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These are notes

Contents

1 Notable references

- KOTO anomaly https://indico.cern.ch/event/769729/contributions/3510939/attachments/1
- first anomaly paper [1].
- KOTO previous searches [2, 3]
- Follow-up paper with FCNC Z' [4].
- Kaon bounds on general Z' scenarios [5].
- Kitahara's talk at KAON19 https://indico.cern.ch/event/783304/contributions/3497938/atta

2 Explaining the $K_L \to \pi^0 \nu\overline{\nu}$ measurement

The KOTO experiment observes an anomalously high number of $K_L \to \pi^0 \nu\overline{\nu}$ events. In fact, a total of 4 events are observed, being only one of them consistent with background expectations. The 3 anomalous events imply a total BR of

$$\mathrm{BR}\left(\mathrm{K_L} \to \pi^0 \nu\overline{\nu}\right) = \left(2.1^{+4.1}_{-1.7}\right) \times 10^{-9} \text{ measured by KOTO.} \tag{2.1}$$

Beyond the new physics scenarios explored in Refs. [1] and [4], one may conceive that additional light new particles may generate such a signal when produced in K_L decays. One general possibility is that of a dark sector composed of two new states: a light X_1 and a heavier X_2 particle. If this particle pair is produced via 2-body decays, then X_2 may decay back to the lightest state, emitting a π^0 plus missing energy, as follows

$$\begin{aligned} K_L \to X_1 \, + \, & X_2 \\ & \searrow X_1 \, + \, \pi^0. \end{aligned} \tag{2.2}$$

Such K_L decays are necessarily FCNC, either generated by the SM NC or by a new current. Here, we focus on the latter case. Before discussing new sources of FCNC, we note that a minimal SM extension that could accommodate our scenario with SM FCNC is the addition of a singlet fermion N, a sterile neutrino. In this case, the decay $K_L \to \nu\overline{\nu}$ is significantly enhanced due to the mass of N [6], and the decay $N \to \nu\pi^0$ has a large BR, especially if N only mixes with the τ flavor. In particular,

$$\mathrm{BR}\left(K_L \to \nu_\alpha \nu_4\right) \approx 3.7 \times 10^{-7} \left[|U_{\alpha 4}|^2 \frac{\alpha^2 G_F^2 \tau_K}{8\pi^3 \sin^4 \theta_{\mathrm{W}}} M_K^3 f_K^2 r_4^2 \left(1 - r_4^2\right) \lambda^{\frac{1}{2}} \left(1, 0, r_4^2\right) \right], \tag{2.3}$$

where $\lambda(a, b, c) = (a - b - c)^2 - 4bc$, and $r_4 = m_4/M_K$. The numerical factor arises from the SM FCNC factors, including CKM matrix elements. It is easy to see that for a sterile neutrino with $m_4 \gtrsim M_K - m_\mu$, such that peak searches can be avoided, the typical BR achieved are of the order of

$$\mathrm{BR}\left(K_L \to \nu_\alpha \nu_4\right) \approx 4.6 \times 10^{-9} |U_{\alpha 4}|^2, \quad \text{and} \quad c\tau_{\nu_4} \approx \frac{20\,\mathrm{cm}}{|U_{\alpha 4}|^2}. \tag{2.4}$$

From this it is clear that even if N only mixes with the τ flavour, this scenario is ruled out as it requires $\mathcal{O}(1)$ values for $|U_{\alpha 4}|^2$, both for production and decay. This suggests that stronger-than-Weak interactions are required to explain the KOTO anomaly in our scenario.

3 Z' models

New sources of FCNC from SM + X EFT New axial-vector forces can be generically built out of mass mixing between the new vector boson X^μ and the SM Z boson. For instance, take the SM + X EFT discussed in Ref. [7], where the effects of the following mass mixing operator were discussed,

$$\mathscr{L} \supset g_X X^\mu i \left(H^\dagger \overleftrightarrow{D}_{\mu X} H \right) \xrightarrow{\text{EWSB}} \epsilon_Z m_Z^2 X_\mu Z^\mu, \quad \text{with} \quad \epsilon_Z = \frac{g_X v}{m_Z}. \tag{3.1}$$

In this EFT, the current above also couples to the quarks at dimension 6 through the operator

$$O_L^{ij} = i \left(H^\dagger \overleftrightarrow{D}_\mu H \right) \left(\overline{d_L^i} \gamma^\mu d_L^j \right), \tag{3.2}$$

where the problem at hand forces us to focus on down-type quark operators. More specifically, we are interested in the $s \leftrightarrow d$ transitions as given by

$$\mathscr{L} \supset C_L^{sd} O_L^{sd} + \text{h.c.} \xrightarrow{\text{EWSB}} g_{sdX} X_\mu \left(\bar{s} \gamma^\mu P_L d \right) + \text{h.c.}, \tag{3.3}$$

where in the SM + X EFT, only the left-handed wilson coefficient is generated radiatively. More specifically, Ref. [7] finds

$$g_{sdX} \approx \frac{g^3 \epsilon_Z}{32\pi^2 c_W} \sum_{q^u} V_{q^u s} V^*_{q^u d} f\left(\frac{m_{q^u}^2}{M_W^2} \right), \tag{3.4}$$

with $f(x)$ a loop function that depends on the cut-off of the theory. For the current purposes, we will assume the form

$$f(x) = -\frac{x}{4} \log \frac{\Lambda^2}{M_W^2},$$

where Λ is the cut-off which we take to be $\Lambda = 10$ TeV for concreteness. This implies that

$$\frac{g_{sdX}}{\epsilon_Z} \approx (3.76 + i\, 1.58) \times 10^{-6}. \tag{3.5}$$

Within a MFV framework, the $b \to s$ is fixed by an analogous equation, and is much larger due to the large value of $V_{tb} V_{ts}^*$. We find

$$\frac{g_{bsX}}{\epsilon_Z} \approx (45.5 - i\, 0.837) \times 10^{-5}. \tag{3.6}$$

We will discuss how such an operators can help us explain the KOTO anomaly below, and later provide a UV completion for the combination of the new particles content and the X EFT.

Bounds on g_{sdX}

$\Delta S = 2$ *observables:* The well-understood process of $K - \overline{K}$ mixing can be used to contrain new sources of FCNC for $s \leftrightarrow d$. Two such observables exist: ϵ_K and the mass difference ΔM_K.

- ΔM_K: weak bound...
- ϵ_K: This provides a stronger bound than the previous one. We follow Ref. [5] and use their Eqs. (2.7) and (2.8) to compute $|\epsilon_K|$ identifying $\Delta_L = g_{sdX}$ and $\Delta_R = 0$, and assuming the only complex phase in g_{sdX} comes from the CKM matrix elements. We find that

$$\begin{aligned}\epsilon_K &= -4.26 \times 10^7 \Bigg[\mathrm{Re}\{g_{sdX}\}\,\mathrm{Im}\{g_{sdX}\} \\ &\qquad + \frac{g^3}{8\pi^2 c_W}\tilde{C}\,(\mathrm{Im}\{\lambda_t\}\,\mathrm{Re}\{g_{sdX}\} + \mathrm{Im}\{g_{sdX}\}\,\mathrm{Re}\{\lambda_t\}) \Bigg] \\ &= -\,(2.16 - 2.35\,\epsilon_Z)\,\epsilon_Z \times 10^{-4},\end{aligned} \tag{3.7}$$

  where $\tilde{C}$ is a loop function given in Eq. (2.10) of Ref. [5]. Using $3.9 \times 10^{-4} < |\epsilon_Z| < 3.7 \times 10^{-4}$, we find

$$-1.19 < \epsilon_Z < 1.31.$$

  Note that the bounds on a right-handed current are more severe. In that case, we identify $\Delta_L = 0$ and $\Delta_R = g_{sdX}$ and obtain

$$-0.301 < \epsilon_Z < 0.259.$$

$\Delta S = 1$ *observables:* The ϵ' observable is an important probe of new sources of direct CP violation in Kaon decays. Since a Lattice result in 2015, it has been under debate due to a discrepancy between the measurement and theoretical prediction at the 2 to 3σ level (recent calculations claim no anomaly exists [8]).

- ϵ' The new physics contribution scales as [5]

$$\frac{\epsilon'}{\epsilon} = -2.64 \times 10^3 B_8^{3/2} \left(\mathrm{Im}\{g_{sdX}\} + \frac{c_W^2}{s_w^2}\,\mathrm{Im}\{g_{sdX}\} \right), \tag{3.8}$$

  where $B_8^{3/2} = 0.76 \pm 0.05$ [9] Revisit this in light of [9]. Despite the discrepancy present in the current theoretical prediction, we can still set a bound on our new physics scenario. Conservatively, we require that $\epsilon'/\epsilon < 1.0 \times 10^{-4}$, which implies that

$$\epsilon_Z < 3.44 \times 10^{-2} \quad \text{for LH currents, and} \quad \epsilon_Z < 1.03 \times 10^{-2} \quad \text{for RH currents.} \tag{3.9}$$

- $K^+ \to \pi^+ \nu\nu$ weak bound

- $K_L \to \mu\mu$ long-distance effects are subtantial and uncertain, but a very weak bound on ϵ_Z can be derived as

$$\epsilon_Z < 1.06 \quad \text{for LH currents, and} \quad \epsilon_Z < 0.283 \quad \text{for RH currents.} \tag{3.10}$$

Also want to comment on the large direct CP violation observed in the charm sector, see [10]. This requires new couplings, however.

Bounds on ϵ_Z

Beyond avoiding bounds on g_{sdX}, we also need to ensure we saisfy the bounds on ϵ_Z itself. These are worked out in detail in Ref. [7], and for a 10 GeV mass, exclude $\epsilon_Z \gtrsim 10^{-2}$. Most stringent bounds come from APV, higgs decays and $\nu - e$ scattering.

4 IDM Scenario

4.1 Estimating $K \to \psi_1\psi_2$

Dirac or Majorana mass model. Need a Lagrangian of the kind

$$\mathscr{L} \supset g_{qqX} X_\mu \bar{q}\gamma^\mu P_L q + g_\psi X_\mu \overline{\psi_2}\gamma^\mu P_L \psi_1. \tag{4.1}$$

Within our EFT, we can re-scale the rate for $K^+ \to \mu^+\nu$ from Ref. [6] to find

$$\mathrm{BR}\left(K_L \to \psi_1\psi_2\right) \approx \frac{|g_{sdX}|^2 g_\psi^2 \tau_K}{16\pi M_X^4} f_K^2 M_K^3 \left[r_1^2 + r_2^2 - \left(r_2^2 - r_1^2\right)^2\right] \lambda^{\frac{1}{2}}\left(1, r_1^2, r_2^2\right). \tag{4.2}$$

This implies that for $g_\psi = 1$, $m_2 = 3\,m_1 = 0.1$ GeV, we require

$$|g_{sdX}| = 5 \times 10^{-9}\left(\frac{M_X}{10\ \text{GeV}}\right)^4 \implies \epsilon_Z = 1.22 \times 10^{-3}.$$

Addendum, MP Let's take an approximation of $\epsilon_K \simeq 0$. Then K_L is identified with $K_L = (K^0 - \bar{K}^0)/\sqrt{2}$. The matrix elements are then

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 d + \bar{d}\gamma_\mu\gamma_5 s|K_L\rangle = 0 \tag{4.3}$$

$$\begin{aligned}\langle 0|\bar{s}\gamma_\mu\gamma_5 d - \bar{d}\gamma_\mu\gamma_5 s|K_L\rangle &= 2^{-1/2}(\langle 0|\bar{s}\gamma_\mu\gamma_5 d|K^0\rangle + \langle 0|\bar{d}\gamma_\mu\gamma_5 s|\bar{K}^0\rangle) \\ &= 2 \times 2^{-1/2} \times p_\mu 2^{1/2} F_\pi = 2F_\pi p_\mu.\end{aligned} \tag{4.4}$$

The last relation is in exact SU(3) limit, and in reality we need to use a $\sim 25\%$ higher value, $F_\pi \to F_K = 1.25F_\pi$, and $F_\pi = 93$ MeV. Notice that the first relation here is what is going to select the CP-violating part of the amplitude, and $\mathrm{Im} g_{sdX}$ as a result of this - at least in the simplest models. MH: Noting that $F_K = f_k/\sqrt{2}$.

We now proceed in getting the K_L decay rate to exotic particles, and I take for my own convenience two Dirac fermions. We need to calculate $K_L \to \psi_1 + \bar{\psi}_2$ together with $K_L \to \psi_2 + \bar{\psi}_1$, starting from the following effective Lagrangian,

$$\mathcal{L} = X_\mu(\bar{\psi}_2\gamma_\mu\psi_1 g_V^d + X_\mu\bar{\psi}_2\gamma_\mu\gamma_5\psi_1 g_A^d + h.c.) + X_\mu i\mathrm{Im}\, g_{sdX}(\frac{1}{2}\bar{s}\gamma_\mu\gamma_5 d - \frac{1}{2}\bar{d}\gamma_\mu\gamma_5 s) \tag{4.5}$$

Re g_{sdX} contributes to the decay of K_S, which is not of interest.

