HW 9

Max Horowitz-Gelb

3/27/17

Q1

a

Let $T_0 = 0$ represent the time at 8 AM. Then the probability that no people show up from 8 to 10 AM is equal to

$$P(T_1 - T_0 > 2) = 1 - F_{exp(3)}(2) = 0.00248$$

b

T_n is a Poisson process with rate 3 so by definition, $T_1 - T_0 \sim exp(3)$

Q2

a

Let T_n be a Poisson process of cars passing through. Then let $T_0 = 0$ be the last time a car passed before the dear crosses the road. Then let $\alpha \geq 0$ be the time at which the dear crosses the road. Then the probability that a car hits the dear is equivalent to

$$P(T_k - \alpha < 1/12)$$

where $T_k = min\{T_n : T_n > \alpha\}$ Because of the memoryless of the exponential distribution this is equal to,

$$\sum_{i>0} p(k=i) F_{exp(6)}(1/12) = F_{exp(6)}(1/12) \sum_{i>0} p(k=i) = F_{exp(6)}(1/12) = 0.39347$$

b

If the deer only needs 2 seconds to cross the road then the problem is the same and the probability of getting hit is,

$$F_{exp(6)}(1/30) = 0.18127$$

Q3

If A is the total number of muons in the day, then A is a sum of 3 Poisson random variables X_1, X_2, X_3,representing different the different signal rates throughout the day.We then have that

$X_1 \sim 8Poisson(240), \qquad X_2 \sim 9Poisson(360), \qquad X_3 \sim 7Poisson(420)$

Then since a sum of Poissons is Poisson,

$$A \sim Poisson(8100)$$

And therefore,

$$Var(A) = 8100$$

Q4

Let $Y_1, ...Y_N$ be a set of iid random variables representing the amount of money withdrawn from each customer, each with mean 30 and variance 400. Then let N be a Poisson random variable with rate 80 representing the number of customers over the span of 8 hours. Finally let $S = \sum_{i=1}^{N} Y_i$ be the total amount of money withdrawn Then since $|E[Y_i]| = 30 < \infty$ and $E[N] = 80 < \infty$

$$E[S] = E[N] \cdot E[Y_i] = 80 * 30 = 2400$$

and since N is Poisson with finite rate then,

$$Var(S) = 80 * E[Y_i^2] = 80 * (Var(Y_i) + E[Y_i]^2) = 80 * (400 + 900) = 104000$$

$$Stdev(S) = 104000^{0.5} = 322.49031$$