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Adventures in Stochastic Processes

with Illustrations



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^{*}This section contains advanced material which may be skipped on first reading by beginning readers.

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