Reducing to the K_L field, and applying equations of motion, we get,

$$\mathcal{L} = \frac{\mathrm{Im}\, g_{sdX} F_K}{m_X^2} \varphi_{K_L} (i\bar{\psi}_2\psi_1 g_V^d (m_2 - m_1) + \bar{\psi}_2 i\gamma_5 \psi_1 g_A^d (m_2 + m_1) + h.c.), \tag{4.6}$$

where φ_{K_L} is the real scalar field of K_L. We now get to the decay rate

$$\Gamma_{12} = \Gamma_{K_L\to\psi_1\bar{\psi}_2} + \Gamma_{K_L\to\psi_2\bar{\psi}_1} = 2 \times \frac{1}{8\pi}\frac{p_1}{m_K^2}|M_{K_L\to\psi_1\bar{\psi}_2}|^2 \tag{4.7}$$

$$= \frac{p_1}{2\pi m_K^2}\frac{F_K^2(\mathrm{Im} g)^2}{m_X^4}\left(g_V^2(\Delta m)^2(m_K^2-(m_2+m_1)^2) + g_A^2(m_2+m_1)^2(m_K^2-(\Delta m)^2)\right)$$

Here p_1 is $(2m_K)^{-1}[(m_K^2-(\Delta m)^2)(m_K^2-(m_2+m_1)^2)]^{1/2}$.

Equating Branching to 2×10^{-9} and taking all other parameters to be the same as before (10 GeV mediator, 100 and 300 MeV particles, and 1 for g_V and g_A), we extract the required value of the coupling constant:

$$\mathrm{Im}\, g_{sdX} \simeq 2.5\times10^{-9}, \tag{4.8}$$

which is a factor of ~ 2 different, but not too far from the value above. In the full calculation I find that this leads to $\mathrm{BR}(KL\to12) = 1.5\times10^{-9}$, so we need a $\sqrt{2.1/1.5}$ factor larger coupling to achieve 2.1×10^{-9}.

However, we need to use $\mathrm{Im}V_{ts}Vtd^*$ rather than the absolute value. Notice that using numbers from the previous section, we get

$$\frac{\mathrm{Im}\, V_{ts}V_{td}^*}{|V_{ts}V_{td}^*|} \simeq \frac{1.45}{\sqrt{1.45^2+3.82^2}} = 0.35 \tag{4.9}$$

Therefore, with my numbers I should come to the estimate of $\epsilon_Z \sim 2\times10^{-3}$

4.2 Estimating $B\to K\psi_1\psi_2$ in the MFV framework (M.P.)

If new physics is initially flavour-blind (e.g. $X-Z$ mixing), and the only source of flavour transition is in Yukawa matrices, then the $b-s$ and $s-d$ transitions are related. For example, if the (e.g. loop-induced) FCNC amplitude is parametrized by $\propto \bar{Q}_L Y_u^2 \gamma_\mu\gamma_5 Q_L X_\mu$, then the values of the couplings are related

$$g_{sdX} = \mathrm{const}\times V_{ts}V_{td}^*;\quad g_{bsX} = \mathrm{same\ const}\times V_{tb}V_{ts}^* \tag{4.10}$$

In the ratio, this constant completely cancels, so in some sense $K_L\to\psi_1+\psi_2$ contains predictions for $B\to K+\psi_1+\psi_2$ processes.

Now, I calculate the $B\to K\psi_{2(1)}\bar{\psi}_{1(2)}$ rate. I again use the Dirac fermions 1 and 2 for my own convenience.

The matrix element consists of $B\to K$ transition given by the vector current of $\bar{b}\gamma_\mu s$,

$$M_{B\to K\psi_1\bar{\psi}_2} = \frac{g_{bsX}}{2m_X^2}\langle K(k)|\bar{s}\gamma_\mu b|B(p)\rangle\bar{\psi}_2(g_V\gamma_\mu + g_A\gamma_\mu\gamma_5)\psi_1 \tag{4.11}$$

where the matrix element is given by a transitional form-factor of the vector current between B and K:

$$\langle K(k)|\bar{s}\gamma_\mu b|B(p)\rangle = (2p-q)_\mu f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2} q_\mu [f_0(q^2) - f_+(q^2)], \tag{4.12}$$

where $q_\mu = p_\mu - k_\mu$, and $q^2 = m_{12}^2$, m_{12} is the invariant mass of the $\psi_1\psi_2$ pair, in our case.

Calculation is simplest in the limit of $m_{1(2)}, m_K \ll m_B$, which we will adopt for a first pass. Then, in that limit all q_μ proportional terms in the form-factor can be dropped, as they lead only to m_1 or m_2 proportional amplitudes. The value of the matrix element squared to $O(m_1^0, m_2^0)$ order is given by:

$$|M_{B\to K\psi_1\bar{\psi}_2}|^2 = |f_+(q^2)|^2 \frac{4|g_{bsX}|^2(g_V^2+g_A^2)}{m_X^4}(2(pp_1)(pp_2) - m_B^2(p_1p_2)), \tag{4.13}$$

where p_1 and p_2 are four-momenta of the "dark" pair particles. Going over to invariant mass notation, and defining $m_{23}^2 = (p_2+p_K)^2$, we get

$$|M_{B\to K\psi_1\bar{\psi}_2}|^2 = |f_+(m_{12}^2)|^2 \frac{2|g_{bsX}|^2(g_V^2+g_A^2)}{m_X^4}[(m_B^2 - m_{23}^2)(m_{23}^2 + m_{12}^2) - m_B^2 m_{12}^2] \tag{4.14}$$

Making use of formula (47.22) from PDG review sections, taking an integral over $m_2 3^2$ within 0 and $m_B^2 - m_{12}^2$, then the integral over m_{12}^2 within 0 and m_B^2, and multiplying everything by 2 to account for the $\psi_2\bar{\psi}_1$ final state, we get to the rate of

$$d\Gamma_{B\to K+12} = 2\frac{1}{(2\pi)^3}\frac{dm_{12}^2}{32m_B^3}|f_+(m_{12}^2)|^2\frac{2|g_{bsX}|^2(g_V^2+g_A^2)}{m_X^4}$$
$$\times((m_B^2 - m_{12}^2)m_{12}^2 m_B^2 + \frac{1}{6}(m_B^2 - m_{12}^2)^3 - m_B^2 m_{12}^2(m_B^2 - m_{12}^2)) \tag{4.15}$$

If the form factor f_+ were a constant (~ 0.33 or so), then the final answer is given by

$$\Gamma_{B\to K+12} = \frac{1}{1536\pi^3}\frac{f_+^2 m_B^5 |g_{bsX}|^2(g_V^2+g_A^2)}{m_X^4} \tag{4.16}$$

In reality, we need to take a realistic form of f_+, and we take the the light cone sum rule derived quantities from the hep-ph/9910221

$$f_+ \simeq 0.32 \times \exp\{c_1\hat{s} + c_2\hat{s}^2 + c_3\hat{s}^3\} \tag{4.17}$$

I am going to take the equation (4.16) for a moment, and calculate the following quantity: $\mathrm{Br}(K_L \to 12)/\mathrm{Br}(B \to k+12)$. If this ratio comes out to be larger than $2\times10^{-9}/(3\times10^{-5})$, ($2\times10^{-9}$ is the kaon branching that we need to satisfy, and 3×10^{-5} is B-branching that we need to avoid) then it is possible to have a new signal in K_L within an MFV approach.

Let's take again $g_V = g_A = 1$ for simplicity. Then we have the following relation

$$\frac{\mathrm{Br}(K_L\to 12)}{\mathrm{Br}(B\to K+12)} = \frac{\tau_{K_L}}{\tau_B}\frac{|\mathrm{Im}V_{td}^* V_{ts}|^2}{|V_{ts}^* V_{tb}|^2}\left(\frac{f_+^2 m_B^5}{768\pi^3}\right)^{-1}\frac{p_1 F_K^2}{2\pi m_K^2}$$
$$\times\left((\Delta m)^2(m_K^2 - (m_2+m_1)^2) + (m_2+m_1)^2(m_K^2 - (\Delta m)^2)\right) = 9\times10^{-4} \tag{4.18}$$

(As advertised, ϵ_K, M_X and loop factors cancel from this ratio.) I find a factor of 5 larger number by explicit computation of the expression above.

This is a good number (if I did not make a mistake!). It implies

$$\mathrm{Br}(B \to K + 12) \simeq 2.5 \times 10^{-6} \times \frac{\mathrm{Br}(K_L \to 12)}{2 \times 10^{-9}}. \tag{4.19}$$

So, at the moment is that an MFV approach to this problem + B-decays do not rule out a $K_L \to \psi_1\psi_2$ hypothesis for KOTO.

4.3 IDM predictions

- Copious production of ψ particles in beam dump and neutrino experiments, mainly through η and heavier mesons. Search for $\psi_2 \to \psi_1 \pi^0$. Note also that ψ_1 upscattering into ψ_2 is also dangerous. Bounds from LDM scattering?

4.4 Model details for IDM Fermions

We will now derive the mass eigenstates of the theory and work out the possible interactions of the dark fermions. We start with

$$\mathscr{L} \supset \overline{\psi_L} i(\not{\partial} + i g_X Q_L \not{X})\psi_L + \overline{\psi_R} i(\not{\partial} + i g_X Q_R \not{X})\psi_R + \mathscr{L}_{\mathrm{mass}}, \tag{4.20}$$

where $\psi_D = \psi_L + \psi_R$, and the mass lagrangian is

$$\begin{aligned} -\mathscr{L}_{\mathrm{mass}} =& \frac{1}{2}(-\mu_L^* \overline{\psi_L^c}\psi_L + \mu_L \overline{\psi_L}\psi_L^c) + \frac{1}{2}(\mu_R \overline{\psi_R^c}\psi_R - \mu_R^* \overline{\psi_R}\psi_R^c) + (M_D \overline{\psi_L}\psi_R + M_D^* \overline{\psi_R}\psi_L) \\ =& \frac{1}{2}\begin{pmatrix} \overline{\psi_L} & \overline{\psi_R^c} \end{pmatrix} \begin{pmatrix} \mu_L & M_D^\psi \\ M_D^\psi & \mu_R \end{pmatrix} \begin{pmatrix} \psi_L^c \\ \psi_R \end{pmatrix} + \mathrm{h.c.}, \end{aligned} \tag{4.21}$$

by virtue of the useful relation [1]

$$\overline{\chi}\psi^c = \overline{\psi}\chi^c. \tag{4.22}$$

Note that all fields are 4-component fields and that the mass parameters may be complex. Having setup the problem, we now diagonalize the mass matrix with a unitary transformation

$$U = \begin{pmatrix} c_\theta & -s_\theta e^{i\delta_\chi} \\ s_\theta e^{-i\delta_\chi} & c_\theta \end{pmatrix}, \quad \tan 2\theta = \frac{2M_D}{\mu_L - \mu_R}. \tag{4.23}$$

such that

$$-\mathscr{L}_{\mathrm{mass}} \supset \frac{1}{2}\overline{\Psi_R^c} M \Psi_R = \frac{1}{2}\overline{X_R^c} U^T M U X_R = \frac{1}{2}\begin{pmatrix} \overline{\chi_L} & \overline{\chi_R^c} \end{pmatrix} \begin{pmatrix} m_2 & 0 \\ 0 & m_1 \end{pmatrix} \begin{pmatrix} \chi_L^c \\ \chi_R \end{pmatrix}, \tag{4.24}$$

[1] Note the minus sign for the hermitian conjugate of the Majorana mass terms as $(\overline{\psi^c}\psi)^\dagger = -\overline{\psi}\psi^c$. Comparing with [11], we see that $\mu_R = -m_R$.

where the physical masses are given by

$$m_{1,2} = \frac{\mu_L + \mu_R}{2} \pm \sqrt{M_D^2 + \left(\frac{\mu_R - \mu_L}{2}\right)^2}, \tag{4.25}$$

such that the physical Majorana states are

$$-\mathscr{L}_{\text{mass}} = \frac{m_1}{2}\overline{\chi_1}\chi_1 + \frac{m_2}{2}\overline{\chi_2}\chi_2 + \text{h.c.}, \quad \chi_{2(1)} = \chi_{\text{L(R)}} + \chi^{\text{c}}_{\text{L(R)}}, \tag{4.26}$$

where $\chi^c_{2(1)} = \chi_{2(1)}$. Now, we can write the interaction states in terms of the physical Majorana states as

$$\begin{aligned} \psi_L^c =& U_{11} P_R \chi_2 + U_{12} P_R \chi_1, \quad \psi_L = U_{11}^* P_L \chi_2 + U_{12}^* P_L \chi_1 \\ \psi_R =& U_{21} P_R \chi_2 + U_{22} P_R \chi_1, \quad \psi_R^c = U_{21}^* P_L \chi_2 + U_{22}^* P_L \chi_1. \end{aligned} \tag{4.27}$$

The dark fermion current is now (using Eq. (4.22))

$$\begin{aligned} J^\mu_\psi =& \overline{\psi_L} i(\partial\!\!\!/ + i g_X Q_L X\!\!\!/)\psi_L - \overline{\psi_R^c} i(\partial\!\!\!/ + i g_X Q_R X\!\!\!/)\psi_R^c \\ =& \left(Q_L|U_{11}|^2 - Q_R|U_{21}|^2\right)\overline{\chi_2}\gamma^\mu P_L \chi_2 + \left(Q_L|U_{12}|^2 - Q_R|U_{22}|^2\right)\overline{\chi_1}\gamma^\mu P_L \chi_1 \\ &+ i\,\text{Im}\left(Q_L U_{11} U_{12}^* - Q_R U_{22}^* U_{21}\right)\overline{\chi_2}\gamma^\mu \chi_1 \\ &- \text{Re}\left(Q_L U_{11} U_{12}^* - Q_R U_{22}^* U_{21}\right)\overline{\chi_2}\gamma^\mu\gamma^5 \chi_1, \end{aligned} \tag{4.28}$$

This shows that the dark fermions have both vector and axial-vector interactions in general. Using the parametrization of U from before and assuming $Q_L = Q_R$, we find

$$\begin{aligned} J^\mu_\psi =& \cos 2\theta\left(\overline{\chi_2}\gamma^\mu P_L \chi_2 - \overline{\chi_1}\gamma^\mu P_L \chi_1\right) + i\sin 2\theta \sin\delta_\chi\, \overline{\chi_2}\gamma^\mu\chi_1 + \sin 2\theta\cos\delta_\chi\, \overline{\chi_2}\gamma^\mu\gamma^5\chi_1 \\ =& \cos 2\theta\left(\overline{\chi_1}\gamma^\mu\gamma^5 \chi_1 - \overline{\chi_2}\gamma^\mu\gamma^5 \chi_2\right) + i\sin 2\theta \sin\delta_\chi\, \overline{\chi_2}\gamma^\mu\chi_1 + \sin 2\theta\cos\delta_\chi\, \overline{\chi_2}\gamma^\mu\gamma^5\chi_1, \end{aligned} \tag{4.29}$$

which agrees with [11] in the limit $\delta_\chi \to 0$ and understanding that our $\chi_L(\chi_R^c)$ is their $\chi_2(\chi_1)$. Note that to recover Eq.3.2 from [12], we expand around $\theta = 45°$, so that $\cos 2\theta$ is small.

