Math 425 Assignment 1

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Lemma A:

Statement: A connected subgraph of n nodes requires at least $n-1$ edges.

Proof: Consider a graph $G = (V, E)$.

Let G_n be the set of all connected subgraphs of G with n nodes. And let G'_n be a subgraph of G_n such that the number of edges in G'_n is minimized. Then let $E'_n = E(G'_n)$ and $V'_n = V(G'_n)$.

First we shall consider the base case of $n = 1$ nodes. Here it is trivial to see by definition of a graph that $E'_1 = 0$.

Now assume that $E'_b = \alpha$. Then it must be true that $E'_{b+1} \geq \alpha + 1$ since for any subgraph in G_b, by definition of a graph, requires at least one new edge to connect a new node.

Therefore $E'_n \geq n + 1$, hence proving our statement. ■