HW 12

Max Horowitz-Gelb

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Q1

(1)

Since $\lambda_i = \sum_{i \neq j} q(i,j)$ then

$$\lambda_i = q(i,i+1) + q(i,i-1) = \lambda + \mathbf{1}_{\{0 \leq i \leq s\}} i\mu + \mathbf{1}_{\{i \geq s\}} s\mu$$

(2)

Then,

$$Q(i,j) = \begin{cases} \lambda & j = i+1 \\ i\mu & 0 \leq i \leq s, \qquad j = i-1 \\ s\mu & i \geq s, \qquad j = i-1 \\ -\lambda_i & i = j \end{cases}$$

Q2

Let λ be the rate of $N(t)$. First note that clearly $q(i,j) = 0$ for $i - j \geq 2$ or $j < i$ then for $j = i+1$

$$q(i,j) = \lim_{t \to 0} \frac{e^{-\lambda t}(\lambda t)^{j-i}}{(j-i)!t} = \lim_{t \to 0} \frac{e^{-\lambda t}\lambda t}{t} = \lambda$$

Then we have

$$Q(i,j) = \begin{cases} \lambda & j = i+1 \\ -\lambda & j = i \\ 0 & else \end{cases}$$