So far we have repeatedly discussed about the consequences of the existence of light neutrino masses to pheromenology, but what is the origin of it?
· Neutrino mass is one of the biggest mysteries of the SM, and the person who formulates the correct mass mechanism for it is 100% sure to get awarded Nobel prize, if he or she is still alive.

Burning question: From where neutrino Majorana mass term originates?

$$-\frac{1}{2}\overbrace{m_{LL}}^{3\times3}\bar{\nu}_L\nu_L^c+h.c.$$

Direct inclusion of such a mass term is forbidden to $SU(2)_L$ gauge invariance. SEESAW MECHANISM assumes such a term does not exist at first, but it can be generated at effective field theory level. Using only SM fields, there is only one possible gauge invariant dimension-5 operator, Weinberg operator

$$\frac{f}{\Lambda}(L^TC^\dagger\varepsilon H)(H^T\varepsilon L)$$

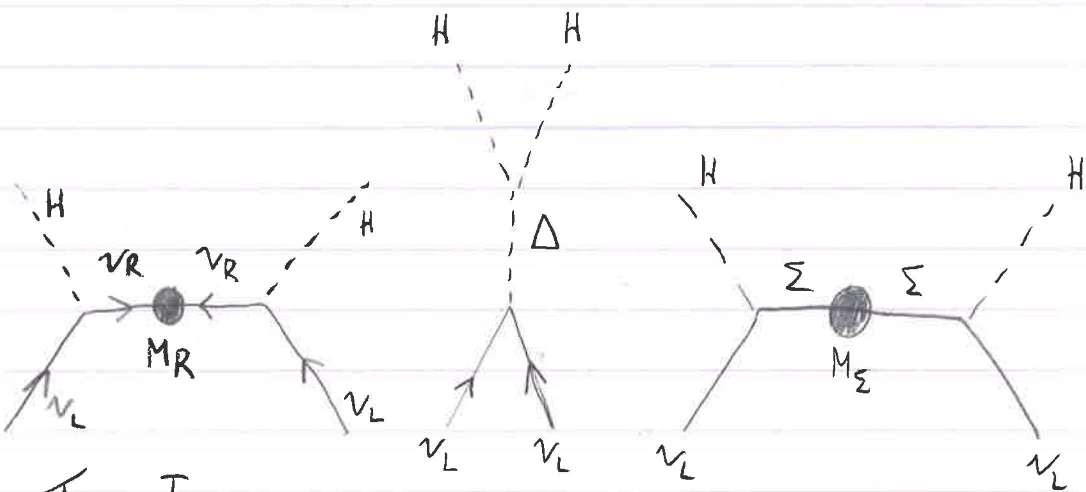
2D Levi-Civita matrix

dimensionless constant

seesaw scale

Charge conjugation operator

Three ways to generate Weinberg operator from tree level:



Type I Type II Type III

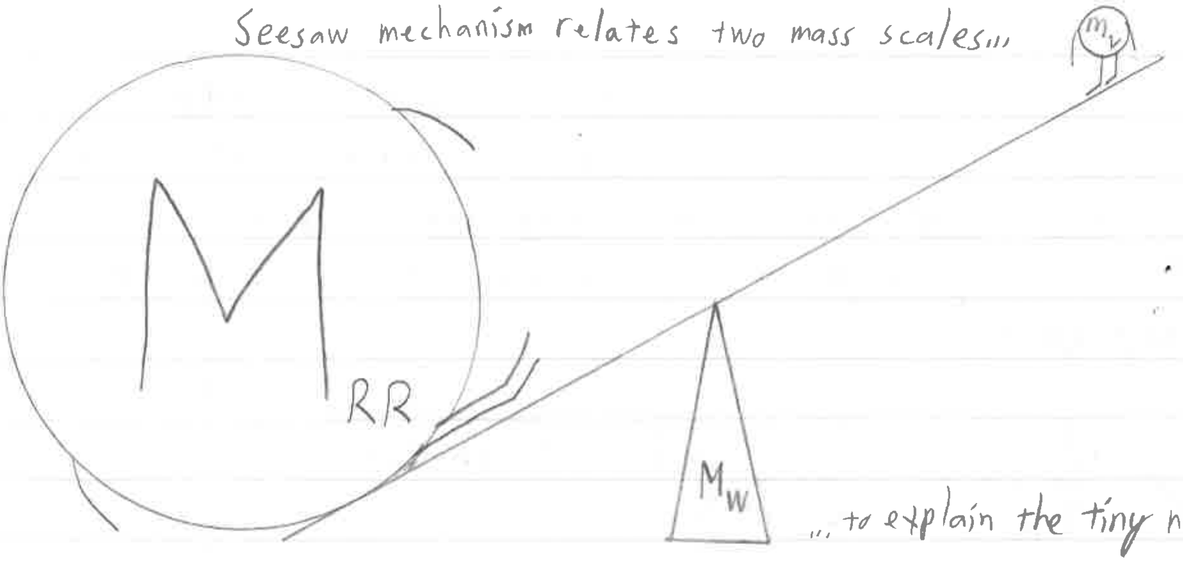
Right-handed Majorana neutrino mass term doesnot break gauge invariance since ν_R are sterile (i.e, singlets with respect to SM gauge symmetries).

$$-\frac{1}{2}\underbrace{M_{RR}}_{3\times3}\overline{\nu_R}\nu_R^c + \text{h.c.}$$

Dirac mass terms can be generated by SM Higgs mechanism:

$$m_{LR}\overline{\nu_L}\nu_R + m_{LR}^T\overline{\nu_R^c}\nu_L^c \quad , \quad \underbrace{m_{LR} = \overset{\mathcal{O}(1)}{Y^\nu}}_{3\times3} v$$

Seesaw mechanism relates two mass scales...



... to explain the tiny neutrino masses:

electroweak scale $\quad v \sim M_W \sim M_Z \sim M_h \sim \mathcal{O}(100)$ GeV

and

new physics scale $\quad \Lambda \sim M_{RR}$.

4.1 TYPE I SEESAW MECHANISM

Dirac and Majorana ν mass terms can be combined in the following (block) matrix form:

$$\left(\overline{\nu_L} \;\; \overline{\nu_R^c}\right)\begin{pmatrix} \overset{3\times3}{0} & \overset{3\times s}{m_{LR}} \\ m_{LR}^T & \underset{s\times s}{M_{RR}} \end{pmatrix}\begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

s = amount of sterile neutrinos

To get the mass matrices of left- and right-handed neutrinos, m_ν, M_N, the mass matrix must be block diagonalized with a unitary transformation matrix

$$U = \begin{pmatrix} A & D^\dagger \\ -C & B^\dagger \end{pmatrix}, \text{ where } \underset{3\times 3}{A}, \underset{5\times 5}{B}, C \text{ and } \underset{5\times 3}{D} \text{ are arbitrary matrices,}$$

such that $$M_\nu = U^T \begin{pmatrix} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix} U = \begin{pmatrix} m_\nu & 0 \\ 0 & M_N \end{pmatrix}$$

Unitarity condition: $U^\dagger U = UU^\dagger = I$

$$\Rightarrow \begin{cases} A^\dagger A + C^\dagger C = AA^\dagger + D^\dagger D = BB^\dagger + DD^\dagger = B^\dagger B + CC^\dagger = I \\ DA - BC = B^\dagger D - CA^\dagger = 0 \end{cases}$$

$$\Rightarrow \begin{pmatrix} -A^T m_{LR} C - C^T m_{LR}^T A + C^T M_{RR} C & A^T m_{LR} B^\dagger - C^T m_{LR}^T D^\dagger - C^T M_{RR} B^\dagger \\ -D^* m_{LR} C + B^* m_{LR}^T A - B^* M_{RR} C & D^* m_{LR} B^\dagger + B^* m_{LR}^T D^\dagger + B^* M_{RR} B^\dagger \end{pmatrix}$$
$$= \begin{pmatrix} m_\nu & 0 \\ 0 & m_N \end{pmatrix}$$

C and D matrices describe active-sterile mixing, which is very small according to nonunitarity bounds. Therefore quadratic terms involving C and D can be ignored.

