

# Johns Hopkins Engineering

## Applied Machine Learning for Mechanical Engineers

Optimization, Part 2, A

# Introduction to Linear Optimization Programming Packages

- By the end of this lecture, you will be able to:
  - Address linear optimization problems such to be compatible with Python and MATLAB optimization packages
  - Describe “scipy.optimize.linprog” package in Python
  - Describe ‘linprog’ package in MATLAB

# Introduction to Linear Optimization Programming Packages

- Formal linear optimization problem with one objective function

$$\begin{aligned} & \text{minimize } f(\mathbf{x}) \\ \text{subject to } & \begin{cases} g_j(\mathbf{x}) \leq 0 & j \in \{1, 2, \dots, J\} \\ h_k(\mathbf{x}) = 0 & k \in \{1, 2, \dots, K\} \end{cases} \quad (2-1) \end{aligned}$$

where  $\mathbf{x} = [x_1, x_2, \dots, x_N]$  include the optimization variables (solution),  $f(\mathbf{x})$ ,  $g_j(\mathbf{x})$ , and  $h_k(\mathbf{x})$  are a linear objective function, linear inequality constraints, and linear equality constraints, respectively, and the rest is similar to Eq. 1-1.

# Introduction to Linear Optimization Programming Packages

- Programming packages for linear programming

$$\begin{aligned} & \text{minimize } \mathbf{C} \cdot \mathbf{x}^T \\ \text{subject to } & \begin{cases} \mathbf{G} \cdot \mathbf{x} \leq \mathbf{A} \\ \mathbf{H} \cdot \mathbf{x} = \mathbf{B} \\ \mathbf{L} \leq \mathbf{x} \leq \mathbf{U} \end{cases} \quad (2-2) \end{aligned}$$

where  $\mathbf{C}$  is the  $N$ -dimensional row vector of multipliers of  $N$  optimization variables in objective function,  $\mathbf{G}$  is a  $J$  by  $N$  matrix of multipliers of  $N$  optimization variables in  $J$  inequality constraints,  $\mathbf{A}$  is a  $J$ -dimensional column vector of equality constants,  $\mathbf{H}$  is a  $K$  by  $N$  matrix of multipliers of  $N$  optimization variables in  $K$  inequality constraints,  $\mathbf{B}$  is a  $K$ -dimensional column vector of equality constants,  $\mathbf{L}$  is an  $N$ -dimensional row vector of lower bounds of  $N$  optimization variables, and  $\mathbf{U}$  is an  $N$ -dimensional row vector of upper bounds of  $N$  optimization variables.

# Introduction to Linear Optimization Programming Packages

- Programming packages for linear programming
  - Different annotations

$$\begin{aligned} & \min_x c^T x \\ & \text{such that } A_{ub} x \leq b_{ub}, \\ & \quad A_{eq} x = b_{eq}, \\ & \quad l \leq x \leq u, \end{aligned}$$

Figure 2-1 Annotations used to address linear optimization problems at  
<https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.linprog.html>

# Introduction to Linear Optimization Programming Packages

- Programming packages for linear programming
  - Python: “scipy.optimize.linprog” at  
<https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.linprog.html>
  - MATLAB: “linprog” at  
<https://www.mathworks.com/help/optim/ug/linprog.html>

# Introduction to Linear Optimization Programming Packages

- Python: “scipy.optimize.linprog”

SciPy.org

SciPy.org Docs SciPy v1.4.1 Reference Guide Optimization and Root Finding (scipy.optimize ) index modules next previous

## scipy.optimize.linprog¶

`scipy.optimize.linprog(c, A_ub=None, b_ub=None, A_eq=None, b_eq=None, bounds=None, method='interior-point', callback=None, options=None, x0=None)` [source]

Linear programming: minimize a linear objective function subject to linear equality and inequality constraints.

Linear programming solves problems of the following form:

$$\begin{aligned} & \min_x c^T x \\ & \text{such that } A_{ub}x \leq b_{ub}, \\ & \quad A_{eq}x = b_{eq}, \\ & \quad l \leq x \leq u, \end{aligned}$$

where  $x$  is a vector of decision variables;  $c$ ,  $b_{ub}$ ,  $b_{eq}$ ,  $l$ , and  $u$  are vectors; and  $A_{ub}$  and  $A_{eq}$  are matrices.

Informally, that's:

minimize:

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# Introduction to Linear Optimization Programming Packages

## ■ MATLAB: “linprog”

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**linprog**

Solve linear programming problems

**R2020a** collapse all in page

**Syntax**

```
x = linprog(f,A,b)
x = linprog(f,A,b,Aeq,beq)
x = linprog(f,A,b,Aeq,beq,lb,ub)
x = linprog(f,A,b,Aeq,beq,lb,ub,options)
x = linprog(problem)
[x,fval] = linprog(___)
[x,fval,exitflag,output] = linprog(___)
[x,fval,exitflag,output,lambda] = linprog(___)
```

**Description**

Linear programming solver

Finds the minimum of a problem specified by

$$\min_x f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub. \end{cases}$$

*f*, *x*, *b*, *beq*, *lb*, and *ub* are vectors and *A* and *Aeq* are matrices

# Introduction to Linear Optimization Programming Packages

- In this lecture, you learned about:
  - General formulations of linear optimization problems in Python and MATLAB optimization packages
  - “scipy.optimize.linprog” programming package in Python
  - ‘linprog’ programming package in MATLAB
- In the next lecture, we will practice these programming packages in Python and MATLAB



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