It should be emphasized that even for large $\Delta m = m_2 - m_1$, one does not need to have large elastic interactions. One way to see this is to rewrite the couplings and masses as

$$\sin 2\theta = -\frac{1}{\sqrt{1+\delta^2}}, \quad \cos 2\theta = -\frac{\delta}{\sqrt{1+\delta^2}}, \quad m_{1,2} = \mu \pm M_D\sqrt{1+\delta^2}, \tag{4.30}$$

where we defined

$$\delta = \frac{\Delta\mu}{M_D}, \quad \Delta\mu = \frac{\mu_L - \mu_R}{2}, \quad \mu = \frac{\mu_L + \mu_R}{2}. \tag{4.31}$$

So for $\sin 2\theta \gg \cos 2\theta$ (inelastic $\gg$ elastic), we could take $\delta \ll 1$ ($\mu_L/\mu_R \sim 1$), while keeping the mass splitting large. In this case, M_D controls the mass difference ($m_2 - m_1 \approx 2M_D$).

4.4.1 Dark Matter annihilation

We now compute the dark matter annihilation into SM particles. Our model realises the inelastic and self-interacting DM scenario advocated in Ref. [13]. There, it is argued that models with SM singlets annihilating into SM particles through a singlet mediator are favoured by small scale structure formation problems and experimental constraints. With a(n) (axial-)vector mediator, we also ought to ensure that s-wave annihilation is forbidden (*i.e.*, velocity-suppressed annihilation cross section), as otherwise CMB and BBN constraints are severe. Conveniently, this is the case if $\psi_{1,2}$ are Majorana fermions, as their s-wave annihilation through a vector boson s-channel is disallowed [2].

Computing the annihilation channel $\chi_1\chi_1 \to ff$ keeping both vector and axial-vector fermion vertices, I find

$$\sigma v_{\rm rel} = a + b\, v_{\rm CM}^2 \tag{4.32}$$

with

$$\begin{aligned} a &= \left(\frac{\epsilon_Z g}{2c_w}\right)^2 \frac{(g_A^\chi g_A^f)^2}{2\pi} \frac{m_f^2}{\left(m_{Z'}^2 - 4m_1^2\right)^2} \lambda^{1/2}(1, m_f^2/m_1^2, 0) \\ b &= \left(\frac{\epsilon_Z g}{2c_w}\right)^2 \frac{(g_A^\chi)^2}{3\pi} \frac{g_V^{f\,2}(m_f^2 + 2m_1^2) + 2g_A^{f\,2} m_1^2}{\left(m_{Z'}^2 - 4m_1^2\right)^2} \lambda^{1/2}(1, m_f^2/m_1^2, 0) \end{aligned} \tag{4.33}$$

which in the limit $m_f \to 0$ agrees with Ref. [14]. As expected, the Majorana nature of DM leads to a velocity-suppressed annihilation cross section. The axial-vector mediator, however, still allows for s-wave annihilation if m_f is sizeable due to the pseudo-scalar longitudinal component of the Z'. For the masses we are considering, $m_1 \lesssim 100$ MeV, the only channel open for DM annihilation is into electrons and neutrinos. For these channels

$$\frac{\sigma^{ee} v_{\rm rel}}{3 \times \sigma^{\nu\nu} v_{\rm rel}} \approx \frac{g_A^{e\,2} m_e^2}{2(g_V^{\nu\,2} + g_A^{\nu\,2}) m_1^2 v_{\rm CM}^2} \approx \frac{g_A^{e\,2}}{4(g_V^{\nu\,2} + g_A^{\nu\,2})} \frac{10^{-1}}{v_{\rm rel}^2} \times 10^{-9}, \tag{4.34}$$

which is tiny during freeze-out, but sub-per-mille today. Clearly, keeping $m_1 < m_\pi$ is a necessity to obtain dominant p-wave annihilation.

4.5 Results

I define

$$\frac{G_X}{\sqrt{2}} = \frac{\epsilon_Z g_\chi g}{4 c_W m_{Z'}^2},$$

where g is the weak coupling and I assume $g_\chi = g_V^\chi = g_A^\chi$. The results for KOTO, and the DM abundance are incompatible in this model.

[2] If the initial Majorana fermions are identical, their state has to be anti-symmetrized to compensate the minus sign from particle exchange. This is only possible for the $S = 0$ state, which does not allow a $L = 0, J = 1$ s-channel annihilation as needed for a s-wave vector boson exchange.

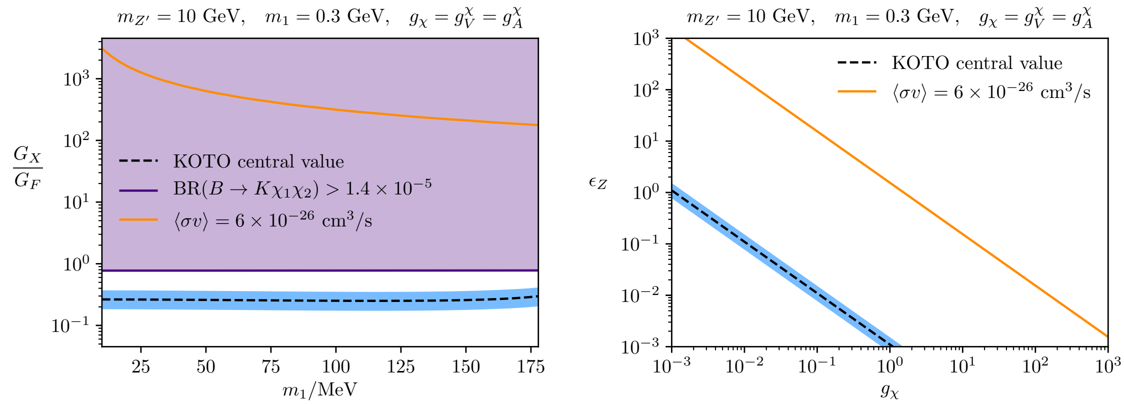

Figure 1: Some results fixing $\epsilon_Z = 10^{-3}$.

Semantics... Note that this suggests $G_X/G_F < 1$. This is why I believe the phrase "stronger-than-weak" is a bit misleading. What we really mean is that the KOTO result requires

$$\frac{\Gamma(K_L \to X^* \to \chi_1\chi_2)}{\Gamma(K_L \to Z^* \to \chi_1\chi_2)} \gg 1 \quad \to \quad \frac{g_{sdX}\, M_Z^2}{g_{sdZ}\, \epsilon_Z\, M_X^2} \approx \frac{M_Z^2}{M_X^2} \gg 1, \tag{4.35}$$

or in the case of neutrinos

$$\frac{\Gamma(K_L \to X^* \to \nu N)}{\Gamma(K_L \to Z^* \to \nu N)} \gg 1 \quad \to \quad \frac{g_{sdX}|U|\, M_Z^2}{g_{sdZ}|U| M_X^2} \approx \frac{M_Z^2}{M_X^2} \gg 1. \tag{4.36}$$

Of course, one may want to use the requirement $\Gamma(K_L \to X^* \to \nu N)/\Gamma(K_L \to \pi^0\nu\nu) \gg 1$, but I think that does not reflect the reason why we added a new mediator. Do you agree? I suppose we may say that KOTO requires either stronger-than-weak interactions (implicitly taking into account the enhancement from 2-body decays), or that KOTO requires weak-strength interactions when two-body decays are allowed.

5 HNL Scenario

Another relatively simpler and more testable model can be obtained by assigning $X_1 = \nu_\alpha$ and $X_2 = N$, and assuming additional interactions in the heavy neutral lepton sector as well as chiral 1st generation quark operators. In short,

$$\mathscr{L} \supset -g_X X_\mu \left[\overline{N}\gamma^\mu N + \epsilon_Z \overline{u}\gamma^\mu P_L u + \epsilon_Z \overline{d}\gamma^\mu P_L d\right] . \tag{5.1}$$

Decays are analogous to before, with appropriate couplings put in.

5.1 HNL Predictions

- $K_L \to N\nu \to \mu^+\mu^-\nu\nu$ with a rate BR of $\approx 5\%$ of measured BR$(K_L \to \pi^0\nu\nu)$.
- $K_L \to N\nu \to e^+e^-\nu\nu$ with a rate BR of $\approx 20\%$ of measured BR$(K_L \to \pi^0\nu\nu)$ (displaced vertex to reduce backgrounds? photons travel...)
- $\tau \to \nu_\ell \ell N \to \nu_\ell \ell \nu \pi^0$ with BR of order $\approx |U_{\tau 4}|^2 \times 17\%$. If N decays promptly, there are no SM backgrounds (surprising!).
- $\nu_\tau h \to N h'$, with h hadronic matter, followed by $N \to \nu_\tau \pi^0$. This is large for light mediator masses. Ignoring thresholds and kinematics, we have that

  $$\frac{\sigma^X_{\rm NC}}{\sigma^Z_{\rm NC}} \approx \epsilon_Z^2 |U_{\tau 4}|^2 \frac{M_Z^4}{M_X^4},$$

  which much below 10% for our parameters of interest.

6 Comments, MP

- The dark photon mediated models do not quite work - but not because of $K_L \to \psi_1\psi_2$, but because $\psi_2 \to \psi_1 + \pi^0$ will be forbidden by parity in this case.
- I like the mass mixing with the Z model, as an example of a theory that can be "semi-UV complete". It has been discussed in the literature before, and UV-completions based on 2HDM exist.
- Comments to the notes above. Transition from K_L to vacuum is only possible because of the imaginary part of $g_{sdZ'}$, because it is a CP-odd transition.
- The dark matter abundance will be achieved using $\psi_1\bar{\psi}_1 \to \pi^+\pi^-$, not $\pi^0\pi^0$ in the final state, simply because one cannot have a vector current out of two π^0
- It is reasonable to be agnostic about the couplings of dark states to the mediator X, and have $X_\mu \bar{\psi}_2\gamma_\mu\psi_1 \times g^d_V + X_\mu\bar{\psi}_2\gamma_\mu\gamma_5\psi_1 \times g^d_A$.

I estimate the decay rate of one dark state to another. Integrating out heavy particles we end up with

$$\mathcal{L} = (\bar{\psi}_2\gamma_\mu\psi_1 g_V^d + \bar{\psi}_2\gamma_\mu\gamma_5\psi_1 g_A^d)\frac{\epsilon_Z}{m_X^2} \times \frac{g}{2\cos\theta_W}(\frac{1}{2}\bar{u}\gamma_\mu\gamma_5 u - \frac{1}{2}\bar{d}\gamma_\mu\gamma_5 d). + h.c. \tag{6.1}$$

where I have dropped the vector current of quarks as it does not mediate the vacuum to π^0 transition. Vector current is important for the freeze-out calculation, though.

The last bracket can be replaced with $F_\pi\partial_\mu\varphi_\pi$ ($F_\pi = 93$MeV), and integrating by part the effective Lagrangian becomes

$$\begin{aligned}\mathcal{L} &= \frac{gF_\pi\epsilon_Z}{2m_X^2\cos\theta_W}\varphi_\pi\partial_\mu(\bar{\psi}_2\gamma_\mu\psi_1 g_V^d + \bar{\psi}_2\gamma_\mu\gamma_5\psi_1 g_A^d) + h.c. \\ &= \frac{gF_\pi\epsilon_Z}{2m_X^2\cos\theta_W}\varphi_\pi(i\bar{\psi}_2\psi_1 g_V^d(m_2 - m_1) + \bar{\psi}_2 i\gamma_5\psi_1 g_A^d(m_2+m_1)) + h.c.\end{aligned} \tag{6.2}$$

Matrix element averaged over initial spin and summed over final for $\psi_2 \to \psi_1 + \pi_0$ is then

$$|M|^2 = \left(\frac{gF_\pi\epsilon_Z}{2m_X^2\cos\theta_W}\right)^2 [g_V^2(\Delta m)^2((m_2+m_1)^2 - m_\pi^2) + g_A^2(m_2+m_1)^2((\Delta m)^2 - m_\pi^2)] \tag{6.3}$$

The total width comes out to be

$$\Gamma_{2\to 1+\pi^0} = \frac{1}{8\pi}|M|^2\frac{p_\pi}{m_2^2} \tag{6.4}$$

where p_π is given by

$$p_\pi = \frac{1}{2m_2}\left((m_2^2 - (m_1+m_\pi)^2)(m_2^2 - (m_1 - m_\pi)^2)\right)^{1/2} \tag{6.5}$$

Taking a point in the parameter space with $g_V = g_A = 1$, and $m_2 = 300$ MeV, $m_1 = 100$ MeV, $m_X = 10$GeV, $\epsilon_Z = 10^{-2}$ we get

$$\Gamma_{2\to 1+\pi^0} \sim 4\times 10^{-6}\text{eV} = 6\times 10^9\text{Hz}, \text{ or } c\tau = 5\text{cm} \tag{6.6}$$

I think this is fast enough to satisfy our purposes, with the KOTO boosts.

