Geostatistical Capture-Recapture Models

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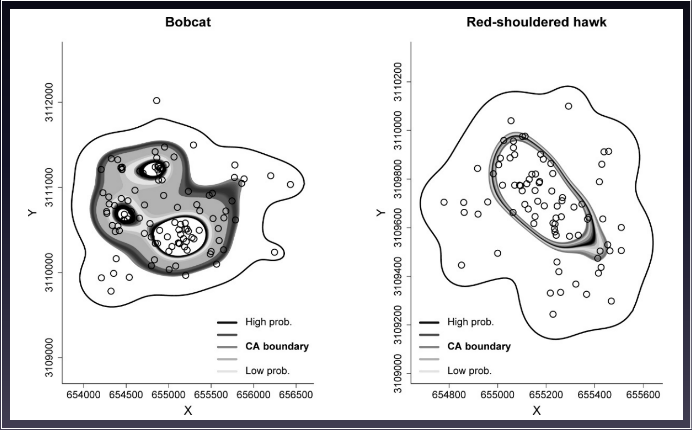
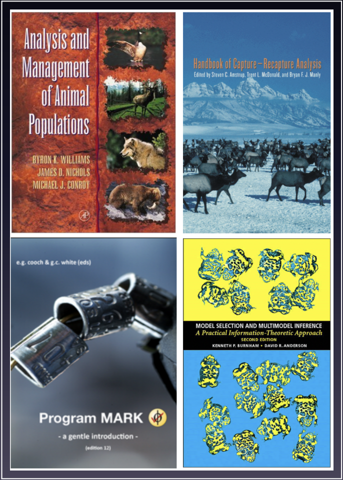
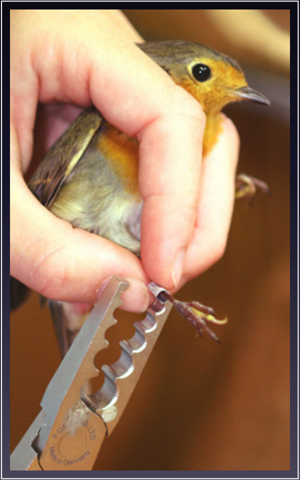


- Wilson, Hooten, Strobel, Shivik (2010). Accounting for individuals, uncertainty, and multi-scale clustering in core area estimation. Journal of Wildlife Management, 74: 1343-1352.

Wildlife Demographic Models





CR Model w/ Data Augmentation

Data Model

$$y_i \sim \begin{cases} \mathbb{1}_{\{y_i=0\}} & , z_i = 0 \\ \text{Binom}(J, p) & , z_i = 1 \end{cases}$$

Process Model

$$z_i \sim \text{Bern}(\psi)$$

Derived Quantity

$$N = \sum_{i=1}^{M} z_i$$

i	$\mathbf{Y}$	y_i
1	111	3
2	100	1
$\vdots$	$\vdots$	$\vdots$
n	101	2
$n+1$	000	0
$\vdots$	$\vdots$	$\vdots$
M	000	0

- Royle, Dorazio, Link. (2007). Analysis of multinomial models with unknown index using data augmentation. J Comput Graph Stat 16:67-85.

Recursive Bayes

Partition Data

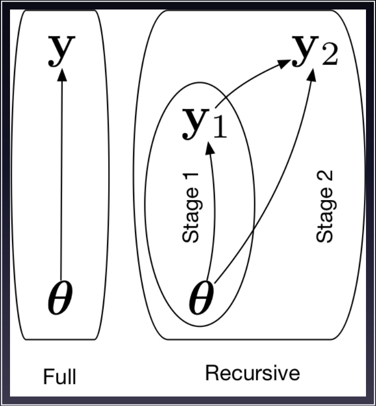
$$\mathbf{y} \equiv (\mathbf{y}_1', \mathbf{y}_2')'$$

Stage 1

$$[\boldsymbol{\theta}|\mathbf{y}_1] \propto [\mathbf{y}_1|\boldsymbol{\theta}][\boldsymbol{\theta}]$$

Stage 2

$$\begin{aligned}[\boldsymbol{\theta}|\mathbf{y}_1, \mathbf{y}_2] &\propto [\mathbf{y}_2|\boldsymbol{\theta}, \mathbf{y}_1][\boldsymbol{\theta}|\mathbf{y}_1] \\ &\propto [\mathbf{y}_2|\boldsymbol{\theta}, \mathbf{y}_1][\mathbf{y}_1|\boldsymbol{\theta}][\boldsymbol{\theta}]\end{aligned}$$



- Hooten, Johnson, Brost. (2021). Making recursive Bayesian inference accessible. The American Statistician, 75: 185-194.

