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1 Preliminaries

Sterile neutrino decays are often discussed in the context of simplified models, which makes it hard to connect to a full theoretical picture of what the decay channels are. In our discussions so far, we have taken the simplified model in Ref. [1], namely

$$-\mathcal{L} \supset g_s \overline{\nu}_s \nu_s \phi + \sum_{a,b=\{e,\mu,\tau,s,\dots\}} m_{ab} \overline{\nu_a} \nu_b. \tag{1.1}$$

Here $\nu_a = \nu_a^L + \nu_a^R$ is a flavour state, where ν_e^L is an active electron-neutrino, and ν_e^R is an sterile electron-neutrino. Note that yet another sterile state, $\nu_s = \nu_s^L + \nu_s^R$, is present and is the only state that couples to ϕ.

In our "benchmark model" of Eq. (1.1), neutrinos are assumed to be Dirac and the new interaction is *parity conserving*. If we want m_{ab} to be arbitrary, then the model requires further UV completion. To clarify any confusion, let us state in most general grounds, the mass terms allowed with the minimum particle content above. Taking $\alpha \in \{e, \mu, \tau\}$ and conserving Lepton number, we have:

$$-\mathcal{L}_0 = \sum_{\alpha\beta} y_{\alpha\beta} (\overline{L}_\alpha \widetilde{H}) \nu_\beta^R + \sum_\alpha y_{\alpha s} (\overline{L}_\alpha \widetilde{H}) \nu_s^R + \sum_\alpha m_{s\alpha}^R \overline{\nu_s^L} \nu_\alpha^R + m_s \overline{\nu_s^L} \nu_s^R \tag{1.2}$$

where I wrote down all chiral fermions explicitly.

Now, I will work with a single generation of active neutrinos, with $\nu_e = \nu_e^L + \nu_e^R$. After EWSB, I define $m_D = y_{ee} v_h / \sqrt{2}$, $m_L = y_{es} v_h / \sqrt{2}$, and $m_R = m_{se}^R$, such that the mass matrix is

$$\begin{pmatrix} \overline{\nu_e^L} & \overline{\nu_s^L} \end{pmatrix} \begin{pmatrix} m_D & m_L \\ m_R & m_s \end{pmatrix} \begin{pmatrix} \nu_e^R \\ \nu_s^R \end{pmatrix} + \text{h.c.} \tag{1.3}$$

Clearly, the statement that neutrino masses and mixing are arbitrary in this model may require that the mass matrix above receives additional corrections from other sector we do not specify here. Nevertheless, we could proceed to find the singular values of an arbitrary mass matrix M, such as the one above, and write

$$\begin{pmatrix} \overline{\nu_1^L} & \overline{\nu_2^L} \end{pmatrix} V_L^\dagger M V_R \begin{pmatrix} \nu_1^R \\ \nu_2^R \end{pmatrix} + \text{h.c.}, \tag{1.4}$$

where $V_L^\dagger M V_R = \text{diag}(m_1, m_2)$. In this way, the Dirac mass eigenstates, the left-chiral, and right-chiral mass states are given by

$$\nu_k = \nu_k^L + \nu_k^R, \quad \nu_a^L = (V_L)_{ak} \nu_k, \quad \nu_a^R = (V_R)_{ak} \nu_k^R. \tag{1.5}$$

The charge current can now be written in its standard format

$$-\mathcal{L}_{CC} \supset \frac{g}{\sqrt{2}} \overline{\nu_e^L} \not{W} e_L + \text{h.c.} = \frac{g}{\sqrt{2}} \overline{\nu} (V_L^\dagger V_L^e) \not{W} P_L e + \text{h.c.} \tag{1.6}$$

where $U^\dagger_{PMNS} \equiv V_L^\dagger V_L^e$ defines the new PMNS matrix, and V_L^e is the charged lepton mixing matrix that is diagonal without loss of generality. Note that only V_L and not V_R appears.

In what follows we will write down a few possibilities for the scalar coupling and study their general properties. This is mostly a trivial exercise, but which will help clarify any confusion about chirality in the scalar decays.

2 Neutrino decays

In all generality, the model we have worked with can be writen as

$$\mathcal{L}_{\phi-\text{int}} = g_S \overline{\nu_s^L} \nu_s^R \phi + g_S^* \overline{\nu_s^R} \nu_s^L \phi, \tag{2.1}$$

with $g_S = |g_S| e^{i\delta}$, allowing for CP violation. We rearrange it into

$$\mathcal{L}_{\phi-\text{int}} = \sum_{i,j} \overline{\nu_i} \left[g_S S_{ij} P_R + g_S^* S_{ji}^* P_L \right] \nu_j \phi, \tag{2.2}$$

where $S_{ij} = (V_L)^*_{si} (V_R)_{sj}$. Note that in all generality, $S^*_{ij} \neq S_{ji}$ since $V_L \neq V_R$.

