Name: _____
Value: 4

I. **Multiple Choice.** Choose the **one** best answer.

1. Which of the following is correct?
   a) $T1(X) = 2X^2$ dominates $T2(X) = 2X$
   b) $T2(X) = 2X$ dominates $T1(X) = 2X^2$
   c) neither $T1(X) = 2X^2$ nor $T2(X) = 2X$ dominates the other

2. Which of the following is correct?
   a) $T1(X) = 5X^2$ dominates $T2(X) = 2X^3$
   b) $T2(X) = 2X^3$ dominates $T1(X) = 5X^2$
   c) neither $T1(X) = 5X^2$ nor $T2(X) = 2X^3$ dominates the other

3. Which of the following is correct?
   a) $T1(Y) = 55Y^{15} + 3Y^4 + 7500$ dominates $T2(Y) = Y^{16} + 2$
   b) $T2(Y) = Y^{16} + 2$ dominates $T1(Y) = 55Y^{15} + 3Y^4 + 7500$
   c) neither $T1(Y) = 55Y^{15} + 3Y^4 + 7500$ nor $T2(Y) = Y^{16} + 2$ dominates the other

4. Which of the following is correct?
   a) $T1(Z) = (Z + 3)(Z + 5)$ dominates $T2(Z) = 250Z$
   b) $T2(Z) = 250Z$ dominates $T1(Z) = (Z + 3)(Z + 5)$
   c) neither $T1(Z) = (Z + 3)(Z + 5)$ nor $T2(Z) = 250Z$ dominates the other

5. Which of the following is correct?
   a) $T1(Z) = (Z + 7)(Z + 9)$ dominates $T2(Z) = 2Z^3$
   b) $T2(Z) = 2Z^3$ dominates $T1(Z) = (Z + 7)(Z + 9)$
   c) neither $T1(X) = (Z + 7)(Z + 9)$ nor $T2(Z) = 2Z^3$ dominates the other

6. Which of the following is correct?
   a) $T1(N) = 37N^2$ dominates $T2(N) = N \log N$
   b) $T2(N) = N \log N$ dominates $T1(N) = 37N^2$
   c) neither $T1(N) = 37N^2$ nor $T2(N) = N \log N$ dominates the other

7. Which of the following is correct?
   a) $T1(N) = 37N^2$ dominates $T2(N) = N^2 \log N$
   b) $T2(N) = N^2 \log N$ dominates $T1(N) = 37N^2$
   c) neither $T1(N) = 37N^2$ nor $T2(N) = N^2 \log N$ dominates the other

8. Which of the following is correct?
   a) $T1(N) = \log_2 N$ dominates $T2(N) = \log_3 N$
   b) $T2(N) = \log_3 N$ dominates $T1(N) = \log_2 N$
   c) neither $T1(N) = \log_2 N$ nor $T2(N) = \log_3 N$ dominates the other

9. Which of the following is correct?
   a) $T1(N) = 84$ dominates $T2(N) = \log_9 N$
   b) $T2(N) = \log_9 N$ dominates $T1(N) = 84$
   c) neither $T1(N) = 84$ nor $T2(N) = \log_9 N$ dominates the other

10. Which of the following is correct?
    a) $T1(X) = \frac{3X+2}{X}$ dominates $T2(X) = 766X$
    b) $T2(X) = 766X$ dominates $T1(X) = \frac{3X+2}{X}$
    c) neither $T1(X) = \frac{3X+2}{X}$ nor $T2(X) = 766X$ dominates the other

11. Which of the following is correct?
    a) $T1(X) = \frac{X^4+X^2-17}{X^3}$ dominates $T2(X) = X^2$
    b) $T2(X) = X^2$ dominates $T1(X) = \frac{X^4+X^2-17}{X^3}$
    c) neither $T1(X) = \frac{X^4+X^2-17}{X^3}$ nor $T2(X) = X^2$ dominates the other

12. Which of the following is correct?
    a) $T1(W) = W^9$ dominates $T2(W) = 9^W$
    b) $T2(W) = 9^W$ dominates $T1(W) = W^9$
    c) neither $T1(W) = W^9$ nor $T2(W) = 9^W$ dominates the other

13. Which of the following is correct?
    a) $T1(W) = W!$ dominates $T2(W) = 67^W$
    b) $T2(W) = 67^W$ dominates $T1(W) = W!$
    c) neither $T1(W) = W!$ nor $T2(W) = 67^W$ dominates the other

14. Which of the following is correct?
    a) $T1(V) = V^V$ dominates $T2(V) = 25^V$
    b) $T2(V) = 25^V$ dominates $T1(V) = V^V$
    c) neither $T1(V) = V^V$ nor $T2(V) = 25^V$ dominates the other