Recursive Bayes for CR

Posterior

$$[p,\psi|\mathbf{y}_{1:n},\mathbf{y}_{(n+1):M},n] \propto \cancel{[\mathbf{y}_{(n+1):M}|p,\psi,\mathbf{y}_{1:n},n]}[p,\psi|\mathbf{y}_{1:n},n]$$
$$\propto \left(\prod_{i=1}^{n}[y_i|p,y_i>0]\right)[n|p,\psi][p][\psi]$$

Stage 1

$$\left(\prod_{i=1}^{n}[y_i|p,y_i>0]\right)[p][\psi]$$

Stage 2

$$r = \frac{[n|p^{(*)},\psi^{(*)}]}{[n|p^{(k-1)},\psi^{(k-1)}]}$$

- Hooten, Schwob, Johnson, Ivan. (2023). Multistage hierarchical capture-recapture models. Environmetrics.

Conditional Model for n

Definition of n

$$n = \sum_{i=1}^{M} \mathbb{1}_{\{y_i > 0\}}$$

Prob. of Detecting Individual

$$\text{Pr}(\mathbb{1}_{\{y_i > 0\}} = 1 | p, \psi) = \psi(1 - (1 - p)^J)$$

Resulting Conditional Model for n

$$[n | p, \psi] = \text{Binom}(M, \psi(1 - (1 - p)^J))$$

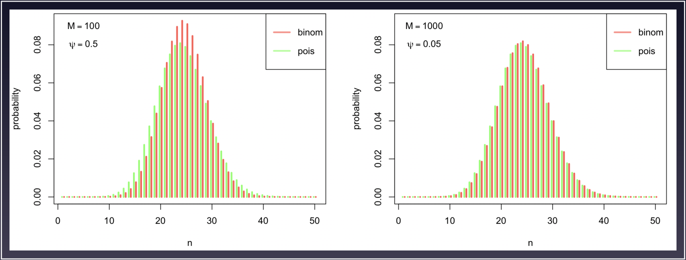
- Hooten, Schwob, Johnson, Ivan. (2023). Multistage hierarchical capture-recapture models. Environmetrics.

Alternative Conditional Model for n

Poisson Model for n

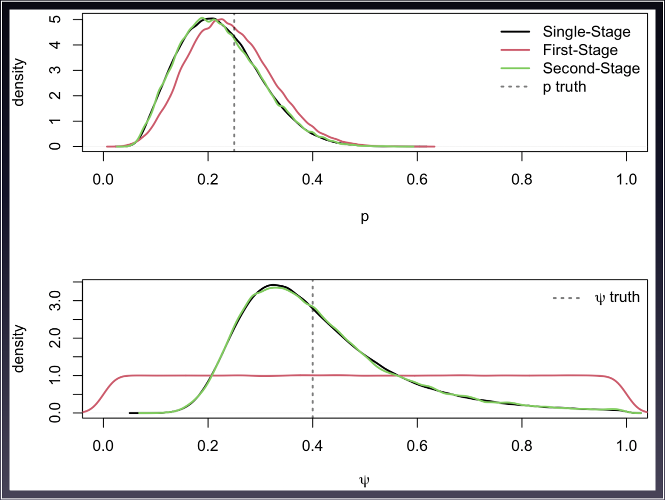
$$[n|p,\psi] = \text{Pois}(M\psi(1-(1-p)^J))$$

for $M \to \infty$ and $\psi \to 0$



- Hooten, Schwob, Johnson, Ivan. (2023). Multistage hierarchical capture-recapture models. Environmetrics.

