Formulating and simulating a hypothesis

STATISTICAL THINKING IN PYTHON (PART 2)

Justin Bois

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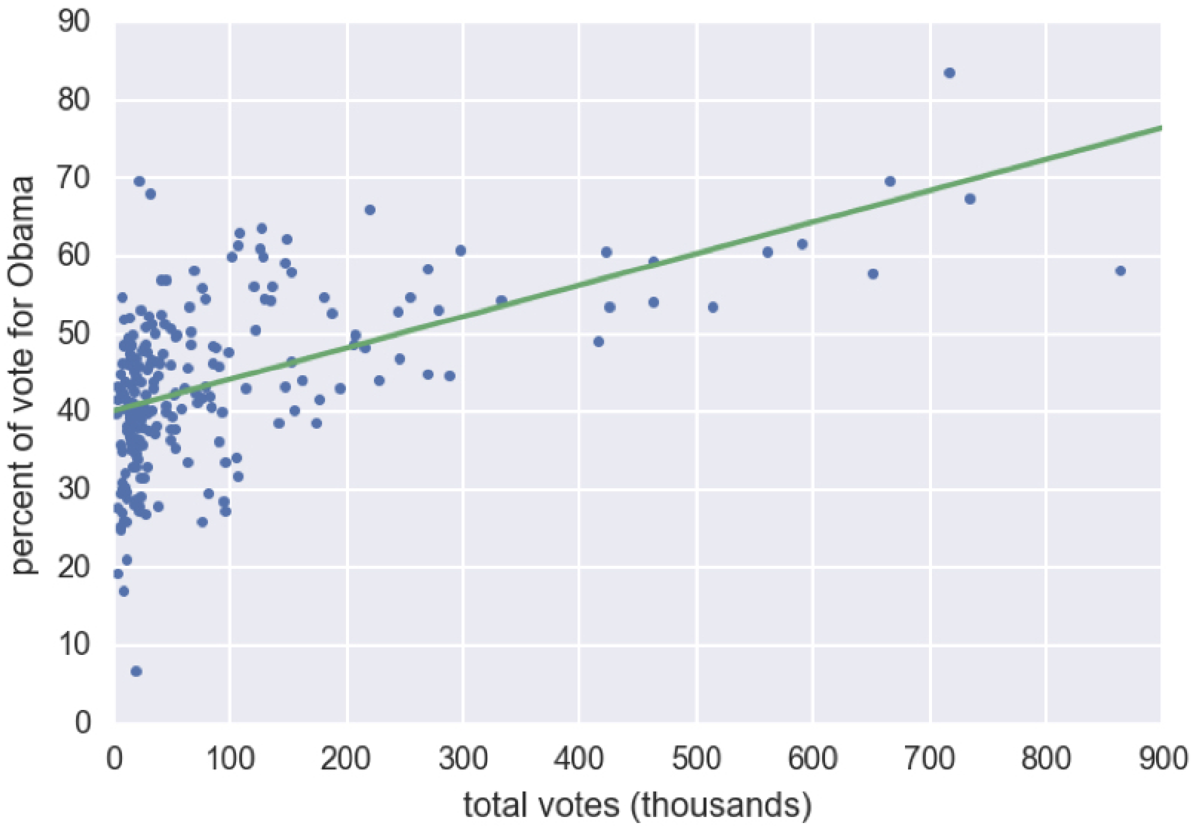
datacamp

2008 US swing state election results



[1] Data retrieved from Data.gov (https://www.data.gov/)

