2D Model of Solar System

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1 Introduction

It has long been an interest of mine to apply my growing understanding of physics to a simple, computer generated model. With the understanding of orbital dynamics developed in PHYS 263 Classical Mechanics and the know how to solve differential equations through computational methods from PHYS 375 Stars, this project has become achievable. This report documents my process, thoughts and ideas for further development.

2 Obtaining the D.E.

2.1 Polar Coordinates

Recall the equations of polar coordinates,

$$\begin{aligned}
r &= \sqrt{x^2 + y^2}, \\
\theta &= atan2(y, x), \\
\hat{r} &= cos(\theta)\hat{x} + sin(\theta)\hat{y}, \\
\hat{\theta} &= -sin(\theta)\hat{x} + cos(\theta)\hat{y},
\end{aligned}$$

and their respective time derivatives,

$$\begin{aligned}
\dot{\hat{r}} &= \dot{\theta}\hat{\theta}, \\
\dot{\hat{\theta}} &= -\dot{\theta}\hat{r}, \\
\boldsymbol{v} &= \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}, \\
\boldsymbol{a} &= (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\hat{r}\dot{\theta})\hat{\theta}.
\end{aligned}$$

See website for further reading.

2.2 Circular Orbit

Beginning from,

$$\boldsymbol{F} = m_1\boldsymbol{a}, \tag{1}$$

where m_1 is the mass of our orbiting body. Considering only Newtonian gravity between the sun and our object,

$$\begin{aligned} -\frac{GM_\odot m_1}{r^2}\hat{\boldsymbol{r}} &= m_1\boldsymbol{a} \\ -\frac{GM_\odot}{r^2} &= a_r \\ -\frac{GM_\odot}{r^2} &= \ddot{r} - r\dot{\theta}^2, \text{ where } \ddot{r} = 0. \end{aligned}$$

Our D.E. is then,

$$\sqrt{\frac{GM}{r^3}} = \dot{\theta}. \tag{2}$$

Assuming we know the initial θ and the semi-major axis of the planets in our Solar system, we can now model their approximate paths around the Sun. Using Euler's method,

$$\theta(t+\Delta t) \approx \theta(t) + \dot{\theta}(t)\Delta t,$$

while r(t) remains constant.

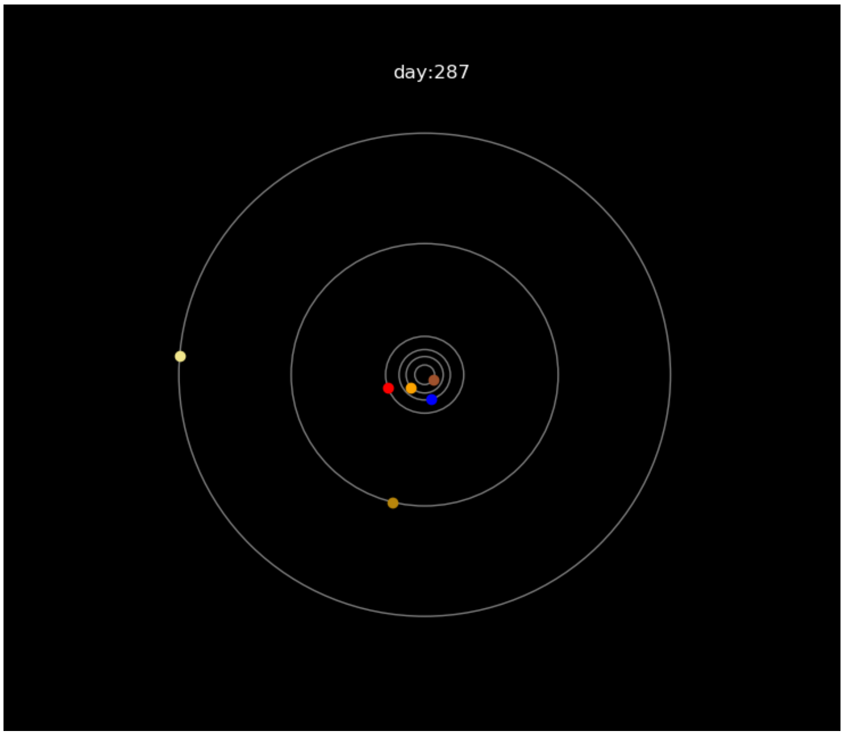

Figure 1: Arrangement of Saturn, Jupiter, Mars, Earth, Venus and Mercury at a random day

2.3 Non-Circular Orbit

The ultimate goal of this project is to predict the motion of any object within the Sun's gravitational influence. This means that it is not necessarily in a circular orbit. Let us approach the same problem without the use of polar coordinates,

$$\boldsymbol{F} = m_1 \boldsymbol{a}, \tag{3}$$

where, in Cartesian coordinates, we obtain,

$$-\frac{GM}{x^2+y^2}\big[\cos(\theta)\hat{x} + \sin(\theta)\hat{y}\big] = \ddot{x}\hat{x} + \ddot{y}\hat{y}.$$

For simplicity, let's consider the $\hat{x}$ and the $\hat{y}$ components individually,

$$\begin{aligned}
-\frac{GM}{x^2+y^2}\cos(\theta) &= \ddot{x}, \\
-\frac{GM}{x^2+y^2}\sin(\theta) &= \ddot{y},
\end{aligned}$$

where θ is defined as $\arctan 2(y, x)$. From here, we can solve for x and y computationally,

$$\begin{aligned}
x(t+\Delta t) &= x(t) + \dot{x}(t)\Delta t \\
\ddot{x}(t) &= -\frac{GM}{x^2(t)+y^2(t)}\cos\left(\arctan 2\big(y(t), x(t)\big)\right) \\
\dot{x}(t+\Delta t) &= \dot{x}(t) + \ddot{x}(t)\Delta t \\
&\& \\
y(t+\Delta t) &= y(t) + \dot{y}(t)\Delta t \\
\ddot{y}(t) &= -\frac{GM}{x^2(t)+y^2(t)}\sin\left(\arctan 2\big(y(t), x(t)\big)\right) \\
\dot{y}(t+\Delta t) &= \dot{y}(t) + \ddot{y}(t)\Delta t.
\end{aligned}$$

Our initial conditions are both components of position and velocity in Cartesian space for a given object.