Off-diagonals imply: $C = M_{RR}^{-1} m_{LR}^T A = (M_{RR}^{-1})^T m_{LR}^T A$
$\Rightarrow M_{RR} = M_{RR}^T$ (symmetric block!)

$$\Rightarrow D = B M_{RR}^{-1} M_{LR}^T$$

$$\Rightarrow M_\nu = \begin{pmatrix} -A^T m_{LR} M_{RR}^{-1} m_{LR}^T A & 0 \\ 0 & B^* (M_{RR} + M_{RR}^{-1} m_{LR}^\dagger m_{LR} + m_{LR}^T m_{LR}^\dagger M_{RR}^{-1\dagger}) B^* \end{pmatrix}$$

We expect m_{LR} be at EW scale and M_{RR} very heavy, so $M_{RR} \gg m_{LR}^2 M_{RR}^{-1}$ and then

$$M_\nu = \begin{pmatrix} -A^T m_{LR} M_{RR}^{-1} m_{LR}^T A & 0 \\ 0 & B^* M_{RR} B^\dagger \end{pmatrix}$$

Note: this procedure works for any matrices A and B. For simplicity we will choose them now to be unit matrices. Now the transformation matrix is

$$U = \begin{pmatrix} I & m_{LR}^* M_{RR}^{*-1} \\ -M_{RR}^{-1} m_{LR}^T & I \end{pmatrix}$$

where the magnitude of off-diagonal matrix block elements gives a prediction on the scale of active-sterile mixing:

$$P(\nu_A \to \nu_s) \sim |U_{\ell i}|^2 \sim \frac{m_{LR}^2}{M_{RR}^2} \sim \frac{m_\nu}{M_N} \sim \frac{m_\nu}{0.1\,eV} \cdot \frac{M_N}{TeV} \cdot 10^{-13}$$

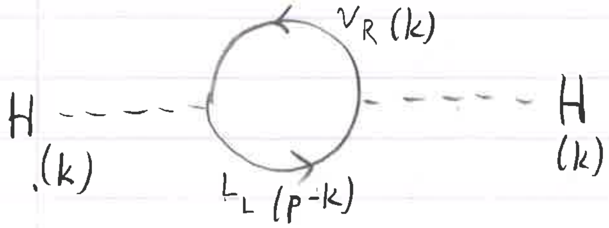
$P_{a\to s}$ inversely proportional to heavy neutrino mass
$\Rightarrow$ oscillation effects not observable if M_{RR} is very heavy! ☹

$$M_\nu = \begin{pmatrix} -m_{LR} M_{RR}^{-1} m_{LR}^T & 0 \\ 0 & M_{RR} \end{pmatrix} \Rightarrow \begin{cases} m_\nu = -m_{LR} M_{RR}^{-1} m_{LR}^T \\ M_N = M_{RR} \end{cases}$$

This is the result usually shown as a result of Type I seesaw mechanism. To remove the negative sign in m_ν, transform light neutrinos: $\nu_L^c \mapsto i\nu_L^c$, which gives the physical masses

$$\tilde{m}_\nu = m_{LR} M_{RR}^{-1} m_{LR}^T = v^2 Y^\nu M_{RR}^{-1} Y^{\nu T}$$

This explains why neutrinos are light and why the mechanism is called "seesaw".



The existance of right-handed neutrino induces loop corrections to SM Higgs mass. We calculate a rough estimate of it using cutoff regularisation.