15. Which of the following is correct?
    a) $T1(X, Y) = 2X^3Y$ dominates $T2(X, Y) = 3XY$
    b) $T2(X, Y) = 3XY$ dominates $T1(X, Y) = 2X^3Y$
    c) neither $T1(X, Y) = 2X^3Y$ nor $T2(X, Y) = 3XY$ dominates the other

16. Which of the following is correct?
    a) $T1(X, Y) = X^2 + 75$ dominates $T2(X, Y) = 7Y$
    b) $T2(X, Y) = 7Y$ dominates $T1(X, Y) = X^2 + 75$
    c) neither $T1(X, Y) = X^2 + 75$ nor $T2(X, Y) = 7Y$ dominates the other

17. Which of the following is correct?
    a) $T1(V, W) = W^3 + W + 74$ dominates $T2(V, W) = V + 18$
    b) $T2(V, W) = V + 18$ dominates $T1(V, W) = W^3 + W + 74$
    c) neither $T1(V, W) = W^3 + W + 74$ nor $T2(V, W) = V + 18$ dominates the other

II. Determine the big-O measure for each of the functions in the lower table. Choose from the various big-O measures in the upper table.

A. $O(1)$	B. $O(X)$	C. $O(\log X)$	D. $O(X \log X)$	E. $O(X^2)$	F. $O(X^3)$
G. $O(Y)$	H. $O(\log Y)$	I. $O(Y \log Y)$	J. $O(Y^2)$	K. $O(Y^3)$	L. $O(Y^{15})$
M. $O(Y^{16})$	N. $O(Z)$	O. $O(Z^2)$	P. $O(Z^3)$	Q. $O(\log N)$	R. $O(N)$
S. $O(N \log N)$	T. $O(N^2)$	U. $O(N^2 \log N)$	V. $O(W^3)$	W. $O(W^9)$	X. $O(9^W)$
Y. $O(W!)$	Z. $O(67^W)$	AA. $O(V)$	BB. $O(V^V)$	CC. $O(25^V)$	DD. $O(XY)$
EE. $O(X^3Y)$					

18.	$T1(X) = 2X^2$	19.	$T2(X) = 2X$
20.	$T1(X) = 5X^2$	21.	$T2(X) = 2X^3$
22.	$T1(Y) = 55Y^{15} + 3Y^4 + 7500$	23.	$T2(Y) = Y^{16} + 2$
24.	$T1(Z) = (Z + 3)(Z + 5)$	25.	$T2(Z) = 250Z$
26.	$T1(Z) = (Z + 7)(Z + 9)$	27.	$T2(Z) = 2Z^3$
28.	$T1(N) = 37N^2$	29.	$T2(N) = N \log N$
30.	$T1(N) = 37N^2$	31.	$T2(N) = N^2 \log N$
32.	$T1(N) = \log_2 N$	33.	$T2(N) = \log_3 N$
34.	$T1(N) = 84$	35.	$T2(N) = \log_9 N$
36.	$T1(X) = \frac{3X+2}{X}$	37.	$T2(X) = 766X$
38.	$T1(X) = \frac{X^4+X^2-17}{X^3}$	39.	$T2(X) = X^2$
40.	$T1(W) = W^9$	41.	$T2(W) = 9^W$
42.	$T1(W) = W!$	43.	$T2(W) = 67^W$
44.	$T1(V) = V^V$	45.	$T2(V) = 25^V$
46.	$T1(X, Y) = 2X^3Y$	47.	$T2(X, Y) = 3XY$
48.	$T1(X, Y) = X^2 + 75$	49.	$T2(X, Y) = 7Y$
50.	$T1(V, W) = W^3 + W + 74$	51.	$T2(V, W) = V + 18$

III. Rank the following big-O measures from greatest to least. (Recall that $O(f) > O(g)$ if and only if f dominates g.)

a. $O(N)$

b. $O(N^3)$

c. $O(4^N)$

d. $O(\log_4 N)$

e. $O(\log_5 N)$

f. $O(N^2)$

g. $O(1)$

h. $O(N \log_3 N)$

i. $O(N^2 \log_3 N)$

52.	53.	54.	55.	56.	57.	58.	59.	60.

IV. Classify each big-O measure in the lower table using the classfications from the upper table. Choose the one best classification.

A. constant	B. linear	C. quadratic	D. cubic
E. logarithmic	F. polynomial	G. exponential	

61.	$O(N)$	62.	$O(N^3)$	63.	$O(4^N)$
64.	$O(\log_4 N)$	65.	$O(\log_5 N)$	66.	$O(N^2)$
67.	$O(1)$	68.	$O(N \log_3 N)$	69.	$O(N^2 \log_3 N)$

V. Determine the order of each function in the lower table using the classfications from the upper table. Choose the one best classification.