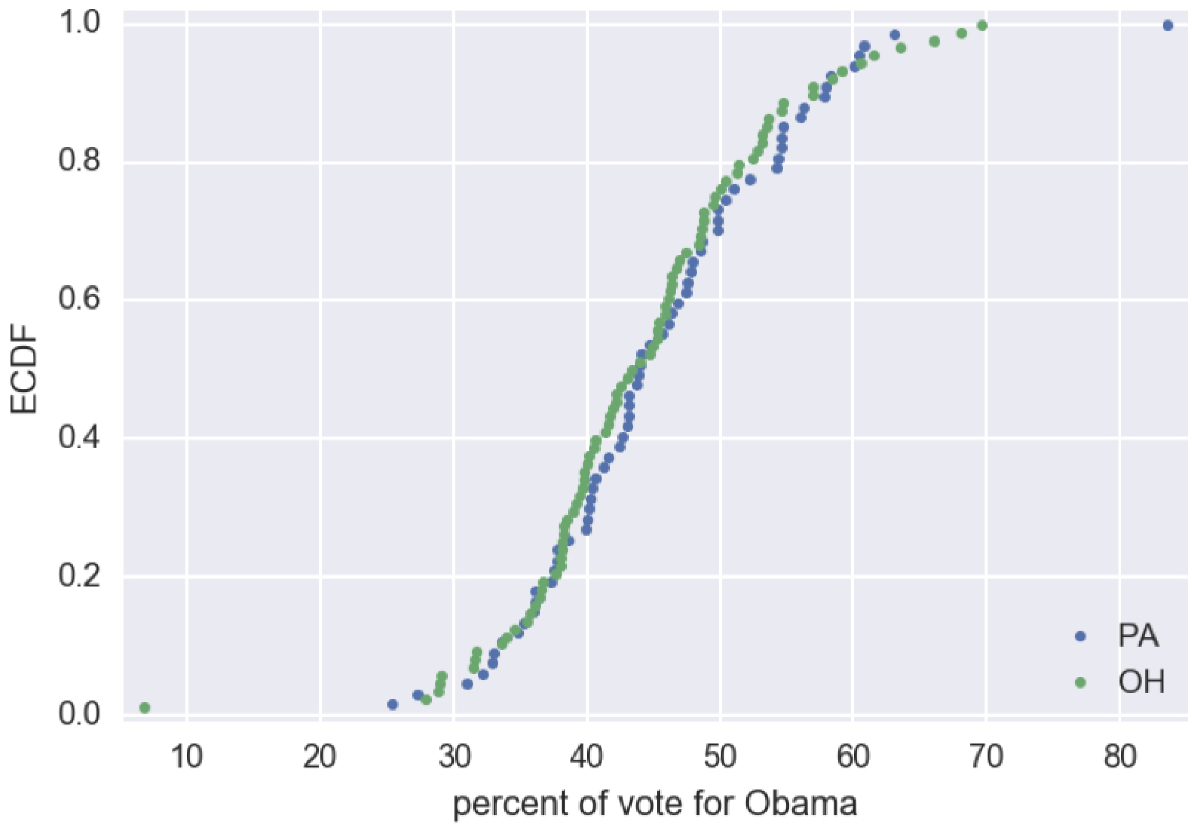
Hypothesis testing

- Assessment of how reasonable the observed data are assuming a hypothesis is true

Null hypothesis

- Another name for the hypothesis you are testing

ECDFs of swing state election results



[1] Data retrieved from Data.gov (https://www.data.gov/)

Percent vote for Obama

	PA	OH	PA — OH difference
mean	45.5%	44.3%	1.2%
median	44.0%	43.7%	0.4%
standard deviation	9.8%	9.9%	—0.1%

[1] Data retrieved from Data.gov (https://www.data.gov/)

Simulating the hypothesis

```
60.08,  40.64,  36.07,  41.21,  31.04,  43.78,  44.08,  46.85,
44.71,  46.15,  63.10,  52.20,  43.18,  40.24,  39.92,  47.87,
37.77,  40.11,  49.85,  48.61,  38.62,  54.25,  34.84,  47.75,
43.82,  55.97,  58.23,  42.97,  42.38,  36.11,  37.53,  42.65,
50.96,  47.43,  56.24,  45.60,  46.39,  35.22,  48.56,  32.97,
57.88,  36.05,  37.72,  50.36,  32.12,  41.55,  54.66,  57.81,
54.58,  32.88,  54.37,  40.45,  47.61,  60.49,  43.11,  27.32,
44.03,  33.56,  37.26,  54.64,  43.12,  25.34,  49.79,  83.56,
40.09,  60.81,  49.81,  56.94,  50.46,  65.99,  45.88,  42.23,
45.26,  57.01,  53.61,  59.10,  61.48,  43.43,  44.69,  54.59,
48.36,  45.89,  48.62,  43.92,  38.23,  28.79,  63.57,  38.07,
40.18,  43.05,  41.56,  42.49,  36.06,  52.76,  46.07,  39.43,
39.26,  47.47,  27.92,  38.01,  45.45,  29.07,  28.94,  51.28,
50.10,  39.84,  36.43,  35.71,  31.47,  47.01,  40.10,  48.76,
31.56,  39.86,  45.31,  35.47,  51.38,  46.33,  48.73,  41.77,
41.32,  48.46,  53.14,  34.01,  54.74,  40.67,  38.96,  46.29,
38.25,   6.80,  31.75,  46.33,  44.90,  33.57,  38.10,  39.67,
40.47,  49.44,  37.62,  36.71,  46.73,  42.20,  53.16,  52.40,
58.36,  68.02,  38.53,  34.58,  69.64,  60.50,  53.53,  36.54,
49.58,  41.97,  38.11
```

