Bead on a Tilted Wire.

Consider a bead of mass m (Fig. 1) sliding along a straight wire inclined at an angle θ with respect to the horizontal. The mass is attached to a spring of stiffness k and relaxed length L_0 and is also acted on by gravity. For simplicity, choose coordinates along the wire so that $x = 0$ occurs at the point closest to the support point of the spring; let a be the distance between this support point and the wire.

- Show that the equilibrium positions of the bead are:

$$mg \sin\theta = kx \left(1 - \frac{L_0}{\sqrt{x^2 + a^2}}\right) \tag{1}$$

- Show that these equilibrium positions can be written in terms of nondimensional variables and parameters as

$$1 - \frac{h}{u} = \frac{R}{\sqrt{1 + u^2}} \tag{2}$$

  for appropriate choices of R, h, and u.

- Give a graphical analysis of the dimensionless equation for the equilibrium points for $R < 1$ and $R > 1$. Discuss your results.

- Define $r = R - 1$ and show that the equilibrium equation reduces to $h + ru - \frac{u^3}{2} = 0$ for small r, h, and u. What is the approximate expression for the saddle-node bifurcations?

- Interpret your results physically, in terms of the original dimensional variables.

- **(Extra question).** Obtain a numerically accurate plot of the bifurcation curves in the (r, h) plane.

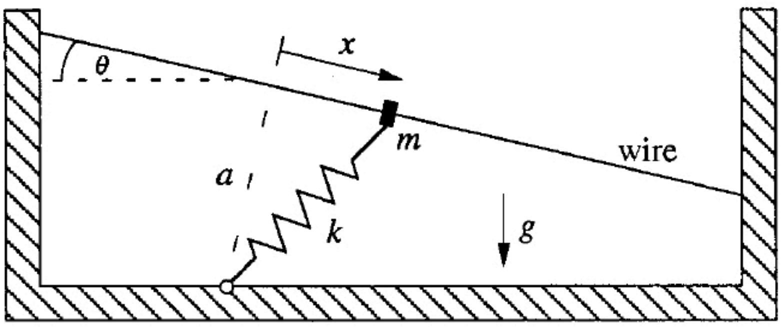

Figure 1: Schematic representation of the mechanical system.