thus, the capital tax is positive even in the long run. When technology is not (yet) advanced enough to permit sustained economic growth through capital accumulation alone, the effects of automation on wages and the labor share of income are worse than that of traditional technical progress. Moreover, once again workers who are in the majority choose to tax capital to transfer resources to themselves from the entrepreneurs. Future research may examine how government's tax and transfer policy may depend on alternative assumptions about the constraints and mechanisms for setting such policies.

References

Daron Acemoglu and Pascual Restrepo. Robots and jobs: Evidence from us labor markets. Working Paper 23285, National Bureau of Economic Research, March 2017. URL http://www.nber.org/papers/w23285.

Daron Acemoglu and Pascual Restrepo. Artificial intelligence, automation and work. Working Paper 24196, National Bureau of Economic Research, January 2018a. URL http://www.nber.org/papers/w24196.

Daron Acemoglu and Pascual Restrepo. Modeling automation. Working Paper 24321, National Bureau of Economic Research, February 2018b. URL http://www.nber.org/papers/w24321.

Philippe Aghion, Benjamin F. Jones, and Charles I. Jones. Artificial intelligence and economic growth. Working Paper 23928, National Bureau of Economic Research, October 2017. URL http://www.nber.org/papers/w23928.

William J Baumol and William G Bowen. *Performing arts, the economic dilemma : a study of problems common to theater, opera, music and dance*. MIT Press, 1966.

Jagdish N Bhagwati, Arvind Panagariya, and Thirukodikaval Nilakanta Srinivasan. *Lectures on international trade*. MIT press, 1998.

Erik Brynjolfsson and Andrew McAfee. *The second machine age: Work, progress, and prosperity in a time of brilliant technologies*. WW Norton & Company, 2014.

Christophe Chamley. Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica: Journal of the Econometric Society*, pages 607–622, 1986.

CNBC. More Americans now support a universal basic income. https://www.cnbc.com/2018/02/26/roughly-half-of-americans-now-support-universal-basic-income.html, 2018.

Avinash Dixit and Victor Norman. *Theory of international trade: A dual, general equilibrium approach*. Cambridge University Press, 1980.

Forbes. Finally, Someone Does Something Sensible: Finland To Bring In A Universal Basic Income. https://www.forbes.com/sites/timworstall/2015/12/06/finally-someone-does-something-sensible-finland-to-bring-in-a-universal-basic-income/#7c9e0cdd6803, 2015.

Martin Ford. *Rise of the Robots: Technology and the Threat of a Jobless Future*. Basic Books, 2015.

Milton Friedman. The case for the negative income tax: A view from the right. https://miltonfriedman.hoover.org/objects/57681/the-case-for-the-negative-income-tax-a-view-from-the-right, 1966.

Gene M Grossman and Elhanan Helpman. *Innovation and growth in the global economy*. MIT press, 1991.

Joao Guerreiro, Sergio Rebelo, and Pedro Teles. Should robots be taxed? Working Paper 23806, National Bureau of Economic Research, September 2017. URL http://www.nber.org/papers/w23806.

Kenneth L Judd. Redistributive taxation in a simple perfect foresight model. *Journal of public Economics*, 28(1):59–83, 1985.

Anton Korinek and Joseph E. Stiglitz. Artificial intelligence and its implications for income distribution and unemployment. Working Paper 24174, National Bureau of Economic Research, December 2017. URL http://www.nber.org/papers/w24174.

Kevin J Lansing. Optimal redistributive capital taxation in a neoclassical growth model. *Journal of Public Economics*, 73(3):423–453, 1999.

Leslie J Reinhorn. On optimal redistributive capital taxation. *Journal of Public Economic Theory*, 21(3):460–487, 2019.

Ludwig Straub and Iván Werning. Positive long-run capital taxation: Chamley-judd revisited. *American Economic Review*, 110(1):86–119, 2020.