Back to basics: The ordering of i, j matters. The amplitude relevant for $\nu_i \to \nu_j \phi$ decay is given by

$$\langle \nu_j \phi | \, \overline{\nu_j} (g_S S_{ji} P_R + g_S^* S_{ij}^* P_L) \nu_i \phi \, | \nu_i \rangle \,, \tag{2.3}$$

and for scalar decay

$$\langle \nu_j \overline{\nu_i} | \, \overline{\nu_j} (g_S S_{ji} P_R + g_S^* S_{ij}^* P_L) \nu_i \phi \, | \phi \rangle \,, \tag{2.4}$$

Nothing particularly new here, but with great care one can find the helicity amplitudes for $\nu_i \to \nu_j \phi$ decay. Neglecting the final state masses,

$$|\mathcal{M}_{\nu_i^L \to \nu_j^L \phi}|^2 = |g_S|^2 |S_{ji}|^2 m_i^2 r_e, \tag{2.5}$$
$$|\mathcal{M}_{\nu_i^L \to \nu_j^R \phi}|^2 = |g_S|^2 |S_{ij}|^2 m_i^2 (1 - r_e), \tag{2.6}$$
$$|\mathcal{M}_{\nu_i^R \to \nu_j^L \phi}|^2 = |g_S|^2 |S_{ij}|^2 m_i^2 (1 - r_e), \tag{2.7}$$
$$|\mathcal{M}_{\nu_i^R \to \nu_j^R \phi}|^2 = |g_S|^2 |S_{ji}|^2 m_i^2 r_e, \tag{2.8}$$
$$|\mathcal{M}_{\phi \to \nu_j^L \overline{\nu}_i^R}|^2 = |g_S|^2 |S_{ij}|^2 m_\phi^2, \tag{2.9}$$
$$|\mathcal{M}_{\phi \to \nu_j^R \overline{\nu}_i^L}|^2 = |g_S|^2 |S_{ji}|^2 m_\phi^2, \tag{2.10}$$

where $r_e = E_j/E_i$ and we denoted by $\nu^{L\,(R)}$ neutrinos with $h = -1\,(+1)$ and $\overline{\nu}^{L\,(R)}$ antineutrinos with $h = +1\,(-1)$. Clearly, the allowed channels depend crucially on S_{ij} and S_{ji}.

We have

$$|S_{ij}|^2 = |(V_L)_{si}|^2 |(V_R)_{sj}|^2, \tag{2.11}$$
$$|S_{ji}|^2 = |(V_R)_{si}|^2 |(V_L)_{sj}|^2. \tag{2.12}$$

Clearly, i denotes a heavy neutrino mass state, and j a light one, then if $|(V_R)_{sj}|^2 = 0$ only *visible* light neutrinos or antineutrinos (namely, ν_j^L and $\overline{\nu}_j^R$) are produced as a result of heavy neutrino decays. On the other hand, if $|(V_L)_{sj}|^2 = 0$, then only *invisible* neutrinos are ever produced. We will see that these two conditions are easy to achieve in a simple model, even if the latter case is useless for SBL phenomenology. Note that if ϕ is lighter than ν_i, then $\phi \to \nu_j \overline{\nu_j}$ is the only allowed decay, and is not allowed at all unless both $|(V_R)_{sj}|^2 |(V_L)_{sj}|^2 \neq 0$. In this case, it is not possible to separate the ϕ decays into visible or invisible channels.

2.1 Decoupling visible and invisible decays

Now, let us check if it is possible to obtain $S_{ij} \neq 0$ with $S_{ji} = 0$, or vice-versa, so that either only visible or invisible neutrinos are ever produced in the heavy neutrino decays. We start by finding some constraints on the mixing matrices V_L and V_R. By virtue of $V_L^\dagger M V_R = \hat{m} \equiv \mathrm{diag}(m_1, m_2, \dots)$, we have

$$V_L^\dagger M M^\dagger V_L = (\hat{m})^2, \quad V_R^\dagger M^\dagger M V_R = (\hat{m})^2. \tag{2.13}$$

For concreteness, let us work with the 2×2 matrix of sub-block matrices in Eq. (1.3). In general

$$M^\dagger M = \begin{pmatrix} m_D^\dagger m_D + m_R^\dagger m_R & m_D^\dagger m_L + m_R^\dagger m_s \\ m_L^\dagger m_D + m_s^\dagger m_R & m_L^\dagger m_L + m_s^\dagger m_s \end{pmatrix}, \tag{2.14}$$

$$M M^\dagger = \begin{pmatrix} m_D m_D^\dagger + m_L m_L^\dagger & m_D m_R^\dagger + m_L m_s^\dagger \\ m_R m_D^\dagger + m_S m_L^\dagger & m_R m_R^\dagger + m_s m_s^\dagger \end{pmatrix}. \tag{2.15}$$

Clearly each one of these different matrices is self-adjoint.

