HW 8

Max Horowitz-Gelb

March 13, 2017

Q1

a.

The probablility the repair takes more than two hours is equal to

$$P_{\lambda=1/2}(X > 2) = 1 - F_{\lambda=1/2}(2) = 0.3678794$$

b.

The probability that it takes more than 5 hours given it takes more than 3 hours is

$$p_{\lambda=1/2}(X > 5|X > 3) = 1 - F(2) = 0.3678794$$

Q2

If $X \sim exp(\lambda)$ then for $t \geq 0$

$$P(X > t) = 1 - e^{-\lambda t}$$

and by definition of conditional for $s \geq 0$

$$P(X > t + s|X > t) = \frac{P(X > t + s \cap X > t)}{P(X > t)} = \frac{P(X > t + s)}{P(X > t)}$$

$$= e^{-\lambda(t+s)}/e^{-\lambda t} = e^{-\lambda s} = P(X > s)$$

Q3

Let X_2, X_2, X_3 all be iid from an exponential with rate 2. Then let T_1, T_2, T_3 be the corresponding order statistics.

Since X_1, X_2, X_3 are iid then

$$P(T_1 > t) = P(X_1 > t) * P(X_2 > t) * P(X_3 > t) = e^{-3\lambda t}$$

Therefore we can see that $T_1 \sim exp(3\lambda)$

Then since the exponential distribution is memoryless we can see clearly that $(T_2 - T_1) \sim exp(2\lambda)$ and $(T_3 - T_2) \sim exp(\lambda)$

Then since $max(X_1, X_2, X_3) = T_1 + (T_2 - T_1) + (T_3 - T_2)$ then

$E[max(X_1, X_2, X_3)] = E[T_1] + E[T_2 - T_1] + E[T_3 - T_2] = 1/6 + 1/4 + 1/2 = 11/12$

Q4

Let $\tau_1 ... \tau_{100}$ be iid from an exponential with rate 3.

Then $T_100 = \sum_{i=1}^{100} \tau_i$. Therefore

$$E[T_100] = \sum_{i=1}^{100} E[\tau_i] = 100/3$$

and since $\tau_1 ... \tau_100$ are iid, then

$$Var[T_100] = \sum_{i=1}^{100} Var(\tau_i) = 100/9$$

Q5

a.

Using the same logic as in Q4

$$E[T_{12}] = 12/\lambda = 4$$

b.

Because of memorylessness of exponentials

$$E[T_{12} | N(2) = 5] = 2 + E[\tau_6 + ... + \tau_12] = 2 + 7/3 = 13/3$$

c.

Since $N(t)$ is a Poisson process, then

$$N(t + s) - N(t) \sim Poisson(3 * s)$$

Therefore,

$$E(N(5)|N(2) = 5) = 5 + E[N(5) - N(2)] = 5 + 3 * 3 = 14$$