Pennsylvania

Ohio

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42.38,  38.10,  43.82,  45.31,  60.81,  54.37,  53.14,  32.97,
61.48,  50.10,  31.75
```

"Pennsylvania"

"Ohio"

Permutation

- Random reordering of entries in an array

Generating a permutation sample

```
import numpy as np
dem_share_both = np.concatenate(
                       (dem_share_PA, dem_share_OH))
dem_share_perm = np.random.permutation(dem_share_both)
perm_sample_PA = dem_share_perm[:len(dem_share_PA)]
perm_sample_OH = dem_share_perm[len(dem_share_PA):]
```

Let's practice!

STATISTICAL THINKING IN PYTHON (PART 2)

Test statistics and p-values

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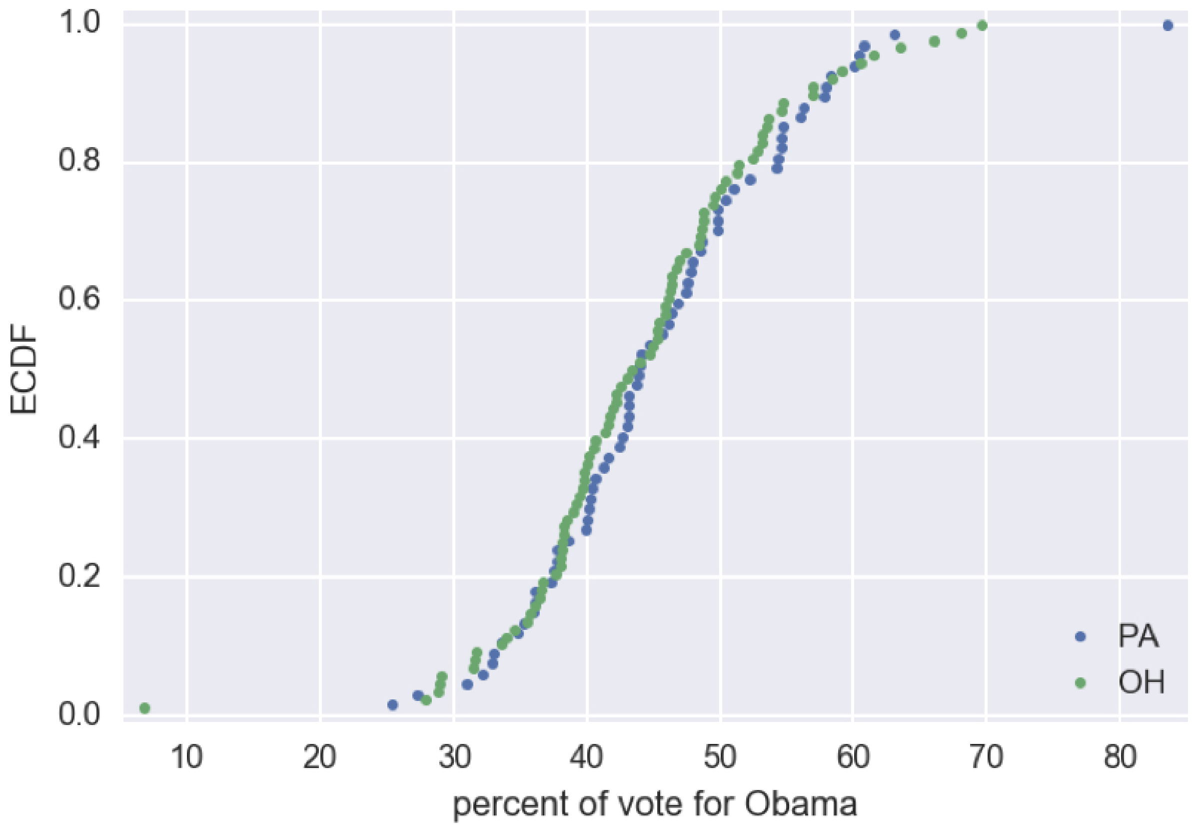
Are OH and PA different?



[1] Data retrieved from Data.gov (https://www.data.gov/)

Hypothesis testing

- Assessment of how reasonable the observed data are assuming a hypothesis is true

Test statistic

- A single number that can be computed from observed data and from data you simulate under the null hypothesis
- It serves as a basis of comparison between the two

Permutation replicate

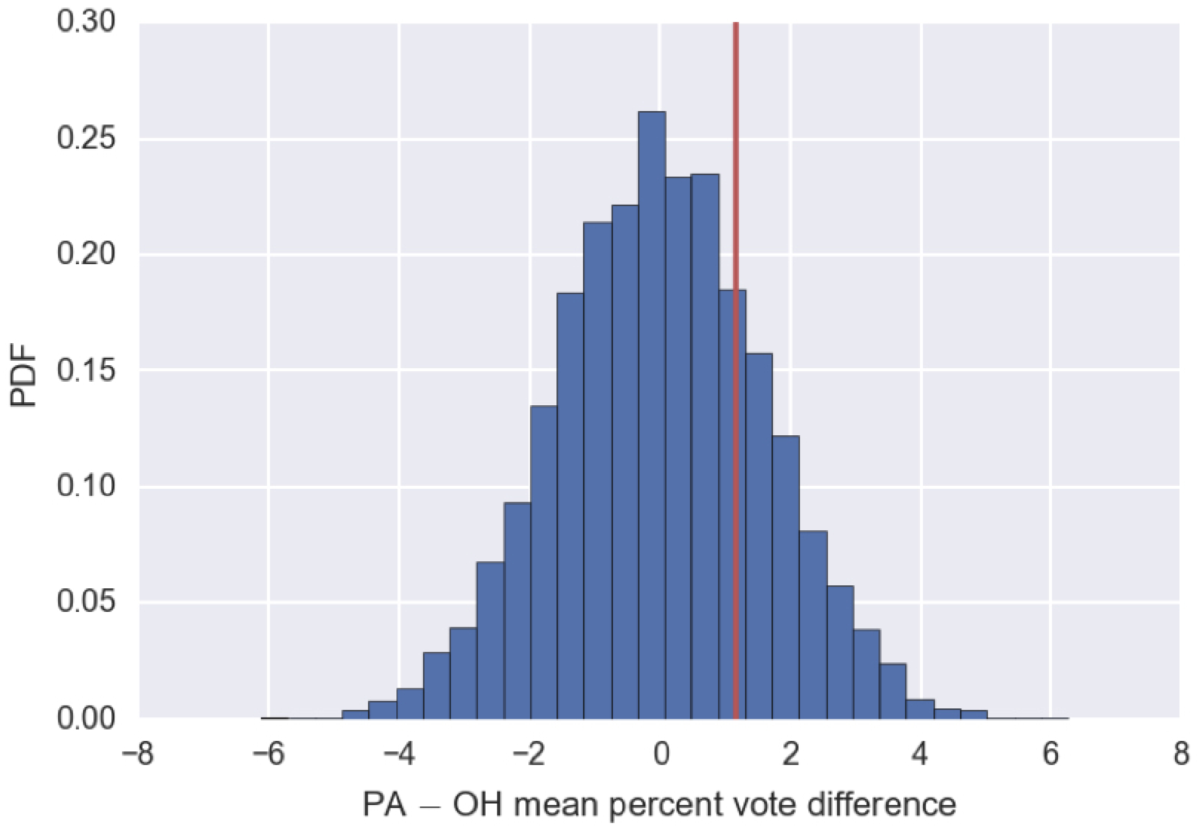
```
np.mean(perm_sample_PA) - np.mean(perm_sample_OH)
```