Amplitude for the above Feynman diagram;

$$A = \underbrace{(-1)}_{\text{Fermion loop}} \underbrace{\tfrac{1}{2}Y^2}_{\text{Vertex}} \underbrace{\int \frac{d^4k}{(2\pi)^4}}_{\text{Loop momentum}} \mathrm{Tr}\, \underbrace{\frac{\not p - \not k + M}{(p-k)^2 - M^2}}_{A} \cdot \underbrace{\frac{\not k + m}{k^2 - m^2}}_{B}$$

Feynman parameterisation: $\dfrac{1}{AB} = \displaystyle\int_0^1 \frac{dx}{(Ax + (1-x)B)^2}$

$$\Rightarrow A = \frac{1}{2}Y^2 \int_0^1 dx \int \frac{d^4 l}{(2\pi)^4}\left(\frac{l^2}{(l^2+\Delta)^2} + \frac{N}{(l^2+\Delta)^2}\right)$$

$$\equiv \frac{1}{2}Y^2 \int_0^1 dx\,(I_1 + I_2)$$

$l = k - xp$

$\Delta = -x^2p^2 + xp^2 + (m^2 - M^2)x - m^2$

$N = x^2p^2 - xp^2 - Mm$

$$I_1 \equiv \int \frac{d^4 l}{(2\pi)^4}\frac{l^2}{(l^2+\Delta)^2} = \int d\Omega_4 \int_M^\Lambda \frac{dl}{(2\pi)^4}\left(l + \frac{2\Delta}{l}\right) + (\text{finite terms})$$

$$I_2 \equiv \int \frac{d^4 l}{(2\pi)^4}\frac{N}{(l^2+\Delta)^2} = N \underbrace{\int d\Omega_4}_{2\pi^2,\ \text{area of 4D unit sphere}} \int_M^\Lambda \frac{dl}{(2\pi)^4}\frac{1}{l} + (\text{finite terms})$$

$$\Rightarrow I_1 + I_2 = \frac{1}{8\pi^2}\left(\Lambda^2 + (2\Delta + N)\ln\frac{\Lambda}{M}\right) + (\text{finite terms})$$

$$\Rightarrow A = \frac{1}{2}Y^2 \int_0^1 \frac{dx}{8\pi^2}\left(\Lambda^2 + (2\Delta + N)\ln\frac{\Lambda}{M}\right)$$

$$= \frac{Y^2}{16\pi^2}\left[\Lambda^2 + \left(\frac{p^2}{6} - m^2 - M^2 - mM\right)\ln\frac{\Lambda}{M}\right]$$

$$\approx \frac{Y^2}{16\pi^2}\left(\Lambda^2 + M^2\ln\frac{\Lambda}{M}\right) = \delta m_h^2$$

large corrections!

ALTERNATE DERIVATION OF TYPE I SEESAW FORMULA

$$\mathcal{L} = \bar{\nu}_L m_{LR} \nu_R + \overline{\nu_R^c} m_{LR}^T \nu_L^c - \overline{\nu_R^c} M_{RR} \nu_R$$

What is the state of ν_R when we approach low energy scale?
In effective field theory (EFT) one way is to extremize the Lagrangian with respect to the heavy field.

$$\frac{\partial \mathcal{L}}{\partial \nu_R} = \bar{\nu}_L m_{LR} - \overline{\nu_R^c} M_{RR} = 0$$

$$\Rightarrow \overline{\nu_R^c} = \bar{\nu}_L m_{LR} M_{RR}^{-1}$$

$$\Rightarrow \nu_R = M_{RR}^{-1T} m_{LR}^T \nu_L^c = M_{RR}^{-1} m_{LR}^T \nu_L^c$$

Substituting the fields $\bar{\nu}_R^c$ and ν_R back to Lagrangian, we get

$$\mathcal{L}_{eff} = \underline{\bar{\nu}_L m_{LR} M_{RR}^{-1} m_{LR}^T \nu_L^c}$$

Light neutrino seesaw formula! More elegant than block diagonalizing. ☺

SOMETHING MUST BE SMALL OR LARGE

$m_\nu = Y_\nu V$ Dirac neutrino Small Yukawa	$m_\nu = \dfrac{Y_\nu^2 v^2}{M_N}$ Majorana neutrino Seesaw mechanism Heavy sterile neutrino	$m_\nu = \dfrac{Y_\nu^2}{M_N} v_\nu^2$ Neutrinophilic model Extended Higgs sector Small VEV

Also many more possibilities...

