HW 7

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Q1

Q2

First without loss of generality we may assume the starting position is at time 1 since otherwise we could simply rename all the states by rotating the clock until the starting position was 1. Also note that all states communicate with state 1 since there is at most a shortest path of length 6 from any state to 1 with a probability of 0.5^6 of occurring. Then using this, we may apply theorem 1.28 and say that if $g(x)$ is the expected number of transitions required to first get to state 1, then $g(1) = 0$ and for $x \neq 1$

$$g(x) = 1 + \sum_y p(x, y)g(y) = 1 + \sum_y r(x, y)g(y)$$

where r is p restricted to $\{2..12\}$. Then we can solve

$$g[2..12] = (I - r)^{-1}\vec{1}$$

g turns out to be ,

$$[0, 11, 20, 27, 32, 35, 36, 35, 32, 27, 20, 11]^T$$

Then since with probability 1 the starting state will transition to 2 or 12, then the expected the expected number of steps it will take X_n to return to the starting position is

$$1 + 0.5 * g(2) + 0.5 * g(12) = 1 + 0.5 * 11 + 0.5 * 11 = 12$$

Q4

First since the problem was not clear on edge cases I assume $p(1, 2) = 1$ and $p(6, 5) = 1$. With that we first realize clearly that all states communicate with state 1 since each contains a shortest path to state 1 of positive probability. So we again may use theorem 1.28 where our exit state is 1. and solve for g

$$g[2..6] = (I - r)^{-1}\vec{1}$$

And it turns out g is equal to

$$g = [0, 2.8755.6258.12510.12511.125]^T$$

So since state 1 transitions to 2 with probability 1 then

$$E_1 T_1 = 1 + g(2) = 3.875$$