```
1.122220149253728
```

```
np.mean(dem_share_PA) - np.mean(dem_share_OH) # orig. data
```

```
1.1582360922659518
```

Mean vote difference under null hypothesis



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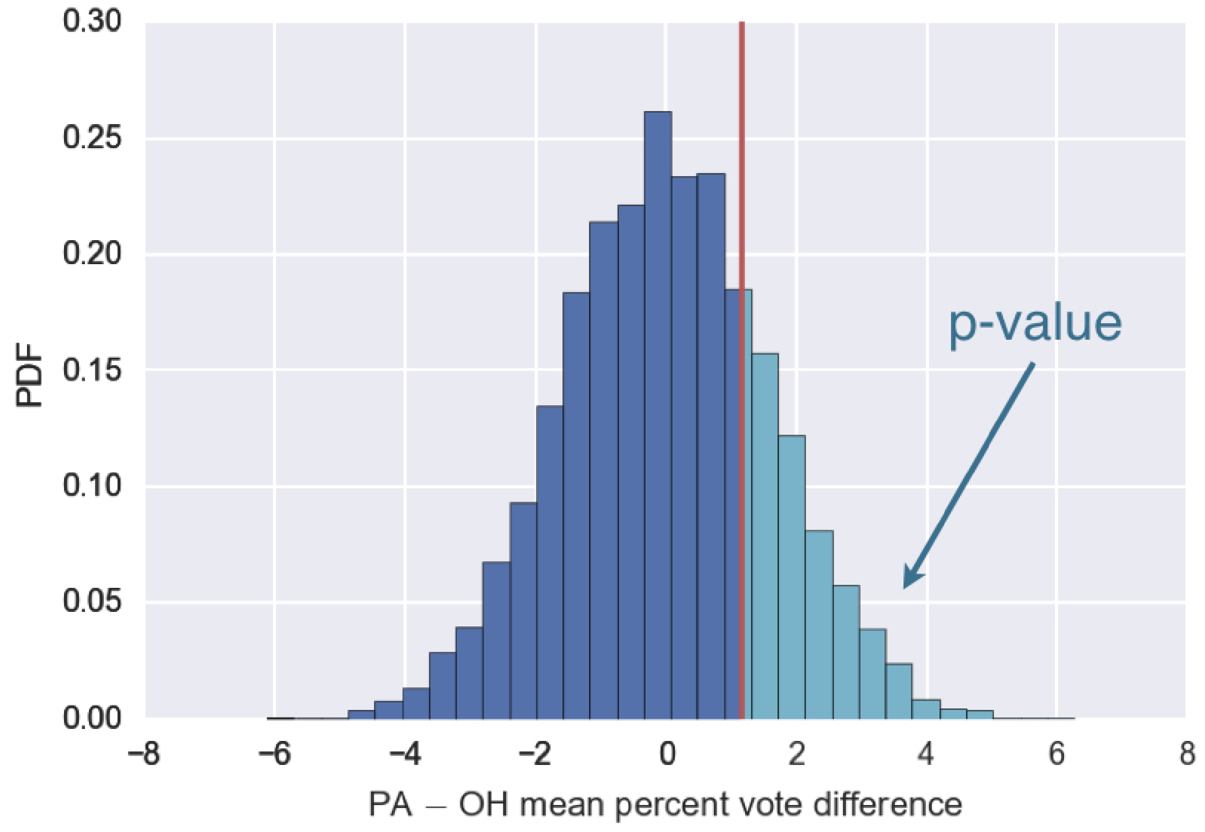
Mean vote difference under null hypothesis



[1] Data retrieved from Data.gov (https://www.data.gov/)

p-value

- The probability of obtaining a value of your test statistic that is at least as extreme as what was observed, under the assumption the null hypothesis is true
- **NOT** the probability that the null hypothesis is true

Statistical significance

- Determined by the smallness of a p-value

Null hypothesis significance testing (NHST)