7 Higgs portal scalars, $K_L \to S_1 S_2 \to S_1\gamma\gamma$

We would like to investigate whether on can achieve a significant rate for $K_L \to S_1 S_2 \to S_1\gamma\gamma$ AND have a "harder" P_T than in the SM neutrino-pi0 rate. We first investigate whether a relatively prompt decay rate of a massive (pseudo)-scalar is possible. Consider a $\pi^0 - \gamma - \gamma$ Lagrangian, $\pi^0 F\tilde{F}\alpha/(4\pi F_\pi)$, and consider now a new scalar S_2 [that I would also call a], that is close in mass to π_0, but with effective F_π that is in the TeV's.

Then we have:

$$c\tau_{S_2} = c\tau_{\pi^0}\times(m_a/m_\pi)^{-3}(F_a/F_\pi)^2 \simeq 2.5\times 10^{-6}\text{cm}\times(m_a/m_\pi)^{-3}(F_a/F_\pi)^2. \tag{7.1}$$

So, a TeV scale $F_a \sim 10^4 F_\pi$ gives a "tolerable" travel distance. so that there is not much penalty for the $K_L \to S_1 S_2$ rate. To UV complete this interaction we can couple a to the vector-like fermion under SM (e.g. some PQ style coupling). Mass m_a is the soft breaking of the PQ - so it is okay.

Now I would like to investigate a topic of whether I can maximize $K_L \to S_1 S_2$ somehow ($S_2 = a$) using *Higgs portal*. The idea is that I can maintain the MFV properties, and hopefully evade other constraints. We will see.

To maximize the rate *without* getting punishing $K^+ \to \pi^+ s$ we use an intermediate mass scalar, S_{int}, - think 10 GeV mass - that has a vertex with the Higgs, and with $S_1 S_2$.

So we have,

$$\mathcal{L} = ...A(H^\dagger H)S_{int} + BS_{int}S_1S_2 \tag{7.2}$$

S_{int} mixes with the Higgs. But the size of the mixing depends on the off-shell-ness. Let me assume the following Hierrarchy:

$$m_K \ll m_{S_{int}} \ll m_h. \tag{7.3}$$

Then at low energy for the K_L decay, I can integrate out both S_{int} and physical h.

I should end up with the following effective Lagrangian

$$\mathcal{L}_{eff} = m_s \bar{s}_R d_L S_1 S_2 \times \frac{3V_{ts}^* V_{td} y_t^2}{16\pi^2} \frac{AB}{m_h^2 m_{S_{int}}^2} + h.c. \tag{7.4}$$

At the same time, A, B and $m_{S_{int}}$ parameters are constrained by Higgs decays and other collider signatures. The Higgs coupling is given by replacing $H^\dagger H \to hv$, so that Higgs decay amplitude to $S_1 + S_2$ is

$$\mathcal{A}_{h\to S_1S_2} = v\frac{AB}{m_h^2 - m_{S_{int}}^2} \simeq \frac{vAB}{m_h^2}, \tag{7.5}$$

and the new decay channel is

$$\Gamma_{h\to S_1S_2} = \frac{1}{16\pi}\frac{v^2A^2B^2}{m_h^5} \tag{7.6}$$

Requiring this to be less than $0.1\Gamma_{h,SM} = 4 \times 10^{-4} GeV$, we get

$$\frac{|AB|}{m_h^2} < 0.0064 \tag{7.7}$$

or so.

It is now clear that with this constraint *we are* going to satisfy $K_L \to S_1 S_2$ rate requirements...

Let's calculate the relevant K_L rate. We use (overall sign or phase would not matter):

$$m_s \bar{s}_R d_L S_1 S_2 = \bar{s}_L \gamma_\mu d_L \times \partial_\mu(S_1 S_2); \quad \langle 0|\bar{s}_L\gamma_\mu d_L - \bar{d}_L \gamma_\mu s_L|K_L\rangle = F_\pi p_\mu. \tag{7.8}$$

Then we can write the amplitude for a decay:

$$\mathcal{A}_{K_L\to S_1S_2} = \frac{3\mathrm{Im}[V_{ts}^* V_{td}] y_t^2}{16\pi^2}\frac{AB}{m_h^2 m_{S_{int}}^2} F_\pi m_K^2, \tag{7.9}$$

plug the constraint from the Higgs decay, and arrive at the maximum possible rate for kaon decay (I took $m_{1,2} \ll m_K$),

$$\text{Max}(BR_{K_L \to S_1 S_2}) = \frac{1}{16\pi m_K \Gamma_K} |\mathcal{A}_{K_L \to S_1 S_2}|^2 = 1 \times 10^{-7} \left(\frac{ABm_h^{-2}}{0.0064} \right)^2 \left(\frac{10\,\text{GeV}}{m_{S_{int}}} \right)^4 \quad (7.10)$$

It means that the combination of parameters AB/m_h^2 can be ~ 30 times smaller, and we seem to be okay.

Unfortunately, the lightest scalar cannot be made into the dark matter. It is unstable. At 3 loop level, there is a decay $S_1 \to \gamma\gamma$, mediated by the loops of top, VL vermion, S_{int} and S_2. This would give a much suppressed effective f_{S1} relative to $f_{S2} = f_a = TeV$ but this is not going to be enough to suppress the coupling to avoid fast decays. Taking this 3-loop suppression to be 10^{-10}, we get to $f_{S2} \propto 10^{13} GeV \sim 10^{14} F_\pi$. This gives the lifetime of 10^{12} seconds, and even if we take m_{S1} to be a $\sim$5 MeV, we are going to get to fast a decay to γ's. Therefore if we decay K_L to two particles, one of which completely decays to the SM, we are not getting, in general, stable dark matter from the other one. The degree of instability looks to be too large. This is *different* from the situation when we have $S_2 \to S_1 + \pi^0$ or $S_2 \to S_1 + \gamma\gamma$, where we have an "$S$"-conservation and the lightest state is stable.

****

Do we need an "intermediate" scalar to maximize the rate. Let's calculate the maximum branching of K_L decay using $\lambda H^\dagger H S_1 S_2$ operator.

The Higgs decay rate is given by

$$\Gamma_h = \frac{1}{16\pi m_h} \lambda^2 v^2. \quad (7.11)$$

Requiring this to be less than 4×10^{-4} GeV we get

$$|\lambda| < 0.64 \times 10^{-2} \quad (7.12)$$

So it is the same limit on λ as in the previous result on AB/m_h^2. Now, the maximum branching for the K_L decay is given by

$$\text{Max}(BR_{K_L \to S_1 S_2}) = \frac{1}{16\pi m_K \Gamma_K} \left| \frac{3\text{Im}[V_{ts}^* V_{td}] y_t^2}{16\pi^2} \frac{\lambda}{m_h^2} F_\pi m_K^2 \right|^2 < 4.5 \times 10^{-12} \quad (7.13)$$

This is too small!

Indeed, the reason we get such a difference with the previous result is that once the Higgs decays are imposed, effectively intermediate scalar gives an enhancement of K_L branching of $(m_h/m_{S_{int}})^4 \to 2.4 \times 10^4$.

We can now investigate whether $K_L \to \bar\psi\psi$ decay, where ψ is some "Higgs-portal-coupled-fermion" gives an acceptable phenomenology. So we have

$$\mathcal{L} = ...A(H^\dagger H) S_{int} + \lambda_\psi S_{int} \bar\psi\psi \quad (7.14)$$

(Or equivalently, two different fermions, $1/2(\bar{\psi}_1\psi_2 + \bar{\psi}_2\psi_1)$, or with $i\gamma_5$ coupling – all the same as long as I neglect masses.

Then we have the Higgs rate to these two fermions as

$$\Gamma_{h\to\psi\psi} = \frac{\lambda_\psi^2 A^2 v^2}{8\pi m_h^3} \tag{7.15}$$

This gives a constraint

$$|\lambda_\psi A/m_h| < 0.9\times 10^{-3}. \tag{7.16}$$

This gives a maximum for the K_L branching as 10^{-13} for intermediate scalar mass of 10 GeV (need to check!), and the reason it does not work is because for the Higgs decay the fermion amplitude squared is $\propto m_h^2$, and for the kaon decay it is $\propto m_K^2$, so essentially $K_L \to \psi\bar{\psi}$ is suppressed relative to $K_L \to S_1 S_2$ by 5 orders of magnitude.

8 Note on matrix elements, C, P, etc for the kaon decay

Summary: we have made significant mistakes previously, and our results have to be modified. The mistakes stem from the treatments of the K_L matrix element.

This is easy to see by examining answers in the Standard Model. For example, we know very well that $K_L \to \pi^0\nu\bar{\nu}$ amplitude is induced by the complexity in the CKM,

$$A_{K_L\to\pi^0\nu\bar{\nu}} \propto \mathrm{Im}(V_{ts}V_{td}^*) \tag{8.1}$$

However, we can examine also the $K_L \to \mu\bar{\mu}$ decay. There is a unitarity contribution from the intermediate $\gamma\gamma$ state. But there is also a real part of the amplitude due to Z-mediation, and W box, for example. It is also quite well-known, see [15], Eq. (9), that this [dispersive] part of the amplitude is

$$A_{K_L\to\pi^0\nu\bar{\nu}} \propto \mathrm{Re}(V_{ts}V_{td}^*) \tag{8.2}$$

This thing alone should convince us that the Z' portal - from the point of its CP properties identical to Z portal - should also be proportional to the real part of the CKM.

We now need to get down to the bottom of the issue.

C,P properties of mesons, and matrix elements

We start from P properties of mesons. It is easy, under P-transformations, all mesons we consider are pseudoscalars,

$$P(\pi^0) = -\pi^0,\ P(K^0) = -K^0,\ P(\bar{K}^0) = -\bar{K}^0. \tag{8.3}$$

We will also use the definition of CP properties in such a way that it turns K^0 into $\bar{K}^0$,

$$CP(\pi^0) = -\pi^0,\ CP(K^0) = \bar{K}^0,\ CP(\bar{K}^0) = K^0,\ CP(K_L) = -1. \tag{8.4}$$

I am going to disregard small ϵ_K parameter, and assume that K_L is an exact linear combination

$$K_L \equiv (K^0 - \bar{K}^0)/\sqrt{2} \tag{8.5}$$

And this also give use the following charge conjugation properties,

$$C(\pi^0) = +\pi^0,\ C(K^0) = -\bar{K}^0,\ C(\bar{K}^0) = -K^0,\ C(K_L) = +K_L \tag{8.6}$$

The last relation is especially important, because it teaches us that only operators that are *even* under charge conjugation will have non-zero matrix elements, for annihilation of K_L or for its transition to π^0. This follow from the fact that strong interactions, responsible for these matrix elements, preserve charge symmetry.

$$\langle 0|O_a|K_L\rangle \neq 0 \Longrightarrow C(O_a) = +O_a;\quad \langle \pi^0|O_t|K_L\rangle \neq 0 \Longrightarrow C(O_t) = +O_t. \tag{8.7}$$

We can now classify what kind of O_t and O_a we can have in BSM and SM. (The C, P properties of fermionic bilinears one can find in Volume IV of Landau-Lifshits series, Section 28.) Before we do that we make yet another clarification: Consider a matrix element that annihilates K^0:

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 d|K^0\rangle = i\sqrt{2}F_K p_\mu \tag{8.8}$$

Consider now the CP conjugation of this matrix element. Let's take $\mu = 0$, and then CP of r.h.s gives us the same result, while $CP(\langle 0|\bar{s}\gamma_0\gamma_5 d|K^0\rangle) = \langle 0| - \bar{d}\gamma_0\gamma_5 s|\bar{K}^0\rangle$. For $\mu = i$, the r.h.s changes sign because $P(\vec{p}) = -\vec{p}$, while the l.h.s is positive. In both cases, we get the same result:

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 s|\bar{K}^0\rangle = -i\sqrt{2}F_K p_\mu. \tag{8.9}$$

This difference in sign is the consequence of our definition for $CP(K^0) = \bar{K}^0$. This defines the matrix element that creates/annihilates K_L from vacuum:

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 s + \bar{s}\gamma_\mu\gamma_5 d|K_L\rangle = 2^{-1/2}(\langle 0|\bar{d}\gamma_\mu\gamma_5 s|K^0\rangle - \langle 0|\bar{d}\gamma_\mu\gamma_5 s|\bar{K}^0\rangle) = 2iF_K p_\mu \tag{8.10}$$

Notice that the sign between these two terms is opposite to what I had wrongly before. The key point here that the current that mediates the transition is C-even: $C(\bar{d}\gamma_\mu\gamma_5 s + \bar{s}\gamma_\mu\gamma_5 d) = +(\bar{d}\gamma_\mu\gamma_5 s + \bar{s}\gamma_\mu\gamma_5 d)$.

