

Homework 2

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Q1

a.

Are transition matrix $p_{i,j} =$

| - | A | B | C |
|---|-----|-----|-----|
| A | 0 | 1/2 | 1/2 |
| B | 3/4 | 0 | 1/4 |
| C | 3/4 | 1/4 | 0 |

b.

To calculate probabilities of events at time 2 we simply calculate $p(i, j)^2$.

$p(i, j)^2 =$

| - | A | B | C |
|---|------|------|------|
| A | 3/4 | 1/8 | 1/8 |
| B | 3/16 | 7/16 | 3/8 |
| C | 3/16 | 3/8 | 7/16 |

So,

$$P(X_2 = A | X_0 = A) = p(A, A)^2 = 3/4$$

$$P(X_2 = B | X_0 = A) = p(A, B)^2 = 1/8$$

$$P(X_2 = C | X_0 = A) = p(A, C)^2 = 1/8$$

Finally we can also calculate

$$\begin{aligned} P(X_3 = B | X_0 = A) &= p(A, A)^2 * p(A, B) + p(A, B)^2 * p(B, B) + p(A, C)^2 * p(C, B) \\ &= 3/4 * 1/2 + 1/8 * 0 + 1/8 * 1/4 = 13/32 \end{aligned}$$

Q2

$$\begin{aligned} &P(X_2 = 3, X_4 = 4 | X_7 = 9, X_6 = 8) \\ &= \frac{P(X_2 = 3, X_4 = 4, X_7 = 9, X_6 = 8)}{P(X_7 = 9, X_6 = 8)}, \text{ using basic definition of conditional probability} \\ &= \frac{P(X_7 = 9 | X_6 = 8) * P(X_6 = 8 | X_4 = 4) * P(X_4 = 4 | X_2 = 3) * P(X_2 = 3)}{P(X_7 = 9 | X_6 = 8) * P(X_6 = 8)}, \text{ using that } X \text{ is a THMC and chain rule} \\ &= \frac{p(8, 9) * p(4, 8)^2 * p(3, 4)^2 * p(1, 3)^2}{p(8, 9) * p(1, 8)^6} \end{aligned}$$