- Another name for what we are doing in this chapter

statistical significance ? practical significance

Let's practice!

STATISTICAL THINKING IN PYTHON (PART 2)

Bootstrap hypothesis tests

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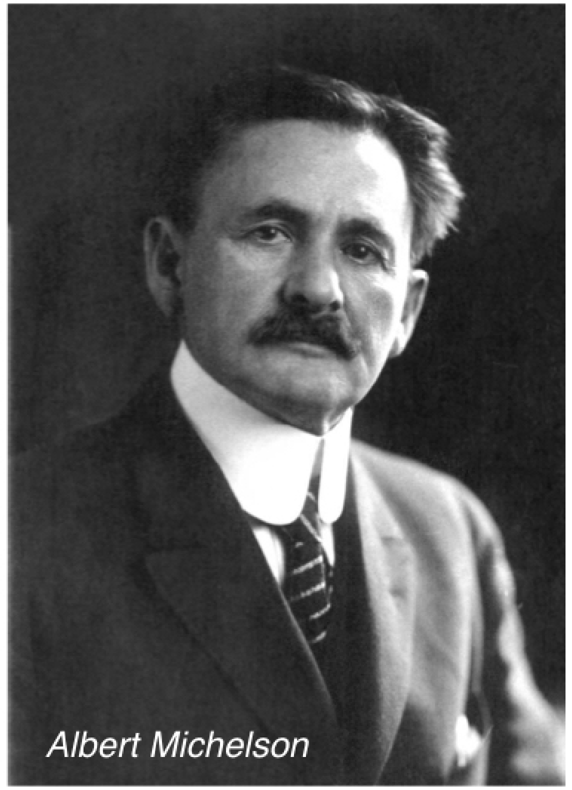
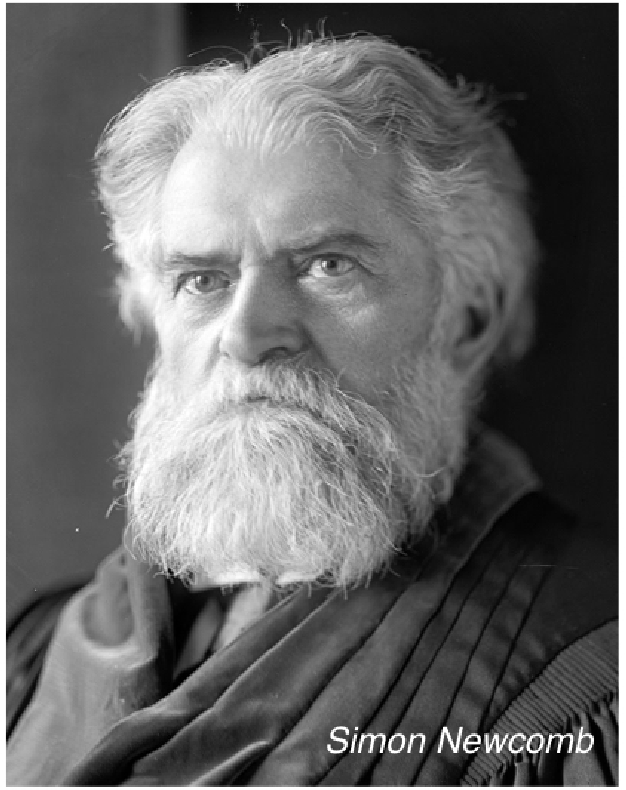
Pipeline for hypothesis testing

- Clearly state the null hypothesis
- Define your test statistic
- Generate many sets of simulated data assuming the null hypothesis is true
- Compute the test statistic for each simulated data set
- The p-value is the fraction of your simulated data sets for which the test statistic is at least as extreme as for the real data

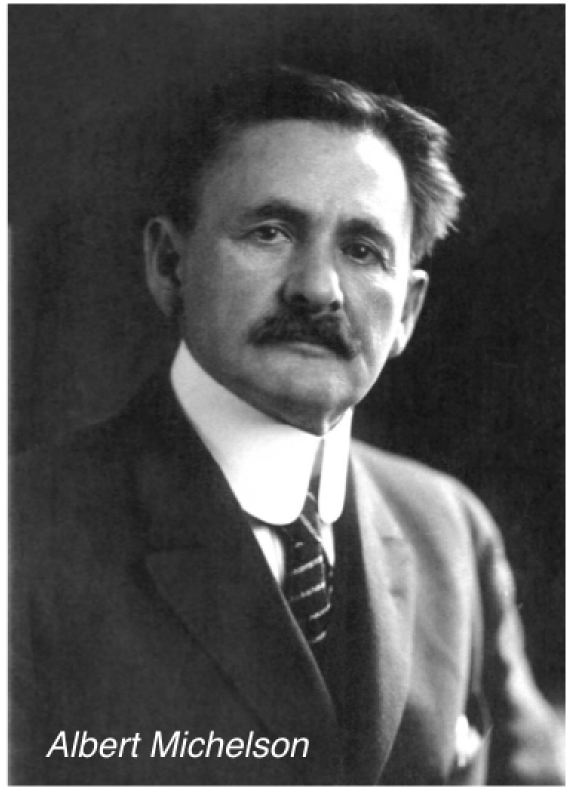
