HW Week 4

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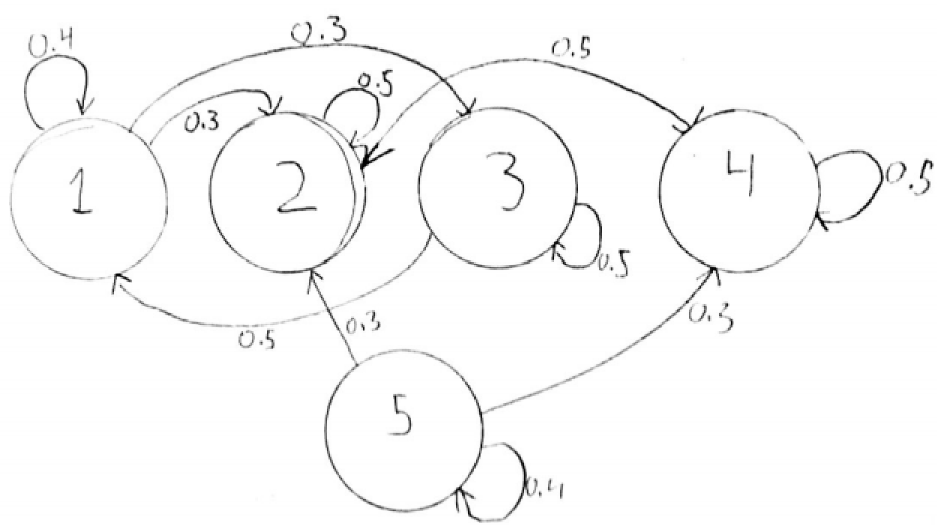
Q1

Let X be a THMC Markhov random variable. Then let our transition matrix be described,

$$p(i,j) = \begin{cases} 1 & j = i+1 \\ 0 & else \end{cases}$$

Clearly then for all $n \geq 1$, $Xn = X_{n-1} + 1$ with probability 1, and for any state i, $\rho(i,i) = 0$. Therefore for any state i, i is transient.

Q2



State 1 has a 0.3 probability of transitioning to state 2 and there is no path from state 2 to state 1. Therefore $\rho(1,1) \leq 0.7 < 1$ and state 1 is transient.

State 3 has a 0.5 probability of transitioning into state 1, and state 1 is the only state besides state 3 that can transition into state 3. Therefore, since state 1 is transient, then state 3 is transient. This is because if state 3 were recurrent, then $\rho(3,3) = 1$, and since $p(3,1) > 0$, then this would imply $\rho(3,1) = 1$. But state 1 has a 0.3 probability of transitioning to state 2 and state 2 has no transition path to state 1 or 3. This would imply that $\rho(1,3) < 1$, which is a contradiction since $\rho(3,1) = 1$ and $\rho(1,3) < 1$ implies that $\rho(3,3) < 1$.

State 5 is transient since there is a 0.6 probability of transitioning from state 5 to state 2 or 4 and there is no transition path from 2 or 4 back to state 5.

Q3

b

States 1, 4, 5 and 6 are in an irreducible closed set. This is because for any pair in that set, there is a transition path between them. As well, there are no transitions out of that set into the remaining states 2 and 3. There is no irreducible closed set that contains either state 2 or 3, since there are transitions out of those states into states 1, 5 and 6, but there are no transition paths from 1, 5 and 6 back to state 2 or 3. Then since there are a finite set of states and 1, 4, 5 and 6 are in a irreducible closed set, then by theorem 1.7, they are all recurrent. And 2 and 3 do not exist in any irreducible closed set, so by the same theorem they must be transient.

d

States 1 and 4 are in an irreducible closed set since they both have transitions to each-other and no outgoing transitions. For the same reasons, states 5 and 2 are in an irreducible closed set. No other states can be places in such a set and since there are a finite number of states, then by theorem 1.7, 1, 4, 5 and 2 are recurrent states, and 3 and 6 are transient.