# Math 425 Assignment 1

Max Horowitz-Gelb

## 1

## 2

**Lemma A:**

**Statement:** A connected subgraph of $n$ nodes requires at least $n-1$ edges.

**Proof:** Consider a graph $G = (V, E)$.

Let $G_n$ be the set of all connected subgraphs of $G$ with $n$ nodes. And let $G'_n$ be a subgraph of $G_n$ such that the number of edges in $G'_n$ is minimized. Then let $E'_n = E(G'_n)$ and $V'_n = V(G'_n)$.

First we shall consider the base case of $n = 1$ nodes. Here it is trivial to see by definition of a graph that $E'_1 = 0$.

Now assume that $E'_b \geq \alpha$. Then it must be true that $E'_{b+1} \geq \alpha + 1$ since for any subgraph in $G_b$, by definition of a graph, requires at least one new edge to connect a new node.

Therefore $E'_n \geq n - 1$, hence proving our statement. ∎

**Lemma 2.2**

**Statement:** Let $G$ be a connected graph with $n$ nodes. Then $G$ is a spanning tree if and only if it has exactly $n-1$ nodes.

**Proof:** Let $S_n$ be the set of all subgraphs of $G$ that are spanning trees. Then let $|E_n|$ be the number of edges of used for each subgraph in $S_n$ assuming they are all the same.

Clearly $|E_1| = 0$ by definition of a graph.

Then if $S_n = n - 1$ then $S_{n+1} = n$. This is because Lemma A implies $S_{n+1} \geq n$.