A. $O(1)$	B. $O(\log N)$	C. $O(N)$	D. $O(N^2)$
E. $O(N^3)$	F. $O(2^N)$	G. $O(3^N)$	H. $O(17^N)$

70.	$T(N) = 3N^2 + N$	71.	$T(N) = 55N^3 + 77N^2 + 99$	72.	$T(N) = 2^N N^2$
73.	$T(N) = 7501$	74.	$T(N) = \log N + 46N$	75.	$T(N) = 3^{(N+1)}$
76.	$T(N) = \frac{N \log N}{2+N}$	77.	$T(N) = \frac{3N^4+4N^3}{5N^2+N}$	78.	$T(N) = \frac{17^N}{N^2}$

VI. Determine the big-O measure for each of the functions in the lower table. Classify each big-O measure using the classfications from the upper table. Choose the one best classification.

A. constant	B. linear	C. quadratic	D. cubic
E. logarithmic	F. polynomial	G. exponential	

79.	$T(N) = 3N^2 + N$	80.	$T(N) = 55N^3 + 77N^2 + 99$	81.	$T(N) = 2^N N^2$
82.	$T(N) = 7501$	83.	$T(N) = \log N + 46N$	84.	$T(N) = 3^{(N+1)}$
85.	$T(N) = \frac{N \log N}{2+N}$	86.	$T(N) = \frac{3N^4+4N^3}{5N^2+N}$	87.	$T(N) = \frac{17^N}{N^2}$

VII. Using big-O notation, estimate the runnning time of each of the following algorithms. You may assume that all variables are of type `int`. Choose the one best big-O measure from the table below.

A. $O(1)$	B. $O(\log X)$	C. $O(X)$
D. $O(X \log X)$	E. $O(X^2)$	F. $O(X^3)$
G. $O(X!)$	H. $O(X^X)$	I. $O(X^2Y)$

88.
```
1  for (i = 1; i <= X; i++)
2    for (j = 1; j <= X; j++)
3      for (k = 1; k <= X; k++)
4      {
5        // Five assignment instructions
6      }
```

89.
```
1  for (i = 10; i <= X; i++)
2  {
3    // Two assignment instructions
4    for (j = 15; j <= X; j++)
5    {
6      for (k = 1; k <= X; k++)
7      {
8        // Five assignment instructions
9      }
10     // Seven assignment instructions
11   }
12 }
```

90.
```
for (i = 1; i <= X; i++)
{
  for (j = 1; j <= X; j++)
  {
    // Twenty assignment instructions
  }
  for (j = 1; j <= X; j++)
    if (j % 2 == 1)
      for (k = 1; k <= X; k++)
      {
        // Five assignment instructions
      }
}
```

91.
```
for (i = 1; i <= X; i++)
{
  for (j = i; j <= X; j++)
  {
    // Six assignment instructions
  }
  if (i % 2 == 1)
  {
    // Four assignment instructions
  }
}
```

92.
```
for (i = 1; i <= X; i++)
  for (j = 1; j <= Y; j++)
    for (k = 1; k <= X; k++)
    {
      // Two assignment statements
    }
```

93.
```
i = 1;
while (i <= X)
{
  // Three assignment instructions
  j = 17;
  while (j <= 100)
  {
    // Two assignment instructions
    j++;
  }
  i++;
}
```

94.
```
i = X;
do
{
  // Three assignment instructions
  j = 1;
  while (j <= X)
  {
    // Two assignment instructions
    j++;
  }
  i--;
} while (i >= 1);
```

95.
```
i = 1;
while (i <= X)
{
  // Three assignment instructions
  j = 1;
  while (j <= X)
  {
    // Two assignment instructions
    j = j * 2;
  }
  i++;
}
```

96.
```
i = X;
while (i >= 1)
{
  // Three assignment instructions
  j = 1;
  while (j <= X)
  {
    // Two assignment instructions
    j = j + 2;
  }
  i = i / 3;
}
```

97.
```
void RecA(int X)
{
  // Some task requiring constant time
  if (X > 0)
    RecA(X - 1);
}
```

98.
```
1  void RecB(int X)
2  {
3    int i;
4
5    for (i = 1; i <= X; i++)
6    {
7      // Some task requiring constant time
8    }
9    if (X > 1)
10     RecB(X - 1);
11 }
```

99.
```
1  void RecC(int X)
2  {
3    int i;
4
5    for (i = 1; i <= X; i++)
6    {
7      // Some task requiring constant time
8    }
9    if (X > 1)
10     RecC(X / 2);
11 }
```

100.
```
1  void RecD(int X)
2  {
3    int i;
4
5    // Some task requiring constant time
6    for (i = 1; i <= X; i++)
7      if (X > 1)
8        RecD(X - 1);
9  }
```