Michelson and Newcomb: speed of light pioneers



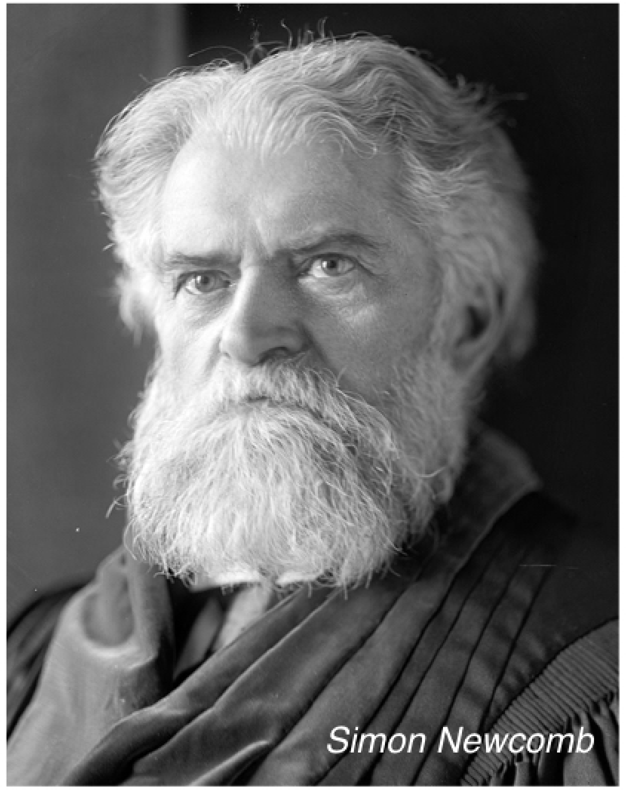


[1] Michelson image: public domain, Smithsonian [2] Newcomb image: US Library of Congress

Michelson and Newcomb: speed of light pioneers



299,852 km/s



299,860 km/s

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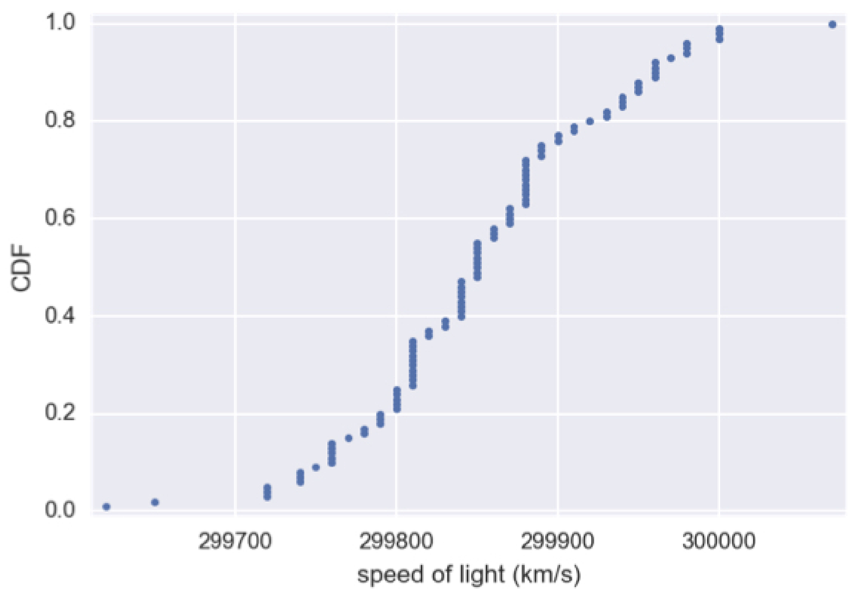
The data we have

Michelson:



Newcomb:

mean = 299,860 km/s

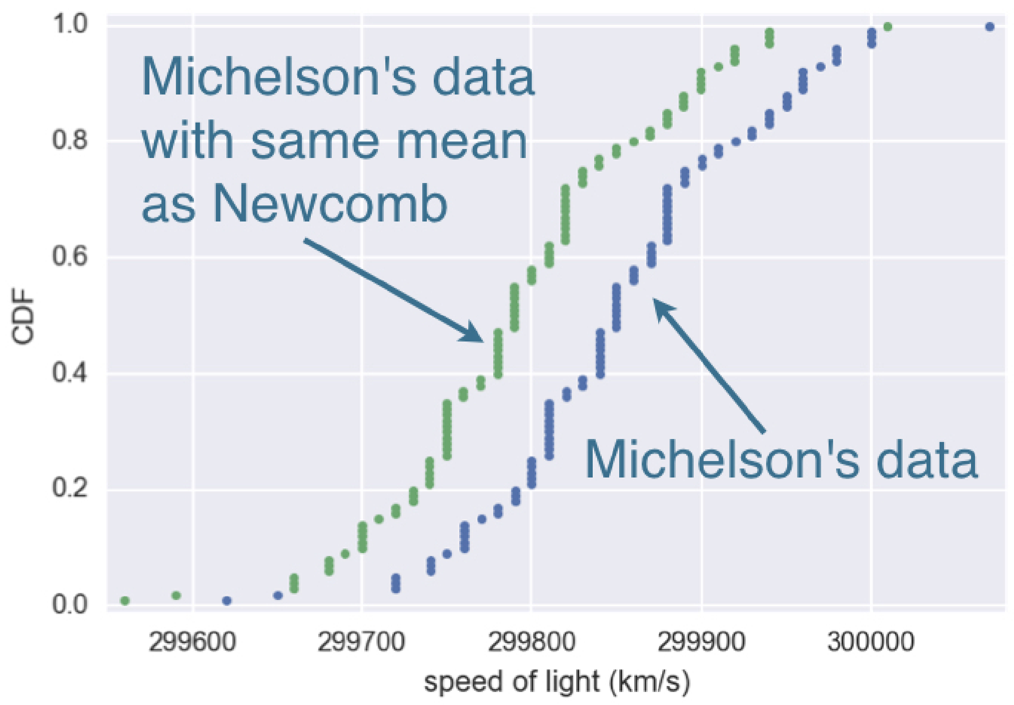
[1] Data: Michelson, 1880

Null hypothesis

- The true mean speed of light in Michelson's experiments was actually Newcomb's reported value

Shifting the Michelson data

```
newcomb_value = 299860  # km/s
michelson_shifted = michelson_speed_of_light \\
        - np.mean(michelson_speed_of_light) + newcomb_value
```



Calculating the test statistic

```
def diff_from_newcomb(data, newcomb_value=299860):
    return np.mean(data) - newcomb_value
```

```
diff_obs = diff_from_newcomb(michelson_speed_of_light)
diff_obs
```

```
-7.5999999999767169
```

Computing the p-value

```
bs_replicates = draw_bs_reps(michelson_shifted,
                             diff_from_newcomb, 10000)
p_value = np.sum(bs_replicates <= diff_observed) / 10000
p_value
```

```
0.16039999999999999
```

One sample test

```
- Compare one set of data to a single number
```

Two sample test

```
- Compare two sets of data
```

Let's practice!

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