Simulation Results

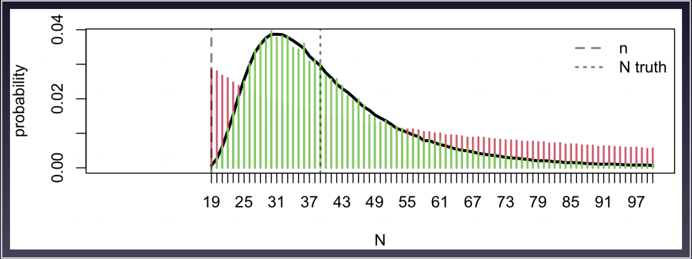


Inference for N

Stage 3: Sample N_0

$$N_0^{(k)} \sim \text{Binom}(M-n, \psi^{(k)}(1-p^{(k)})^J/(\psi^{(k)}(1-p^{(k)})^J+1-\psi^{(k)}))$$

$$\text{for } k=1,\ldots,K \text{ and let } N^{(k)} = n + N_0^{(k)}$$



SCR Model w/ Data Augmentation

Data Model

$$y_{i,l} \sim \begin{cases} \mathbb{1}_{\{y_{i,l}=0\}} & , z_i = 0 \\ \text{Binom}(J, p_{i,l}) & , z_i = 1 \end{cases}$$

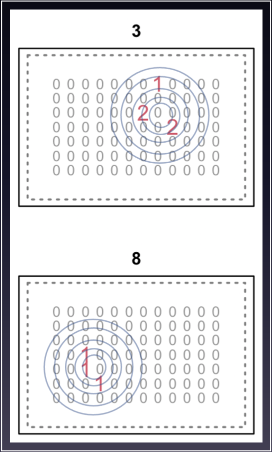
Process Model

$$z_i \sim \text{Bern}(\psi)$$

Detection Probability

$$g(p_{i,l}) = \beta_0 + \beta_1 ||\mathbf{c}_i - \mathbf{x}_l||_2^2$$

$$\mathbf{c}_i \sim \text{Unif}(\mathcal{S})$$



- Royle and Young. (2008). A hierarchical model for spatial capture-recapture data. Ecology, 89: 2281–2289.

GCR Model w/ Data Augmentation

Data Model

$$y_{i,l} \sim \begin{cases} \mathbb{1}_{\{y_{i,l}=0\}} & , z_i = 0 \\ \text{Binom}(J, p_{i,l}) & , z_i = 1 \end{cases}$$

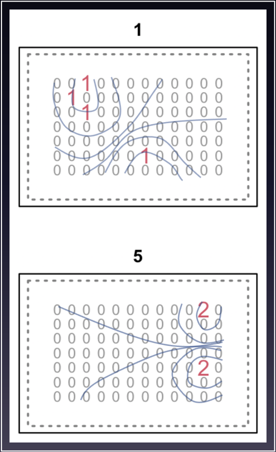
Process Model

$$z_i \sim \text{Bern}(\psi)$$

Detection Probability

$$\mathbf{v}_i = g(\mathbf{p}_i) \sim \mathbf{N}(\mu\mathbf{1}, \boldsymbol{\Sigma})$$

$$\boldsymbol{\Sigma} = \sigma^2 \exp(-\mathbf{D}^2/\theta^2)$$



- Hooten, Schwob, Johnson, Ivan. (2023). Geostatistical capture-recapture models. arXiv:2305.04141.

GCR Distributions

Posterior

$$[\mu, \sigma^2, \theta, \psi | \mathbf{Y}_{1:n}, n] \propto \left(\prod_{i=1}^{n} [\mathbf{y}_i | \mu, \sigma^2, \theta, \mathbf{y}_i' \mathbf{1} > 0] \right) \times [n | \mu, \sigma^2, \theta, \psi][\mu][\sigma^2][\theta][\psi]$$

Integrated model for $\mathbf{y}_i$

$$[\mathbf{y}_i | \mu, \sigma^2, \theta, \mathbf{y}_i' \mathbf{1} > 0] = \int [\mathbf{y}_i | \mathbf{p}_i, \mathbf{y}_i' \mathbf{1} > 0][\mathbf{v}_i | \mu, \sigma^2, \theta, \mathbf{y}_i' \mathbf{1} > 0] d\mathbf{v}_i$$

Conditional model for $\mathbf{y}_i$

$$[\mathbf{y}_i | \mathbf{p}_i, \mathbf{y}_i' \mathbf{1} > 0] = \frac{\mathbb{1}_{\{\mathbf{y}_i' \mathbf{1} > 0\}} [\mathbf{y}_i | \mathbf{p}_i]}{1 - \prod_{l=1}^{L} (1 - p_{i,l})^J}$$

