HW 5

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Q1

For a two state THMC with transitions probabilities,

$$\begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$$

Our stationary distribution is simply,

$$[\frac{b}{a+b}, \frac{a}{a+b}] = [\frac{1}{7}, \frac{6}{7}]$$

Q2

We must solve for

$$\pi p = p$$

This gives us the set of equations

$$\pi_1 = \pi_1$$

$$0.4\pi_2 + 0.1\pi_3 = \pi_2$$

$$0.6\pi_2 + 0.9\pi_3 = \pi_3$$

$$\pi_1 + \pi_2 + \pi_3 = 1$$

$$\pi_1, \pi_1, \pi_3 \in [0, 1]$$

Solving for these equations you get that π_1 is free $\pi_2 = (1/7)(1 - \pi_1)$and $\pi_3 = (6/7)(1 - \pi_1)$. So the the staitionary distributions are

$$[\pi_1, (1/7)(1 - \pi_1), (6/7)(1 - \pi_1)] \text{ for any } \pi_1 \in [0, 1]$$

Q3

If

$$v = vp$$

then,

$$(-1/10)v = -(1/10)vp$$

therefore

$$[1/10, 2/10, 3/10, 4/10]$$

is also a stationary measure, and since its elements sum to 1 and are positive then it is a stationary distribution.

Q4

a

For a single step it is simply,

	RR	RS	SR	SS
RR	0.6	0.4	0	0
RS	0	0	0.6	0.4
SR	0.6	0.4	0	0
SS	0	0	0.3	0.7

b

For two steps we must calculate two transitions which gives us

	RR	RS	SR	SS
RR	0.6 * 0.6	0.6 * 0.4	0.4 * 0.6	0.4 * 0.4
RS	0.6* 0.6	0.6 * 0.4	0.4* 0.3	0.4 * 0.7
SR	0.6 * 0.6	0.6 * 0.4	0.4 * 0.6	0.4 * 0.4
SS	0.3*0.6	0.3 * 0.4	0.7 * 0.3	0.7 * 0.7

which is,

	RR	RS	SR	SS
RR	0.36	0.24	0.24	0.16
RS	0.36	0.24	0.12	0.28
SR	0.36	0.24	0.24	0.16
SS	0.18	0.12	0.21	0.49

c

The probability it will rain on Wednesday given it did not rain on Sunday or Monday is equal to

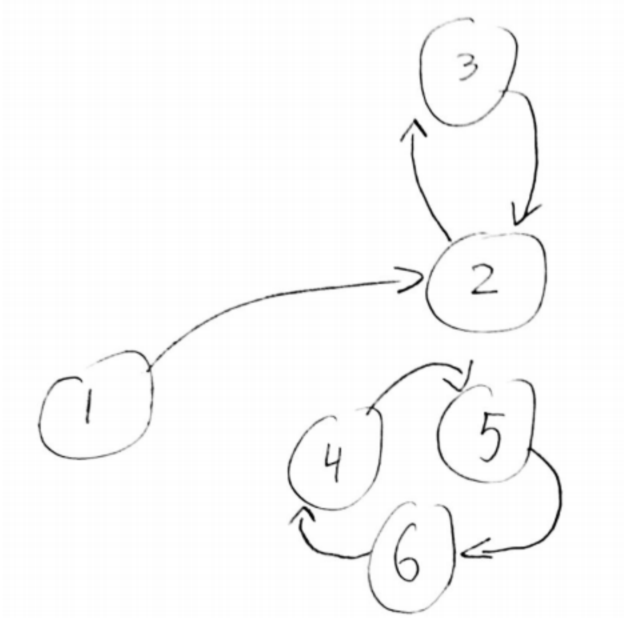
$$P(SS \to RR) + P(SS \to SR) = 0.18 + 21 = 0.39$$

Q5

Let our set of states be $1, 2$ with tranition probability matrix $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$. Then state 1 is transient and state 2 is recurrent. Clearly state 1 is transient since with probability 1 it will transition from state 1 to 2, and there is no way to transition back to 1. Clearly state 2 is recurrent since it will always transition back to itself with proability 1.

Q6

For a state set of 1,2,3,4,5,6 we define the transition probabilities graphically,



State 1 is clearly transient since it only has an outgoing edge.

States 2 and 3 both have period 2 since both will travel to the other and then travel back to theirselves with probability 1, which takes two transitions.

States 4,5 and 6 all have period 3, since all of them with probability 1 will travel once to each of the other nodes before returning to theirselves which takes three transitions.

Q7

State -2 starts in the triangle, so it will return to itself after at least once going through the triangle and some number of times through the square so, $I_{-2} = \{3a + 4b : a \in \mathbb{N}, b \in \mathbb{N} \cup \{0\}\}$

3 and 7 exist in I_{-2} and the gcd of these is 1 so state -2 has period 1.

State 2 starts in the square, so it will return to itself after at least once going through the square and some number of times throught the triangle so, $I_2 = \{3a+4b : a \in \mathbb{N} \cup \{0\}, b \in \mathbb{N}\}$ 4 and 7 are in I_2 and they have a gcd of 1 so state 2 has a period of 1.