Wired. Free Money: The Surprising Effects of a Basic Income Supplied by Government. https://www.wired.com/story/free-money-the-surprising-effects-of-a-basic-income-supplied-by-government/, 2017.

Joseph Zeira. Workers, machines, and economic growth. *The Quarterly Journal of Economics*, 113(4):1091–1117, 1998.

A Omitted Proofs

Proof of Lemma

We would like to prove that the marginal product of capital (r_t) is increasing in L_t. This amounts to proving that the derivative $F_K(K, L)$ is increasing in L, where:

$$F(K, L) = \max_{\alpha} \left\{ \left[\alpha^{\frac{1}{\sigma}} (A(\alpha) \cdot K)^{1-\frac{1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} (\Gamma(\alpha) \cdot L)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}$$

We begin by proving the following lemmas.

Lemma 3. *Optimal capital task share α is decreasing in labor/capital ratio L/K.*

Proof. The equilibrium condition that holds at α can be written as:

$$\begin{cases} Z(i) \geq X(\alpha) \cdot \frac{L}{K} & \text{for } i < \alpha \\ Z(i) \leq X(\alpha) \cdot \frac{L}{K} & \text{for } i > \alpha \end{cases}$$

where:

$$X(\alpha) = \frac{\int_0^{\alpha} (a(i))^{\sigma-1} di}{\int_{\alpha}^{1} (\gamma(i))^{\sigma-1} di}$$
$$Z(i) = \left[\frac{a(i)}{\gamma(i)} \right]^{\sigma}$$

(This condition is found by substituting the expressions for r and w into the condition for α, and substituting in the expressions for A and Γ). By assumption, $Z(i)$ is decreasing in i. Furthermore, since $a(i), \gamma(i) \geq 0$, it is clear that $X(\alpha)$ is increasing in α, since:

$$X'(\alpha) = X(\alpha) \cdot \left(\frac{(a(\alpha))^{\sigma-1}}{\int_0^{\alpha} (a(i))^{\sigma-1} di} + \frac{(\gamma(\alpha))^{\sigma-1}}{\int_{\alpha}^{1} (\gamma(i))^{\sigma-1} di} \right) > 0$$

Now suppose that $L_2/K_2 > L_1/K_1$. We show that $\alpha_2 < \alpha_1$ by contradiction. Suppose not, so that $\alpha_2 > \alpha_1$. Take some $i \in (\alpha_1, \alpha_2)$. Since $i > \alpha_1$, it follows that:

$$Z(i) \leq X(\alpha_1) \cdot \frac{L_1}{K_1}$$

and since $i < \alpha_2$, it follows that:

$$Z(i) \geq X(\alpha_2) \cdot \frac{L_2}{K_2}$$

Therefore:

$$X(\alpha_1)\cdot\frac{L_1}{K_1}\geq X(\alpha_2)\cdot\frac{L_2}{K_2}$$

Since by assumption $L_2/K_2 > L_1/K_2$, it follows that:

$$X(\alpha_1) > X(\alpha_2)$$

Since X is increasing in α, this implies that

$$\alpha_1 > \alpha_2$$

which contradicts our assumption. Therefore $\alpha_2 \leq \alpha_1$, which proves the first part of the claim. $\square$

Lemma 4. *The ratio $w/r = F_L(K,L)/F_K(K,L)$ is increasing in $\kappa = K/L$.*

Proof. We know that:

$$\begin{cases} \frac{a(i)}{\gamma(i)} \geq \frac{r}{w} & \text{for } i < \alpha \\ \frac{a(i)}{\gamma(i)} \leq \frac{r}{w} & \text{for } i > \alpha \end{cases}$$

and that α is decreasing in L/K, and therefore increasing in κ. We further know that a/γ is decreasing in i.

Now consider what happens if there is an increase in L/K. We can consider two cases. One case is that a/γ is continuously decreasing at α. In this case, $r/w = a/\gamma$ holds with equality in the neighborhood of α. Therefore an increase in L/K will cause α to decline, and therefore $a/\gamma = r/w$ will increase.