So the Lagrangian relevant for the transition of K_L to a muon pair, in the SM has the following form, where I suppress all dependences other than the CKM and package it into some real constant B:

$$\mathcal{L}_a^{SM} = B(V_{td}^* V_{ts}\bar{d}\gamma_\nu\gamma_5 s + V_{td}V_{ts}^*\bar{s}\gamma_\nu\gamma_5 d) \times \bar{\mu}\gamma_\nu\gamma_5\mu. \tag{8.11}$$

$$\to B\mathrm{Re}(V_{td}^* V_{ts})(\bar{d}\gamma_\nu\gamma_5 s + \bar{s}\gamma_\nu\gamma_5 d) \times \bar{\mu}\gamma_\nu\gamma_5\mu \tag{8.12}$$

The Im part is important only for the K_S transition, which is not useful experimentally. It is absolutely clear that the Z' mediation into some $X_1 - X_2$ current, can only be proportional to the $\mathrm{Re}(V_{td}^* V_{ts})$, in complete analogy to $K_L \to \mu\bar{\mu}$.

Consider now Higgs/real scalar mediation. We first consider the Lagrangian in the SM that leads to the K_L and h mixing. Taking derivative on both side of (8.10) we find that

$$\langle 0|m_s\bar{d}i\gamma_5 s + m_s\bar{s}i\gamma_5 d|K_L\rangle = 2F_K m_K^2 \tag{8.13}$$

(was not following the overall sign here)

Such an operator is clearly CP-odd, and relevant flavour changing portal to the Higgs is

$$\mathcal{L}_h \propto \mathrm{Im}(V_{td}^* V_{ts})(m_s \bar{d} i\gamma_5 s + m_s \bar{s} i\gamma_5 d)(h/v) \tag{8.14}$$

If instead of the Higgs, we would have an ALP particle a that couples to the $\bar{t} i\gamma_5 t$, then

$$\mathcal{L}_h \propto \mathrm{Re}(V_{td}^* V_{ts})(m_s \bar{d} i\gamma_5 s + m_s \bar{s} i\gamma_5 d)(a/f_a) \tag{8.15}$$

So, for the H-portal $K_L \to X_1 X_2$ transition we will have $\propto \mathrm{Im}(V_{td}^* V_{ts})$.

Why is such difference between Z' and Higgs portal? It comes from the fact that it is effectively the longitudinal part of Z' that mixes with K_L. Longitudinal part of Z' is of opposite CP properties compared to Higgs.

Finally, transitional matrix element, K_L to π^0 in the SM that is relevant is

$$\langle \pi^0 | \bar{d}\gamma_\mu s - \bar{s}\gamma_\mu d | K_L \rangle \propto p_K + p_\pi \tag{8.16}$$

Again, the operator on the inside has to be C even, and this is what achieved with " $-$ " sign between $s-d$ and $d-s$ terms for the vector current. Clearly then, the SM interaction that lead to $\nu\nu\pi^0$ mode has to be

$$\mathcal{L} = B' i \mathrm{Im}(V_{td}^* V_{ts})(\bar{d}\gamma_\mu s - \bar{s}\gamma_\mu d)\bar{\nu}\gamma_\mu \nu \tag{8.17}$$

which is of course well-known in the SM.

Finally, lets look at the scalar transition,

$$\langle \pi^0 | \bar{d}s + \bar{s}d | K_L \rangle \propto (F_\pi F_K)^{-1} \langle 0 | \bar{q}q | 0 \rangle \tag{8.18}$$

So, if the Higgs portal generates $K_L - \pi^0 - h$ vertex, it must come from this Lagrangian

$$\mathcal{L} = B'' i \mathrm{Re}(V_{td}^* V_{ts})(m_s \bar{d}s + m_s \bar{s}d)(h/v), \tag{8.19}$$

which is consistent with Daniel's paper. Notice that there is come internal consistency indeed: for the Higgs portal K_L to π^0 amplitude contains real part, and for K_L to vacuum, imaginary part of CKM matrix element product.

9 Mass Mixing UV Completion

While kinetic mixing is a fairly general feature of additional $U(1)'$ extensions of the SM, mass mixing between the the Z and a new Z_d boson remains a much more model dependent feature, which, generically speaking, appears in the presence of additional sources of EWSB. A popular choice to generate such couplings is the Type-I two-Higgs-Doublet-Model (2HDM), where only one out of two $SU(2)_L$ doublets couples to fermions. This model successfully avoids constraints from EW and FCNC observables, the latter constraints requiring only one doublet to couple to quarks, and the former a hierarchy between the doublets vevs. As we will see, to achieve the Z' masses and small mixing we are interested in while at the same time avoiding constraints from SM Z and h decays, a new complex SM singlet that also breaks the $U(1)'$ is required.

Take the following scalar field content

$$H_1 \sim (\mathbf{2}, 1/2, Q_X), \quad H_2 \sim (\mathbf{2}, 1/2, 0), \quad \varphi_d \sim (\mathbf{1}, 0, Q_X), \tag{9.1}$$

where all scalars get a vev with $v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2$ and arbitrary v_d. With our convention,

$$\tan 2\beta = \frac{v_2}{v_1}, \tag{9.2}$$

and we will work in the limit where $\tan 2\beta \gg 1$. The potential reads

$$V = m_1^2|H_1|^2 + m_2^2|H_2|^2 + \lambda_1|H_1|^4 + \lambda_2|H_1|^4 \tag{9.3}$$

$$+ \lambda_3|H_1|^2|H_2|^2 + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) + V_\varphi + V_{\text{mix}} \tag{9.4}$$

where

$$V_\varphi = m_\varphi^2\varphi^2 + \lambda_\varphi\varphi^4 + \lambda_{1\varphi}|H_1|^2|\varphi|^2 + \lambda_{2\varphi}|H_2|^2|\varphi|^2, \tag{9.5}$$

$$V_{\text{mix}} = \lambda_{\varphi 1}|H_1|^2|\varphi|^2 + \lambda_{\varphi 2}|H_2|^2|\varphi|^2 + \mu_\varphi\left(\varphi(H_2^\dagger H_1) + \text{h.c.}\right) \tag{9.6}$$

Out of the 10 scalar degrees of freedom, 3 get eaten by the SM bosons, 1 by the Z' boson, and 1 is the SM higgs. We are left with 2 CP-even scalars, H' and φ', 1 charged scalar H^+ (2 d.o.f.) and 1 CP-odd scalar A.

The additional scalars from the second $SU(2)_L$ doublet contribute to mass mixing as well as kinetic mixing at loop level. In the alignment limit, where only Φ_2 constributes to EWSB and $v_1 \to 0$

$$\kappa_\gamma = \frac{g_X e}{16\pi^2}\left(\frac{1}{3}\log\left(\frac{m_A^2}{m_{H^+}^2}\right) - \frac{1}{6}\frac{m_A^2 - m_H^2}{m_A^2}\right), \tag{9.7}$$

$$\kappa_Z = \frac{g_X e c_W}{16\pi^2 s_W}\left(\frac{1}{3}\log\left(\frac{m_A^2}{m_{H^+}^2}\right) - \frac{1}{6}\frac{m_A^2 - m_H^2}{m_A^2}\right), \tag{9.8}$$

where A is the pseudoscalar in the Φ_d direction. This shows that depending on the mass hierarchies of the heavy scalars and assuming $g_X = 1$, we may have $\mathcal{O}(10^{-3})$ couplings, even if their tree-level values vanish.

10 Some comments and reminders (KK)

For a sake of reminders, I am listing some comments, particularly on the DM implementation. In the following, $X_i^{(F)}$ and $X_i^{(S)}$ indicate fermionic and bosonic $X's$, respectively.

1. de-excitation scenario
   (a) In a model with $\{X_1^{(F)}, X_2^{(F)}, Z'\}$, $X_1^{(F)}$ is not a good DM candidate, because $X_1^{(F)} X_1(F) \to Z' \to f\overline{f}$ is p-wave, so large Z' coupling is needed, which is excluded by $B \to K X_1 X_2$.
   (b) We cannot replace Z' by a scalar mediator S, because $K_L \to S \to X_1^{(F)} X_2^{(F)}$ is suppressed by K_L mass and cannot explain the K_L branching.
   (c) Models with $\{X_1^{(B)}, X_2^{(B)}, Z'\}$ and $\{X_1^{(B)}, X_2^{(B)}, S\}$ may work for the K_L branching. However, in a model with $\{X_1^{(B)}, X_2^{(B)}, Z'\}$, a minimul UV completion with $\{H_1, H_2, \varphi\}$ with $X_1^{(B)}$, identified as φ_R or φ_I, being DM does not work, since such a light φ accompanied by a light neutral component in H_2 (charged under $U(1)_X$) is not allowed due to $Z \to Z' h_2$. (should be refined, if necessary)
   (d) An UV completion to a model with $\{X_1^{(B)}, X_2^{(B)}, S\}$ would be worth checking, because $X_1^{(B)}$ may still be a good DM candidate.
2. dipole scenario
   (a) A mediator should be Z' to avoid m_K suppression.
   (b) To make $X_1^{(F)}$ a DM, we need additional scalar (φ), so that we have an annihilation channel such as $X_1^{(F)} X_1^{(F)} \to \varphi Z'$ which is s-wave.
3. π^0 impostor
   (a) DM is impossible because of absence of a conserved charge.

11 Plots of kinematics

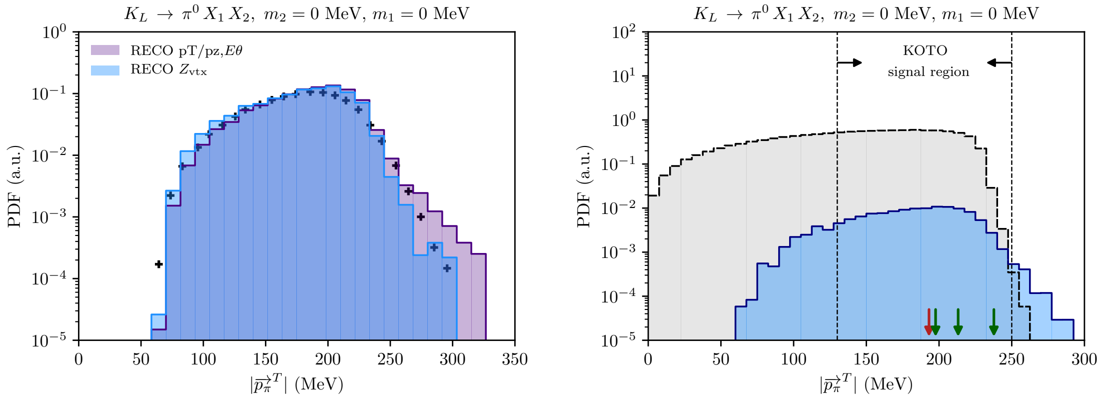

Figure 2: Validation plots for $K^+ \to \pi^+ \nu\bar{\nu}$.

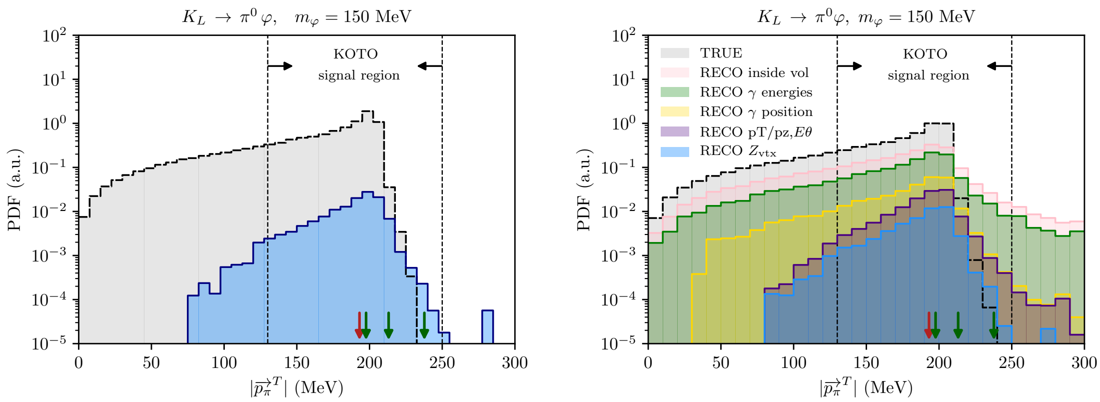

Figure 3: Prediction for singlet scalar model.

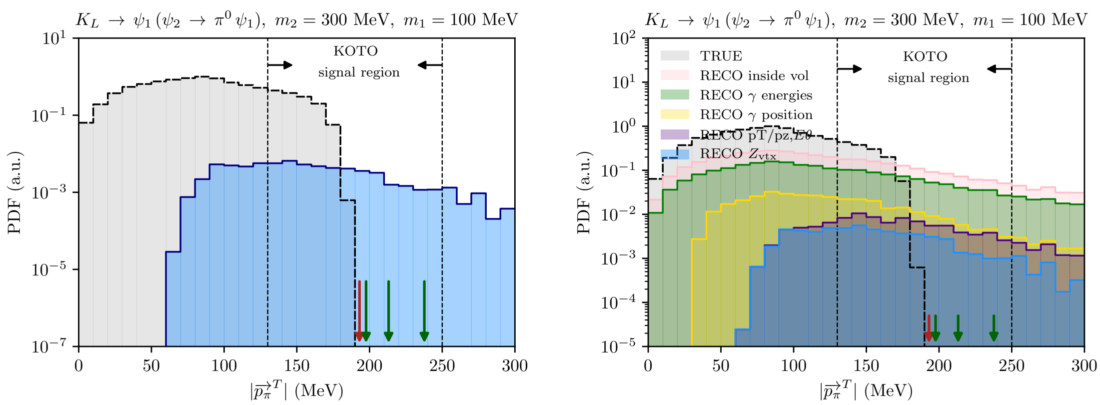

Figure 4: Prediction for pair of Majorana fermions.