4.2 TYPE II SEESAW MODEL

Instead of right-handed neutrinos, one may postulate the existence of extended Higgs sector. Since neutrino masses belong to completely different scale than other SM particles, one may presume different parts of Higgs sector are responsible for different particles on different mass scales.

$$\mathcal{L}_m^{\nu,\text{eff}} \sim \bar{\nu}_L \,(\,???\,)\, \nu_L^c$$

$Y = -1$

The scalar would have $Y = 2$

$$\mathcal{L}_{SCALAR} = (D_\mu H)^\dagger (D^\mu H) + \mathrm{Tr}\left((D_\mu \Delta)^\dagger (D^\mu \Delta)\right) + Y^\nu L_L^T C i\sigma_2 \Delta L_L - V(H,\Delta)$$

A triangle TRIplet (Get it?)

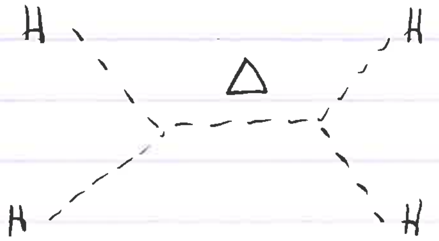
$$\Delta = \frac{1}{\sqrt{2}} \sigma_i \Delta_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \Delta_3 & \Delta_1 - i\Delta_2 \\ \Delta_1 + i\Delta_2 & -\Delta_3 \end{pmatrix} \equiv \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{pmatrix}$$

Three scalar fields $\Delta_1, \Delta_2, \Delta_3$.

$$D_\mu \Delta = \partial_\mu \Delta + i g_2 [\tau \cdot W_\mu, \Delta] + i g_1 \underbrace{Y_\Delta}_{2} B_\mu \Delta / 2$$

$$V(H,\Delta) = -m_H^2 H^\dagger H + \frac{\lambda_H}{4} (H^\dagger H)^2 + M_\Delta^2 \mathrm{Tr}(\Delta^\dagger \Delta)$$
$$+ \left(\underbrace{\lambda_{H\Delta}}_{\text{dimension -1 coupling}} H^T i \sigma_2 \Delta^\dagger H + h.c. \right)$$

Corresponds to SM Higgs self-interaction



Electric charges of triplet fields

We use Gell-Mann–Nishijima formula, $Q = T_3 + \frac{Y}{2}$

For bidoublet form, $Q\Delta = [T_3, \Delta] + \frac{Y_\Delta}{2}\Delta$

For 1×3 representation, $Q\Delta = \left(T_3 + \frac{Y_\Delta}{2}\right)\Delta$

↑ Third SU(2) group generator in 3×3 representation!

We need to find the "3×3" Pauli matrices T_1, T_2, T_3 obeying the Lie algebra defined by commutation relation

$$[T_i, T_j] = i\,\varepsilon_{ijk} T_k$$

(Exercise)

$$\Rightarrow T_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix}$$

or in bidoublet form $Q\Delta = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \odot \Delta$ ← Matrix Hadamard product

$$\equiv \begin{pmatrix} Q(\Delta_{11}) & Q(\Delta_{12}) \\ Q(\Delta_{21}) & Q(\Delta_{22}) \end{pmatrix} \odot \Delta$$

