HW 9

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Q1

a

Let $T_0 = 0$ represent the time at 8 AM. Then the probability that no people show up from 8 to 10 AM is equal to

$$P(T_1 - T_0 > 2) = 1 - F_{exp(3)}(2) = 0.00248$$

b

T_n is a Poisson process with rate 3 so by definition, $T_1 - T_0 \sim exp(3)$

Q2

a

Let T_n be a Poisson process of cars passing through. Then let $T_0 = 0$ be the last time a car passed before the dear crosses the road. Then let $\alpha \geq 0$ be the time at which the dear crosses the road. Then the probability that a car hits the dear is equivalent to

$$P(T_k - \alpha < 1/12)$$

where $T_k = min\{T_n : T_n > \alpha\}$ Because of the memoryless of the exponential distribution this is equal to,

$$\sum_{i>0} p(k=i) F_{exp(6)}(1/12) = F_{exp(6)}(1/12) \sum_{i>0} p(k=i) = F_{exp(6)}(1/12) = 0.39347$$

b

If the deer only needs 2 seconds to cross the road then the problem is the same and the probability of getting hit is,

$$F_{exp(6)}(1/30) = 0.18127$$

Q3

If A is the total number of muons in the day, then A is a sum of 3 Poisson random variables X_1, X_2, X_3,representing different the different signal rates throughout the day.We then have that

$X_1 \sim 8Poisson(240), \qquad X_2 \sim 9Poisson(360), \qquad X_3 \sim 7Poisson(420)$

Then since a sum of Poissons is Poisson,

$$A \sim Poisson(8100)$$

And therefore,

$$Var(A) = 8100$$

Q4

Let $Y_1, ...Y_N$ be a set of iid random variables representing the amount of money withdrawn from each customer, each with mean 30 and variance 400. Then let N be a Poisson random variable with rate 80 representing the number of customers over the span of 8 hours. Finally let $S = \sum_{i=1}^{N} Y_i$ be the total amount of money withdrawn Then since $|E[Y_i]| = 30 < \infty$ and $E[N] = 80 < \infty$

$$E[S] = E[N] \cdot E[Y_i] = 80 * 30 = 2400$$

and since N is Poisson with finite rate then,

$$Var(S) = 80 * E[Y_i^2] = 80 * (Var(Y_i) + E[Y_i]^2) = 80 * (400 + 900) = 104000$$

$$Stdev(S) = 104000^{0.5} = 322.49031$$

Q5

Let $X_1, ...X_N \in \{E, \bar{E}\}$ be a set of independent random variables representing whether each person is enthusiastic or not where N is a Poisson random variable with mean 60.

Then let $N_E = |\{X_i : X_i = E\}|$ and $N_{\bar{E}} = |\{X_i : X_i = \bar{E}\}|$. Then since $X_1, ...X_N$ are iid, then $N_E \sim Poisson(40)$ and $N_{\bar{E}} \sim Poisson(20)$

Then let $Y_{E,1}, ...Y_{E,N_E}$ be the random variables of work done by enthusiastic workers and let $S_E = Y_{E,1} + ... + Y_{E,N_E}$.

Then since $Y_{E,i}, N_E$ have finite mean and variance.

$$E[S_E] = E[N_E] \cdot E[Y_{E,i}] = 40 * 10 = 400$$

$$Var[S_E] = E[N_E] * E[Y_{W,i}^2] = 40 * (25 + 100) = 5625$$

Then let $Y_{\bar{E},1}, ..., Y_{\bar{E},N_{\bar{E}}}$ be the random variables of work done by lazy workers and let $S_{\bar{E}} = Y_{\bar{E},1} + ... + Y_{\bar{E},N_{\bar{E}}}$.

Then since $Y_{\bar{E},i}, N_{\bar{E}}$ have finite mean and variance.

$$E[S_{\bar{E}}] = E[N_{\bar{E}}] \cdot E[Y_{\bar{E},i}] = 20 * 3 = 60$$

$$Var[S_{\bar{E}}] = E[N_{\bar{E}}] * E[Y_{\bar{E},i}^2] = 20(4 + 9) = 260$$

Then let $S = S_E + S_{\bar{E}}$ be the total number of cans picked up. Then since $S_E \perp S_{\bar{E}}$

$$E[S] = E[S_E] + E[S_{\bar{E}}] = 400 + 60 = 460$$

$$Var(S) = Var(S_E) + Var(S_{\bar{E}}) = 5625 + 260 = 5885$$

$$Stdev(S) = 76.71375$$

Q6

Let $N_T(t), N_S(t)$ be the number of trout and salmon caught after time t respectively. Since the probability of each catch being a trout or a salmon is independent of other catches, then $N_T(t)$ and $N_S(t)$ are two independent Poisson process, with

$N_T(t) \sim Poisson(0.6 * 2 * t), \qquad N_S(t) \sim Poisson(0.4 * 2 * t)$

So,

$$P(N_T(2.5) = 2, N_S(2.5)) = f_{Poisson(3)}(2) * f_{Poisson(2)}(1) = 0.06064$$

Q7

a

Since whether she writes a speeding or a DWI ticket is independent of all other tickets. We can represent the tickets she writes by two independent Poisson processes. Let $N_S(t), N_D(t)$ be two independent Poisson processes of the number of speeding and DWI tickets written after time t respectively. Then

$$N_S(t) \sim Poisson((2/3) * 6 * t) \qquad N_D(t) \sim Poisson((1/3) * 6 * t)$$

Then let $S_S(t) = 100 * N_S(t)$ be the total revenue made from speeding tickets after time t.

Then,

$$E[S_S(1)] = 100 * E[N_S(1)] = 100 * 4 = 400$$

and,

$$Var(S_S(1)) = 100^2 * Var(N_S(1)) = 100^2 4 = 40000$$

Then let $S_D(t) = 400 * N_D(t)$ be the the the total revenue made from DWI tickets after time t,

Then,

$$E[S_D(1)] = 400 * E[N_D(t)] = 400 * 2 = 800$$

and,

$$Var(S_D(1)] = 400^2 Var(N_D(t)) = 400^2 * 2 = 320000$$

Then let $S(t) = S_D(t) + S_S(t)$ be the total revenue made after time t.

Then since $S_D(t) \perp S_S(t)$,

$$E[S(1)] = E[S_D(1)] + E[S_S(1)] = 400 + 800 = 1200$$

and,

$$Var(S(1)) = Var(S_D(1)) + Var(S_S(1)) = 40000 + 320000 = 360000$$

$$Stdev(S(1)) = 600$$

b

Since $N_D(t) \perp N_S(t)$ then the probability that between 2AM and 3AM that she writes exactly 5 speeding tickets and 1 DWI ticket is equal to,

$$p(N_S(1) = 5) * p(N_D(1) = 1) = f_{Poisson(4)}(5) * f_{Poisson(2)}(1) = 0.04230$$

Q8

First for notation sake, let $N(t)$ be the number of tickets written since the start and let $N = N(1)$.

Then,

$$P(A) = P(N(0.5) = 0) = f_{Poisson(3)}(0) = 0.04979$$

$$\begin{aligned} P(A|N = 5) &= \frac{P(A, N = 5)}{P(N = 5)} \\ &= \frac{P(A) * P(N = 5|A)}{P(N = 5)} \\ &= \frac{f_{Poisson(3)}(0) * f_{Poisson(3)}(5)}{f_{Poisson(6)}(5)} = 0.03125 \end{aligned}$$

So $P(A) \neq P(A|N = 5)$