GCR Distributions (cont)

Conditional Model for $\mathbf{v}_i$

$$[\mathbf{v}_i|\mu,\sigma^2,\theta,\mathbf{y}_i'\mathbf{1}>0] = \frac{(1-\prod_{l=1}^{L}(1-p_{i,l})^J)[\mathbf{v}_i|\mu,\sigma^2,\theta]}{\int(1-\prod_{l=1}^{L}(1-p_l)^J)[\mathbf{v}|\mu,\sigma^2,\theta]d\mathbf{v}}$$

Collapsed Integrated Model for $\mathbf{y}_i$

$$\left[\mathbf{y}_i|\mu,\sigma^2,\theta,\mathbf{y}_i'\mathbf{1}>0\right] = \frac{\int \mathbb{1}_{\{\mathbf{y}_i'\mathbf{1}>0\}}[\mathbf{y}_i|\mathbf{p}_i][\mathbf{v}_i|\mu,\sigma^2,\theta]d\mathbf{v}_i}{\int(1-\prod_{l=1}^{L}(1-p_l)^J)[\mathbf{v}|\mu,\sigma^2,\theta]d\mathbf{v}}$$

Integrated Conditional Model for n

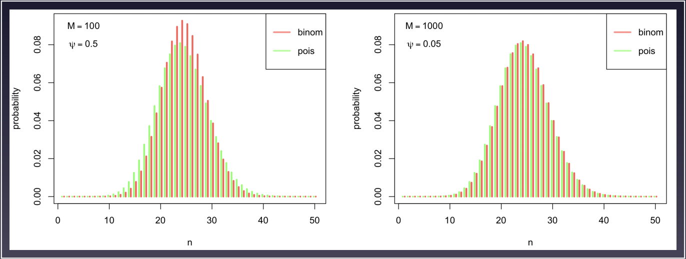
$$[n|\mu,\sigma^2,\theta,\psi] = \int [n|\mathbf{P},\psi][\mathbf{V}|\mu,\sigma^2,\theta]d\mathbf{V}$$

Conditional Model for n

Poisson Model for n

$$[n|\mathbf{P},\psi] = \text{Pois}\left(\sum_{i=1}^{M}\psi\left(1-\prod_{l=1}^{L}(1-p_{i,l})^{J}\right)\right)$$

for $M \rightarrow \infty$ and $\psi \rightarrow 0$



Recursive Bayes Perspective

Posterior

$$[\mu,\sigma^2,\theta,\psi|\mathbf{Y}_{1:n},n] \propto \left(\prod_{i=1}^{n}[\mathbf{y}_i|\mu,\sigma^2,\theta,\mathbf{y}_i'\mathbf{1}>0]\right) \times [n|\mu,\sigma^2,\theta,\psi][\mu][\sigma^2][\theta][\psi]$$

Stage 1: Fit model to only observed individuals

$$\left(\prod_{i=1}^{n}[\mathbf{y}_i|\mu,\sigma^2,\theta,\mathbf{y}_i'\mathbf{1}>0]\right)[\mu][\sigma^2][\theta][\psi]$$

Stage 2: Update using information from n

$$r = \frac{[n|\mu^{(*)},\sigma^{2,(*)},\theta^{(*)},\psi^{(*)}]}{[n|\mu^{(k-1)},\sigma^{2,(k-1)},\theta^{(k-1)},\psi^{(k-1)}]}$$

Inference for N

Stage 3: Sample N_0

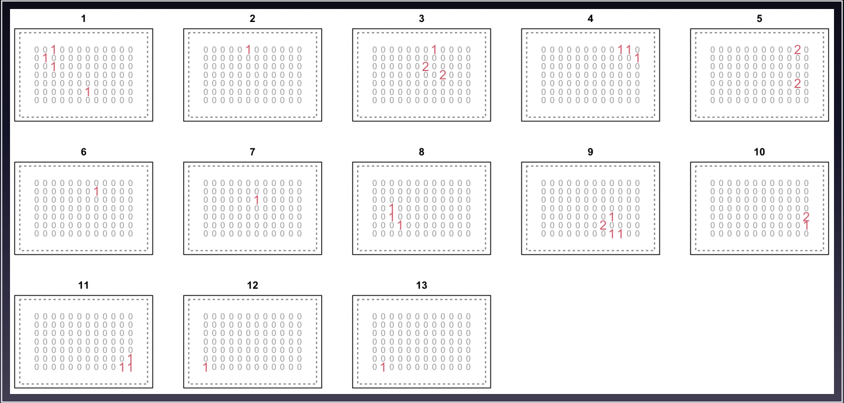
$$N_0^{(k)} \sim \text{Pois}((M-n)\bar{\psi}^{(k)})$$