$$\Rightarrow \begin{cases} \frac{1}{\sqrt{2}}(\Delta_1 - i\Delta_2) = \Delta^{++} \\ \frac{1}{\sqrt{2}}(\Delta_1 + i\Delta_2) = \Delta^{0} \\ \Delta_3 = \Delta^{+} \end{cases}$$

$$\Rightarrow \Delta = \begin{pmatrix} \Delta^{+}/\sqrt{2} & \Delta^{++} \\ \Delta^{0} & -\Delta^{+}/\sqrt{2} \end{pmatrix} \xmapsto{SSB} \begin{pmatrix} 0 & 0 \\ v' & 0 \end{pmatrix}$$

Finding the VEV of Δ

$$V\left(\tfrac{v}{\sqrt{2}},\Delta\right) = -\tfrac{1}{2} m_H^2 v^2 + \frac{\lambda_H v^4}{16} + M_\Delta^2 \,\mathrm{Tr}(\Delta^\dagger \Delta)$$

$$+\left[\lambda_{H\Delta} \frac{1}{\sqrt{2}} (0\ \ v) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} \Delta^+ & \sqrt{2}\Delta^{++} \\ \sqrt{2}\Delta^0 & -\Delta^+ \end{pmatrix}^\dagger \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} + h.c. \right]$$

$$= M_\Delta^2 \left(|\Delta^{++}|^2 + |\Delta^+|^2 + |\Delta^0|^2 \right) - \tfrac{1}{2} v^2 \lambda_{H\Delta} \Delta^{0*} - \tfrac{1}{2} v^2 \lambda_{H\Delta} \Delta^0$$

+ constant

$$\Rightarrow \frac{\partial V(\frac{v}{\sqrt{2}},\Delta)}{\partial \Delta^{0*}} = 0$$

$$\Rightarrow \langle \Delta^0 \rangle \equiv v' = \lambda_{H\Delta} \frac{v^2}{2M_\Delta^2}$$

Tree-level correction to electroweak ~~gauge~~ boson masses

$\mathrm{Tr}\left[(D_\mu \Delta)^\dagger (D^\mu \Delta) \right]$ ⟵ Let us pick only the part which contribute to the masses of EW ~~gauge~~ bosons.

$$\left(i g_2 [\underbrace{\vec{\tau}\cdot\vec{W}_\mu, \Delta}] + i g_1 Y_\Delta B_\mu \Delta/2 \right)^\dagger \left(i g_2 [\vec{\tau}\cdot\vec{W}^\mu, \Delta] + i g_1 Y_\Delta B_\mu \Delta/2 \right)$$

$$= \left[\frac{1}{2} \begin{pmatrix} W_3 & W_1 - iW_2 \\ W_1 + iW_2 & -W_3 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v' & 0 \end{pmatrix} \right] = \frac{v'}{2\sqrt{2}} \begin{pmatrix} \sqrt{2}W^- & 0 \\ -2W_3 & \sqrt{2}W^- \end{pmatrix}$$

$$= \frac{v'}{\sqrt{2}} \begin{pmatrix} W^- g_2/\sqrt{2} & 0 \\ -Z\sqrt{g_1^2+g_2^2} & -W^- g_2/\sqrt{2} \end{pmatrix} \cdot (h.c.)$$

$$\Rightarrow \mathrm{Tr} = \frac{v'^2}{2} \left(g_2^2 W_\mu^+ W^{-\mu} + (g_1^2 + g_2^2) Z_\mu Z^\mu \right)$$

$$\Rightarrow m_W^2 = g_2^2 \left(\tfrac{1}{4} v^2 + \tfrac{1}{2} v'^2 \right), \quad m_Z^2 = (g_1^2 + g_2^2) \left(\tfrac{1}{4} v^2 + \tfrac{1}{2} v'^2 \right)$$

Electroweak precision measurements have measured

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1,00037 \pm 0,00023 \qquad (\rho_{SM} = 1)$$

$$\Rightarrow v' \lesssim 4,3 \text{ GeV}$$

v' could be naturally small to account for lightness of neutrinos. Mass matrix for neutrinos is

$$m_\nu = -Y^\nu v' = -Y^\nu \lambda_{H\Delta} \frac{v^2}{2M_\Delta^2}$$

Small m_ν from
- smallness of Yukawa? ← not pretty
- smallness of $\lambda_{H\Delta}$?
- heaviness of Δ?