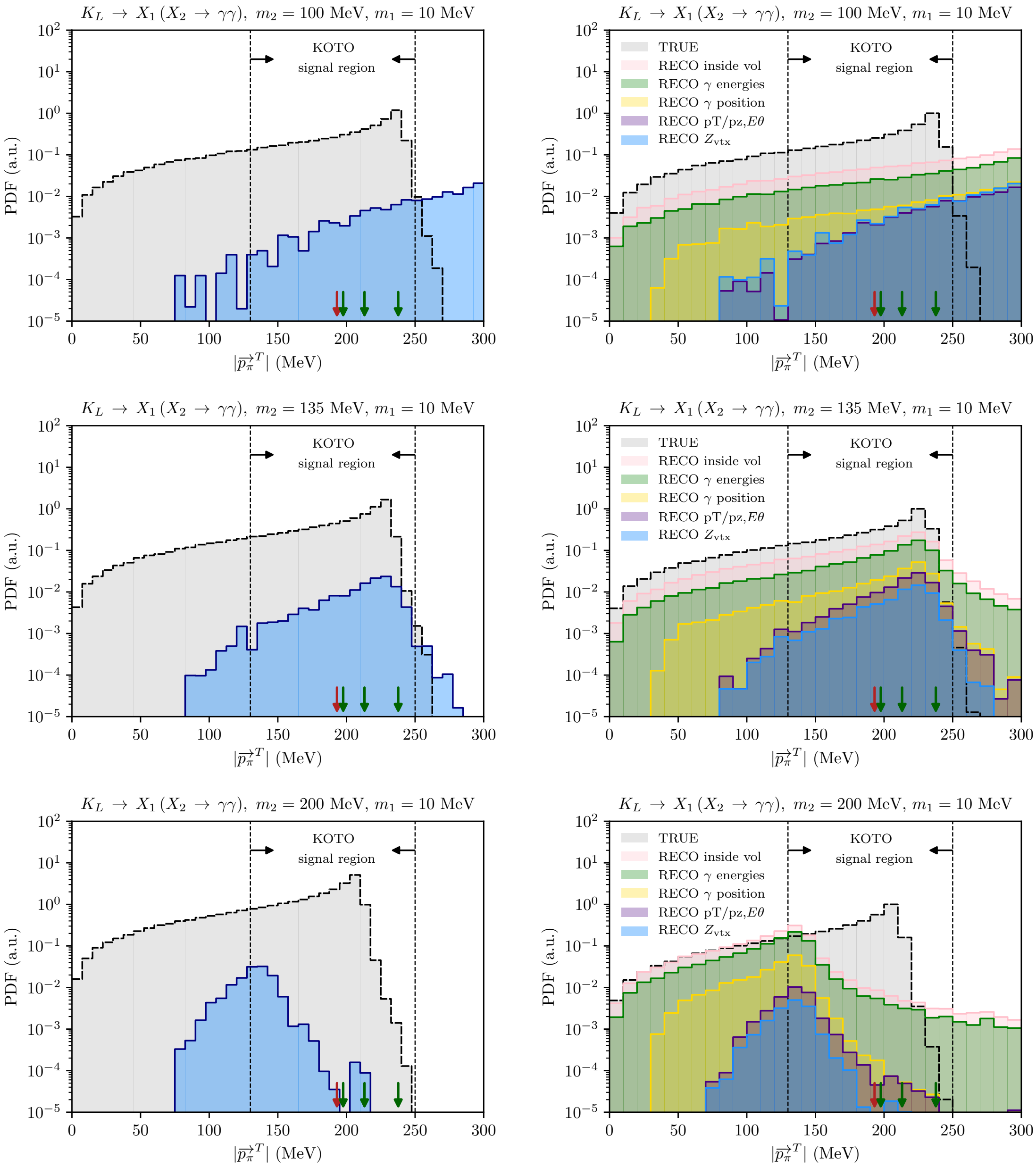

Figure 5: Prediction for pion impostor model assuming a prompt signal.

A A minimal UV completion with $\mathrm{U}(1)_X$

We introduce two SM singlets ψ and ψ^c whose $\mathrm{U}(1)_X$ charge is assigned to be q_X and $-q_X$, respectively. Note that they are defined as left-handed spinors. A complex scalar φ with $\mathrm{U}(1)_X$ charge $-2q_X$ breaks $\mathrm{U}(1)_X$ by acquiring a VEV. The particle content of the hidden sector is summarized in Table 1. Then the relevant part of the Lagrangian is given as

$$\mathcal{L} = \overline{\psi} i \not{D} \psi + \overline{\psi^c} i \not{D} \psi^c - m \psi^{cT} \psi - \frac{1}{2} y \varphi \psi^T \psi - \frac{1}{2} y^c \varphi^* \psi^{cT} \psi^c + h.c., \tag{A.1}$$

where $D_\mu = \partial_\mu - i g_X Q_X X_\mu$ with Q_X representing a $\mathrm{U}(1)_X$ charge, m is a mass parameter, and y and y^c are Yukawa couplings. By defining $\mu \equiv y\langle\varphi\rangle$ and $\mu^c \equiv y^c\langle\varphi^*\rangle$, the mass term may be written as

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}(\psi^T \; \psi^{cT}) \begin{pmatrix} \mu & m \\ m & \mu^c \end{pmatrix} \begin{pmatrix} \psi \\ \psi^c \end{pmatrix} + h.c. = -\frac{1}{2}(\chi_1^T \; \chi_2^T) \begin{pmatrix} m_1 & \\ & m_2 \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} + h.c., \tag{A.2}$$

where we define $(\chi_1 \; \chi_2)^T = U(\psi \; \psi^c)^T$ with U being a unitary matrix that diagonalizes the mass matrix. In terms of four-component spinors $\psi_i = (\chi_1 \; \overline{\chi}_1)^T$ with $i = 1, 2$, the Lagrangian becomes

$$\mathcal{L} = \overline{\psi}_i i \not{\partial} \psi_i - \frac{1}{2} m_i \overline{\psi}_i \psi_i + \frac{1}{2} g_X q_X X_\mu \overline{\psi}_i \left[i\mathrm{Im}(\Gamma_{ij})\gamma^\mu + \mathrm{Re}(\Gamma_{ij})\gamma^\mu\gamma_5 \right] \psi_j, \tag{A.3}$$

where $\Gamma_{ij} \equiv U_{i1}U_{j1}^* - U_{i2}U_{j2}^*$. Thus, we have both vector and axial-vector coupling in the hidden gauge sector, i.e., $g_V^d X_\mu \overline{\psi}_2 \gamma^\mu \psi_1 + g_A^d X_\mu \overline{\psi}_2 \gamma^\mu \gamma_5 \psi_1$ with $g_V^d = (i/2) g_X q_X \mathrm{Im}(\Gamma_{21})$ and $g_A^d = (1/2) g_X q_X \mathrm{Re}(\Gamma_{21})$. For instance, when we parametrize

$$U = \begin{pmatrix} \cos\theta & e^{i\delta}\sin\theta \\ -e^{-i\delta}\sin\theta & \cos\theta \end{pmatrix}, \tag{A.4}$$

we obtain $\Gamma_{21} = e^{i\delta}\sin 2\theta$, and thus $g_V^d = (i/2) g_X q_X \sin\delta \sin 2\theta$ and $g_A^d = (1/2) g_X q_X \cos\delta \sin 2\theta$. Cross checked this derivation, but my g_V and g_A are different by a factor of 2. Maybe you have 21 and 12 terms in eq. 5.3? My final result with real parameters agrees with [11]

KK: maybe we need some additional interaction to obtain the mixing between X_μ and Z_μ, such as $\mathrm{U}(1)_X$ charged SU(2) doublet Higgs.

field	$\mathrm{U}(1)_X$
ψ	q_X
ψ^c	$-q_X$
φ	$-2q_X$

Table 1: Particle content of the hidden sector.

B 2HDM + 2 scalars

See also Ref. [16]. In this case, we do not need additional neutral fermions for the decay $X_2 \to X_1 \pi^0$.

The following model construction does not work for us, but we add this for a future reference. We consider 2 doublet and 1 complex scalars whose quantum numbers are given in the table.

field	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_X$
H_1	2	1/2	q_X
H_2	2	1/2	0
φ	1	0	$-q_X/2$

Table 2: Particle content of 2HDM+2 scalars model.

The scalar potential is given by

$$\begin{aligned} V = & -m_{11}^2|H_1|^2 - m_{22}^2|H_2|^2 + \frac{1}{2}\lambda_1(H_1^\dagger H_1)^2 + \frac{1}{2}\lambda_2(H_2^\dagger H_2)^2 \\ & +\lambda_3|H_1|^2|H_2|^2 - \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) \\ & +m^2|\varphi|^2 + (\lambda\varphi^2 H_1^\dagger H_2 + h.c.), \end{aligned} \tag{B.1}$$

where we assume all of the couplings are real, and neglect the Higgs portal couplings, such as $H_1|^2|\varphi|^2$ and $|H_2|^2|\varphi|^2$, for simplicity. We expand the fields as

$$H_i = \begin{pmatrix} w_i \\ v_i + \frac{h_i^0 + i a_i}{\sqrt{2}} \end{pmatrix}, \tag{B.2}$$

$$\varphi = \frac{1}{\sqrt{2}}(\varphi_R + i\varphi_I), \tag{B.3}$$

and define $t_\beta = v_2/v_1$, and $v^2 \equiv v_1^2 + v_2^2$. In the following we denote $s_x \equiv \sin x, c_x \equiv \cos x$, and $t_x \equiv \tan x$.

The mass matrix for (h_1^0, h_2^0) is obtained as

$$M_{h^0}^2 = \begin{pmatrix} 2\lambda_1 v^2 c_\beta^2 & (\lambda_3 - \lambda_4)v^2 s_{2\beta} \\ (\lambda_3 - \lambda_4)v^2 s_{2\beta} & 2\lambda_2 v^2 s_\beta^2 \end{pmatrix}, \tag{B.4}$$

which can be diagonalized by

$$U_\alpha M_{h^0}^2 U_\alpha^T = \begin{pmatrix} m_h^2 & \\ & m_H^2 \end{pmatrix}, \tag{B.5}$$

$$U_\alpha = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix}, \tag{B.6}$$

$$m_h^2 = v^2[\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2 + \sqrt{(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2) + (\lambda_3 - \lambda_4)^2 s_{2\beta}^2}], \tag{B.7}$$

$$m_H^2 = v^2[\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2 - \sqrt{(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2) + (\lambda_3 - \lambda_4)^2 s_{2\beta}^2}], \tag{B.8}$$

$$t_{2\alpha} = \frac{(\lambda_3 - \lambda_4) s_{2\beta}}{\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2}. \tag{B.9}$$

Note that a_1 and a_2 are massless, which correspond to NG bosons for Z and Z'.

Mass matrix for (w_1, w_2) is obtained as

$$M_w^2 = \begin{pmatrix} \lambda_4 v^2 s_\beta^2 & -\frac{1}{2}\lambda_4 v^2 s_{2\beta} \\ -\frac{1}{2}\lambda_4 v^2 s_{2\beta} & \lambda_4 c_\beta^2 \end{pmatrix}, \tag{B.10}$$

which can be diagonalized in the same manner by using U_β. Then, we obtain

$$m_{w_1}^2 = |\lambda_4| v^2, \quad m_{w_2}^2 = 0 \tag{B.11}$$

where w_2 is a NG boson for $W^\pm$.

Next, we consider the mass mixing between Z and Z'. We may write the covariant derivative as

$$D_\mu H_i = (\partial_\mu + ig' Y[H_i] B_\mu + ig T_3[H_i] W_{3\mu} + ig_X X[H_i] Z'^0_\mu) H_i \tag{B.12}$$

where $X[H_i]$ denotes an $\mathrm{U}(1)_X$ charge of H_i. Then, using $W_{3\mu} = c_W Z_\mu^0 + s_W A_\mu$ and $g_Z \equiv g c_W + g' s_W$, we obtain

$$|D_\mu H_1|^2 + |D_\mu H_2|^2 \supset \frac{1}{2} m_{Z^0}^2 Z_\mu^0 Z^{0\mu} - \Delta^2 Z_\mu^0 Z'^{0\mu} + \frac{1}{2} m_{Z'^0}^2 Z'^0_\mu Z'^{0\mu}, \tag{B.13}$$

$$m_{Z^0}^2 = \frac{1}{2} g_Z^2 v^2, \tag{B.14}$$

$$m_{Z'^0}^2 = 2 q_X^2 g_X^2 v^2 c_\beta^2, \tag{B.15}$$

$$\Delta^2 = q_X g_X g_Z v^2 c_\beta^2. \tag{B.16}$$

The mass matrix for (Z_μ^0, Z'^0_μ) given as

$$M_{ZZ'}^2 = \begin{pmatrix} m_{Z^0}^2 & \Delta^2 \\ \Delta^2 & m_{Z'^0}^2 \end{pmatrix} \tag{B.17}$$

can be diagonalized by

$$U_\gamma = \begin{pmatrix} c_\gamma & -s_\gamma \\ s_\gamma & c_\gamma \end{pmatrix} \tag{B.18}$$

such that

$$U_\gamma M_{ZZ'}^2 U_\gamma^T = \begin{pmatrix} m_Z^2 & \\ & m_{Z'}^2 \end{pmatrix}, \tag{B.19}$$

$$m_Z^2 = \frac{1}{2}[m_{Z^0}^2 + m_{Z'^0}^2 + \sqrt{(m_{Z^0}^2 - m_{Z'^0}^2)^2 + 4\Delta^4}] \simeq m_{Z0}^2, \tag{B.20}$$

$$m_{Z'}^2 = \frac{1}{2}[m_{Z^0}^2 + m_{Z'^0}^2 - \sqrt{(m_{Z^0}^2 - m_{Z'^0}^2)^2 + 4\Delta^4}] \simeq m_{Z'^0}^2 - \frac{\Delta^4}{m_{Z^0}^2}, \tag{B.21}$$

$$t_{2\gamma} = \frac{2\Delta^2}{m_{Z^0}^2 - m_{Z'^0}^2} \simeq 2\frac{\Delta^2}{m_Z^2}. \tag{B.22}$$