for $k = 1, \ldots, K$ and let $N^{(k)} = n + N_0^{(k)}$

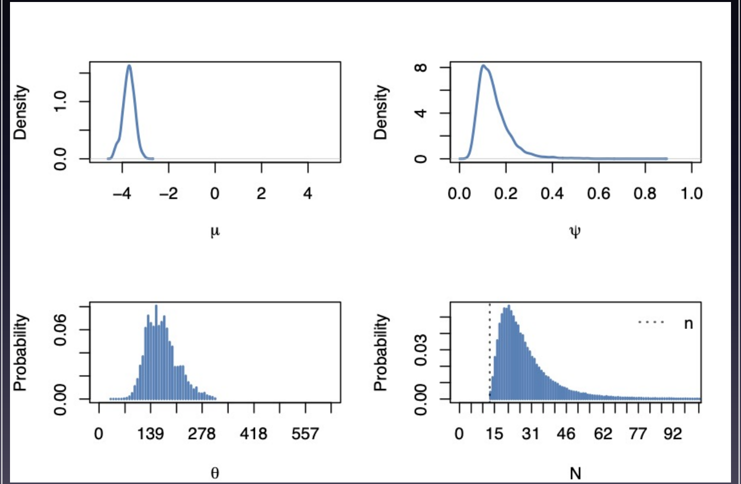
Membership Probability for Undetected Individuals

$$\bar{\psi}^{(k)} = \int \left(\frac{\psi^{(k)} \prod_{l=1}^{L} (1-p_l)^J}{\psi^{(k)} \prod_{l=1}^{L} (1-p_l)^J + 1 - \psi^{(k)}} \right) [\mathbf{v} | \mu^{(k)}, \sigma^{2(k)}, \theta^{(k)}] d\mathbf{v}$$

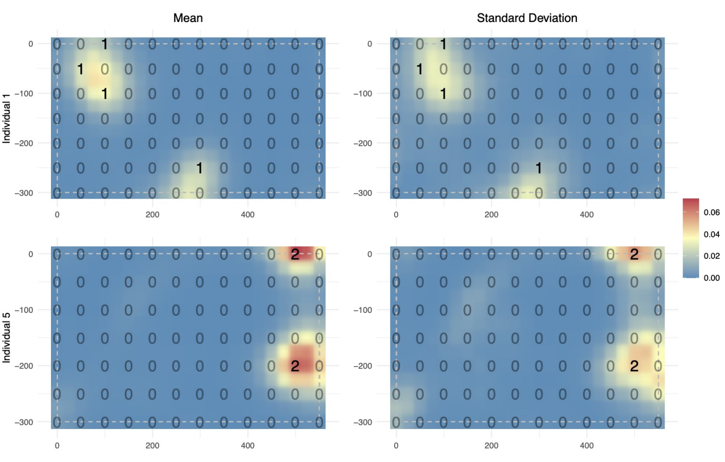
Snowshoe Hare Data



Snowshoe Hare Parameters



Snowshoe Hare Detection Function



References

- Hooten, Schwob, Johnson, Ivan. (2023). Geostatistical capture-recapture models. arXiv:2305.04141.
- Hooten, Schwob, Johnson, Ivan. (2023). Multistage hierarchical capture-recapture models. In Press. Environmetrics, arXiv:2205.04453.
- Hooten and Hefley. (2019). Bringing Bayesian Models to Life. CRC Press.
- Hooten, Johnson, and Brost. (2021). Making recursive Bayesian inference accessible. The American Statistician, 75: 185-194.
- McCaslin, Feuka, and Hooten. (2021). Hierarchical computing for hierarchical models in ecology. Methods in Ecology and Evolution, 12: 245-254.
- Wilson, Hooten, Strobel, Shivik (2010). Accounting for individuals, uncertainty, and multi-scale clustering in core area estimation. Journal of Wildlife Management, 74: 1343-1352.