# HW1

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## Q1

Let $X_1...X_40$ be a set of random variables such that $X_i = 1$ is the event that box $i$ is empty after all 80 balls have been placed, and $X_i = 0$ otherwise. Then the number of boxes empty after all balls are placed, which let's call $T$, is

$$T = \sum_{i=1}^{40} X_i$$

Then,

$$E[T] = E[\sum_{i=1}^{40} X_i] = \sum_{i=1}^{40} E[X_i]$$

The expected value of any $X_i$ is the same and can be calculated as,

$$E[X_i] = 1 * P(X_i = 1) + 0 * P(X_i = 0) = P(X_i = 1) = \frac{39}{40}^{80} = 0.13193780538$$

Therefore,

$$E[T] = \sum_{i=1}^{40} E[X_i] = 40 * 0.13193780538 = 5.27751221548$$

A similar procedure is used to find the vaiance of $T$. We know that the variance of $T$ is equal to $E[T^2] - E[T]^2 = E[T^2] - 27.85213518454$. Then,

$$E[T^2] = E[\sum_{i=1}^{40} \sum_{j=1}^{40} X_i X_j] = \sum_{i=1}^{40} \sum_{j=1}^{40} E[X_i X_j]$$