If Δ has lepton number -2, only term violating lepton number conservation would be proportional to $\lambda_{H\Delta}$. Since at limit $\lambda_{H\Delta} \to 0$ the symmetry of the theory would be enhanced, $\lambda_{H\Delta}$ may be small in 't Hooft sense:

't Hooft's naturalness criterion (1980)

"At any energy scale μ, a set of parameters, $\alpha_i(\mu)$ describing a system can be small, if and only if, in the limit $\alpha_i(\mu) \to 0$ for each of these parameters the system exhibits an enhanced symmetry."

Direct collider searches: $M_\Delta \gtrsim 750$ GeV
Peskin–Takeuchi limits: $|m_{\Delta^{++}} - m_{\Delta^+}| \lesssim 40$ GeV

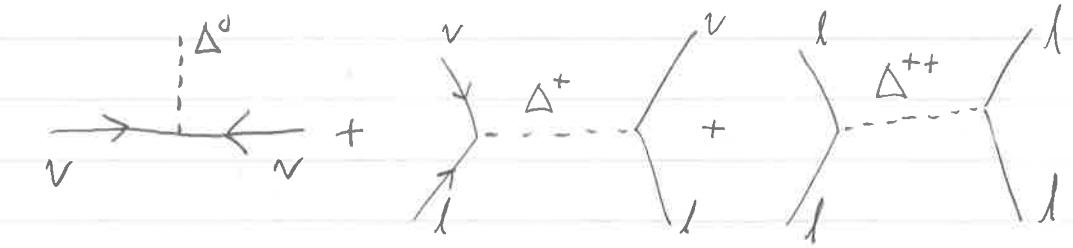
$\Rightarrow$ mass degeneracy of triplet scalars is a reasonably good approximation.

Type II seesaw Yukawa sector

$$\mathcal{L} = Y^{\nu}_{\alpha\beta} L^{T}_{\alpha L} C i\sigma_2 \Delta L_{\beta L} + h.c.$$

$$= Y^{\nu}_{\alpha\beta}\left(\Delta^0 \overline{\nu^c_{\alpha R}} \nu_{\beta L} - \frac{1}{\sqrt{2}}\Delta^+ \left(\overline{l^c_{\alpha R}} \nu_{\beta L} + \overline{\nu^c_{\alpha R}} l_{\beta L}\right) - \Delta^{++} \overline{l^c_{\alpha R}} l_{\beta L}\right)$$

$+ h.c.$

$=$ 

Yukawa matrix elements arbitrary $\Rightarrow$ no hints or texture for ν mass or mixing matrices

Corresponding low-energy Lagrangians are

$$\mathcal{L}^m_\nu = \frac{Y_{\alpha\beta}\lambda_{H\Delta} v^2}{M_\Delta^2}\left(\overline{\nu^c_{\alpha R}}\nu_{\beta L}\right) = - m^\nu_{\alpha\beta}\overline{\nu^c_{\alpha R}}\nu_{\beta L} \Rightarrow \text{neutrino mass}$$

$$\mathcal{L}_{NSI} = \frac{Y_{\sigma\beta} Y^\dagger_{\alpha\rho}}{M_\Delta^2}\left(\overline{\nu_{\alpha L}}\gamma_\mu \nu_{\beta L}\right)\left(\overline{l_{\rho L}}\gamma^\mu l_{\sigma L}\right) \Longrightarrow \text{nonstandard interactions}$$

$$\mathcal{L}_{4l} = \frac{Y_{\sigma\beta} Y^\dagger_{\alpha\rho}}{M_\Delta^2}\left(\overline{l_{\alpha L}}\gamma_\mu l_{\beta L}\right)\left(\overline{l_{\rho L}}\gamma^\mu l_{\sigma L}\right) \Longrightarrow \text{CLFV decays}$$