We readily calculate the $h - Z - Z'$ coupling where h is the lightest neutral Higgs boson dominantly arising from H_1. Note that the SM-like Higgs boson is the one mainly arising from H_2. By writing $\mathcal{L} \subset C_{hZZ'} h Z_\mu Z'^\mu$, we obtain

$$C_{hZZ'} = \frac{m_Z}{\sqrt{2}v}[c_\alpha s_\beta s_\gamma c_\gamma - s_\alpha c_\beta (c_\gamma + 2s_\gamma q_X g_X/g_Z)(s_\gamma - 2c_\gamma q_X g_X/g_Z)]. \tag{B.23}$$

Using $2q_X g_X c_\beta/g_Z \simeq m_{Z'}/m_Z$ and $m_{Z'} \ll m_Z$, we have

$$C_{hZZ'} \simeq \frac{m_Z m_{Z'}}{\sqrt{2}v} c_{\alpha-\beta} s_\beta. \tag{B.24}$$

As discussed in [17] the invisible Z decay gives a stringent constraint, whose decay width is given as

$$\Gamma(Z \to hZ') \simeq C_{hZZ'}^2 \frac{m_Z}{64\pi m_{Z'}^2}\left(1 - \frac{m_h^2}{m_Z^2}\right)^3 \simeq \frac{m_Z^3}{128\pi v^2} c_{\alpha-\beta}^2 s_\beta^2, \tag{B.25}$$

which should be smaller than 2 MeV, leading to $t_\beta \lesssim 0.25$ when $\alpha = \beta$. Thus, we obtain a lower bound on m_h, which is

$$m_h = \sqrt{\frac{\lambda_1}{\lambda_2}} t_\beta^{-1} m_H \gtrsim \sqrt{\frac{\lambda_1}{\lambda_2}} \times (1 \text{ TeV}). \tag{B.26}$$

So, by taking $\lambda_1 \ll \lambda_2$ m_h can be arbitrary small. In that case, however, $H \to hh$ gives a stringent constraint [18], and s_α is required to be close to unity, which excludes this model.

B.1 Further comments

MH: Let me expand on $\alpha \to \pi/2$ limit. The model above does not contain a new singlet φ with charge q_X. This turns out to be crucial to send $v_1/v_2 \to 0$ while keeping $M_{Z'}$ at 10 GeV and ε_Z at 10^{-3}. In this case, a new small parameter appears:

$$\frac{1}{\tan\beta_d} = \frac{v_d}{v_2}, \tag{B.27}$$

and

$$\delta \simeq \frac{1}{\tan\beta \tan\beta_d}, \quad \varepsilon_Z = \frac{m_{Z'}}{m_Z}\delta, \quad m_{Z'} \simeq \frac{g_X v \cos^2\beta}{\delta} \tag{B.28}$$

Let us identify possible issues with the limit $s_\alpha \to 1$. As we will see, a light higgs is only possible in that case with a large fine-tuning and h will become very difficult to detect through fermion, gauge boson and scalar couplings.

- First point to note is that

$$\tan 2\alpha = \frac{2(\lambda_3-\lambda_4)v_1v_2}{\lambda_1 v_1^2-\lambda_2 v_2^2} \xrightarrow[v1/v2\ll 1]{} \frac{2(\lambda_4-\lambda_3)}{\lambda_2}\tan^{-1}\beta, \tag{B.29}$$

so that $\alpha\to\pi/2$ ($s_\alpha\to 1$) is merely a consequence of the hierarchy of the vevs. Note that λ_4 has to remain large for $m_{H^\pm} > v$.

- Decays into fermions $h\to ff$ constrains $(c_\alpha/s_\beta)^2$. This can be arbitrarily small as it is quadratic in a doubly suppressed coupling combination.

- $Z\to hZ'$ decays constrains $C_{hZZ'} = \cos^2(\beta-\alpha)(\delta\tan\beta)^2$. I find

$$\Gamma(Z\to hZ') \simeq \left(\delta\frac{M_Z M_{Z'}}{v}t_\beta c_{\beta-\alpha}\right)^2 \frac{M_Z}{64\pi m_{Z'}^2}\left(1-\frac{m_h^2}{m_Z^2}\right)^3 \sim \delta^2 c_{\beta-\alpha}^2\frac{m_Z^3}{64 v^2\pi}, \tag{B.30}$$

so that for $v = 246$ GeV and requiring $\Gamma < 2$ MeV, implies $\delta^2 < 3\times 10^{-2}$, which is easily satisfied for $\delta = 10^{-2}$.

- Now the Higgs decays to dark bosons. These are given by

$$\Gamma(H\to ZZ') \simeq \frac{g^2}{64\pi}\frac{(m_H^2-m_Z^2)^3}{m_H^3 m_Z^2}(\delta\tan\beta)^2\sin^2(\beta-\alpha) \tag{B.31}$$

$$= 0.43\ \text{MeV}\frac{(\delta\tan\beta)^2}{10^{-2}}\sin^2(\beta-\alpha) \tag{B.32}$$

$$\Gamma(H\to Z'Z') \simeq \frac{g^2}{128\pi}\frac{m_H^3}{m_Z^2}(\delta\tan\beta)^4\left(\frac{c_\beta s_\alpha}{t_\beta}+t_\beta\sin_\beta c_\alpha\right)^2 \tag{B.33}$$

$$= 0.047\ \text{MeV}\times\frac{(\delta\tan\beta)^4}{10^{-4}}\left(\frac{c_\beta s_\alpha}{t_\beta}+t_\beta\sin_\beta c_\alpha\right)^2, \tag{B.34}$$

which is far below 10% of the SM Higgs width of 4.1 MeV with $\alpha\to\beta$ and $c_\alpha\ll 1$.

- Now we face $H\to hh$, which is a strong constraint on α. The total rate depends on $\lambda_{Hhh} = c_{\beta-\alpha}s_{2\alpha}(2m_h^2+m_H^2)/2M_W s_{2\beta}$. It is given by

$$\Gamma(H\to hh) = \frac{M_H\lambda_{Hhh}^2}{32\pi}\left(1-4\frac{m_h^2}{M_H^2}\right)^{1/2} \tag{B.35}$$

$$\simeq 1.14\times 10^7\ \text{MeV}\left(\frac{c_{\beta-\alpha}s_{2\alpha}}{s_{2\beta}}\right)^2 \tag{B.36}$$

$$\simeq 1.14\times 10^7\ \text{MeV}\left(\frac{\lambda_3-\lambda_4}{\lambda_2}\right)^2, \tag{B.37}$$

which requires a fine-tuning of $(\lambda_3-\lambda_4)/\lambda_2 = 2\times 10^{-4}$ to achieve a rate below 10% of the SM value. Note that the only constraint on these parameters from the stability of the potential is $\lambda_3-\lambda_4 > -\sqrt{\lambda_1\lambda_2}$, which is easily satisfied. Note, however, that λ_4 is large to keep $m_{H^\pm}$ large. The question at the moment is, how comfortable are we with $s_\alpha = 1$, exactly. Loop corrections to λ_3 vs λ_4 could easily spoil this alignment.

	H_1	H_2	φ	S
$\mathrm{SU}(2)_L$	**2**	**2**	**1**	**1**
$\mathrm{U}(1)_Y$	1	1	0	0
$\mathrm{U}(1)_X$	Q	0	$Q/2$	0

Table 3: The scalar content of the UV completion with a type-I 2HDM, an additional complex scalar φ, and a real singlet S. In this case, the imaginary part of φ constitutes a dark matter candidate.

- The charged higgs decays remain to be investigated. But these rely on producing them at the LHC, which may or may not be excluded for large mass $H^\pm$. In fact, the BR into $H^\pm \to hW^\pm$ dominates for large $\tan\beta$ (*n.b.*, $\Gamma(H^\pm \to tb) \propto \tan^{-1}\beta$), which for light and boosted h is not such a striking signature as it leads to a lot of missing energy.

C An alternative

Finally, let me remark that we do not need to live with such light scalars. A valid solution to the DM problem is to work in the alignment limit of the 2HDM, and find a dark sector particle Y such that

$$X_1X_1 \to (Z')^* \to Y_1Y_2, \qquad \text{with } m_2 < 3\times m_1 \text{ and } m_{Y_1} + m_{Y_2} < 2m_1, \tag{C.1}$$

ensuring that $X_2 \to X_1Y_1Y_2$ cannot happen, and that the annihilation is allowed (no need for forbidden DM). An extra ingredient need to be introduced to ensure $Y_1 \to SM$ sufficiently fast.

Probably a $Y_1 = \varphi_R$ and $Y_2 = \varphi_I$ can do the job, where $(D_\mu\varphi)^\dagger(D^\mu\varphi) \supset g_X Z'_\mu i(\varphi_R\partial_\mu\varphi_I - \varphi_I\partial_\mu\varphi_R)$ gives a off-diagonal coupling to the Z', and $|\varphi|^2|H|^2$ a Higgs portal coupling that is not too large, but sufficiently large for φ_R decay to fermions. Note that φ_R is the lightest in this case. We may need some phase-space suppression in annihilation for $g_X = 1$, and it is guaranteed to be p-wave.

D Moved from main text, dark matter part

We focus on the latter case by exploring a residual $\mathcal{Z}_2$ symmetry in the theory and assigning Y to be a new real scalar particle S.

While kinetic mixing is a fairly general feature of additional $U(1)'$ extensions of the SM, mass mixing between the the Z and a new Z' boson remains a much more model dependent feature, which, generically speaking, appears in the presence of additional sources of EWSB. A popular choice to UV complete the operator in Eq. (**??**) is the Type-I two-Higgs-Doublet-Model (2HDM), where only one out of two $SU(2)_L$ doublets couples to fermions []. This model successfully avoids constraints from EW and FCNC observables, and displays a decoupling limit where all additional scalar degrees of freedom are heavy with EWSB arising from mostly one of the vevs. In the notation of Ref. [], $\sin(\alpha-\beta) \ll 1$. As already discussed [], to achieve the Z' masses and small mixings we are interested in, while at the same time avoiding constraints from SM Z and h decays, a new complex SM singlet that also breaks the $U(1)'$ is required. We then readily to introduce

$$H_1 \sim (\mathbf{2}, 1/2, Q_X), \quad H_2 \sim (\mathbf{2}, 1/2, 0), \quad \varphi_d \sim (\mathbf{1}, 0, Q_X/2), \tag{D.1}$$

where only the doublets get a vev with $v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2$. We work in the large $\tan\beta = v_2/v_1$ limit, achieving the alignment *and* decoupling limit when the lightest neutral component of the doublets is SM-like. In practice, this means that all non-SM-like scalars from the doublets are heavy and can be integrated out. In the usual notation of Ref. [17], this choice corresponds to setting $\alpha = 0$, such that $\cos(\beta - \alpha) \ll 1$. In full, our Lagrangian reads

$$\mathcal{L} \supset \mathcal{L}_{\rm 2HDM} + \mathcal{L}_{\rm X_\mu} + \left(\partial_\mu\varphi^\dagger + ig_X X_\mu \varphi^\dagger\right)\left(\partial^\mu\varphi - ig_X X^\mu\varphi\right) + \frac{1}{2}\left(\partial^\mu S\partial_\mu S - m_S^2 S^2\right) \tag{D.2}$$

$$- m_\varphi^2|\varphi|^2 - \lambda_\varphi|\varphi|^4 - \left(\kappa\,\varphi^2 H_2^\dagger H_1 + \text{h.c.}\right) - \mu S|H_2|^2 - \mu' S|\varphi|^2, \tag{D.3}$$

where κ, μ, and μ' are taken to be real and we ignored additional mixing terms for simplicity. The pieces $\mathcal{L}_{\rm 2HDM}$ and $\mathcal{L}_{\rm X_\mu}$ are, respectively, the Type-I 2HDM+$U(1)_X$ symmetry Lagrangian and the Proca Lagrangian for X_μ extended to include a Stueckelberg mass m_X. We expand on those in Appendix **??** with additional comments on the heavy scalar spectrum. Note that κ will be responsible for lifting the degeneracy between the real and imaginary components of φ. In addition, we assume $\lambda \ll \kappa$, and foreseeing an analogy with the kaon decay models, we write

$$\varphi = \left(\frac{S_2 + iS_1}{\sqrt{2}}\right), \text{ with } m_{1,2}^2 = m_\varphi^2 \mp \kappa\frac{\sin(2\beta)v^2}{4} + \mathcal{O}(\lambda s_\beta^2 v^2). \tag{D.4}$$

After diagonalizing the gauge bosons and scalar spectrum, one finds the massive gauge bosons (A_μ, Z_μ, Z'_μ), with Z'_μ mostly in the direction of X_μ, and

$$\varepsilon_Z = \frac{m_{Z'}}{m_Z}\frac{1}{\tan\beta}, \quad m_{Z'}^2 \simeq m_X^2 + \left(\frac{g_X v\sin(2\beta)}{2}\right)^2. \tag{D.5}$$

(KK: I obtained $m_{Z'}^2 \simeq m_X^2 \sin^2\beta$) You're right. Note that in my notation m_X is a Stueckelberg mass and the second term is a correction to that from the vev which I presume you take to be $g_X v_2$. We have to careful here, as I cannot recover a SM higgs with such large values of tan(beta). I think we need cannot go to a decoupling limit and we have to live with EW scale dark scalars. The Kaon decay hypothesis for KOTO is then realised when we identify $X_1 = S_1$ and $X_2 = S_2$.

