Harvesting a Single Natural Population.

The simplest model for the growth of a population of organisms in a given environment (Model I) is $\dot{n(t)} = rn(t)$, where $n(t)$ represents the number of individuals at time t and r is the population growth rate. This model, however, presents some inconvenients and is usually replaced by an improved version (Model II) of the form:

$$\dot{n}(t) = r\, n(t) \left(1 - \frac{n(t)}{K}\right), \tag{1}$$

where K is usually called the carrying capacity of the environment.

- What are the pathological behaviors shown by Model I? How do you interpret the quadratic term in Eq. (1)
- Obtain the fixed points of Model I and Model II and discuss the stability of both of them (analytically and performing a graphical analysis).

Although mathematically very simple, Model II is the basis for more elaborated models of population dynamics. Next, we will consider that n represents a population of fishes that can be harvested by humans. The effect of harvesting can be modeled by modifying Eq. (1) in various ways. We will consider here (Model III),

$$\dot{n}(t) = r\, n(t) \left(1 - \frac{n(t)}{K}\right) - E\, n(t), \tag{2}$$

where En is the harvesting yield per unit of time and E is a measure of the effort expended.

- Discuss the effect of harvesting on the steady state of the fishery as a function of the harvesting efforts (graphically and analytically).
- What is the harvesting yield in the steady state and how would you maximize it (maximum sustained yield)? Which population size should we expect if the fishery is harvested at such maximum sustained yield?

Extra bonus. Consider now that your steadily harvested fishery is perturbed by a (small) external factor that takes it away from the stationary state. How and how fast will the population react to that perturbation? How is this reaction compared to the one by a non-harvested population? Discuss the effect of the harvesting yield on this ratio between recovery times.