We know $Y = -\dfrac{2M_\Delta^2}{v^2\lambda_{H\Delta}}(m^\nu)^{-1}$. Substituting this to $\mathcal{L}_{NSI}$, one gets

$$\varepsilon^{\rho\sigma L}_{\alpha\beta} = -\frac{\sqrt{2}M_\Delta^2\, m^\nu_{\sigma\beta}\, m^{\nu\dagger}_{\alpha\rho}}{G_F v^4 \lambda^2_{H\Delta}}$$

$\Rightarrow$ bounds from NSI can be translated to bounds of Type II seesaw, giving $\dfrac{M_\Delta}{|\lambda\phi|} < \mathcal{O}(10^{12})$ from oscillations and $< \mathcal{O}(10^9)$ from CLFV decays.

CMS: $M_\Delta \gtrsim 750$ GeV $\Rightarrow \lambda_{H\Delta} \gtrsim 31$ meV.

4.3 TYPE III SEESAW MECHANISM

Similar to Type I and II mechanisms, Type III is a simple mechanism, adding a hyperchargeless fermion triplet field $\Sigma = (\Sigma^1, \Sigma^2, \Sigma^3) \sim (3, 0)$ to the SM.

$$\mathcal{L} = \mathrm{Tr}(\bar{\Sigma} i \not{D} \Sigma) - \frac{1}{2}\mathrm{Tr}(\bar{\Sigma} M_\Sigma \Sigma^c + \bar{\Sigma}^c M_\Sigma^* \Sigma)$$
$$- H'^{\dagger} \bar{\Sigma} \sqrt{2}\, Y_\Sigma L + \text{h.c.}$$

Kinetic Yukawa Majorana mass

$$\Sigma = \frac{1}{\sqrt{2}} \sigma_i \Sigma_i = \frac{1}{\sqrt{2}} \begin{bmatrix} \Sigma_3 & \Sigma_1 - i\Sigma_2 \\ \Sigma_1 + i\Sigma_2 & -\Sigma_3 \end{bmatrix} \equiv \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

Let's determine the charges using Gell-Mann-Nishijima formula:

$$Q\Sigma = [T^3, \Sigma] + \frac{Y}{2}\Sigma = \begin{pmatrix} 0 & \Sigma_{12} \\ -\Sigma_{21} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \odot \Sigma$$

$$\Rightarrow \Sigma_3 \equiv \Sigma^0, \quad \Sigma_1 \mp i\Sigma_2 = \sqrt{2}\,\Sigma^{\pm}$$

- Kinetic Lagrangian couples Σ to gauge bosons. Therefore Σ can be produced by colliders. Direct search bounds imply $M_\Sigma > 840\,\mathrm{GeV}$.

- Yukawa Lagrangian induces $\Sigma^{\pm} - \ell^{\pm}$ mixing and $\Sigma^0 - \nu$ mixing, with mixing strength $\frac{Y_\Sigma v}{M_\Sigma}$, generating flavour violating vertices, producing LFV decays, like $\mu \to e\gamma$.

$$\Rightarrow \text{NSI!} \quad \varepsilon^S = \frac{v^2}{2} Y_\Sigma^{\dagger} (M_\Sigma^{\dagger} M_\Sigma)^{-1} Y_\Sigma \ll 1$$
$$\Rightarrow M_\Sigma > 200\,\mathrm{TeV}$$

- Neutrino mass is generated similar to Type I case, with $M_{RR} \mapsto M_\Sigma$:

$$m_\nu = \frac{Y_\Sigma^2 v^2}{M_\Sigma}$$, mass terms breaks lepton number