Note that S_1 is stable due to a residual $\mathcal{Z}_2$ symmetry. In addition, the trilinear coupling with S allows t-channel annihilation with a velocity averaged cross section of

$$\langle \sigma v \rangle (S_1 S_1 \to SS) = \frac{\mu' \ldots}{\ldots}. \tag{D.6}$$

After S is produced, it decays via its trilinear coupling to the SM-Higgs...

This applies if we had fermions coupled to the Z' in a neutrino portal decay for Y, for instance. To avoid that, Y cannot be too light, being constrained to a relatively narrow mass range. For instance, if Y particles couple to the vector portal and are produced via s-channel, we have

$$m_2 - m_1 < m_{Y_1} + m_{Y_2} < 2m_1, \quad \text{so that} \quad X_1 X_1 \to (Z')^* \to Y_1 Y_2 \quad \text{but} \quad X_2 \not\to X_1 Y_1 Y_2. \tag{D.7}$$

Note that by default, $X_2 \to 3X_1$ decays are also disallowed, and the kaon decay hypothesis remains unaltered.

E Conclusions

Phenomenal success of the flavour program at high-luminosity e^+e^- and hadron colliders provided precision tests of the CKM paradigm, and put strong constraints on models of new physics. One of the most stringent tests, anticipated for many decades, is the neutrino pair-production channels in the decay of the B and K mesons. So far, only one of such modes, the three-body decay of K^+ to $\pi^+\nu\bar{\nu}$ has been observed before, with limited statistics. Next generation of experiments with K^+, K_L and B-mesons will detect more modes, and increase precision.

The missing energy decays of B mesons have been used in the past to set limits on the pair-production of dark matter states and on the single production of the Higgs-like particles. In this paper, we have addressed a possibility that K_L decays can provide an additional probe. Central to our paper is the idea that K_L can produce a pair of dark states, $X_{1(2)}X_2$ in the two-body decay. If one or both of these states is unstable with respect to the decay to photons, or to π^0, such sequential decays will partially or completely mimic the signature of $K_L \to \pi^0\nu\bar{\nu}$. The two-body nature of the decay means that the use of K_L is more sensitive to $X_{1(2)}X_2$ final states than K^+ decays, in contrast with models that modify the effective $d - s - \nu\nu$ vertex using the short distance physics [19].

To be more specific, we provide several models where the $X_{1(2)}X_2$ states appear as a consequence of the Z' and Higgs portals, interacting with generic dark sectors. For all models in this paper, we adopt the MFV approach, thus minimizing the number of free parameters, and relating amplitudes in B and K decays. For both types of portals, we have found models with either scalar or fermionic X states in the final states where $K_L \to X_{1(2)}X_2$ will exceed, sometimes significantly, the corresponding SM $\nu\bar{\nu}$ mode. In some models, the same type of "dark" vertex, *e.g.* $Z' - X_1 - X_2$ governs both $K_L \to X_{1(2)}X_2$ and $X_2 \to X_1\pi^0$ decays.

Having identified the new classes of models that give enhanced K_L missing energy decay signatures, we investigate the plausibility of these scenarios vis-a-vis the reported excess of events in the KOTO experiment. We concentrate on some benchmark case studies, choosing models where one and the same parameter (analogue of G_F for the Z' mediation) governs the decays of K_L, B mesons, and subsequent decay of X_2. By explicitly simulating the KOTO experiment, we were able to show that all pair production models can lead to large $|\vec{p}_\pi^{\,T}|$ events that fully populate the signal region and successfully explain all observed events. This is possible even for processes with relatively small phase space, where the X_2 decay away from the beam is misreconstructed as having larger $|\vec{p}_\pi^{\,T}|$. (It is evident, however, that the future progress may come only from a deeper experimental investigation into the nature of events seen at KOTO.)

Finally, we address the possibility that one of the states emerging from the K_L decay may be a DM particle. We find

F An old UV completion

field	$SU(2)_L$	$U(1)_X$
H_1	**2**	0
H_2	**2**	Q
φ_3	**1**	Q
φ_4	**1**	$2Q$
ν_D	**1**	Q
ν'_D	**1**	$-Q$
ψ_L	**1**	Q
ψ_R	**1**	Q

Table 4: Particle content of the anomaly-free UV completion of both scenarios.

Now I attempt to provide a UV completion of the inelastic DM as well as the neutrino models, while explaining DM and neutrino masses. In fact, the UV completion of the two scenarios could be the same and is given by the model in [20]. Of course, one may not want to involve neutrinos in the picture at all, but even then, the UV completion still requires four scalars as far as I can tell.

The most general scalar potential for the particle content in Table 4 reads

$$V = V_H + V_\varphi + V_{\text{tadpoles}} + V_{\text{mixing}} \tag{F.1}$$

where

$$\begin{aligned} V_H &= -m_1^2|H_1|^2 + \lambda_1|H_1|^4 + m_2^2|H_2|^2 + \lambda_2|H_2|^4 \\ V_\varphi &= -m_3^2|\varphi_3|^2 + \lambda_3|\varphi_3|^4 + m_4^2|\varphi_4|^2 + \lambda_4|\varphi_4|^4 \\ V_{\text{tadpole}} &= \frac{\mu}{2}\varphi_3 H_2^\dagger H_1 + \frac{\mu'}{2}\varphi_3^2\varphi_4^* + \frac{\alpha}{2}H_1^\dagger H_2\varphi_3\varphi_4^* + \text{h.c.} \\ V_{\text{mixing}} &= \sum_{i>j}\lambda_{ij}|s_i|^2|s_j|^2, \quad \text{with} \quad s_i = \varphi_i, H_i \end{aligned} \tag{F.2}$$

Out of the 12 scalar degrees of freedom, 3 get eaten by the SM bosons, 1 by the Z' boson, and 1 is the higgs. We are left with 3 CP-even scalars, h', s, and s', 1 charged scalar H^+ (2 d.o.f.) and 2 CP-odd scalar a and a'. The masses of the pseudo-scalars are induced by the vevs of H_1 and φ_3.

We now compute the scalar properties under the simplification $V_{\text{mixing}} = 0$, which amount to setting $\lambda_{ij} = 0$ for all i, j.

Neutrino masses The neutrino sector is merely

$$-\mathscr{L} \supset y_\alpha \overline{L_\alpha}\tilde{H}_2\nu_D^c + y\varphi_4^*\overline{\nu_D^c}\nu_D + y'\varphi_4\overline{(\nu'_D)^c}\nu'_D + \Lambda\overline{\nu_D^c}\nu'_D + \text{h.c.}, \tag{F.3}$$

which after SSB and in more familiar notation, gives

$$\mathscr{L} \supset -\frac{1}{2}\left(\overline{\nu_L}\ \overline{\nu'_D}\ \overline{\nu_D}\right)\begin{pmatrix} 0 & m_D & 0 \\ m_D & \mu' & \Lambda \\ 0 & \Lambda & \mu \end{pmatrix}\begin{pmatrix} \nu_L^c \\ (\nu'_D)^c \\ \nu_D^c \end{pmatrix} + \text{h.c.}, \tag{F.4}$$

which realises the inverse seesaw at tree-level with

$$m_\nu = \frac{m_D^2}{\Lambda^2}\mu, \tag{F.5}$$

where a term proportional to μ' appears only at one-loop level.

Dark Matter The dark matter sector is analogous to the above. We have

$$-\mathscr{L} \supset y_L \varphi_4^* \overline{\psi_L^c}\psi_L + y_R \varphi_4^* \overline{\psi_R^c}\psi_R + M_D \overline{\psi_L}\psi_R + \text{h.c.}, \tag{F.6}$$

where after SSB, we recover

$$-\mathscr{L} \supset \frac{1}{2}\left(\overline{\psi_L}\ \overline{\psi_R^c}\right)\begin{pmatrix} \mu_L & M_D \\ M_D & \mu_R \end{pmatrix}\begin{pmatrix} \psi_L^c \\ \psi_R \end{pmatrix} + \text{h.c.}, \tag{F.7}$$

In this way, we recover the two scenarios we discussed above, where one can choose to drop all DM sector, or all the HNL sector without significant consequences.

Meson couplings and decays

We have the following effective Lagrangian due to $Z - Z'$ mixing and exchange

$$\mathcal{L} = \frac{g_X \epsilon_Z}{m_{Z'}^2}\frac{g}{2\cos\theta_W} J^\mu_{dark}\left(\frac{1}{2}\bar{u}\gamma_\mu\gamma_5 u - \frac{1}{2}\bar{d}\gamma_\mu\gamma_5 d - \frac{1}{2}\bar{s}\gamma_\mu\gamma_5 s\right) = \tag{F.8}$$

$$\sqrt{2}G_X J^\mu_{dark}\left(\frac{1}{2}\bar{u}\gamma_\mu\gamma_5 u - \frac{1}{2}\bar{d}\gamma_\mu\gamma_5 d - \frac{1}{2}\bar{s}\gamma_\mu\gamma_5 s\right) \tag{F.9}$$

I do not follow the overall sign.

I claim that this current can be replaced with (using exact SU(3))

$$\left(\frac{1}{2}\bar{u}\gamma_\mu\gamma_5 u - \frac{1}{2}\bar{d}\gamma_\mu\gamma_5 d - \frac{1}{2}\bar{s}\gamma_\mu\gamma_5 s\right) \to F_\pi p_\mu(\pi^0 + \frac{1}{\sqrt{3}}\eta), \tag{F.10}$$

where η is η_8.

The proof is like this. We split the week current into the singlet, and t_3 and t_8 part that is then identified with the $F_\pi \partial_\mu \varphi_a$, and π^0 is the t_3 part and η is the t_8 part. So,

$$\left(\frac{1}{2}\bar{u}\gamma_\mu\gamma_5 u - \frac{1}{2}\bar{d}\gamma_\mu\gamma_5 d - \frac{1}{2}\bar{s}\gamma_\mu\gamma_5 s\right) = -\frac{1}{6}\bar{q}I\gamma_\mu\gamma_5 q + \bar{q}t_3\gamma_\mu\gamma_5 q + \frac{1}{\sqrt{3}}\bar{q}t_8\gamma_\mu\gamma_5 q, \tag{F.11}$$

where $I = diag(1,1,1)$, $t_3 = diag(1/2,-1/2,0)$ and $t_8 = (1/\sqrt{3}) \times diag(1/2,1/2,-1)$. So, the amplitude with η is related with the amplitude of π^0 as

$$\mathcal{M}_\eta = (1/\sqrt{3})\mathcal{M}_{\pi^0} \tag{F.12}$$

and that is independent of dark current.

We can separately check that $\langle 0|\frac{1}{2}\bar{u}\gamma_\mu\gamma_5 u - \frac{1}{2}\bar{d}\gamma_\mu\gamma_5 d|\pi^0\rangle = p_\mu F_\pi$ by taking a derivative from both parts, and reducing π^0 using soft-pion theorem etc, arriving at $(m_u + m_d)\langle\bar{q}q\rangle = -F_\pi^2 m_\pi^2$.

So, the scalar case has:

$$\mathcal{L} = \sqrt{2}G_X \partial_\mu(J^\mu_{dark})(\pi^0 + \frac{1}{\sqrt{3}}\eta) \tag{F.13}$$

So, the three cases we have are:

$$\mathcal{L} = \sqrt{2}G_X(\pi^0 + \frac{1}{\sqrt{3}}\eta)\left((m_2 - m_1^2)S_1S_2\right. \tag{F.14}$$

$$\left.+c_V[(m_2-m_1)\bar{\psi}_1\psi_2 + h.c.] + c_A[(m_2+m_1)\bar{\psi}_1 i\gamma_5\psi_2]\right) \tag{F.15}$$

I'll address the fermionic decay to π^0 first, as this is most sensitive stuff. The matrix element squared appropriately averaged and spin-summed is:

$$|\mathcal{M}_{\psi_2\to\psi_1\pi^0}|^2 = 2G_X^2F_\pi^2c_V^2(m_2-m_1)^2((m_1+m_2)^2 - m_\pi^2). \tag{F.16}$$

$$\Gamma_{\psi_2\to\psi_1\pi^0} = \frac{1}{8\pi m_2}G_X^2F_\pi^2c_V^2(m_2-m_1)^2((m_1+m_2)^2 - m_\pi^2). \tag{F.17}$$

I agree with the numerics, as I also get 16 cm for the inverse rate. For g_A we need to invert the sign for one of the masses.

Now the decay of eta,

$$|\mathcal{M}_{\eta\to\psi_2\psi_1}|^2 = \frac{1}{3}\times 4G_X^2F_\pi^2c_V^2(m_2-m_1)^2(m_\eta^2 - (m_1+m_2)^2) \tag{F.18}$$

$$\Gamma_\eta = 2\Gamma_{\eta\to\bar{\psi}_1\psi_2} = \frac{1}{6\pi m_2}G_X^2F_\pi^2c_V^2(m_2-m_1)^2(m_\eta^2 - (m_1+m_2)^2) \tag{F.19}$$

1/3 comes from SU(3) relations (see above), and 2 comes from two modes (particle-1 antiparticle 2, and vice versa), and another 2 from the spin sum (relative to $\psi_2 \to \psi_1\pi^0$).

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