Conversely, suppose that a/γ is discontinuous at i. Then a marginal change in L/K will not change α. But then from:

$$\frac{r}{w} = \left(\frac{\int_0^\alpha (a(i))^{\sigma-1}\,di}{\int_\alpha^1 (\gamma(i))^{\sigma-1}\,di}\right)^{\frac{1}{\sigma}}\left(\frac{L}{K}\right)^{\frac{1}{\sigma}}$$

we immediately get that r/w must increase.

Thus in either case a marginal increase in L/K causes a marginal increase in r/w. $\square$

Now we are in position to prove our final lemma:

Lemma 5. *The marginal product of capital $F_K(K,L)$ is increasing in L, since $F_{KL}(K,L) > 0$.*

Proof. We can rewrite the production function as:

$$F(K,L) = \max_{\alpha}\left\{\left(\alpha^{\frac{1}{\sigma}}(A)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}}\left(\Gamma\frac{L}{K}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} K\right\}$$

This implies that the choice of α that maximizes production for a given (K, L) depends only on the ratio $\kappa = K/L$.

Next we calculate the derivatives of the aggregate production function. These are:

$$F_K(K,L) = \left(\frac{\alpha A^{\sigma-1}Y}{K}\right)^{\frac{1}{\sigma}} = \left(\alpha A^{\sigma-1}\right)^{\frac{1}{\sigma}}\left(\alpha^{\frac{1}{\sigma}}(A)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}}\left(\Gamma\frac{L}{K}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}}$$
$$F_L(K,L) = \left(\frac{(1-\alpha)\Gamma^{\sigma-1}Y}{L}\right)^{\frac{1}{\sigma}} = \left((1-\alpha)\Gamma^{\sigma-1}\right)^{\frac{1}{\sigma}}\left(\alpha^{\frac{1}{\sigma}}\left(A\frac{K}{L}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}}(\Gamma)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}}$$

The key thing to observe is that both equations depend only on $\kappa = K/L$, and not on K or L independently. Therefore we can write each as:

$$F_K(K,L) = r(\kappa)$$
$$F_L(K,L) = w(\kappa)$$

Moreover, from lemma 2 we know that:

$$\frac{w(\kappa)}{r(\kappa)} = \frac{F_L(K,L)}{F_K(K,L)} = \left(\frac{(1-\alpha)\Gamma^{\sigma-1}}{\alpha A^{\sigma-1}}\kappa\right)^{\frac{1}{\sigma}}$$

is increasing in κ.

Now we can prove the claim. Note that changes in L and K affect F_K and F_L only by altering κ. Therefore we have:

$$F_{KL} = \frac{d}{dL}(F_K) = -r'(\kappa)\cdot\frac{K}{L^2}$$
$$= \frac{d}{dK}(F_L) = w'(\kappa)\cdot\frac{1}{L}$$

Therefore:

$$-r'(\kappa)\cdot\kappa = w'(\kappa)$$

This implies that $w'(\kappa)$ and $r'(\kappa)$ have opposite signs.

In lemma 2 we showed that w/r is increasing in κ. Therefore:

$$\frac{d}{d\kappa}\left(\frac{w(\kappa)}{r(\kappa)}\right)=\frac{w}{r}\left(\frac{w_\kappa}{w}-\frac{r_\kappa}{r}\right)\geq 0$$

Now, since r_κ and w_κ have opposite signs, the only way that:

$$\frac{w_\kappa}{w}\geq\frac{r_\kappa}{r}$$

is if $w_\kappa \geq 0$ and $r_\kappa \leq 0$. From this we conclude that $r_\kappa \leq 0$, and therefore:

$$F_{KL}=-r'(\kappa)\cdot\frac{K}{L^2}\geq 0$$

That is, the return on capital is increasing in L/K, and therefore in L. $\square$