

Pendulum connected to a torsion spring.

Consider a damped pendulum connected to a torsional spring (make sure you know what a torsional spring is) and subject to a constant external torque. The dynamics of the system is given by,

$$mL^2\ddot{\theta} + b\dot{\theta} + mgL \sin \theta = \Gamma - k\theta \quad (1)$$

- Make a pretty sketch of the system you are studying.
- Discuss the meaning of each of the terms in Eq. (1) and the limit in which the system can be approximated to the ones we have studied in the course so far.
- What are the possible behaviors of the pendulum in the long term? Discuss the bifurcation of the system as k is varied from 0 to ∞ . Discuss your results (keep in mind the physical meaning of k). Hint: obtain a nondimensional form of the system before doing your calculations.
- **Extra question:** construct the bifurcation diagram of the system using k (in its non-dimensional form) as the control parameter.