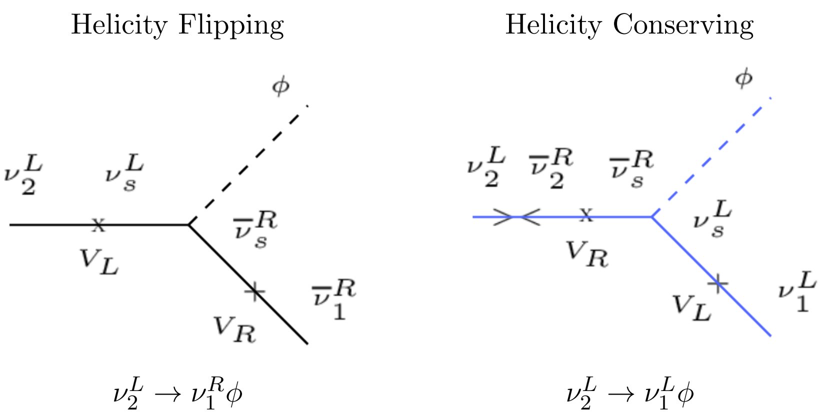

Figure 1: The two decay modes for polarized ν_2 states shown in terms of chiral fields. An additional mass insertion in the right diagram conserves the initial helicity.

Visible decays only: Taking $V_R = \mathbb{1}$ is a solution to $|(V_R)_{sj}|^2 = 0$ when $j = 1$, for instance. In that case, it follows that $M^\dagger M = (\hat{m})^2$. If we treat the entries of Eq. (2.14) as numbers, then the condition $V_R = \mathbb{1}$ with non-trivial $V_L \neq \mathbb{1}$ is simply

$$m_L m_D + m_R m_s = 0, \quad m_R m_D + m_L m_s \neq 0. \tag{2.16}$$

A perfectly reasonable solution to this can be found for $m_D = m_R = 0$, with $m_L \neq 0$ and $m_s \neq 0$, so that $m_1^2 = 0$ and $m_2^2 = |m_L|^2 + |m_s|^2$. In this case, only visible neutrino decays are allowed. By virtue of $S_{11} = 0$, the scalar ϕ remains stable, unless $m_2 < m_\phi$.

Invisible decays only: On the other hand, had we chosen to set $V_L = \mathbb{1}$ with $V_R \neq \mathbb{1}$, then there exists a trivial solution with $m_D = 0 = m_L = 0$, and $m_1 = 0$ and $m_2 = |m_R|^2 + |m_s|^2$. In this case, however, ν_2 cannot be produced in weak interactions, as $U_{e2} = 0$. If other active flavours are involved, then one may be able to produce ν_2 in pion decays via mixing with the muon, for instance, but ν_2 would still not decay to visible neutrinos. This is not so useful in the context of SBL anomalies.

2.2 Alternative Dirac models with ν_s non-conservation

What if instead we had not preserved ν_s number, and written down the following Lagrangian,

$$\mathcal{L}_{\phi-\text{int}} = g_S \overline{\nu_s^L} \nu_e^R \phi + g_S^* \overline{\nu_e^R} \nu_s^L \phi \tag{2.17}$$

Then, everything else follows as above, with the substitution $S_{ij} \to R_{ij} \equiv (V_L)^*_{si} (V_R)_{ej}$. In this case, setting $V_R = \mathbb{1}$ leads to an *invisible* decay model. This makes sense, since $\nu_1^R = \nu_e^R$, and

Similar considerations can be made when UV completing operators such as

$$\frac{1}{\Lambda}(LH)\nu_s^R \phi, \quad \frac{1}{\Lambda}(LH)\nu_e^R \phi, \quad \text{or} \quad \frac{1}{\Lambda^2}(LH)(LH)\phi. \tag{2.18}$$

3 Conclusion

In the 2 neutrino model we studied, we were able to separate the visible and invisible ν_2 decays depending on the choice of the mass terms of the theory. We found a simple model with purely visible decays, in which the helicity flipping channels are *not* present. This also leads to the curious scenario where ϕ is stable even though it is heavier than ν_1.

