HW 13

Max Horowitz-Gelb

4/24/17

Q1

First note that for $i \neq j$

$$q(i,j) = \lim_{t \to 0} \frac{p_t(i,j)u(i,j)}{t}$$

Then by definition of p' and the Chapman-Kolmogorov equation,

$$p'_t(i,j) = \lim_{h \to 0} \frac{p_{t+h}(i,j) - p_t(i,j)}{h}$$

$$= \lim_{h \to 0} \frac{\sum_k \big(p_t(i,k)p_h(k,j)\big) - p_t(i,j)}{h}$$

$$= \lim_{h \to 0} \frac{\sum_{k \neq j} \big(p_t(i,k)p_h(k,j)\big) + p_t(i,j)(p_h(j,j) - 1)}{h}$$

$$= \lim_{h \to 0} \frac{\sum_{k \neq j} \big(p_t(i,k)p_h(k,j)\big)}{h} + \lim_{h \to 0} \frac{p_t(i,j)(p_h(j,j) - 1)}{h}$$

$$= \sum_{k \neq j} p_t(i,k)q(k,j) - \lim_{h \to 0} \frac{p_t(i,j)(\sum_{k \neq j} p_h(j,k)}{h} \qquad \text{By definition of } q$$

$$= \sum_{k \neq j} p_t(i,k)q(k,j) + p_t(i,j)(-\sum_{k \neq j} q(j,k))$$

$$= \sum_{k \neq j} p_t(i,k)q(k,j) + p_t(i,j)(-\lambda_j)$$

$$= p_t(i,\cdot)Q(\cdot,j)$$

Q2

Note

$$\frac{\delta}{\delta t}p_t = \frac{\delta}{\delta t}e^{Qt}$$

$$= Q\sum_{n=1}^{\infty}\frac{nQ^{n-1}t^{n-1}}{n!}$$

$$= Q\sum_{n=1}^{\infty}\frac{Q^{n-1}t^{n-1}}{(n-1)!}$$

$$= Qp_t \qquad \text{by the givien solution of KBE}$$

Q3

$$(\tilde{\pi}\tilde{p})_i = \frac{1}{\sum_{k\in S}\pi_k\lambda_k}\sum_{k\neq i}\pi_k\lambda_k\tilde{p}(k,i)$$

$$= \frac{1}{\sum_{k\in S}\pi_k\lambda_k}\sum_{k\neq i}\pi_k\lambda_k\frac{q(k,i)}{\lambda_k}$$

$$= \frac{1}{\sum_{k\in S}\pi_k\lambda_k}\sum_{k\neq i}\pi_k q(k,i)$$

We know that $\pi Q = 0$ so This is equal to

$$\frac{-\pi_i Q_{i,i}}{\sum_{k\in S}\pi_k\lambda_k}$$

$$= \frac{\pi_i\lambda_i}{\sum_{k\in S}\pi_k\lambda_k}$$

so $\tilde{\pi}$ is a stationary measure, and clearly by the normalizing term it is also a distribution.