HW Week 3

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Q1

We have shown in class that this probability can be rewritten as for arbitrary $n \geq 1$ as,

$$P(X_{n+2} = 3, X_{n+4} = 4 | X_{n+1} = 9, X_n = 8)$$

Then since X is a THMC this can be written simply as,

$$p(9,3) * p(3,4)^2$$

Q2

a

If $V = \max\{T, U\}$ then V is a stopping time. This is because there are only two cases.

Case 1: $T \geq U$

Since T and U are stopping times, we can check this with only $X_1...X_U$, which implies then that we can tell this with only $X_1...X_T$. Then in this case $V = T$ and since T is a stopping time, V can be determined with only $X_1...X_V$.

Case 2: $T < U$

Since T and U are stopping times, we can check this with only $X_1..X_T$ which implies we can tell this with $X1...X_U$. Then in this case $V = U$ and since U is a stopping time, V can be determined with only $X_1...X_V$.

Then by definition V is a stopping time.

b

If $V = \min\{T, U\}$ then again there are the two same cases.

Case 1: $T \geq U$

Again we can tell this with only $X_1...X_U$. Then $V = U$ and since U is a stopping time, we can know V with only $X_1...X_V$.

Case 2: $T < U$.

Again we can determine if this inequality is true with only $X_1...X_T$. And $V = T$, and since T is a stopping time, then we can determine V with only $X_1...X_V$.

Therefore V is a stopping time.

Q3

Let n be given. First note that by definition of a THMC

$$P(X_{T+2} = j|X_T = i, T = n) = P(X_{n+2} = j|X_n = i, past) = p(i,j)^2$$

implies,

$$\forall n' P(X_{T+2} = j|X_T = i, T = n') = p(i,j)^2$$

Then note that $P(X_{T+2} = j|X_T = i)$ can be rewritten as,

$$\sum_{n'} P(X_{T+2} = j|X_T = i, T = n') * P(T = n')$$

Which by what we've shown above can be rewritten as,

$$= p(i,j)^2 * \sum_{n'} P(T = n')$$

$$= p(i,j)^2$$

Q4

Let $X_0 = E$, and $T = minn \geq 1 : X_n = E$ Then

$$P(T = \infty) = P(E,N) * \lim_{n \to \infty} \prod_{i=2}^{n} p(N,N)$$

$$= 0.9 * \lim_{n \to \infty} \prod_{i=2}^{n} 0.2$$

$$= 0$$