Clearly, whenever ϕ can decay to light neutrinos, there will be a signature to search for at Borexino etc. However, it is possible to forbid ϕ decays without resorting to kinematics. These models require a case by case inspection. In other words,

A Some technicalities of helicity amplitudes

Let me review the current picture in the model of Eq. (1.1). Heavy neutrinos are produced inside the Sun via the Weak interactions, which couple only to the left-handed *chiral* neutrino fields. In the limit of vanishing mass, the neutrinos produced in $W^+ \to e^+ \nu$, for instance, are all polarized. This implies that $\Gamma(W^+ \to e^+ \nu_i^{h=1}) \ll \Gamma(W^+ \to e^+ \nu_i^{h=-1})$.

This can actually be directly computed. By virtue of Eq. (1.6), we have

$$\mathcal{M}_{W^+ \to e^+ \nu_i^h} = -\frac{ig}{\sqrt{2}} \overline{u_{\nu_i}^r}(k_2) P(h,s) \gamma^\mu P_L v_e^s(k_1), \tag{A.1}$$

where r, s are the spin of the particles. Following the notation of Romão and Silva, we decompose the Dirac spinors as

$$u^{r=+1} \simeq P_L \frac{m}{2E}\sqrt{E}\begin{pmatrix}\chi_\uparrow \\ -\chi_\uparrow\end{pmatrix} + P_R\sqrt{E}\begin{pmatrix}\chi_\uparrow \\ \chi_\uparrow\end{pmatrix}, \tag{A.2}$$

$$u^{r=-1} \simeq P_L\sqrt{E}\begin{pmatrix}\chi_\downarrow \\ -\chi_\downarrow\end{pmatrix} + P_R \frac{m}{2E}\sqrt{E}\begin{pmatrix}\chi_\downarrow \\ \chi_\downarrow\end{pmatrix}, \tag{A.3}$$

where we expanded on m/E. Clearly, in the massless limit the CC amplitude in Eq. (A.1) picks up only the $u^{r=-1}$ spinor, with a subleading m/E suppressed contribution from the $u^{r=+1}$ spinor. This is the mathematical reason behind the statement "neutrinos produced in Weak interactions are mostly left-handed". A comment about antiparticles is in order. If we take the antiparticle spinor solutions v, then the *helicity* of antifermions is defined as minus the *chirality* of the field. In practice, this means that in the massless limit $P_L P(-, s)v = 0$.

Helicity of a particle with momentum p^μ is defined as

$$h = \frac{\vec{s}\cdot\vec{p}}{|\vec{p}|}. \tag{A.4}$$

Note that for antiparticles, $\vec{s} \to -\vec{s}$. In computing helicity amplitudes, it is useful to define the spin-direction 4-vector,

$$s^\mu = (\gamma\beta, \gamma\hat{\beta}), \quad \hat{\beta} = \vec{\beta}/\beta, \quad \beta = \vec{p}/p^0, \quad \gamma = m/p^0, \tag{A.5}$$

where $s^2 = -1$, and $s \cdot p = 0$. Then, one finds

$$P(h, s) = \frac{1 + h\gamma^5 \not{s}}{2}. \tag{A.6}$$

This allows to compute the decay amplitudes by inserting $P(h, s)$ in the matrix element. Specifically, in computing $\nu_i \to \phi\nu_j$, we took

$$\mathcal{M} = \overline{u}(k_j)\left(\frac{1 - h_j\gamma^5}{2}\right)\left(S_{ji}g_S P_R + S^c_{ij}g_S^* P_L\right)\left(\frac{1 - h_j\gamma^5 \not{s}_i}{2}\right)u(P), \tag{A.7}$$

where we compute the decay in the CM with $(s \cdot p_1) = -|\vec{k}_j^{\mathrm{CM}}|\cos\theta_{\mathrm{CM}}$, defining θ_{CM} as the angle between $\vec{k_j}$ and the direction of the (infinitesimal) boost or velocity of ν_i. Direct computation of these amplitudes and the analogous ones for ϕ leads to the amplitudes in 2.5.

B Kaon decays

One can compute kaon decays $K^+ \to \mu^+\nu_i\phi$ explicitly:

$$M = \cdots \sum_k u(k_i) i(g_S S_{ik} P_R + g_S^* S^*_{ki} P_L)\frac{i(\not{p} + m_k)}{p^2 - m_k^2} U^*_{\mu k}\frac{g}{\sqrt{2}}\gamma^\mu P_L v(k_\mu)\ldots \tag{B.1}$$

In the massless limit, one picks up only the term $\sum_k^{\mathrm{all}}(V_L)_{sk}(V_R)^*_{si}U^*_{\mu k} = \sum_k^{\mathrm{all}}(V_R)^*_{si}(V_L)_{sk}(V_L)^*_{\mu k} = 0$. Other terms are much suppressed. The matrix V_R does not do much here, as expected.

References

[1] M. Dentler, I. Esteban, J. Kopp, and P. Machado, Phys. Rev. D **101**, 115013 (2020), arXiv:1911.01427 [hep-ph] .